Defence Science Journal, Vol. 63, No. 5, September 2013, pp. 515-520, DOI : 10.14429/dsj.63.3773

Multifractal Modelling of Aircraft Echoes from Low-resolution Radars Based on Structural Functions

Qiu Sheng Li[*,!] and Wei Xin Xie[*]
[!]*Intelligent Information Institute, Shenzhen University, Shenzhen-518 060, China*
[*]*School of Physics and Electronic Information, Gannan Normal University, Ganzhou-341 000, China*
[*]*E-mail: bjliqiusheng@163.com*

ABSTRACT

As a kind of complex targets, the nonrigid vibration and attitude change of an aircraft as well as the rotation of its rotating parts will induce complex nonlinear modulation on its echo from low-resolution radars. If one performs the multifractal analysis of measures on an aircraft echo, it may offer a fine description of the dynamic characteristics which induce the echo structure. On basis of introducing multifractal theory based on structural functions, the paper models real recorded aircraft echo data from a low-resolution radar by using the random walk process and the incremental process respectively, and investigates the application of echo multifractal characteristics in aircraft target classification with low-resolution radars. The analysis shows that aircraft echoes from low-resolution radars have clear multifractal characteristics, and one should take an aircraft echo series as a random walk process to perform the multifractal analysis. The experimental results validate the classification method based on multifractal signatures.

Keywords: Low-resolution radar, multifractal, random walk process, incremental process, target recognition

1. INTRODUCTION

Aircraft is a kind of nonrigid targets with complex shapes. The nonrigid vibration or attitude change of aircraft relative to the observation radar will induce complicated nonlinear modulations on the echo amplitude and its phase[1]. In addition, the jet engine modulation (JEM) induced by the rotation of the aircraft rotating parts, such as the rotor, empennage, propeller, turbine fan, etc., is also a typical nonlinear modulation, which embodies in the echo characteristics such as amplitude, phase, frequency, and polarization[1-3]. These kinds of nonlinear modulations reflect the complicated micro-motion modulation effects of various parts of aircraft and contain target attribute information such as the geometric structure, material composition, etc.[4-5]. Generally speaking, different types of aircraft often have different structure and rotating parts, and have different nonrigid vibration and JEM characteristics. So if these nonlinear modulation signatures which reflect the physical characteristics of an aircraft target can be extracted effectively, then one may apply them to target classification and recognition directly[6-7].

So far, some scholars have proposed several theoretical models for aircraft echoes from low-resolution radars[8-12]. However, due to the complexity of the nonlinear modulation induced by the nonrigid vibration or attitude change, most models have paid more attention to the modeling of the JEM echo section, and simplified the modeling of the airframe echo section; so in some cases they are unsatisfactory in analyzing the nonlinear modulation characteristics of aircraft echoes from low-resolution radars. In recent years, some fractal geometry methods, such as mono-fractal, fuzzy fractal, multifractal, etc., have been introduced into the characteristic analysis of aircraft echoes from low-resolution radars[6-7,13-14]. However, it has not been reported that multifractal theory has been applied to the modeling of real-recorded aircraft echo data from low-resolution radars so far. Therefore, the paper plans to take multifractal theory based on structural functions as the tool to model aircraft echoes from low-resolution radars. On basis of introducing multifractal theory based on structural functions, the text models aircraft echoes from low-resolution radars by using the random walk process and the incremental process respectively[15], judges their multifractal characteristics, and investigates the application of echo multifractal signatures in aircraft target classification.

2. MULTIFRACTAL THEORY BASED ON STRUCTURAL FUNCTIONS

Multifractal models based on structural functions, such as the random walk process, incremental process, etc., may explain the multifractal properties of a fractal object preferably, from which one can see the relationships between multifractal and mono-fractal easily.

Multifractal analysis based on structural functions is mainly composed of a series of power law proof-tests of different orders of statistical momenta. Let $\boldsymbol{X} = \{X_i, i = 1, 2, \ldots, N\}$ be a generalized stationary random series with the mean μ and variance σ^2, then the analysis process is as follows.

(a) Zero the mean of $\boldsymbol{X}$, then the series can be expressed as

$$\mathbf{x} = \{x_i = X_i - \mu, \quad i = 1, 2, \cdots, N\} \tag{1}$$

Received 26 January 2013, revised 26 May 2013, online published 25 September 2013

(b) Calculate the sum of the preceding k items of $\boldsymbol{x}$, then one can construct a new series $\boldsymbol{y} = \{y_k, k = 1, 2, \ldots, N\}$, viz.

$$y_k = \sum_{i=1}^{k} x_i \quad (2)$$

$\boldsymbol{y}$ is called a random walk process of $\boldsymbol{x}$, and $\boldsymbol{x}$ itself is known as an incremental process of $\boldsymbol{y}$ correspondingly.

(c) Check up whether the following power law relationship is tenable or not:

$$F^{(q)}(m) = \left\langle \left| y(k+m) - y(k) \right|^q \right\rangle^{1/q} \propto m^{H(q)}, \quad (3)$$

where m denotes the analysis scale, q is a real number, and $H(q)$ is a function of q. It can be seen from Eqn. (3), the negative values of q highlight the lesser absolute increments of $\boldsymbol{y}$, while the positive values of q highlight the larger absolute increments of $\boldsymbol{y}$. If the power law relationship shown by Eqn. (3) is tenable, then $\boldsymbol{y}$ is a fractal process. Moreover, if $H(q)$ is a constant, then $\boldsymbol{y}$ is a mono-fractal process; contrarily, if $H(q)$ is not a constant, then $\boldsymbol{y}$ is a multifractal process.

When q equals 2, the analysis process shown by Eqn. (3) is often called fluctuation analysis (FA), which depicts the correlation characteristics of the investigated process. In this case, the autocorrelation function of $\boldsymbol{y}$ satisfies the following power law relationship

$$R_y(m) \propto m^{2H(2)} \quad (4)$$

where $H(2)$ is the so-called Hurst exponent, which is often noted as H and satisfies $0 < H < 1$. When H equals 0.5, the process is memoryless or called a short-range correlation process, and Brownian motion is its typical model. For a natural or man-made system, its Hurst exponent H is often unequal to 0.5, and fractional Brownian motion (FBM) is its typical model. If $H > 0.5$, the process is called a durative correlation process; However, if $H < 0.5$, the process is called a non-durative correlation process.

According to the Wiener-Khinchin theorem[16]

$$S_P(\omega) = F\left[R_P(m)\right], \quad (5)$$

where $\boldsymbol{P}$ is a stationary stochastic process, $R_p(m)$ is its autocorrelation function, $S_p(\omega)$ is its power spectrum density function, and $F[\ ]$ denotes the Fourier transform, if the stochastic process $\boldsymbol{y}$ is a fractal object, then one can see from Eqn. (4), the power spectrum density function of $\boldsymbol{y}$ has the following form:

$$S_y(\omega) \propto 1/\omega^{2H+1}, \quad (6)$$

where ω is the radian frequency, and that of the corresponding incremental process $\boldsymbol{x}$ has the form of

$$S_x(\omega) \propto 1/\omega^{2H-1}. \quad (7)$$

Therefore, the investigated process is also called a $1/\omega^{\alpha}$ noise. Generally speaking, a $1/\omega^{\alpha}$ process is nonstationary[17] when $1 < \alpha < 3$. For example, a Gaussian process ($\alpha = 0$) is stationary, however a Brown motion process ($\alpha = 2$) is nonstationary.

3. MULTIFRACTAL MODELING OF AIRCRAFT ECHOES

The echo data used in the text are recorded from a surveillance radar, and they are from two different types of aircraft targets with the one fighter aircraft and the other civil aircraft. There into, the radar operates in the VHF band with its PRF 100 Hz and pulse width 25 μs, and the flight attitude of both types of aircraft targets contains two kinds: towards the radar station and off the radar station. In the following analysis, since it is difficult to obtain the prior information to determine whether an aircraft echo is a random walk process or an incremental process, therefore, both models will be used to analyze the real recorded echo data from aircraft targets.

3.1 Modelling Using Random Walk Process

Here one can substitute the normalized aircraft echo series for $y(k)$ in Eqn. (3). Figure 1 shows the $lbF^{(q)}(m)$~lbm curves ($q = 2$) of a group of normalized echo data from both types of aircraft targets when they fly off the radar station. It can be seen from the figure, with the scale m ranging from 2^0 to 2^3 or from 2^3 to 2^6, both $lbF^{(2)}(m)$~lbm curves can be approximated as straight lines. It comes to light that, aircraft are man-made targets, they can be depicted by some regular geometric cells, and their surface is very slippery. When the analysis scale m is lesser, the fractal characteristics are mainly determined by the relationship among scattering points with closer distance, and the smoothness of small areas plays a main role. However, when the analysis scale m is larger, the fractal characteristics are mainly determined by the relationship among scattering points with longer distance, and the irregularity among different areas gradually takes the place of the smoothness of small areas, and thus the large-scale fractal characteristics may better reflect the difference among the physical characteristics of different types of aircraft targets. Therefore, here the scale range 2^3 ~ 2^6 is selected, and one can fit the curves within this range with beelines and obtain the estimate of the Hurst exponent H. Also it can be seen from Fig. 1, the civil aircraft has more distinct fractal characteristics than the fighter aircraft as a whole, because a civil aircraft often has intenser nonrigid vibration and attitude change than a fighter aircraft, and its JEM effect is also more distinct than that of the fighter aircraft. However, along with the farther increase of the analysis scale (m is more than 2^6), the difference between their fractal characteristics will reduce, here the fractal characteristics are mainly determined by the noise, and that is the reason that there is a sudden overlap in the data of Fig. 1 for civil *vs* fighter aircraft.

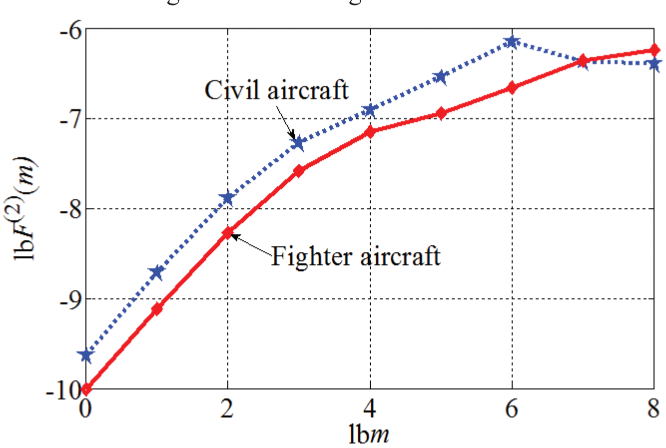

Figure 1. lb$F^{(2)}$(m) ~ lb$m$ curves.

Next, judge the multifractal characteristics of the echo data. Figure 2 presents the $lbF^{(q)}(m)$~lbm curves in different

q values of a group of echo data from both types of aircraft targets when they fly off the radar station. As can be seen from Fig. 2, $lbF^{(q)}(m)$ of the echo data from both types of aircraft targets show significant nonlinear relationships with lbm, so they are multifractal process, especially the echo data from the civil aircraft behave more obviously. Calculate the sample mean and standard deviation of $H(q)$, then Fig. 3 shows the variational curves of the mean of $H(q)$ with q, where the vertical line is the standard deviation from the corresponding mean. From Fig. 3 one can see, in the conditions of a lesser q value, $H(q)$ has better discrimination ability for different types of aircraft targets.

Figure 4 shows the distributing circumstances of the 2-D signatures composed of the $H(0.5)$ and $H(1)$ of echo data from both types of aircraft targets, with '*' and 'o' denoting the fighter aircraft and the civil aircraft respectively. Among them, the group numbers of echo data from both types of aircraft are all 2560, with the echo group numbers for each flight attitude 1280. It can be seen from the figure, although there are some overlaps between the 2-D signatures of both types of aircraft targets, as a whole, the signatures belonging to different types of aircraft separate from each other distinctly. Therefore, if one combines the two characteristic parameters together to identify different types of aircraft targets, it is hopeful to obtain a better performance. Also it can be seen from Fig. 4, most of the Hurst exponents H of echo data from both types of aircraft targets are located between 0.1 and 0.7, their α values are generally in the range from 1 to 3, and thus according to the previously described, one can know that echo data from both types of aircraft targets have the nonstationary characteristics.

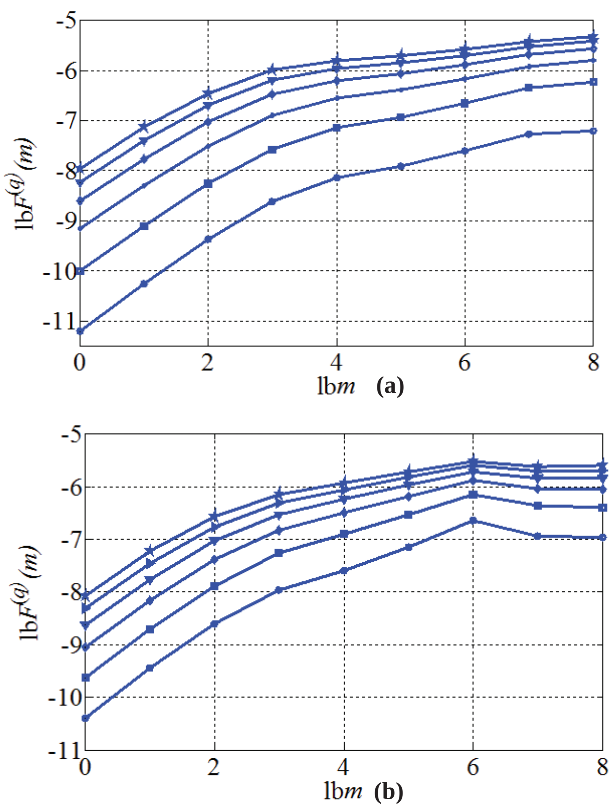

Figure 2. **lb$F^{(q)}$(m) ~ lb$m$ curves with different q with q = 1, 2,…, 6 from bottom to top. (a) Fighter aircraft (b) Civil aircraft**

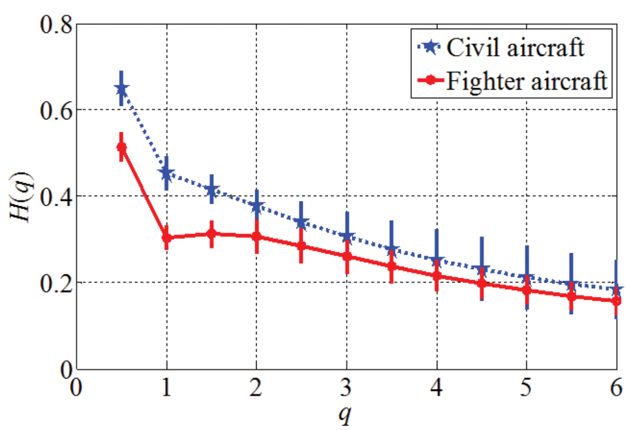

Figure 3. ***H*(*q*) ~ *q* curves.**

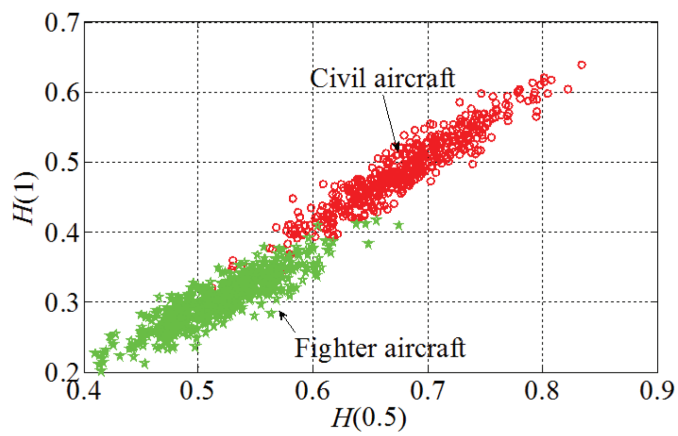

Figure 4. **Distributing circumstances of 2-D signatures composed of *H*(0.5) and *H*(1) of echo data from both types of aircraft.**

What may be expected is that, if PRF is increased and pulse width is lowered, the multifractal characteristics of aircraft echo data will be more distinct, the characteristic difference between different types of aircraft targets will be more prominent, and thus it is hopeful to identify different types of aircraft targets more effectively in the domain of multifractal features.

3.2 Modeling using the Incremental Process

In this case, firstly one should zero the mean of the investigated aircraft echo series, form its partial sum series, and then substitute the new series for $y(k)$ in Eqn. (3). Figure 5(a) presents the $lbF^{(2)}(m)$~lbm curves of a group of echo data from both types of aircraft targets when they fly off the radar station, and the corresponding $H(q)$ ~ q curves are given by Fig. 5(b). It can be seen from the figure, the $lbF^{(2)}(m)$~lbm curves hold a good linear relationship within a wide scale range, and $H(q)$ in different q values still have certain discrimination abilities for different types of aircraft targets. However, by reason that $H(q)$ is very close to 1, the effect by using fractal scale analysis to distinguish echo data from different types of aircraft targets is greatly reduced. If taking $H(2)$ as the example, then Fig. 6 shows its probability density curves for both types of aircraft targets. It is obvious that there are serious overlaps between

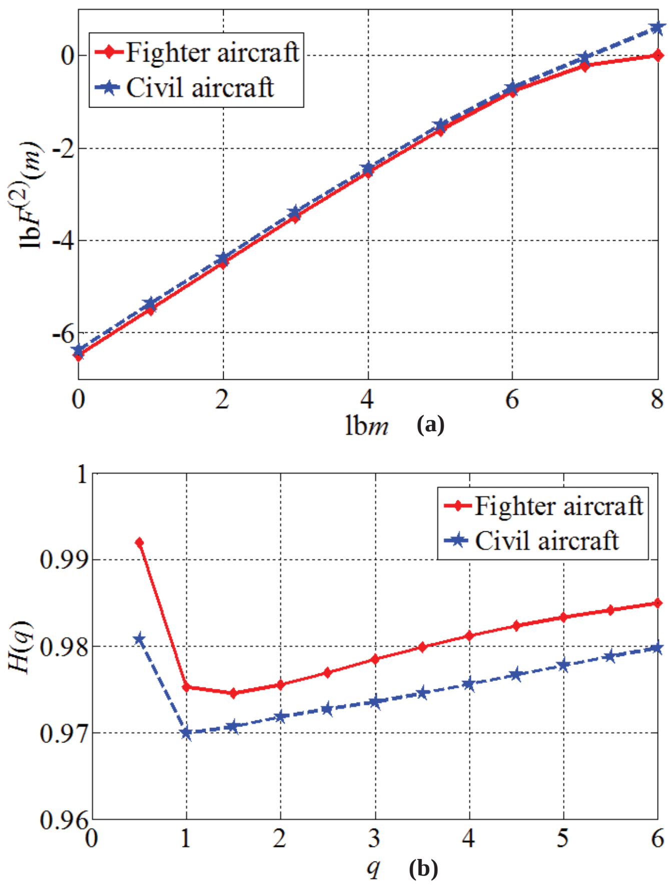

Figure 5. (a) lb$F^{(2)}(m)$ ~ lb$m$ (b) $H(q)$ ~ q curves.

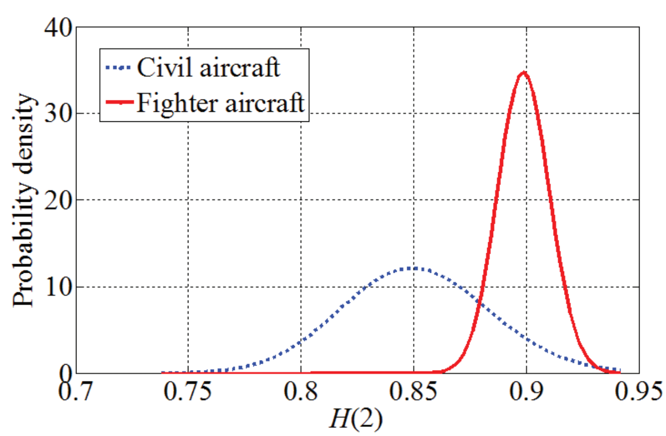

Figure 6. Probability density curves of H(2) of echo data from both types of aircraft targets.

H(2) values of both types of aircraft targets.

The reason for this situation is that the maximum Hurst exponent got by FA is 1. The concrete analysis is as follows. If $y(k)\propto k^{\alpha}$, $\alpha > 1$, then one has

$$\left\langle \left| y(k+m) - y(k) \right|^2 \right\rangle \propto \left\langle \left[(k+m)^{\alpha} - k^{\alpha} \right]^2 \right\rangle . \quad (8)$$

If $k >> m$, then $(k + m)^{\alpha} \approx k^{\alpha}(1+\alpha m/k)$. Substitute it into Eqn. (7), and further one can get

$$\left\langle \left| y(k+m) - y(k) \right|^2 \right\rangle \propto m^2 , \quad (9)$$

i.e., $H(2) = 1$. It is obvious that, in this case, Eqn. (8) is mainly subject to the control of such items with larger k values. This is called the 'saturation' phenomenon in FA. Therefore, when the Hurst exponent is close to 1, one should consider the aircraft echo series as a random walk process rather than an incremental process.

4. AIRCRAFT TARGET CLASSIFICATION BASED ON MULTIFRACTAL SIGNATURES

The previous section shows that the random walk process can model an aircraft echo from low-resolution radar effectively. As pointed out in the introduction, echo data from different types of aircraft targets often have different nonlinear characteristics, and thus they certainly will appear different multifractal signatures. Therefore, it provides a probability for aircraft target classification and recognition with low-resolution radars. Based on the foregoing real recorded aircraft echo data, below the paper will investigate the application of multifractal signatures in aircraft target classification with low-resolution radars.

4.1 Echo Data Preprocessing

Due to complexities of the actual target state and the environment, the target attitude, distance, background, etc. often change, which makes the raw target echo data can not be directly used for feature analysis and extraction, and therefore one must do some data preprocessing to reduce the influence of these factors. Here the following two kinds of preprocessing will mainly be done: one is attitude partition, the other is energy normalization.

(a) *Attitude Partition*

Attitude is an important factor which influences the performance of classification methods for aircraft targets, especially for high-resolution range profile (HRRP) recognition. For conventional low-resolution radars, aircraft targets are generally considered as point targets, and their JEM phenomena can be observed within the range -30° ~ 30° from the front or back view[9]. However, on the one hand, it is very difficult to obtain the accurate description of the target scattering characteristics varying with the attitude in practical applications; on the other hand, it is also unnecessary to divide the attitude meticulously. So here the attitude partition processing method is adopted, and the attitude is divided into three sections: towards the radar station, off the radar station, and in side direction. Generally speaking, only the former two sections are useful for target classification.

(b) *Energy Normalization*

Due to the different distance or azimuth relative to the observation radar, the echo intensity of an aircraft target varies within a wide range. However, what a target signature describes is not the absolute change of the echo amplitude or intensity, but its relative change, therefore, in order to analyze and extract echo signatures more reliably, one must eliminate the influence induced by the intensity difference. Considering the echo characteristic analysis is often performed in various transform domains, here the energy normalization processing is chosen, which is also a commonly used method.

Let $\{x_k\}$, $k = 0, 1, \cdots, N-1$ be the target echo series, then its signal energy can be written as

$$E_x = \sum_{k=0}^{N-1} |x_k|^2 \quad (10)$$

Thus the normalized series can be expressed as

$$\tilde{x}_k = x_k \big/ \sqrt{E_x}\,, \quad (11)$$

i.e., one can get

$$E_{\tilde{x}} = \sum_{k=0}^{N-1} |\tilde{x}_k|^2 = 1\,. \quad (12)$$

4.2 Classification Experiment

Based on the difference between the distributing circumstances of the 2-D signatures of echo data from both types of aircraft targets shown by Fig. 4, here $H(0.5)$ and $H(1)$ are chosen as the characteristic parameters for target classification. Compared to other classifiers, support vector machine (SVM) has stronger generalization abilities and a faster convergence rate[18], so in the experiment SVM using the Gaussian kernel function is taken as the classifier, and the kernel function parameters are selected rationally without going beyond the calculation burden.

Table 1 shows the classification results of the two types of aircraft targets, and as a contrast, the results using the raw echo data without performing any preprocessing are also presented. Among them, the group numbers of echo data from both types of aircraft targets are the same as those in Fig. 4, and for each type of aircraft targets, the signature data extracted from 512 groups of echo data are chosen as training samples (the group numbers for each of the two flight attitudes useful for classification are 256), with the rest signature data as testing samples. As can be seen from Table 1, the average correct classification rate (CCR) is more than 98 per cent, and the data preprocessing obtains a classification gain more than one percent. Therefore the classification effect is satisfactory. What should be pointed out is that the signature dimension reduction processing has been done in the classification experiment. If the whole signatures are made full use of, the average CCR could still have an increase to a certain extent.

Table 1. Classification results

	Using raw echo data (%)	Using preprocessed echo data (%)
Fighter aircraft	96.34	98.47
Civil aircraft	97.62	97.68
Average CCR	96.97	98.07

5. CONCLUSIONS

Based on the complex nonlinear modulation characteristics induced by the nonrigid vibration and attitude change of aircraft targets along with the JEM effect, the paper models aircraft echoes from low-resolution radars from the viewpoint of multifractal. On basis of introducing multifractal theory based on structural functions, it models the real recorded aircraft echo data from low-resolution radars with the random walk process and incremental process respectively, and investigates the application of aircraft echo multifractal signatures in target classification with low-resolution radars. The experimental results show that:

(i) It is an effective method to model aircraft echoes from low-resolution radar using a multifractal model, and one should consider the aircraft echo series as a random walk process rather than an incremental process to perform the multifractal analysis;

(ii) If one performs the multifractal analysis of measures on an aircraft echo, it is hopeful to reveal its internal dynamics evolution mechanism;

(iii) Multifractal characteristic parameters of aircraft echoes can be used as effective signatures for aircraft target classification with low-resolution radars.

ACKNOWLEDGEMENTS

Firstly, we'd like to thank Professor Huang Jianjun at Shenzhen University for offering the experiment data and many useful discussions about target classification and recognition with radars. Secondly, we wish to thank the National Natural Science Foundation of China (Grant: 61271107) and the Specialized Research Fund for the Doctoral Program of Higher Education of China (Grant: 20124408110002). Finally, we also wish to thank the anonymous reviewers for their help in improving this paper.

REFERENCES

1. Huang, P.K.; Yin, H.C. & Xu, X.J. Radar target characteristics. Publishing House of Electronic Industry, Beijing, 2005.
2. Nalecz, M.; Andrianik, R.R. & Wojtkiewicz A. Micro-Doppler analysis of signal received by FMCW radar. *In* the Proceedings of International Radar Symposium, 2003, 231-235.
3. Ding, J. J. & Zhang, X. D. Automatic classification of aircraft based on modulation features. *J. Tsinghua Univ.* (*Sci. Technol.*), 2003, **43(7)**, 887-890.
4. Chen, V.C.; Li, F.Y.; Ho, S.S. & Wechsler, H. Micro-Doppler effect in radar: phenomenon, model, and simulation study. *IEEE Trans. AES,* 2006, **42(1)**, 2-21.
5. Zhuang, Z. W.; Liu, Y. X. & Li, X. The achievements of target characteristic with micro-motion. *Acta Electronica Sinica*, 2007, **35(3)**, 520-525.
6. Li, Q.S.; Xie, W.X. & Luo, C. Identification of aircraft targets based on multifractal spectrum features. *In* the Proceedings of IEEE International Conference on Signal Processing, 2012, **3**, 1821-1824.
7. Li, Q.S. & Xie, W. X. Target classification with low-resolution surveillance radars based on multifractal features. *Prog. Electromagn. Res. B*, 2012, **45**, 291-308.
8. Martin, J. & Mulgrew, B. Analysis of the effects of blade pitch on the radar return signal from rotating aircraft blades. *In* the Proceedings of IET International Conference on Radar, 1992, 446-449.
9. Bell, M.R. & Grubbs, R. A. JEM modeling and measurement for radar target identification. *IEEE Trans. AES*, 1993, **29(1)**, 73-87.
10. Pizza, E. Radar signals analysis and modellization in the presence of JEM application in the civilian ATC radars.

IEEE AES Magazine, 1999, **14(1)**, 35-40.

11. Ding, J.J.; Zhang, X.D. & Lv, J.J. Modeling of the modulation signature for aircraft returns in the conventional radar. *Syst. Eng. Electron.*, 2003, **25(11)**, 1407-1410.
12. Chen, F.; Liu, H.W.; Du, L. & Bao, Z. Target classification with low-resolution radar based on dispersion situations of eigenvalue spectra. *Science China: Inf. Sci.*, 2010, **53(7)**, 1446-1460.
13. Ni, J.; Zhang, S.Y.; Miao, H.F. & Zhang, B. Target classification of low-resolution radar based on fractional Brown feature. *Modern Radar*, 2011, **33(6)**, 46-48.
14. Li Q.S. & Xie, W.X. Classification of aircraft targets with surveillance radars based on fuzzy fractal features. *Prog. Electromagn. Res. M*, 2013, **29**, 65-77.
15. Guan, J.; Liu, N. B.; Huang, Y.; Chen X. L.; Zhang J. & Li, X. Y. Fractal theory for radar target detection and its application. Publishing House of Electronics Industry, Beiijng, 2011.
16. Shen, Y.; Liang, Z.Z.; Xu, L.H. & Cai, Q.Q. Applied mathematical handbook. Science Press, Beijing, 2004.
17. Trevilo, G.; Hardin, J. & Douglas, B. Current trends in nonstationary analysis. World Scientifc, Singapore, 1996.
18. Duda, R.O.; Hart, P.E. & Stork, D.G. Pattern classification. Ed 2nd. John Wiley and Sons, New York, 2001.

CONTRIBUTORS

Mr Q.S. Li, has received his bachelor degree in automatic control from Tongji University, Shanghai, in 1998 and master degree in signal and information processing from Beihang University, Beijing, in 2004. Currently pursuing his PhD from Shenzhen University, Shenzhen, China and is with the School of Physics and Electronic Information, Gannan Normal University, Ganzhou, China. His fields of interests includes: Intelligent information processing, automatic target recognition and tracking.

Prof W.X. Xie has received the bachelor degree from Xidian University, Xi'an, in 1965. Currently working in the College of Information Engineering, Shenzhen University, Shenzhen, China. His fields of interests includes: Intelligent information processing, fuzzy information processing, image processing, pattern recognition, etc.