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Analytical and Experimental Investigation of Flow Through a Turbine Vane Cascade

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ABSTRACT

Present day military aero-gas turbines demand higher stage loadings for turbines so as to meet the growing need for higher thrust/power with lower fuel consumption. This calls for improved methods of blade element profiles. Details of a computer code developed for the design of blade elements for a prescribed distribution of surface velocity (Mach number) based on Stanitz's inverse methods are presented in this paper. Effect of boundary layer growth on the blade surface has also been incorporated in this code. Turbine vane was designed making use of this program and a four-bladed cascade was fabricated. It was tested in a blow down wind tunnel for different blowing pressures and stagger angles. Mach number distribution was determined from measured static pressure on the suction and pressure surfaces of the blade. Based on stream filament technique a computer code was developed to predict the characteristics of flow through a blade cascade. Results of this study show reasonable agreement between experimentally obtained Mach number distribution and the initially prescribed as well as analytically predicted Mach number distributions.

NOMENCLATURE

C	curvature of stream lines, 1/m
H	static enthalpy, kJ/kg
n	orthogonal length from suction surface, m
n_o	total length of orthogonal, m
n,s	orthogonal directions in natural coordinate system
P	static pressure, kg/cm^2
p	n/n_o
R	streamline radius of curvature, m
dS_x, dS_y	grid length in x, y directions
T	static temperature, K
V	velocity, m/s
W	mass flow rate, kg/s
β	air angle
γ	ratio of specific heats
ρ	static density, kg/m^3

Suffixes

cr	critical
mid	mid-channel line
0	stagnation values
x,y	orthogonal coordinate directions

1. INTRODUCTION

Modern day aero-gas turbines (military/civil) call for higher stage loadings for both compressors and turbines in order to have high thrust-to-weight ratio and fuel economy. This calls for efficient design of blade elements. Normally direct methods of profile design are resorted to. In this paper an attempt is made to design the blade profiles by an inverse method and also to investigate the flow in a cascade of blade elements so designed.

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Profiles of blade elements of a turbomachine can be designed by either direct or inverse methods. Direct method makes use of approximate geometric construction features based on existing available empirical data. Alternately, inverse methods require a desired/prescribed distribution of surface velocities for the design of blade element profiles. However, final profile cannot be arrived at without subjecting these profiles to analytical and experimental investigations on the blade cascade. But, there is not much published literature available on inverse methods of blade profile design as well as about the analytical and experimental studies done on such blade cascades. Hence, the present work was undertaken to design a turbine nozzle vane profile, making use of a computer code based on inverse design method[1]. The blade surface Mach number distribution was prescribed as suggested by Stanitz[2]. Initially, the blade profile was obtained making use of Stanitz ir rse methods. The profile thus obtained was corrected for boundary layer growth as described by Venkatakrishnaiah[1].

To test the validity of the procedure followed for the profile design i.e., whether the physical blade shape obtained by it gives the prescribed surface velocities or not, a twin approach was resorted to: (a) make an analytical study of the blade cascade by (i) stream filament method, and (ii) time marching method; and (b) to experimentally test the blade cascade in a high speed cascade tunnel.

By making use of these methods, Mach number distribution over the suction and pressure surfaces of the blade was obtained. In this paper, these Mach number distributions are compared with the one prescribed at the design stage.

2. ANALYTICAL APPROACH

2.1 Stream Filament Analysis

In this method, an attempt was made to calculate the velocity distribution in the covered channel of the cascade (Fig. 1) which satisfies the mass flow continuity at various channel orthogonals. The assumptions are made, viz. (a) the flow is steady, compressible, nonviscous and irrotational, (b) the fluid has constant enthalpy and entropy along the orthogonals, and c) the rvature or radius of curvature vary linearly along the onals from suction surface point to pressure int.

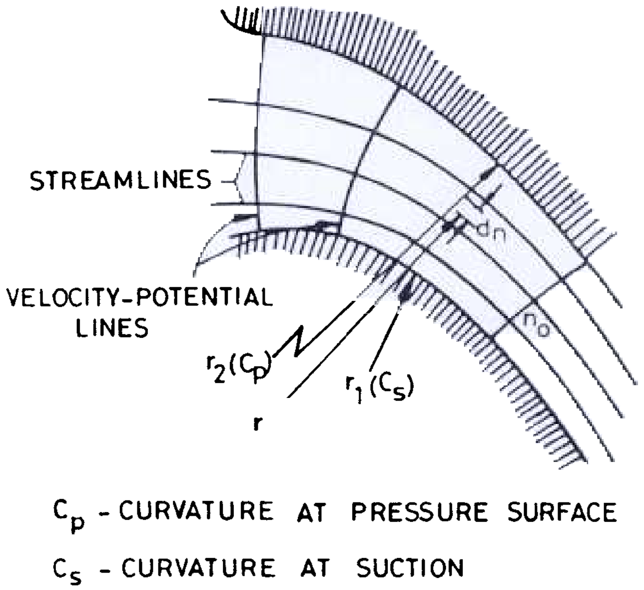

Figure 1 The flow of a gas in the channel between curved surfaces.

2.1.1 Method of Solution

Referring to Fig. 2 and considering the flow in the orthogonal grid as consisting of potential and streamlines and using natural coordinate system[3], one can write the following equation for the 2-D case :

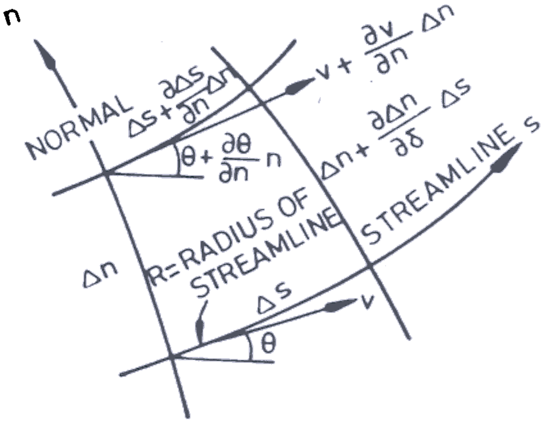

Figure 2. Natural coordinates.

Mass flow continuity

$$dW = \rho V\, dn \tag{1}$$

Momentum conservation in s and n directions

$$\rho V\ (\partial V/\partial s) = -\ \partial P/\partial s) \tag{2}$$

$$\rho V^2/R = -(\partial P/\partial n) \qquad (3)$$

Thermodynamic relation :

$$TdS = dH - dP/\rho \qquad (4)$$

Energy equation .

$$H_0 = H + V^2/2 \qquad (5)$$

Using the above equations, one can get the following equation

$$T\Delta S + V((\partial V/\partial n) - V/R) = \Delta H_0 \qquad (6)$$

If entropy and enthalpy gradients are neglected along orthogonals, the above equation reduces to

$$\partial V/\partial n = V/R = CV \qquad (7)$$

Then curvature at any point at distance *n* from the suction surface on any given orthogonal is given by

$$C = C_s + (C_p - C_s \quad n/n_0 \qquad (8)$$

Where n_0 is the total length of the orthogonal (Fig. 1) On integration of Eqn (7) and using Eqn (8) one can get

$$V \quad V_{mid} = \exp[\,n_0(C_s(0.5-p)+ 0.5(C_s-C_p)(p^2-0.25)\,] \qquad (9)$$

Where V_{mid} is mid-streamline velocity (Fig. 3).

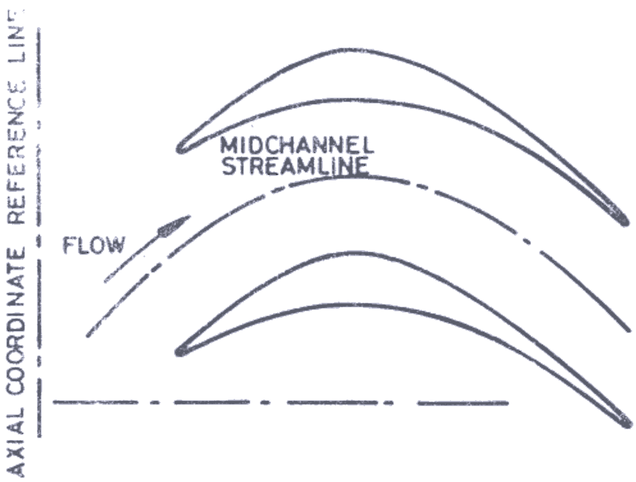

Figure 3. Blade channel.

2.1.2 Generation of Orthogonal Mesh

This is done in two steps. Firstly the blade channel is laid out from the blade profile data for any given stagger angle. The meridional distance between nose and tail boundaries is divided into equal intervals. At every meridional position the channel width is divided into a known number of equal intervals, thus obtaining an array of points. The lines joining the array points in the streamwise direction represent the horizontal orthogonals.

In the second step, the vertical orthogonals are constructed between each pair of streamwise orthogonals starting from a known point on the suction surface as shown in Fig. 4. The figure also illustrates this procedure. The desired vertical orthogonal is the average point as indicated in this figure. Using this point, the process is repeated for the next pair of streamwise orthogonals and thus the vertical orthogonal is obtained from suction surface to pressure surface. The mesh generated by this procedure is shown in Fig. 5. The curvature at the vertical orthogonal ends is also estimated by fitting a cubic spline and using numerical differentiation technique.

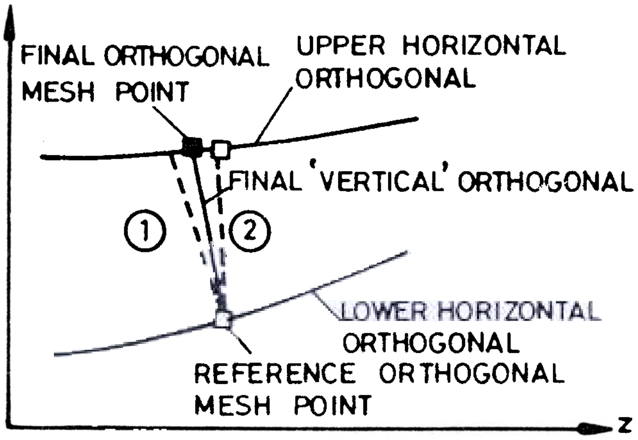

Figure 4. Calculation procedure for a vertical orthogonal link.

2.1.3 Evaluation of Velocity on Orthogonals

The velocity distribution along any orthogonal in a cascade of high solidity is determined by Eqn (9), coupled with the mass flow continuity equation as under

$$\xi = (V/V \quad \Big[\quad (\gamma - 1)/(\gamma + 1)) \quad (V/V_{cr})^2 \Big]^{(1\ /\ (\gamma -}$$

$$W_{cal} = \rho_0 V_{cr} \int_0 \xi \, dn \qquad 10)$$

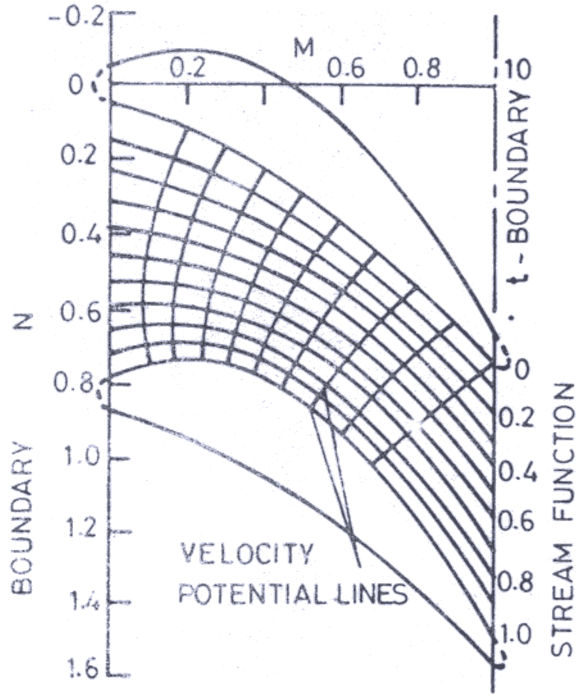

Figure 5. Orthogonal mesh generated.

The values [$(\rho V/\rho_0 V_{cr})\cos\beta$] at inlet and exit of the cascade to be specified as input to the calculation.

2.1.4 Calculation Procedure

For a given orthogonal of known length and end curvatures, the following steps are involved in the calculation of flow across it.

(a) Starting with a guess value of mid-channel streamline velocity V_{mid}/V_{cr}, the velocity distribution along the orthogonal is calculated using Eqn (9).

(b) Mass flow across the orthogonal is calculated using Eqn (10), and a check is made for the required mass flow.

(c) If the calculated mass flow does not agree with the required value, then the value of V_{mid}/V_{cr} is changed suitably and the calculation is repeated for convergence. This procedure is repeated for all orthogonals and thus the distribution of surface velocities is obtained.

2.2 Analysis by Time Marching Method

This method makes use of the unsteady Euler equations of motion as follows :

$$(\partial/\partial t)[F] + (\partial/\partial X)[G] + (\partial/\partial Y)[H] = 0$$

Where

$$[F] = \begin{matrix} \rho \\ \rho V_x \\ \rho V_y \end{matrix} \quad [G] = \begin{matrix} \rho V_x \\ P + \rho V_x^2 \\ \rho V_x V_y \end{matrix} \; ; [H] = \begin{matrix} \rho V_y \\ \rho V_x V_y \\ P + \rho V_y^2 \end{matrix} \tag{11}$$

The flow field is divided into a finite number of volumes. Then for each volume ΔV and time step Δt Eqn (11) can be written in conservation form as :

Mass :

$$\Delta V . \Delta \rho = \Delta t \; \Sigma \, (\rho V_x dS_x + \rho V_y dS_y) \tag{12}$$

x momentum

$$\Delta V \; \Delta (\rho V_x) = \Delta t \; \Sigma \, [(P + \rho V_x^2) dS_x + \rho V_x V_y dS_y] \tag{13}$$

y momentum

$$\Delta V \; \Delta (\rho V_y) = \Delta t \; \Sigma \, [\rho V_x V_y dS_x + (P + \rho V_y^2) dS_y] \tag{14}$$

The energy equation is not considered as the flow is assumed to be at constant enthalpy.

2.3 Calculation Procedure

This involves three steps: (i) divide the flow field into a finite number of volumes; (ii) for each volume ΔV and time step Δt write Eqn (11) in conservation form (energy equation is not considered as the flow is assumed to be at constant enthalpy); and (iii) solve these equations for each point of the grid (Fig. 6) for each time step by summing up the mass and momentum fluxes over all the faces and updating the properties accordingly.

This process starts from initial guess based on inlet and outlet conditions of the cascade and marches

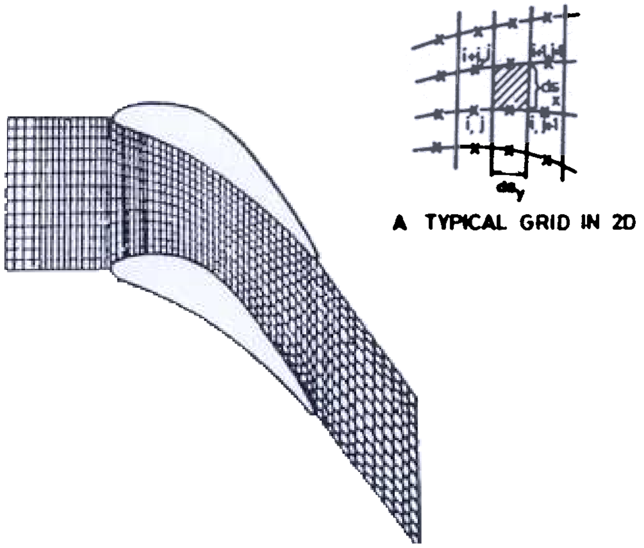

Figure 6. Grid used for analysis.

forward in time, until steady state solution is reached. In the steady state, mass, momentum and energy are conserved and thus the method can deal with transonic flows. A 2-D computer programme code was developed based on Denton[4] and Soundarnayagam[5], and was used to analyse the flow over the cascade.

3. EXPERIMENTAL FACILITY

The experimental facility used for the present study is described elsewhere[6]; a summary is given here. Full scale blades having 80 mm total chord and aspect ratio 1.8 were fabricated out of teak wood using a two piece template. The instrumented blade has 11 static pressure taps (of 0.5 mm dia) on suction and pressure surfaces each. The instrumented blade was mounted on to a circular disc, which is graduated to enable reading the stagger angle setting directly. The tunnel test cross-section is rectangular (251 × 138 mm). It carries a worm gear arrangement as shown in Fig. 7.

This is used for (a) varying the stagger angle of the blades simultaneously, and (b) holding the blades in the secured position during the testing. The instrumented blade is connected to a 25 limb multi-tube mercury manometer through a freezing mechanism.

4. RESULTS AND DISCUSSION

4.1 Comparison of Analysis with Design

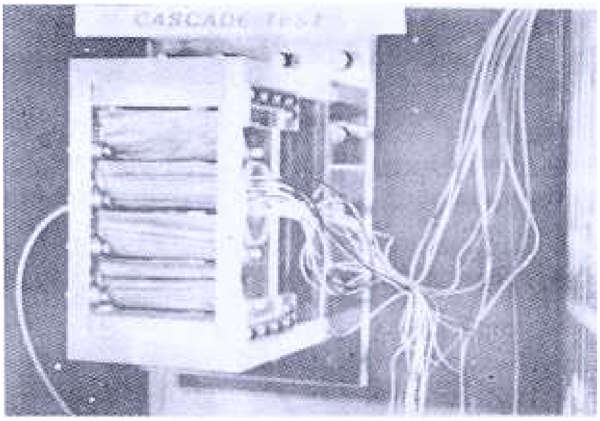

Figure 7. Test section.

A computer code was developed for orthogonal grid generation and for stream filament cascade flow analysis as described in Section 2.1. A plot of the orthogonal grid generated is shown in Fig. 5. The surface Mach number distribution obtained from the stream filament analysis for the design flow/pressure ratio is plotted in Fig. 8 with respect to axial chord. A computer code was developed for the flow analysis using time marching method. The grid used for the analysis is shown in Fig. 6. Results of this analysis for the design pressure ratio of 1.52 (for which isentropic average cascade exit Mach number is 0.8) are shown in Fig. 8. In this figure the plot of prescribed Mach number distribution on the pressure and suction surfaces of the blade is also shown for the sake of comparison.

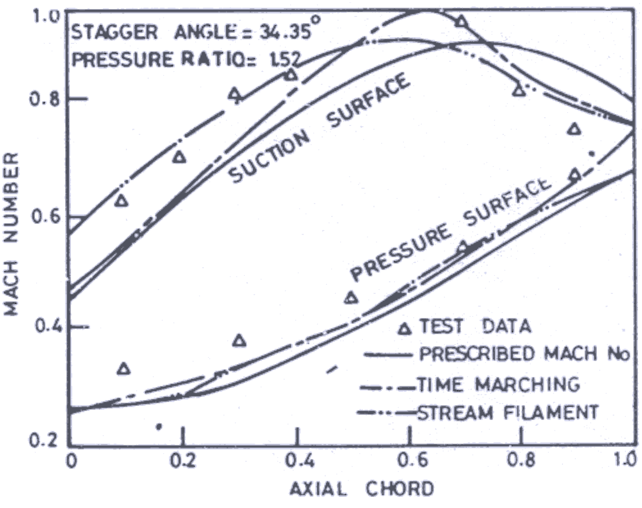

Figure 8. Axial chord vs mach number.

It can be observed from Fig. 8 that both stream filament analysis and time marching analysis give rise to higher Mach numbers on the suction surface from leading edge to the axial position of 0.7. However, Mach numbers are lower thereafter up to the trailing edge compared to the prescribed values. The lower values of Mach numbers in the uncovered portion of the channel on the suction surface may be attributed to the inadequacy of the approximate inverse methods used here to handle the complex flow in this region. Mach numbers on the pressure surface obtained from the two analytical methods are close to the prescribed values.

4.2 Experimental Study of Cascade Flow

Experiments were conducted for the study of flow in a 2-D cascade at the designed stagger angle setting of 34.35 degrees at four blowdown pressures. The blowdown pressure ratios (defined as the ratio of gas inlet total pressure to exit static pressure) were 1.31, 1.37, 1.42 and 1.52 (design value).

Figure 9 shows the measured isentropic Mach number distribution on the suction and pressure surfaces of the blade (plotted with respect to the axial chord excluding the leading and trailing edge radii). The same obtained by time marching technique for these pressure ratios is also plotted for comparison. From Fig. 9 (top) it is seen that in the forward region of the suction surface (0.0 to 0.35), the measured Mach numbers are slightly higher than the estimated values. Close agreement can be seen between 0.4 and 0.7 of the axial chord, while from 0.75 to trailing edge, the measured Mach numbers are lower than the estimated values. This can be attributed to the following factors :

(a) Both design and analytical methods assume inviscid flow and therefore do not consider the blade element pressure loss over the profile; and

(b) Boundary layer growth on the suction surface is more than that predicted at the design stage especially due to the presence of adverse pressure gradient.

On the pressure surface, the measured Mach numbers are higher than the predicted values, though close agreement exists near the trailing edge region. The same trend is noticeable at other blowdown pressure ratios as seen from Fig. 9.

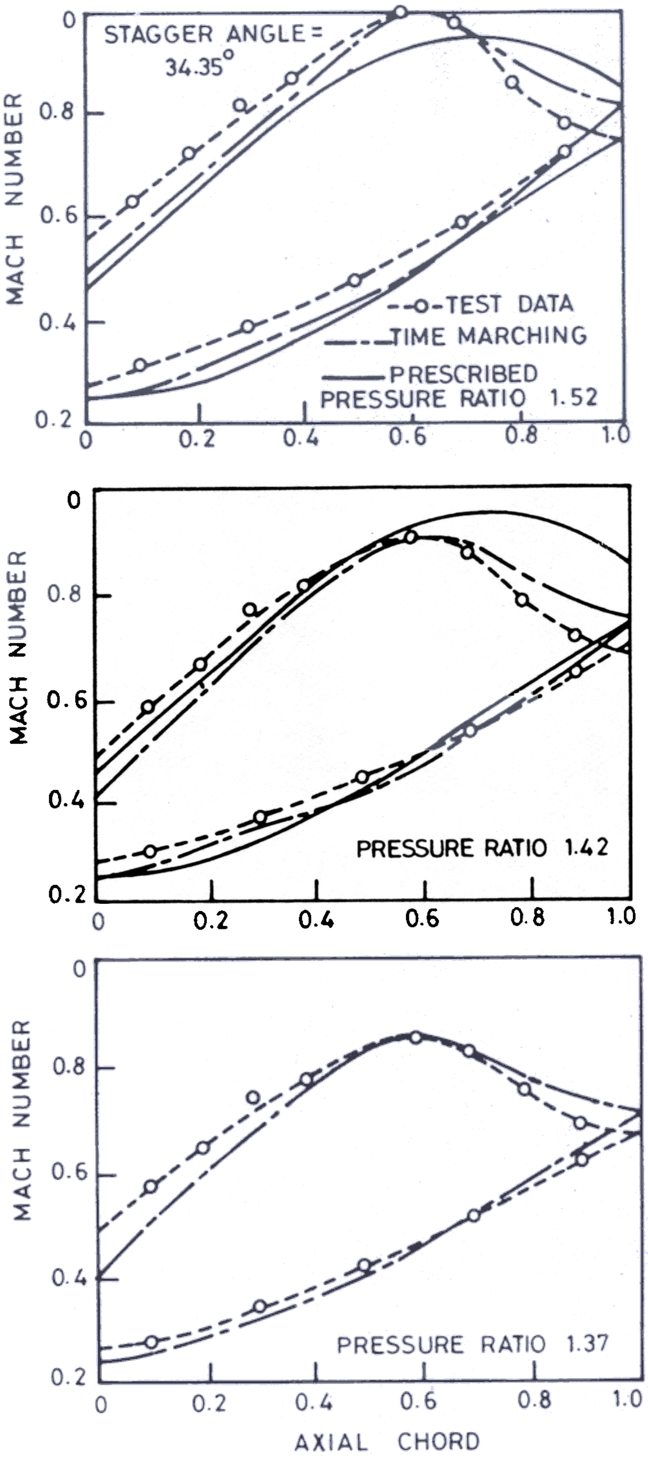

Figure 9. Axial chord vs mach number.

5. CONCLUSION

In the present study, an attempt has been made to compare the design of blade profile elements by inverse methods by analytical and experimental investigations. Results of the study indicate that both stream filament analysis and time marching analysis give higher Mach numbers (from leading edge to axial chord position of 0.7) compared to the prescribed values. The peak Mach number computed by the time marching analysis and experimental measurements are higher than the prescribed value on the suction surface. Also, beyond this point, the Mach numbers on the suction surface are lower. Mach numbers obtained by both the analytical methods show close agreement with prescribed values on the pressure surface.

Experimental results on the blade cascade indicate that Stanitz's inverse methods used for arriving at the blade contour requires improvement to take into account viscous effects as seen from Fig. 8. Also, measured Mach numbers on the uncovered region of the channel on suction surface are lower than the analytical values indicating the difference between the real flow and the inviscid 2-D calculations.

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