

## Astrophysics Contributions of Indian Scientists

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### ABSTRACT

A glimpse of astronomy and astrophysics from the earliest times to the end of the nineteenth century is given. This is followed by important contributions to astrophysics in the twentieth century with detailed accounts of the work of MN Saha, DS Kothari and a few others. This review ends with a brief account of the astrophysical activities in which a large number of scientists are presently involved.

### INTRODUCTION

Astronomy had an hoary past in India with the oldest astronomical text Vedanga Jyotisha dated around 1400 BC<sup>1</sup>. The contribution to mathematical or Siddhantic astronomy, due to the interaction with Greece in the post-Alexander period, made an impact owing to the work of Aryabhata I, Bhaskara I, Bramagupta, Aryabhata II, Bhaskara II, and others, starting at the end of the fifth century and going up to the beginning of the twelfth century. The big event after this was the five masonry observatories built by Raja Jai Singh Sawai in the first half of the eighteenth century; however, these observatories remained architectural show pieces as the invention of telescope had already taken place in Europe by then. The astronomical activities once again started at the end of the nineteenth century, when the Europeans, mostly the British, came to India for observing the solar eclipses of 1868, 1871 and 1872; this set the ball rolling for astrophysical activities in India, culminating in the setting up of the Kodaikanal Observatory in 1899 (this has now become the Indian Institute of Astrophysics with its headquarters at Bangalore). This resulted in some important discoveries from India like the discovery of the helium line during the total eclipse of 1868 by the British and the French scientific team<sup>2</sup>, the discovery of F-corona by Janssen<sup>3</sup> and the discovery of the Evershed<sup>4</sup> effect by John Evershed when he found

the radial motion of solar gases in the vicinity of sunspots.

However, it was the theory of thermal ionization and its application to the understanding of the stellar spectra by Megh Nad Saha<sup>5,6</sup> (1893-1956) in 1920 and 1921 which ushered India not only to the forefront of modern astrophysics, but also left a stamp for all times to come. Important contributions of MN Saha, DS Kothari, S Chandrasekhar and a few others are discussed here, and a brief account of the present-day astrophysical activities is given.

### 2. MODERN ASTROPHYSICS

Megh Nad Saha was a multifaceted personality who contributed not only to theoretical astrophysics but also to atomic and molecular spectroscopy, nuclear physics, propagation of radio waves, age of rocks, Indian calendar reform, water management and national planning as well. Though he did his MSc in applied mathematics, he was inspired to astrophysical problems after he came across two popular books on the Sun and the stars by Agnes M Clarke.

Saha's<sup>7</sup> first work on astrophysics, a short note on 'Radiation pressure and quantum theory—a preliminary work' which dealt with the formulation of the concept of selective radiation pressure was very significant though it has not got the credit it deserves; the fact that

the detailed paper<sup>8</sup> on the subject was published in an obscure journal may be the only cause. The main conclusion of this paper, in his own words, was "that radiation-pressure may exert an effect on the atoms and molecules which are out of all proportion to their actual sizes. It also shows that the radiation-pressure exerts a set of sifting actions on the molecules, driving the active ones radially outward along the direction of the beam. The cumulative effect of the pulses may be sufficiently great to endow the atoms with a large velocity—the velocity with which the tops of solar prominences are observed to shoot up". Olin C Wilson<sup>9</sup> tried this idea to explain mass loss from cool stars, and this idea is very much there in explaining the mass loss from early type stars as well.

The theory of thermal ionization was proposed by Saha in an innocuously titled paper 'Ionization in the solar chromosphere'<sup>5</sup>. This was elaborated in two other papers, and the fourth paper<sup>6</sup> dealt with application to the interpretation of stellar spectra. All these four papers were prepared between February and September 1919 and communicated from India for publication in *Philosophical Magazine* between August and September; however, the fourth paper, on the advice of Professor A Fowler whose laboratory MN Saha was visiting, was withdrawn, completely revised, updated using the latest laboratory spectroscopic data and published in the *Proceedings of the Royal Society*.

In trying to understand the solar spectra, he used the law of mass action or chemical equilibrium, though Saha called it Nerst's formula of 'Reaction-isobar' and extended it to ionization in place of chemical dissociation. If  $x$  is the degree of ionization of a given species at temperature  $T$  and total pressure  $P$ , then

$$\log K = \log \frac{x^2}{1-x^2} P = \frac{U}{4.571T} + \frac{5}{2} \log T - 6.5$$

where  $K$  is the chemical equilibrium constant or 'Reaction-isobar' and  $U$ , the ionization potential or heat of ionization (in place of heat of dissociation) in calories. The important thing to note is that the degree of ionization depends not only on temperature but on pressure as well. The above equation, as given by Saha, has been extended to include the effect of excited states of atoms and ions, to multiple ionization, to a gaseous mixture undergoing ionization, inclusion of interparticle interaction and treating electron having a Fermi-Dirac

rather than a Boltzmann distribution. It is used not only in astrophysical applications but in other fields as well. Using this equation, Saha was not only able to explain many aspects of the solar spectrum, but also the varied nature of stellar spectra and give physical interpretation to the spectral sequence.

The Henry Draper (HD) system of spectral classification<sup>10</sup> introduced by Pickering, Miss Maury and Miss Cannon was simply an arrangement of spectra in order of decreasing ratio of intensities of the Balmer lines of hydrogen to intensities of a number of other lines. More than 99 per cent of stars fell into a small number of groups: *O, B, A, F, G, K, M*; of the small minority, *P, R* and *N, P* refers to spectra of gaseous nebula and types *R* and *N* overlap with *K* and *M*. Some stars had a small *c* suffix, which was found to be for stars of higher luminosity or lower density. Saha looked into the appearance and disappearance of various lines at different temperatures and pressures, and explained the sequence. Milne extended it by considering where the peak or maximum of lines occurs rather than the marginal appearance/disappearance which Saha adopted; Milne's method is to be preferred for broad classification but Saha's approach is better for further subclassification of spectra. Saha would have further carried his theory of thermal ionization but for the lack of observational stellar spectra at his disposal, whereas Henry Norris Russell (1877-1957), the doyen of American astrophysicists, fully realized the impact of the equation and exploited it by his regular pilgrimage to the Mount Wilson Observatory.

Note that though Saha was a theoretician, he kept his work close to observations and experimental results and whenever he felt the need, he persuaded others, including some of his students, to conduct the experiment. In this connection, he proposed<sup>11</sup> at the Harvard College Observatory that a stratosphere solar observatory be set up to observe the Sun in the ultra-violet below 2900 Å. This unique proposal was more than twenty years ahead of time.

Saha<sup>12</sup> was the first to think of some of the observed interstellar lines being molecular in origin.

Though H C van de Hulst<sup>13</sup> in the Netherlands was the first to predict the 21 cm line of atomic hydrogen arising from the hyperfine structure in 1944, Saha<sup>14</sup> independently predicted it. The line was observed by three groups in 1951.

Though Saha contributed to the growth of astrophysics in various ways, his equation of thermal ionization has become an indispensable part of physics and astrophysics.

Daulat Singh Kothari (1906-1993), a student of Professor MN Saha at Allahabad University and of Professor RH Fowler at Cambridge University, made many important contributions to the development of science and education in India, besides making many notable contributions to astrophysics. DS Kothari along with RC Mazumdar<sup>15</sup> computed opacity coefficients for electron degenerate matter using a rigorous quantum mechanical treatment and showed<sup>16</sup> later that the energy flow in the degenerate stellar cores is mainly by the thermal conduction and not by the radiative process. This is still true though somewhat improved coefficients are being used now.

Though Kothari<sup>17</sup> did not visualize the neutron stars, he was the first to suggest neutrons in the interior of cold degenerate bodies like white dwarfs by inverse beta decay by a very simple, back-of-the-envelope calculation, using the difference in mass between neutron and the combined mass of proton and electron and the nuclear volume.

Kothari's<sup>18</sup> theory of pressure ionization and its application to planetary masses is a landmark paper of his. In this paper, he derived a relation for the degree of ionization in the non-relativistic degenerate matter, using virial theorem, and applied it to spherical bodies at zero temperature under their own gravitational force. The important conclusions of this paper can be summarized as:

- a. The ionization takes place at high densities, even at zero temperature, owing to pressure. It is called pressure ionization in contrast to Saha's thermal ionization.
- b. It is possible for hydrogen to be in metallic form at the core of cold dense bodies composed mostly of hydrogen, where pressures greater than  $10^{11}$  dyn  $\text{cm}^{-2}$  may exist. This may be the case for planets like Jupiter and Saturn.
- c. A mass-radius relation was found for cold non-relativistic electron degenerate masses, whose maximum radius occurred at mass  $M_{\text{critical}}$  one-hundredth of the solar mass.
- d. The masses of all the (known) planets in our solar system are less than the critical mass.

A great deal of work<sup>19</sup> has been done but the last conclusion has defied any satisfactory answer so far.

In a study (jointly with BN Singh<sup>20</sup>) of the Bose-Einstein statistics and degeneracy, Kothari considered non-equilibrium radiation with spectrum proportional to  $x^3 dx / [\exp(x+p)-1]$  and called it non-degenerate radiation; they conjectured that non-degenerate radiation may be partly responsible for the departures from black-body radiation in astrophysical studies. Twenty-five years later, Wildt<sup>21</sup> found a similar distribution while studying the thermodynamics of gray atmosphere.

Kothari had looked into the problem of fragmentation in 1 and 3 dimensions, along with his collaborators. He applied it, with RS Kushwaha<sup>22</sup> in understanding the distribution of stars in the Hydes cluster.

Subrahmanyam Chandrasekhar (1910), Nobel Laureate, left for Cambridge for higher studies in the summer of 1930 and later on settled in the USA. However, he had already started his astrophysical research before leaving India, and the work leading now to the famous 'Chandrasekhar limit', which predicts a finite mass limit for a relativistic degenerate body like white dwarfs, was in the making when he left the shores of India<sup>23-25</sup>. This, along with his work on vibrational and secular instabilities, was specifically singled out in his 1983 Noble citation. His other contributions are well known through his more than half a dozen monographs and a few review articles that he has authored that we will not discuss here.

Though MN Saha and Satyendra Nath Bose (1894-1974) were interested in Albert Einstein's General Theory of Relativity, yet the honour goes to Vishnu Vasudeva Narlikar (1908 - 1991) to establish a school of relativity at the Benaras Hindu University in the 1930s. Through this school, this discipline spread to other centres in India as well. One stream of workers was mainly interested in General Relativity purely from a mathematical point of view, whereas, others used it as a tool in relativistic astrophysics and cosmology. Concentrating on the latter stream, the work of Prahlad Chunnilal Vaidya (1918) and of Amal Kumar Raychaudhuri (1923) stands out.

PC Vaidya<sup>26</sup> considered a spherical radiating mass surrounded by a finite non-static envelope of radiation zone, using the metric, known by his name now, and

found it to be surrounded by a radial field of gravitational energy becoming weaker and weaker as it runs away from the central body until at last the field is flat at infinity. This remained buried in the literature, till it was resurrected owing to the advent of relativistic astrophysics by Lindquist, Schwartz and Misner<sup>27</sup>, and Vaidya's metric is now being extensively used.

The Raychaudhuri<sup>28</sup> equation in cosmology governs the evolution of a fluid, especially the expansion, when the rotation and vorticity are also included. This is basic to the singularity theorem of Hawking and Penrose<sup>29</sup>.

Manali Kallat Vainu Bappu (1927-82), who transformed the Kodaikanal Observatory to the Indian Institute of Astrophysics, discovered, jointly with Olin C Wilson<sup>30</sup>, a linear relation between the width of the  $K_2$  emission core in the resonance line of Ca II at  $\lambda$  3933Å, observed in late-type stars, and the absolute magnitudes of the stars; this Wilson-Bappu effect is valid over a range of 16 magnitudes or more than a million in luminosity of the star. A similar relation was later found using ultraviolet Mg II resonance line  $h$  and  $k$  emission cores around  $\lambda$  2800Å. In trying to understand this relation, Bappu found that in the Sun, the principal contributor to the emission is the bright fine mottle of size 1-2 arcsec and has connected this to the evolution of stars from subgiant to giant phase.

Shiv Sharan Kumar<sup>31</sup> (1939) found that a proto-star of mass less than or equal to 0.08 solar mass undergoing gravitational contraction never attains high enough central temperature for nuclear burning to take place owing to the onset of degeneracy. This protostar never reaches the main sequence and becomes black dwarf, or as it now called, brown dwarf. Recent evidence suggests that these brown dwarfs may mitigate the problem of the missing matter in the universe.

### 3. PRESENT ASTROPHYSICAL ACTIVITIES

After a lull of several decades except for a few individual efforts here and there, astrophysical activities were revived starting from fifties and sixties in India and have made very good progress. The Uttar Pradesh State Observatory (UPSO) was established in 1954. The Radio Astronomy Group was formed around 1963 at the Tata Institute of Fundamental Research (TIFR) which has now developed into the National Centre for Radio Astrophysics; the Theoretical Astrophysics Group was also added there a few years later, along

with shifting of many of the cosmic ray physicists into x-ray,  $\gamma$ -ray and infrared astronomy. The group working at the Kodaikanal Observatory grew and developed into the Indian Institute of Astrophysics (IIA) in 1971. The Raman Research Institute (RRI) became a national institute for research in basic sciences in 1972 with a core group in theoretical astrophysics, general relativity and gravitation, and radio astronomy. Theoretical astrophysics, x-ray, infrared and radio astronomy were started at the Physical Research Laboratory (PRL). The Udaipur Solar Observatory, now under PRL, was set up in 1975. And to top it all, a Joint Astronomy Programme was started at the Indian Institute of Science (IIS) in collaboration with the Indian Institute of Astrophysics, the Indian Space Research Organization, the Physical Research Laboratory, the Raman Research Institute and the Tata Institute of Fundamental Research to train young students in astrophysics to cater to the needs of the various centres, as Osmania University (OU) was the only other place with a full-fledged Astronomy Department.

Most of these centres are pure research institutes but to inculcate the excitement of astrophysics into the young budding scientists, one has to take astrophysics to the university arena itself. With this aim, the Inter-University Centre for Astronomy and Astrophysics (IUCAA) was established in 1988 by the University Grants Commission.

It is neither possible nor perhaps feasible to describe the large amount of research work being done by scientists at all these centres. Therefore, it is best to describe only selective work going on in various disciplines of astrophysics, concentrating mostly on theoretical activities.

#### 3.1 Solar System

KS Krishna Swamy (TIFR) and his collaborators have been trying to understand the spectra and other properties of comets; he explained (with CR O'Dell) the puzzle of the relative intensities of the Swan bands of  $C^2$ . B Buti (PRL) has looked into the cometary ionic tail. SK Chakrabarti (TIFR) has been looking into the number distribution of comets in the Oort cloud and their rate of escape due to stellar interaction. KB Bhatnagar (Delhi University) and others have been carrying out studies in celestial mechanics including restricted three body-problem and the phenomenon of chaos. JC Bhattacharyya (IIA) and K Kuppaswamy

(IIA) found independently Saturn-like ring around Uranus. R Rajamohan (IIA) discovered a new asteroid, now named Ramanujan.

### 3.2 Solar and Stellar Physics

The study of global solar oscillation to understand the interior of the Sun has been undertaken by SM Chitre (TIFR) and his coworkers; they are also part of the GONG (Global Oscillation Network Group) project along with A Bhatnagar (PRL) and others from India. MC Pande (UPSO) and collaborators have been studying various aspects of solar physics. KR Shivaraman (IIA) and his group have been analysing Sun using *H* and *K* lines of Ca II. HM Antia, SM Chitre and D Narasimha (all from TIFR) have been studying solar and stellar convection and overshooting.

KD Abhyankar (Osmania University) has done some very good work on radiative transfer of moving atmosphere. A Peraiah (IIA) and his students and coworkers have been involved in radiative transfer studies including the calculation of scattering functions for the Rayleigh phase matrix, with special reference to spherical geometry; they have applied these to planetary and stellar atmospheres.

MS Vardya (TIFR) proposed the idea of pressure dissociation in the context of stellar atmospheres and estimated its effect; he considered the importance of a large number of sources of opacity in late-type stars and computed march of molecular abundances with depth in cool stars. SP Tarafdar (TIFR) formulated a universal expression for mass loss in stars and MS Vardya found that rotation enhances stellar rate of mass loss. SP Tarafdar, along with KS Krishna Swamy (TIFR) and MS Vardya, discovered the CO molecule in hot *B* supergiants and several *Be* stars. MVK Apparao (TIFR) and SP Tarafdar are trying to understand several unexplained properties of *Be* stars. N Kameswara Rao (IIA) has been studying hydrogen deficient R Cor Bor type stars and N Parthasarthy (IIA) has been studying post-asymptotic giant branch stars, among other things.

### 3.3 Interstellar Medium

SP Tarafdar and his associates have been studying various aspects of star formation. SS Prasad (Jet Propulsion Laboratory, Pasadena)-Tarafdar mechanism, in which cosmic ray excitation of the Lyman and Wener systems of the hydrogen molecule is

incorporated, is found to produce significant levels of ultraviolet photon flux in dense interstellar clouds. SP Tarafdar and associates constructed dynamical models of low mass stars from interstellar clouds using processes such as excitation, ionization, cooling and chemical reactions. NC Rana (IUCAA, earlier at TIFR) has been working on stellar and primordial nucleosynthesis and the overall chemical evolution of the Galaxy.

### 3.4 High Energy Astrophysics

A large number of research workers are active in this field. Some of the principal scientists along with their affiliations and main areas of interest are: PC Agarwal (TIFR; x-ray astronomy); MVK Apparao (TIFR; Cosmic ray astrophysics and x-ray astronomy); SK Chakrabarti (TIFR; Accretion discs around compact objects); R Cowsik (IIA and TIFR; Cosmic ray astrophysics); B Datta (IIA; Neutron stars); P Ghosh (TIFR; Neutron stars and accretion discs); Gopal Krishna (TIFR; Radio source modelling); V Krishna (IIA; Plasma astrophysics); R Nityananda, (RRI; Pulsars); N Panchapakesan (Delhi University; Pulsars); V Radhakrishnan (RRI; Pulsars); A Ray (TIFR Pre-supernova stages, neutron stars); PV Ramanamurthy (TIFR; x-ray astronomy and pulsars); BV Sreekantan (TIFR; x-ray astronomy); G Srinivasan (RRI; Pulsars); SC Tonwar (TIFR; Pulsars); M Vivekanand (TIFR; Accretion discs).

### 3.5 Extragalactic Astronomy and Cosmology

This is a very active area of research not only internationally but nationally as well, and a very large number of scientists are working on several aspects of this field. The field got a big boost when JV Narlikar, who had earlier contributed significantly with F Hoyle at Cambridge University, joined the Tata Institute of Fundamental Research in the early 1970s and the Ooty Radio Telescope, under the guidance of G Swarup, became operational. Some of the leading players in the field are: SM Alladin (Osmania University; Interacting and merging galaxies); S Banerjee (Burdwan University; Quasars); SM Chitre (TIFR; Gravitational lensing); R Cowsik (IIA and TIFR; Dark matter, dwarf elliptical galaxies); N Dadhich (IUCAA; Black holes); SV Dhurandhar (IUCAA; Gravitational waves); PS Joshi (TIFR; Cosmic censorship, global hyperbolicity and causality); P Joshi (Utkal University; Quasars); S Kandaswamy (TIFR; Gravitational lensing, galactic

dynamics); VK Kapahi (TIFR; Cosmology, physics of radio sources); RK Kochhar, (IIA; Interacting and merging galaxies); JV Narlikar (IUCAA, earlier at TIFR; Quantum gravity; steady state cosmology, quasars, gravitational lensing); D Narsimha (TIFR; Gravitational lensing and large scale structure). R Nityananda (RRI; Gravitational lensing and large-scale structure); T Padmanabhan (IUCAA, earlier at TIFR; Quantum gravity, gravi-thermal instabilities, cosmic microwave background); N Panchapakesan (Delhi University; Early universe); A R Prasanna (PRL; Black holes and accretion); TP Singh (TIFR; Quantum gravity and large-scale structure); G Swarup (TIFR; Protogalaxies); CV Vishveshwara (IIA, earlier at RRI; Black holes, Vaidya's matrix).

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