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Two-Dimensional Unsteady Free Convective Flow of a Viscous Incompressible Fluid through a Rotating Porous Medium

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ABSTRACT

An exact analysis of the effects of unsteady two-dimensional free convective flow during the motion of a viscous incompressible through a highly porous medium was undertaken. The porous medium is bounded by a vertical plane surface of constant temperature. The surface absorbs the fluid with a constant velocity and the free stream velocity of the fluid vibrates at a mean constant value. The analytical expressions for the velocity of the fluid are presented in the paper. The effects of rotation and permeability parameters on the axial and transverse components of velocity are discussed with the help of graphs.

1. INTRODUCTION

Rotating flows are of considerable interest to engineers and meteorologists. Besides, an extensive survey of this type of flows and their various applications have been given by Rott and Lewellen[1]. Cheng and Minkowycz[2] have obtained solutions for the free convective flow in a porous medium adjacent to a vertical plate with wall temperature being a power function of distance from the leading edge. A theoretical analysis of a two-dimensional free convective flow of a visc...s incompressible fluid through a porous medium, bounded by a porous and isothermal plate has been presented by Raptis, *et al*[3] who have also studied[4] the steady convective flow and the mass transfer through a very porous medium bounded by an infinite vertical plate. Raptis, *et al*[5,6] have extended their studies to unsteady free convective flow through a porous medium under different aspects. For constant flow, Raptis and Singh[7] have studied the influence of the free convective steady flow of the viscous fluid through the porous medium. Later on Raptis and Perdikis[8] have discussed the oscillatory flow in the presence of free convective flow through a porous medium.

The present paper aims to study the effects of unsteady two-dimensional free convective flow during the motion of a viscous incompressible fluid through a highly porous medium which occupies major portion of the porous medium so that viscous effects are important. The porous medium is bounded by a vertical surface of constant temperature. This surface absorbs the fluid with a constant velocity and the free stream velocity of the fluid vibrates at a mean constant value about z-axis in unison with the infinite vertical plate.

2. MATHEMATICAL ANALYSIS

We consider the unsteady two-dimensional flow through a highly porous medium which is bounded by a vertical infinite plane surface. We assume that the fluid is viscous and incompressible, that the surface absorbs the fluid with a constant velocity, and that the velocity of the fluid far away from the surface vibrates at a mean value with a direction parallel to the x'-axis. All the fluid properties are assumed to be constant, excepting that the influence of the density variation with temperature is considered only in the body force term. The x'-axis is taken along the plane surface with a direction opposite to the direction of the gravity and the y'-axis is taken normal to the surface.

The Equations

The equations which govern the problem when the velocities and temperature are the functions of y' and

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time t' are given below

Continuity Equation

$$\frac{\delta v'}{\delta y'} = 0 \qquad (1)$$

Momentum Equations

$$\frac{\delta u'}{\delta t'} + v \frac{\delta u'}{\delta y'} - 2\Omega v' = \quad \frac{\delta p}{\rho \quad \delta x} \quad g$$

$$\frac{\delta^2 u'}{\delta y'^2} \quad \frac{}{k'} u \qquad (2)$$

$$\frac{\delta v'}{\delta t'} + v' \frac{\delta v}{\delta y'} + 2\Omega u' = v \frac{\delta^2 v'}{\delta y'^2} \quad k \quad v \qquad (3)$$

Energy Equation

$$\frac{\delta T'}{\delta t'} + v' \frac{\delta T'}{\delta y'} = \frac{k}{\rho c_p} \frac{\delta^2 T'}{\delta y'^2} \qquad (4)$$

2.2 Boundary Conditions

For $Y' = 0,\ u' = 0,\ v \ = -v =$ constant

$$T' = T'_w$$

When $Y' \rightarrow \infty,\ u' = U' \rightarrow U(1 + \varepsilon e^{i\omega' t'}),$

$$T' \rightarrow T'_\infty \qquad (5)$$

where u' and v' are the components of the velocity, which are parallel to x' and y' axes respectively; ρ, the density of the fluid; P, the pressure; g, the acceleration due to gravity; μ the viscosity; k', the permeability of the porous medium; T', the temperature of the fluid; T'_w, the temperature of the surface; T'_∞, the temperature of the fluid far away from the surface; k, the thermal conductivity of the fluid; c_p, the specific heat of the fluid at constant pressure; U, the constant velocity; ω', the frequency of vibration of the fluid; Ω, the rotation parameter of the fluid; and ε ($<$ 1), the constant quantity.

Equation (2) for the free stream is reduced to

$$\frac{\delta U'}{\delta t'} = - \quad \frac{}{\rho} \frac{\delta p}{\delta x'} - \frac{\rho_\infty}{\rho} g - \quad k \qquad (6)$$

On eliminating $\frac{\delta p}{\delta x'}$ from :qns (2) and (6) we have

$$\frac{\delta u'}{\delta t'} + v' \frac{\delta u'}{\delta y'} - 2\Omega' v' = \frac{dU'}{dt'} + \frac{g}{\rho}$$

$$(\rho_\infty - \rho) + v \frac{\delta^2 u'}{d y'^2} \quad \frac{}{k'} (U' - u \qquad (7)$$

$$\frac{\delta v'}{\delta t'} + v' \frac{\delta v'}{\delta y'} + 2\Omega' u', = v \frac{\delta^2 v'}{dy'^2} - \frac{v}{k'} v' \qquad (8)$$

After using the complex function in the Eqns (7) and (8) it becomes

$$f' = u' + iv' \qquad (9)$$

$$\frac{\delta f'}{\delta t'} + v' \frac{\delta f'}{\delta y'} + 2\Omega i f \quad \frac{\delta U'}{dt'} + g\beta(T' - T'_\infty)$$

$$+ \frac{\delta^2 f'}{dy'^2} \quad \frac{v}{k} (U' - f' \qquad (10)$$

Now we use the constitutive equation

$$\rho_\infty - \rho = \beta\rho(T' - T'_\infty) \qquad (1$$

where β is the volumetric coefficient of thermal expansion and ρ_∞ the density of the fluid far away from the surface.

Since the surface absorbs the fluid with a constant velocity, the continuity Eqn (1) gives $v' = -v_0 =$ constant. Equation (10) becomes

$$\frac{\delta f'}{\delta t'} \quad v_0 \frac{\delta f'}{\delta y} - + 2\Omega' i f' = \frac{dU'}{dt'}$$

$$+ g\beta(T \quad T'_\infty \quad + v \frac{\delta^2 f'}{dy'^2} + \frac{v}{k'} (L \quad f' \qquad 2)$$

We introduce the non-dimensional quantities

$$u = \frac{u}{U} \quad v = \quad L \quad t = \frac{t' v_0^2}{v}$$

$$y = \frac{y' v_0}{} , \quad U^* \quad U \quad \omega = \frac{v\omega'}{v_0^2}$$

$$\Omega = \frac{2\Omega' v}{v_0^2} \quad T \quad \frac{T' - T'_\infty}{T'_w - T'_\infty} \qquad 3)$$

Prandtl number $P \quad \frac{\rho v c_p}{k}$

Grashof number $Gr \quad \frac{v g\beta(T'_w - T'_\infty)}{U v_0^2}$

Permeability parameter, $K = \frac{v_0^2}{\nu^2} k$

μ

ρ

where ν is the kinematic viscosity.

With the help of the above non-dimensional variables, the Eqns (4) and (12) are reduced to the non-dimensional equations

$$P\left(\frac{\delta T}{\delta t} - \frac{\delta T}{\delta y} \right) = \frac{\delta^2 T}{\delta y^2} \quad (14)$$

$$\frac{\delta f}{\delta t} - \frac{\delta f}{\delta y} + i\Omega f = \frac{dU^*}{dt} + GrT + \frac{\delta^2 f}{dy^2}$$

$$\frac{1}{k}(U^* - f) \quad (15)$$

by taking into account from Eqn (5) that U' $U(1 + \varepsilon e^{i\omega' t'})$.

The conditions given in Eqn (5) are reduced to

$$= 0, \; f = 0, \; T = \quad (16)$$

when

$$y \to \infty, \; f \to 1 + \varepsilon e^{i\omega t}, \; T \to 0 \quad (17)$$

In order to solve the differential Eqns (14) and (15), we assume that

$$f(y,t) = f_0(y) + \varepsilon e^{i\omega t} f_1(y) + \quad (18)$$

$$T(y,t) = T_0(y) + \varepsilon e^{i\omega t} T_1(y) + \quad (19)$$

On substituting the Eqns (18) and (19) in Eqns (14) and (15) we get the following system of differential equations

$$f_0'' + f_0' - \left(\frac{1}{k} + i\Omega\right) f_0 = - \frac{1}{k} - GrT_0 \quad (20)$$

$$f_1'' + f_1' \left[\frac{1}{k} + i(\omega + \Omega)\right] f =$$

$$\left(\frac{1}{k} + i\omega\right) \; GrT_1 \quad (21)$$

$$T_0'' + PT_0' \quad 0 \quad (22)$$

$$T_1'' + PT_1' - i\omega PT_1 = 0 \quad (23)$$

The conditions in Eqns (16) and (17) become

At $y = 0$, $f_0 = 0$, $f_1 = 0$, $T_0 =$, $T_1 = 0$ (24)

and as

$y \to \infty$ $f_0 \to$, $f_1 \to$ $T_0 \to 0$

and $T_1 \to 0$

By solving the differential Eqns (17)–(20) under the conditions given in Eqn (24) and by substituting the obtained solutions in Eqns (18) and (19), we have

$$f = \left(\frac{Gr}{(P + R_1)(P + R_2)} - \frac{}{1 + i\Omega k} \right)$$

$$+ \frac{1}{\cdots} \quad \frac{Gr\, e^{-py}}{(P + R_1)(P + R_2)}$$

$$+ \varepsilon e^{i\omega t}(1 - e^{R_3 y}) - i\Omega \varepsilon e^{i\omega t}(1 - e^{R_3 y}) \quad (26)$$

$$T = e^{-py}$$

where

$$R_1 = \frac{-1 - \sqrt{(1 + 4/k) + i4\Omega}}{2}$$

$$R_2 = \frac{1 + \sqrt{(1 + 4/k) + i4\Omega}}{2}$$

$$R_3 = \frac{-1 - \sqrt{(1 + 4/k) + 4i(\omega + \Omega)}}{2} \quad (28)$$

Substituting

$$f_0(y) = u_1 + iv_1 \quad \text{and} \quad f_1(y) = u_2 + iv_2$$

and using the Eqn (24), we have

$$u = u_1 + \varepsilon(u_2 \cos\omega t - v_2 \sin\omega t) \quad (30)$$

$$v = v_1 + \varepsilon(v_2 \cos\omega t - u_2 \sin\omega t) \quad (31$$

If τ_x and τ_y are the axial and transverse components of the skin-friction in the non-dimensional form, we obtain

$$\tau_x + i\tau_y = -\frac{\delta f}{\delta y} \qquad y = 0$$

$$R_1\left[\frac{1}{} - \frac{Gr}{(P+R_1)(P+R_2)}\right]$$

$$\frac{1}{} - \frac{Gr}{(P+R_1)(P+R_2)}$$

$$+\ \varepsilon e^{i\omega t} R_3 (1 - i\Omega) \qquad (32)$$

3. DISCUSSION

Figures 1 and 2 give the primary and secondary velocity profiles for various values of Grashof number (Gr), permeability parameter (K) and rotation parameter (Ω), considering $P = 0.71$, $\omega t = \pi/2$, $\omega = 5$ and $\varepsilon = 0.2$. It is evident from Fig. 1 that the primary velocity increases in magnitude with an increase of permeability parameter and Grashof number, but it decreases with an increase in rotation parameter. Figure 2 reveals that the secondary velocity component decreases in magnitude with an increase in rotation parameter. Further, it increases in magnitude with an increase in permeability parameter and Grashof number.

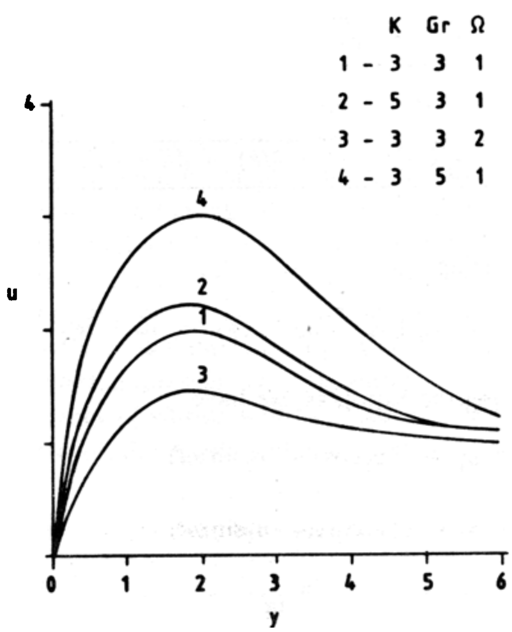

Figure 1. Primary velocity profiles at $P = 0.71$, $\varepsilon = 0.2$ and $\omega = 5$.

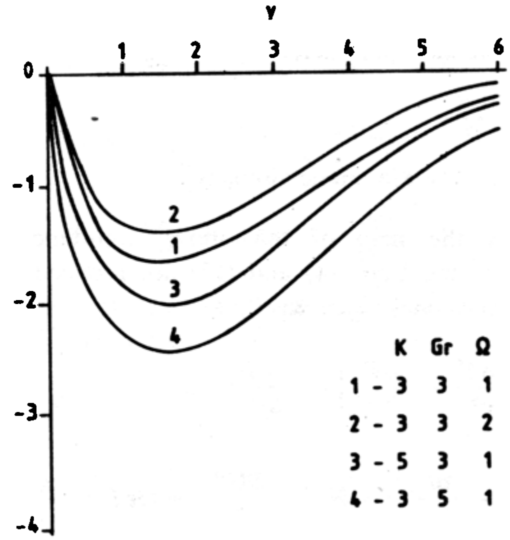

Figure 2. Secondary velocity profiles $P = 0.71$, $\varepsilon = 0.2$ and $\omega = 5$.

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