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New Method For Calculating The Input Impedance of Rectangular Patch Antenna

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ABSTRACT

The cavity model has been modified to account for the impedance boundary condition at the edges of a rectangular microstrip antenna. Results have been compared with those from the old cavity model and experimental data.

1. INTRODUCTION

Cavity model being used to find the input impedance of a rectangular patch antenna, treats the antenna as a cavity with ideal magnetic walls at the boundary[1,2]. However, the boundary is not ideal and possesss finite impedance. In practice, power is lost from the cavity formed by microstrip patch due to radiation from the fringing fields at the periphery. Effects of fringing fields are accounted for by extending the boundary. In the present work, the impedance boundary condition, $\vec{H} = Y_w\,(\hat{n} \times \vec{E})$, suggested by Carver[2] is used to account for radiation. The wall admittance, Y_w will, in general, have different values on the walls along Y-axis than on those along X-axis. With these assumptions the cavity model has been modified and the results of the proposed model are compared with the original cavity model for a rectangular patch antenna.

2. THEORY

Figure 1 shows a rectangular patch antenna of length l, width w, on a substrate of thickness h and dielectric constant $\in_r$. For electrically thin substrate (i.e. $h << \lambda_o$) the Z-directed electric field will be independent of Z, under the patch. The modes will be TM_{mn} so that

$$E_z(x,y) = \sum_m \sum_n A_{mn} e_{mn}(x,y) \tag{1}$$

where, A_{mn} are the mode amplitude coefficient and e_{mn} are the orthonormal electric field mode vectors, expressed as

$$e_{mn}(x,y) = \chi_{mn}\left(e^{-jK_m x} + R_m e^{jK_m x}\right)\left(e^{-jK_n y} + R_n e^{jK_n y}\right) \tag{2}$$

where R_m and R_n are the reflection coefficients along the boundaries parallels to X and Y-axes, respectively. Using normalisation condition for $e_{mn}(x,y)$, it is found that

$$\chi_{mn} = \frac{\delta_m \delta_n}{2\sqrt{lw}} \tag{3}$$

$$\delta_p = \left(1 + \cos p\pi \times \frac{\sin k_p x}{k_p x}\right)^{-1/2} \tag{4}$$

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with $p = m$ when $x = w$ and $p = n$ when $x =$

$$k_m = \frac{m\pi}{w_e} \text{ and } k_n = \frac{n\pi}{l_e}$$

where l_e and w_e are the effective length and width, respectively due to the fringing field[3], expressed as

$$l_{eq} = l + 2\delta_l$$

$$\delta_l = 0.412h \frac{(\epsilon_{eff} + 0.3)(\frac{w}{h} + 0.264)}{(\epsilon_{eff} - 0.258)(\frac{w}{h} + 0.813)}$$

$$\epsilon_{eff} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2}\left(1 + \frac{10h}{w}\right)^{-1/2}$$

By interchanging l with w in the above equation, the effective width can be calculated.

2.1 Determination of Reflection Coefficient

Maxwell's equation states

$$\vec{H} = \frac{j}{\omega\mu_o}\vec{\nabla} \times \vec{E} = \frac{j}{\omega\mu_o}\left(\hat{x}\frac{\partial E_z}{\partial y} - \hat{y}\frac{\partial E_z}{\partial x}\right) \quad (5)$$

From this equation, the $\vec{H}$ field on the

(i) Y-surface at y = 0 and y = l will be of the form

$$H_y = \frac{-j}{\omega\mu_o}\frac{\partial E_z}{\partial x} \quad (6)$$

(ii) X-surface at $x = 0$ and $x = w$ will be of the form

$$H_x = \frac{-j}{\omega\mu_o}\frac{\partial E_z}{\partial y} \quad (7)$$

The impedance boundary condition on these surfaces is

$$\vec{H} = Y_w(\hat{n} \times \vec{E}) \quad (8)$$

For the above equation

at $y = 0$, $\hat{n} = -\hat{y}$; $y = l$, $\hat{n} = -\hat{y}$

$x = 0$, $\hat{n} = -\hat{x}$; $x = w$, $\hat{n} = -\hat{x}$

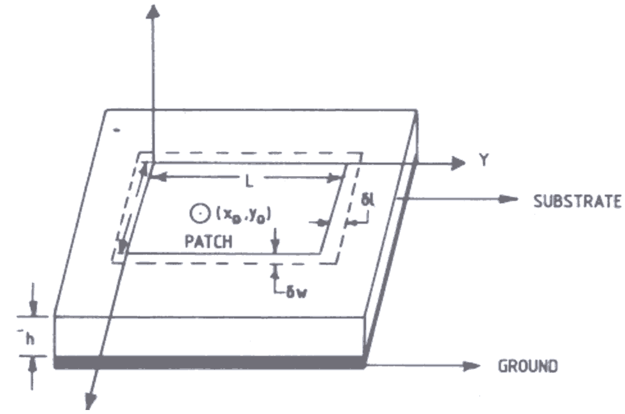

Figure 1. Coax-fed microstrip antenna with coordinate system used.

Equating the mn^{th} terms in the series for H_x from Eqns (7) and (8) at $y = 0$ and $y = l$, one gets

$$\frac{k_n}{\omega\mu_0}[1 - R_n] = -Y_w[1 + R_n] \quad (9)$$

$$\frac{k_n}{\omega\mu_0}[e^{-jk_n l} - R_n e^{jk_n l}] = Y_w[e^{-jk_n l} + R_n e^{jk_n l}] \quad (10)$$

Dividing Eqn (10) by Eqn (9) and solving for R_n, one gets

$$R_n = e^{-jk_n l} \quad (11)$$

Similarly,

$$R_m = e^{-jk_m w} \quad (12)$$

2.2 Determination of Modal Coefficient

For a coaxial feed, considering the effect of Z-directed current I_0 on the probe of small circular cross-section of diameter d at (x_0, y_0) the coefficient of each electric mode vector are

$$A_{mn} = \frac{j\omega\mu_o}{k_d^2 - k_{mn}^2}\iiint J.e^*_{mn}\, dv \quad (13)$$

Here, J is the current density. To account for the losses, it is assumed tha $k_d^2 = \omega^2\mu\epsilon_0\epsilon_{eff}(1 - j\delta_{eff})$ where δ_{eff} is the effective loss tangent.

For compatibility in definitions, we also take $k_{mn}^2 = \omega_{mn}^2 \mu_0 \in_0 \in_{eff} = k_m^2 + k_n^2$.

Using basic calculus with simple algebraic manipulations, Eqn (13) reduces to

$$A_{mn} = \frac{j\omega I_0}{\in_0 \in_{eff} [\omega^2 (1 - j\partial_{eff}) - \omega_{mn}^2]} e^*_{mn}(x_0, y_0) \times G_{mn} \quad (14)$$

with

$$G_{mn} = \frac{\sin(k_m d/2)}{} \times \frac{\sin(k_n d/2)}{} \quad (15)$$

and I_0 is the Z-directed current on the feeding probe.

2.3 Determination of Input Impedance

Now by substituting Eqn (14) in Eqn (1)

$$E_z(x,y) = \frac{j\omega I_0}{\in_0 \in_{eff}} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{e_{mn}(x,y) \times e^*_{mn}(x_0, y_0)}{\omega^2 (1 - j\delta_{eff}) - \omega_{mn}^2} \times G_{mn}$$

The voltage at the feed point is given by

$$V_{in} = -\int_0^h E_z \, dz = -h E_z(x_0, y_0) \quad (17)$$

$$\frac{-j\omega I_0 h}{\in_0 \in_{eff}} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{e_{mn}(x,y) \times e^*_{mn}(x_0, y_0)}{\omega^2 (1 - j\delta_{eff}) - \omega_{mn}^2} \times G_{mn} \quad (18)$$

Using the expressions for R_m and R_n, the input impedance is calculated as

$$Z_{in} = \frac{j16\omega h}{\in_0 \in_{eff}} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{|\chi_{mn}|^2 \cos^2\left[\frac{m\pi}{w_e}(x_0 - \frac{w}{2})\right] \cos^2\left[\frac{n\pi}{l_e}(y_0 - \frac{l}{2})\right]}{\omega^2_{mn} - (1 - j\delta_{eff})\omega^2} \quad (19)$$

The resonant resistance in the dominant TM_{01} mode is

$$R_r = \left| \frac{2hG_{01}}{} \right| \left\{ \cos \frac{\pi(y_0 - \frac{l}{2})}{l_e} \right\}^2 \times \left\{ 1 - \frac{\sin(\frac{\pi l}{l_e})}{\frac{\pi l}{l_e}} \right\}^{-1} \quad (20)$$

3. RESULTS & DISCUSSION

The resonant resistance is calculated using both old cavity model and the proposed cavity model. The results are compared with the measured values[4] in Table 1. The present method gives accurate results for the resonant resistance in comparison to the old method in all the cases. Resonant resistance are mostly within an error of 6 per cent. The observed discrepancy can be explained in the following manner:

When the physical dimensions of the antenna are small, the tolerance effect is high[5], i.e., the feed

Table 1. Comparison of input resistance (experimental and theoretical results) of printed rectangular patches for both old and proposed methods

						Measured[4]		Old model[2]			Proposed model		
$\in_r$	h (mm)	l (mm)	w (mm)	Y_0 (mm)	d (mm)	f_r (GHz)	R_r (Ω)	f_r (GHz)	R_r (Ω)	Error (%)	f_r (GHz)	R_r (Ω)	Error (%)
10.2	1.27			6.5	1.19	2.26	335		343	+2.33	2.31		-1.79
10.2	1.27			3.2	1.19	4.43	339		389	+14.75	4.49		+5.84
10.2	2.54			6.5	2.38	2.18	363		394	+8.50	2.29		+0.08
2.22	0.79			4.0	2.42	3.92	136		136	0.0	3.92		-0.59
2.22	0.79			2.0	2.42	7.56	152		153	+0.65	7.61		-0.39
2.22	1.52			4.0	4.66	3.82	119		153	+28.57	3.82		+27.2
2.22	1.52			2.0	4.66	7.72	69		147	+113.0	7.55		+108.0

position may be slightly offset giving an H-plane error. This can result in excitation of stronger higher order modes. These higher order modes are responsible for shift in resonant resistances in all the cases, and the shifts are prominent as the physical dimensions are reduced increasing the H-plane error.

For antennas with dielectric substrate which are electrically thick and/or have high dielectric constant, excitation of surface wave is strong. This is also responsible for higher discrepancies between the theoretical and the experimental results.

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