Defence Science Journal, Vol 45, No. 4, October 1995, pp 341-347

MASS TRANSFER EFFECTS ON UNSTEADY FREE CONVECTIVE FLOW PAST AN INFINITE, VERTICAL POROUS PLATE WITH VARIABLE SUCTION

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ABSTRACT

A two-dimensional unsteady flow of a viscous incompressible dissipative fluid past an infinite, vertical porous plate with variable suction, is studied. Approximate solutions to the coupled non linear equations governing the flow are derived and expressions for the fluctuating parts of the velocity, the transient velocity, temperature and concentration, the amplitude and the phase of the skin-friction, and the rate of heat transfer, are derived. The effects of ω (frequency), Gr (Grashof number), Gc (modified Grashof number), Sc (Schmidt number), P (Prandtl number) and A (variable suction parameter), on the above physical quantities are calculated numerically and presented in figures and table. Problems of this nature find place in ablative cooling, transpiration and film cooling of rocket and jet engines.

1. INTRODUCTION

Engineering applications of convective heat and mass transfer are extremely varied. Calculation of temperature of a closed turbine blade or the throat of a rocket nozzle involves convective heat transfer alone but if a fluid is injected through the surface, the problem is a mass transfer one. Mass transfer finds its place in ablative cooling (sudden decrease in the temperature of space vehicles during their re-entry into the atmosphere), transpiration and film cooling of rocket and jet engines. Further, the effects of variable suction on the flow past an infinite, vertical plate in the presence of free convection currents, will be found useful in the study of aircraft response to atmospheric gusts, in airfoil lift hysteresis at the stall, in flutter phenomena involving wing and in the prediction of flow through turbomachinery blade cascades.

Ever since the pioneering effort by Lorenz[1], the analysis of free or natural convection has been of considerable interest to engineers and scientists. Most studies in this field have been concerned solely with thermal convection; however, as Gebhart and Pera[2] have pointed out, buoyancy effects resulting from concentration gradients in multicomponent mixtures can be just as important in generating fluid motion as can temperature gradients.

The problem of thermal convection, a situation in which the buoyancy forces are generated only by temperature gradients, has been considered by many researchers. It is only quite recently, Eichhorn[3], Sparrow, Minkowycz and Eckert[4], Gebhart and Pera[2] and many others[5] have considered the problem of free convection, which takes into account the buoyancy effects, has been presented in the case of semi-infinite plates without considering the effects of suction and neglecting the dissipative heat, on the free convective flow. But Gebhart[5], and Gebhart and Mollendrof[6] have shown that the viscous dissipative heat is important when the natural convective flow field is of extreme size or the flow is at extremely low temperature or in high gravity field. Further, the presence of suction is more appropriate from the technological point of view.

The steady free convective flow of a dissipative incompressible fluid past an infinite, vertical porous plate with suction, was analysed by Soundalgekar[7]. Subhashini, *et al*,[8] have studied the mass transfer effects

Received 06 September 1994

on the flow past a vertical porous plate without considering the viscous dissipative heat. Hence this paper attempts to present an analytical analysis of mass transfer effects of unsteady free convective flow past an infinite, vertical porous plate with variable suction, on considering the heat due to viscous dissipation.

2. MATHEMATICAL ANALYSIS

An unsteady two-dimensional flow of an incompressible, viscous, dissipative fluid past an infinite, vertical porous plate is considered. Here, the origin of the coordinate system is taken to be at any point of the plate. The x' and y' axes are chosen in the upward direction along the plate and normal to the plate, respectively. All the fluid properties are assumed to be constant except that the influence of the density variations with temperature considered only in the body force term does not affect the other terms of the momentum and energy equations. The variations of expansion coefficient with temperature is assumed to be negligible. Since the plate is infinite, all the physical variables are functions of y' and t' only. The species concentration, C' is assumed to be small and hence Soret and Dufour effects are negligible.

From the continuity equation, we can see that v' is either a constant or a function of time. So, assuming the suction velocity to be oscillating about a non-zero constant mean, then

$$v' = -v'_w = -v_0(1 + \varepsilon A e^{i\omega' t'}$$

where v_0 is the mean suction velocity and ϵ, A are small, such that $\epsilon A << 1$. The negative sign indicates that the suction velocity is directed towards the plate.

However, the suction velocity is related to the concentration at the plate by the mass balance equation

$$v'_w(\rho - C'_w) = -D\left(\frac{\partial C'}{\partial y'}\right)_{y'=0}$$

Here, species concentration at the plate is assumed to be constant, as in most cases (Eckert & Drake[9]).

Then the flow is governed by the following system of non-dimensional equations:

$$\frac{1}{4}\frac{\partial u}{\partial t} - (1 + \varepsilon A e^{i\omega t})\frac{\partial u}{\partial y} = Gr\,T \quad + Gc\,C + \frac{\partial^2 u}{\partial y^2} \qquad (1)$$

$$\frac{P}{4}\frac{\partial T}{\partial t} - P(1 + \varepsilon A e^{i\omega t})\frac{\partial T}{\partial y} = \frac{\partial^2 T}{\partial y^2} + PE\left(\frac{\partial u}{\partial y}\right. \qquad (2)$$

$$\frac{Sc}{4}\frac{\partial C}{\partial t} - Sc(1 + \varepsilon A e^{i\omega t})\frac{\partial C}{\partial y} = \frac{\partial^2 C}{\partial y^2} \qquad (3)$$

The corresponding boundary conditions are

$$u = 0,\ T = T_w(t) = \quad + \varepsilon e^{i\omega t},\ C = \quad \text{at } y = 0$$
$$u \to 0,\ T \to 0,\ C \to 0 \text{ as } y \to \qquad (4)$$

The non-dimensional quantities are

$$y = \frac{v_0 y'}{\nu},\ t = \frac{v_0^2 t'}{4\nu},\ \omega = \frac{4\nu\omega'}{v_0^2}$$

$$u = \frac{u'}{v_0},\ T = \frac{T' - T'_\infty}{T'_w - T'_\infty}$$

$$v = \frac{v'}{v_0},\ Gr = \frac{\nu g_x \beta (T'_w - T'_\infty)}{v_0^3},$$

$$P = \frac{\mu C_p}{k},\ E = \frac{v_0^2}{C_p(T'_w - T'_\infty)}$$

$$Gc = \frac{\nu g_x \beta^*(C'_w - C'_\infty)}{v_0^3},$$

$$Sc = \frac{\nu}{D} \quad \text{and} \quad C = \frac{C' - C'_\infty}{C'_w - C'_\infty}$$

Here, u' is the velocity component in the x – direction, t' the time, g_x the acceleration due to gravity, β the coefficient of volume expansion, β^* the coefficient of thermal expansion with concentration, T' the temperature of the fluid, T'_∞ the temperature of the fluid far away from the plate, C' the species concentration, C'_∞ the species concentration in the fluid far away from the plate, T'_w the plate temperature, C'_w the species concentration near the plate, ν the kinematic viscosity, ρ' the density, C_p the specific heat at constant pressure, k the thermal conductivity, D the chemical molecular diffusivity and μ the coefficient of viscocity.

All the physical quantities have their usual meaning. The second term in the right hand side of Eqn. (2) represents the viscous dissipative heat and because of retaining this term in equation of energy, the problem is governed by the coupled non-linear equations.

To solve these coupled non-linear equations following Lighthill[10], we assume that the equations in the neighbourhood of the plate as

$$u(y) = u_0(y) + \varepsilon e^{i\omega t} u_1(y)$$
$$T(y) = T_0(y) + \varepsilon e^{i\omega t} T_1(y) \quad (5)$$
$$C(y) = C_0(y) + \varepsilon e^{i\omega t} C_1(y)$$

where, ϵ is a small quantity.

Substituting Eqn. (5) in Eqns (1) – (3), equating harmonic and non-harmonic terms, neglecting the coefficients of ϵ^2, the results are

$$u_0'' + u_0' = -Gr\,T_0 - Gc\,C_0 \quad (6)$$

$$u_1'' + u_1' - \frac{i\omega}{4} u_1 = -Gr\,T_1 - Gc\,C_1 - Au_0' \quad (7)$$

$$T_0'' + PT_0' = -PEu_0'^2 \quad (8)$$

$$T_1'' + PT_1' - \frac{i\omega P}{4} T_1 = -PAT_0' - 2PEu_0'u_1' \quad (9)$$

$$C_0'' + Sc\,C_0' = 0 \quad (10)$$

$$C_1'' + Sc\,C_1' - \frac{i\omega Sc}{4} C_1 = -Sc\,AC_0' \quad ($$

Here, the primes denote the differentiation w.r.t. y. The corresponding boundary conditions are

$$u_0 = 0,\ u_1 = 0,\ T_0 = \ ,\ T_1 = 1,\ C_0 = 1,\ C_1 = 0 \text{ at } y = 0 \quad (12)$$

$$u_0 \to 0,\ u_1 \to 0,\ T_0 \to 0,\ T_1 \to 0,\ C_0 \to 0,\ C_1 \to 0 \text{ as } y \to \infty$$

Solving Eqns (10) and (11) with the corresponding boundary conditions, the results obtained are

$$C = e^{-scy} - i\varepsilon e^{i\omega t} \frac{4ASc}{\omega} (e^{-my} - e^{-scy}) \quad (13)$$

The Eqns (6) – (9) are still coupled non-linear equations, whose exact solutions are not possible. So, u_0, u_1, T_0 and T_1 are expanded in powers of E, as the Eckert number for the incompressible fluids is always very small. Hence

$$u_0(y) = u_{01}(y) + Eu_{02}(y)$$
$$u_1(y) = u_{11}(y) + Eu_{12}(y) \quad (14)$$
$$T_0(y) = T_{01}(y) + ET_{02}(y)$$
$$T_1(y) = T_{11}(y) + ET_{12}(y)$$

Substituting these equations in Eqns (6) – (9), equating the coefficients of E and neglecting the terms in E^2 and higher order, the results are

$$u_{01}'' + u_{01}' = -Gr\,T_{01} - Gr\,C_0$$

$$u_{02}'' + u_{02}' = -Gr\,T_{02}$$

$$u_{11}'' + u_{11}' - \frac{i\omega}{4} u_{11} = -Gr\,T_{11} - Gc\,C_1 - Au_{01}' \quad (17)$$

$$u_{12}'' + u_{12}' - \frac{i\omega}{4} u_{12} = -Gr\,T_{12} - Au_{02}' \quad (18)$$

$$T_{01}'' + PT_{01}' = 0$$

$$T_{02}'' + PT_{02}' = -Pu_{02}'^2$$

$$T_{11}'' + PT_{11}' - \frac{i\omega P}{4} T_{11} = -PAT_{01}'$$

$$T_{12}'' + PT_{12}' - \frac{i\omega P}{4} T_{12} = -PAT_{02}' - 2Pu_{01}'u_{11}' \quad (22)$$

The corresponding boundary conditions are

$$u_{01} = 0,\ u_{02} = 0,\ u_{11} = 0,\ u_{12} = 0,$$
$$T_{01} = 1,\ T_{02} = 0,\ T_{11} = 1,\ T_{12} = 0 \text{ at } y = 0$$

$$u_{01} \to 0,\ u_{02} \to 0,\ u_{11} \to 0,\ u_{12} \to 0,$$
$$T_{01} \to 0,\ T_{02} \to 0,\ T_{11} \to 0,\ T_{12} \to 0 \text{ as } y \to \infty$$

Solving Eqns (15) – (22) under these boundary conditions and using Eqn. (14), u_0, u_1, T_0 and T_1 are obtained.

On substituting the expressions of u_0, u_1, T_0 and T_1 in Eqn. (5), the expressions for velocity and temperature are obtained. Taking $u_1(y) = Mr + i\,Mi$, $T_1(y) = Tr + i\,Ti$, and $C_1(y) = Cr + i\,Ci$, the expressions for velocity, temperature and concentration in terms of the fluctuating parts of the unsteady part can be expressed as

$$u(y, t) = u_0(y) + \varepsilon(Mr \cos \omega t - Mi \sin \omega t)$$

$$T(y, t) = T_0(y) + \varepsilon(Tr \cos \omega t - Ti \sin \omega t)$$

$$C(y, t) = C_0(y) + \varepsilon(Cr \cos \omega t - Ci \sin \omega t)$$

Hence, the expressions for the transient velocity, transient temperature and transient concentration respectively, for $\omega t = \pi/2$ can be obtained as

$$u(y, \pi/2\omega) = u_0(y) - \varepsilon Mi$$
$$T(y, \pi/2\omega) = T_0(y) - \varepsilon Ti$$
$$C(y, \pi/2\omega) = C_0(y) - \varepsilon Ci$$

The graphs of Mr, Mi, $u(y, \pi/2\omega)$, $T(y, \pi/2\omega)$ and $C(y, \pi/2\omega)$ are shown in Figs 1–5.

Knowing the velocity profiles, the expression for the skin-friction in the non-dimensional form can be derived as

$$\left.\frac{\partial u}{\partial y}\right|_{y=0}$$

The expression for skin-friction can also be written in terms of the amplitude and phase as

$$\tau = \tau_0 + \varepsilon \; B \; \cos(\omega t + \alpha) \quad \text{where} \quad \tau_0 = \left.\frac{\partial u_0}{\partial y}\right|_{y=0}$$

$$B = Br + i\,Bi = \left.\frac{\partial u_1}{\partial y}\right|_{y=0} \quad \text{and} \quad \tan\alpha = Bi/Br$$

where τ denotes the skin-friction, $|B|$ is the amplitude of the skin-friction and $\tan\alpha$ is the phase of the skin-friction. The numerical values of $|B|$ and $\tan\alpha$ are listed in Table 1.

Knowing the temperature profiles, the rate of heat transfer can also be written in terms of the amplitude and phase as

$$q = q_0 + \varepsilon \mid Q \mid \cos(\omega t + \alpha_1), \quad \text{where} \quad q_0 = \left.\frac{\partial T_0}{\partial y}\right|_{y=0}$$

$$Q = Qr + i\,Qi = \left.\frac{\partial T_1}{\partial y}\right|_{y=0} \quad \text{and} \quad \tan\alpha_1 = Qi/Qr$$

where q is the rate of heat transfer, $|Q|$ is the amplitude of the rate of heat transfer and $\tan\alpha_1$ is the phase of the rate of heat transfer. The numerical values of $|Q|$ and $\tan\alpha_1$ are listed in Table 1.

Table 1

Gr	Gc	Sc	P/ω	5	10	15	20	25
Values of $\|B\|$								
1	2	0.24	0.71	5.17283	9.79185	13.0088	15.5919	17.80152
2	2	0.24	0.71	28.57432	44.13287	55.84778	65.6458	67.32452
1	3	0.24	0.71	5.39406	10.70973	14.35073	17.2565	19.73518
1	2	0.60	0.71	6.82349	12.03309	16.09664	19.4738	22.41338
1	2	0.24	0.7	8.10869	10.81332	13.05743	14.9809	16.68419
Values of tan α								
1	2	0.24	0.71	-1.67031	-1.67077	-1.71957	-1.7655	-1.80549
2	2	0.24	0.71	6.66379	5.96360	5.36593	4.9784	4.67893
1	3	0.24	0.71	1.53471	1.82365	-1.99968	-2.1367	-2.25045
1	2	0.60	0.71	0.62302	0.30071	0.22261	0.1869	0.16630
1	2	0.24	7	1.81339	1.33682	1.20477	1.1429	1.10702
Values of $\|Q\|$								
1	2	0.24	0.71	0.84078	1.22144	1.44636	1.8320	2.96467
2	2	0.24	0.71	0.67368	0.91204	1.38541	2.3757	4.33294
1	3	0.24	0.71	0.64581	1.03991	1.52646	2.9943	6.07317
1	2	0.60	0.71	0.85351	1.18808	1.46661	2.6897	5.61688
1	2	0.24	7	46.33764	42.50719	44.04672	46.1727	48.86398
Values of tan α_1								
1	2	0.24	0.71	0.00658	0.27395	0.00495	-0.4484	-1.22706
2	2	0.24	0.71	-1.09843	-0.27032	-0.55980	-1.2476	-2.29649
1	3	0.24	0.71	-0.34321	0.00289	-0.49910	-1.5278	-3.07026
1	2	0.60	0.71	0.18815	0.23544	-0.28417	-1.3139	-2.91402
1	2	0.24	7	-0.56849	-0.15609	-0.00371	-0.0048	-0.16403

3. RESULTS AND DISCUSSION

To get a physical insight into the problem, numerical calculations are carried out for different values of ω, *Gr*, *Gc*, *Sc*, *P* and *A* and are represented in figures and table. The motion being free convective, *E* is very small and its value is chosen as 0.001, as this will be more appropriate from the practical point of view. Here, the values of *Gr*, *Gc*, *E*, *A* and ω are arbitrarily chosen, whereas those of *Sc* and *P* are chosen in such a way that they represent realistic case. Thus, the values of *Sc* are chosen such that they represent hydrogen ($Sc = 0.24$) and water ($Sc = 0.6$) and the values of *P* are chosen as 0.71 and 7 for air and water, respectively.

From Fig.1, it is observed that *Mr* increases as *Gr* increases (I,III), whereas it decreases as ω increases (I,II). But, *Mr* first increases near the plate and then

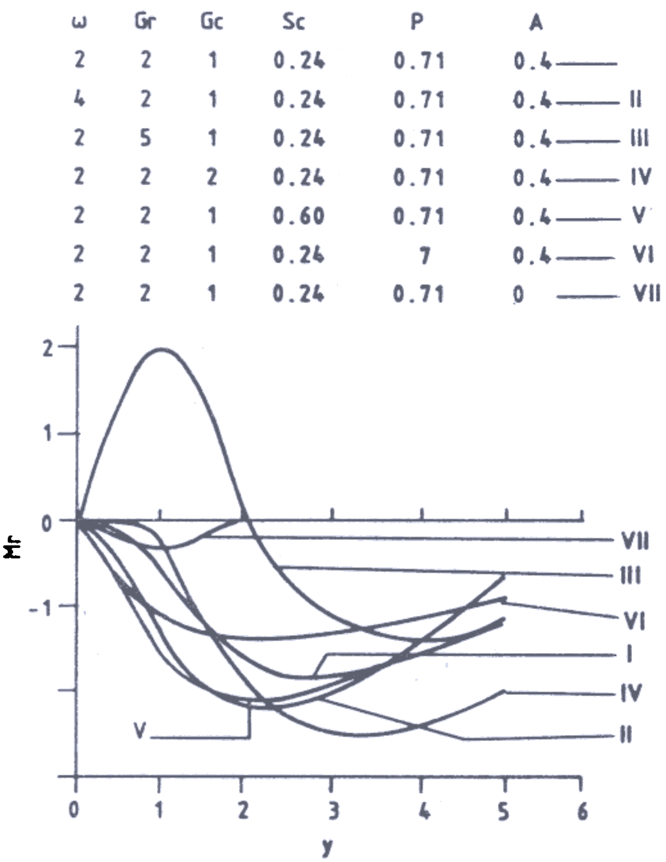

Figure 1. Fluctuating part, Mr $|\epsilon| = 0.2$, E= 0.001, T = .4.

decreases as *Gc* increases (I,IV), whereas it first decreases and then increases as *Sc* or *P* increases ((I,V), (I,VI)). Also, it is observed that in the absence of variable suction parameter, *Mr* increases (I,VII).

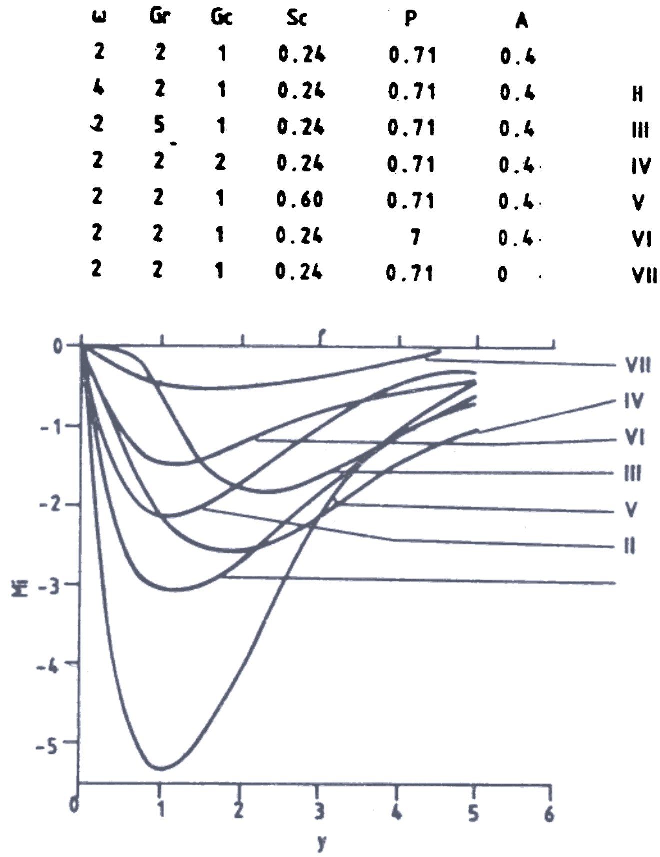

Figure 2. Fluctuating part, Mi $|\epsilon| = 0.2$, E= 0.001, T = 0.4.

From Fig. 2, it is noticed that *Mi* increases as ω or *Gr* or *P* increases ((I,II), (I,III), (I,VI)). But, *Mi* first increases and then, away from the plate, it decreases as *Gc* increases (I,IV), whereas this trend gets reversed as *Sc* increases (I,V). Also, it is noticed that in the absence of variable suction parameter, *Mi* increases (I,VII).

From Fig. 3, it is seen that the velocity *u* increases with the increase of *Gr* or *Gc* ((I,III), (I,IV)), whereas it decreases with the increase of ω or *Sc* or *P* ((I,II), (I, V), (I,VI)). It is also seen that in the absence of variable suction parameter, the velocity decreases (I,VII).

From Fig. 4, it is observed that the temperature *T* increases with the increase of ω (I,II), whereas it decreases with the increase of *Gr* or *Gc* or *P* ((I,III), (I,IV), (I,VI)). But, there is no significant change in *T* as *Sc* increases (I,V). In the absence of variable suction parameter, it is observed that *T* increases (I,VII).

From Fig. 5, it is noticed that the concentration *C* increases as ω increases (I,III). It is also noticed that in the absence of variable suction parameter, *C* increases (I,IV).

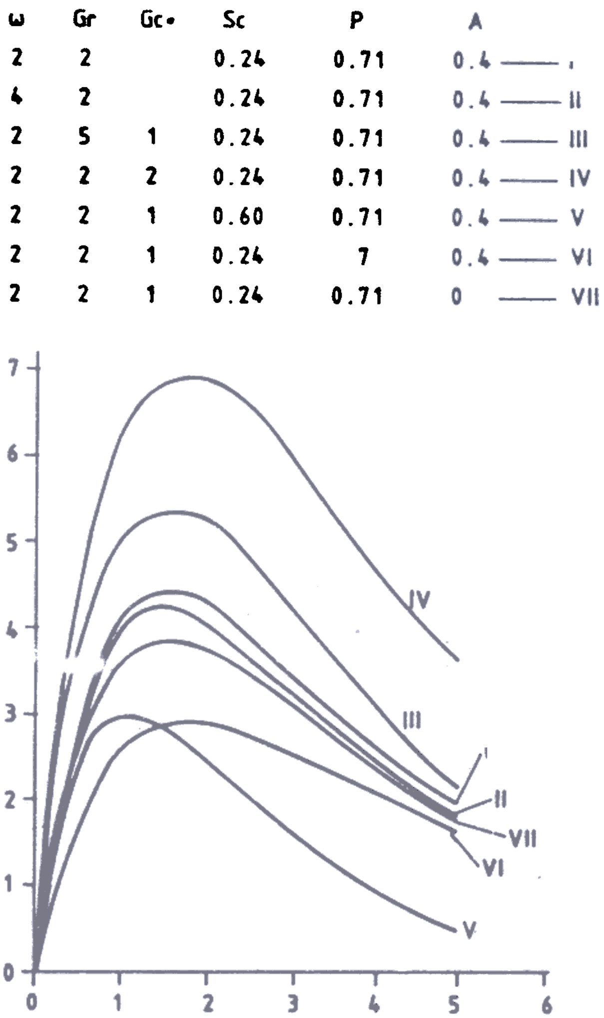

Figure 3. Transient velocity profiles, $\epsilon = 0.2$, E = 0.001, T = 0.4.

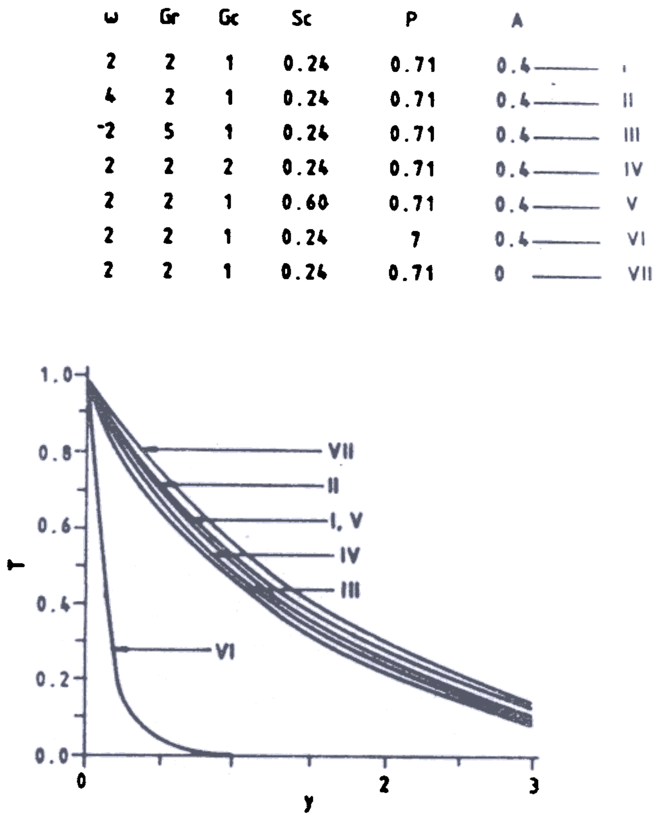

Figure 4. Transient temperature profiles, $\epsilon = 0.2$, E = 0.001, T = 0.4.

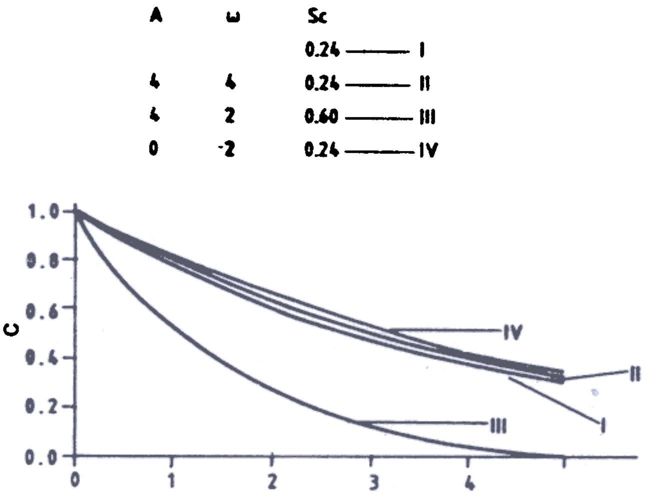

Figure 5. Transient concentration profiles.

From Table 1, it is noticed that the amplitude of skin-friction $|B|$ increases with the increase of ω or *Gr* or *Gc* or *Sc*. But as *P* increases, $|B|$ increases for small values of ω and it decreases for large values of ω (say ω = 20, 25).

Further, in the case of skin-friction, it is noticed that there is a phase lead throughout except when *Gr* is small, say *Gr* = 1 and *Gc* is large, say *Gc* = 3.

From Table 1, it is also seen that the amplitude of rate of heat transfer $|Q|$ increases with the increase of ω or *P*. But for large *P*, say *P* = 7, $|Q|$ first decreases and then increases as ω increases. As *Gr* or *Gc* increases, $|Q|$ decreases for small values of ω and then increases for large values of ω. But as *Sc* increases, $|Q|$ increases for all values of ω except at $\omega = 10$.

Further, in the case of rate of heat transfer, it is seen that there is phase lead for small values of ω, when *Gr* = 1 and *Sc* = 0.6. But, in all other cases there is a phase lag.

4. CONCLUSION

An increase in *Gr* or *Gc* leads to an increase in the velocity, whereas the velocity decreases with the increase of *Sc* or *P*. The temperature decreases with the increase of *Gr* or *Gc* or *P*. The concentration decreases as *Sc* increases. The amplitude of skin-friction increases with the increase of *Gr* or *Gc* or *Sc*, whereas it first increases and then decreases with the increase of *P*. The amplitude of rate of heat transfer first decreases and then increases with the increase of *Gr* or *Gc* and it increases as *Sc* or *P* increases. Further, in general, there is a phase lead in the case of skin-friction and there is a phase lag in the case of rate of heat transfer.

REFERENCES

1 Lorenz, L. Uber das leitungsvermogen der metalle fur warme and electricitat. *Widedmanns Annelen*, 1981, **13**, 582-606.

2. Gebhart, B. & Pera, L. The nature of vertical natural convection flows resulting from the combined buoyancy effects of thermal and mass diffusion. *Int. J. Heat Mass Transfer*, 1971, **14**, 2025-50.

3. Eichhorn, R. The effects of mass transfer of free convective flow. *Trans. ASME J. Heat Transfer*, 1960, **82c**, 260.

4. Sparrow, E.M.; Minkowycz, W.J. & Eckert, E.R.G. Transpiration induced buoyancy and thermal diffusion-diffusion thermo in helium-air free convection boundary layer. *Trans. ASME J. Heat Transfer*, 1964, **86**, 508-20.

5. Gebhart, B. Effects of viscous dissipation in natural convection. *J. Fluid Mech.*, 1962, **14**, 225.

6. Gebhart, B. & Mollendorf, J. Viscous dissipation in external natural convective flows. *J. Fluid Mech.*, 1969, **38**, 97.

7. Soundalgekar, V.M. Effects of mass transfer on free convective flow of a dissipative incompressible fluid past an infinite vertical porous plate with suction, *Proceedings of the Indian Academy Sciences*, 1976, **84A**, No. 5, 194-203.

8. Subhashini, A.; Bhaskara Reddy, N. & Ramana Kumari, C.V. Mass transfer effects on the flow past a vertical porous plate. *J. Energy, Heat & Mass Transfer*, 1993, **15**, 221-26.

9. Eckert, E.R.G. & Drake, R.M. Heat and mass transfer. McGraw-Hill Publishing Company Ltd., New Delhi, 1974.

10. Lighthill, M.J. The response of laminar skin-friction and heat transfer to fluctuations in the stream velocity. Proceedings of the Royal Society, 1954, **1**, 224.