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Probabilistic Analysis of Anti-ship Missile Defence Effectiveness

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ABSTRACT

Effective missile defence systems are primary requirement for naval ships to counter lethal anti-ship cruise missile attacks in today's naval warfare scenario. Anti-ship ballistic missiles would further add worry to ship missile defence. The paper discusses a probabilistic analysis of missile defence system effectiveness by considering a simple scenario of a single ship defence with multiple interceptors against a single non-maneuvering missile attack. The ship's interceptor hard kill lethality is taken as the measures of effectiveness in the analysis. The paper discusses effect of different firing policies, multi-sensor and layered defence to achieve maximum ship survivability.

Keywords: Anti-ship cruise missile, lethality, survivability, kill probability, anti-ship missile defence system, layered defence

NOMENCLATURE

ASMD	Anti-ship missile defence
ASCM	Anti-ship cruise missile
D_r	Target denial range
h_{radar}	Radar height
h_{ASCM}	ASCM flight altitude
α	Number of interceptor launchers
L	Number of layers in layered defence
m	Number of engagements
n	Number of interceptors fired
μ	Minimum number of interceptors required to fire
p_d	Detection probability of ASMD
$p_{k\|d}$	Kill probability of ASMD after detection.
$p_{k\|d}(m)$	Cumulative kill probability of m engagements after detection.
δ_ι	Detection probability of the i^{th} sensor
p_k	Interceptor's single shot kill probability (SSKP)
p_{kj}	SSKP at j^{th} engagement
p'_{kl}	Kill probability of the target at l^{th} layer in layered defence
p	Overall kill probability of the target
p_s	Survival probability = 1-p
p_a	Acceptable leakage probability
r_{L1}	Range to launch the first interceptor
r_d	Target detection range
r_{min}	Min. interceptor launch range
xr_e	Max. effective interception range
nr_e	Min. effective interception range
r_j	Interceptor launch range at the j^{th} engagement
R_j	Interception range at the j^{th} engagement
R_{max}	Max. radar detection range
R_{hor}	Radar horizon range
σ	Radar cross section
S	Number of sensors
t_L	Inter-firing time
t_k	Kill assessment time
T_D	Engagement duration
T_j	Time to j^{th} interception
τ	Initial reaction time
t_f	Time of flight of the interceptor
V_{ASCM}	ASCM velocity
V_{int}	Interceptor velocity
w	Salvo size

1. INTRODUCTION

Since Falkland war, naval war scenarios have undergone sea changes with the successful operation of sea skimming anti-ship missiles. Today, the principal threat to naval ships is anti-ship cruise missiles. These missiles are either flying in subsonic speed with very low altitude, almost touching sea surface, maneuvering or in supersonic speed giving very less reaction time thus posing significant threat to target ships.

A new variant of ballistic missiles for anti-ship role has been under development in China that can engage moving ships thousand miles away from the shore. This anti-ship ballistic missile (ASBM) with maneuvering re-entry vehicle and homing seeker is designed to keep the US Naval ships at bay in the event of any conflict[1]. The US is also gearing up its existing sea based ballistic missile defence system to counter ASBM threats[2,3]. The sequence of events i.e. detection, classification, identification and engagement that the anti-ship missile defence (ASMD) system follows against both ASCM and ASBM are similar although technologies involved are

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different; therefore, approaches for effectiveness analysis for both the cases are also similar. This paper concentrates on the ship defence against an anti-ship cruise missile only.

The effectiveness of a weapon system is a quantitative measure of the level up to which the system meets its objective. The evaluation of effectiveness is usually complex since it depends on number of factors affecting the performance of the system[4].

Analysis of ASMD effectiveness helps the analyst to understand impact of different factors like speed, altitude and radar cross section of the threat missile, speed and number of interceptors, detection range, environment etc. to overall system effectiveness and hence helps to take necessary measures for improvements of naval tactics or defence systems. There are many complex physical and combatant interactions in a ship defence and ASCM attack scenario. Modeling & simulation is useful in analysing over a wide range of complex engagement conditions. However, analytical approach is better for understanding insight of the system reasonably in simplified form.

Bradford[5] has discussed a probabilistic assessment of single ship defence effectiveness against a stream or wave of missile attacks. Eric[6], *et al.* described a simple methodology for determining how to allocate resources among layers of a multi-layered missile defence in cost effective way. Hideto[7] discussed measure of effectiveness of an air defence system against an attacking missile in littoral environment. Dowan and Chang-Kyung[8] suggested defence strategy logic of single ship against multiple ASCMs using closed range anti-air missiles. Roy[9] studied ship survivability from ASCM attack using kill-chain analysis.

In the present paper, a probabilistic analysis of anti-ship missile defence system effectiveness is discussed by considering a scenario of a single ship defence with multiple interceptors firing against a single non-maneuvering missile attack. Survivability of a ship against an anti-ship missile threat depends upon the effectiveness of its missile defence system that includes both hard-kill and soft-kill defence means. However, hard kill lethality is considered as the measure of effectiveness in this analysis. The paper discusses effect of different firing policies undertaken by the defender to achieve maximum ship survivability, expected number of interceptors to be fired to achieve desired outcome, effect of multi-sensor and layered defence environment.

2. METHODOLOGY OF EFFECTIVENESS EVALUATION

Lethality, i.e. the ability to encounter, engage and killing a target, may be considered as the measure of effectiveness of the ASMD. The sequence of operations in lethality assessment is probabilistic in nature. The measure of effectiveness of lethality may therefore be expressed in terms of kill probability as

$$p = p_d \, p_{k|d} \tag{1}$$

The capability of ASMD to destroy targets depends on the efficiency and effectiveness of the system[10]. The factor p_d of Eqn. (1) is considered as detection probability that refers to efficiency of the system and concerns to the front end of the ASMD that includes detection, acquisition, command & control and communication. It describes how quickly targets can be detected, acquired and engaged. Probabilities related to all these events here are combined to detection probability. The factor $p_{k|d}$ is considered as the kill probability of the missile once it is detected and it relates to kill mechanism of the system. In this case, probabilities associated with tracking accuracy, fuzing and warhead impact are multiplicative in nature and are combined to kill probability $p_{k|d}$.

Depending upon the effectiveness of an interceptor against a threat in terms of kill probability, there is a need to determine the number of interceptors to be fired at the incoming threat to achieve a given level of defence effectiveness. On the other hand, if the number of interceptors allocated per threat is taken as fixed due to resource consideration, then it is to be known what kill probability of the interceptor that should be achieved with fixed numbers for the desired level of defence. The value of p, which represents level of technological or tactical requirements of the ASMD system, needs to be analysed.

To increase the lethality p (in Eqn. (1)),the possible options are-to increase either (i) p_d or, (ii) $p_{k|d}$, or (iii) both p_d and $p_{k|d}$. The ASMD may fire multiple interceptors to increase $p_{k|d}$ by following shoot-shoot (SS) firing policy or shot-look-shot (S-L-S) firing policy. In SS firing policy, a salvo of interceptors are fired towards the threat missile, where as in S-L-S firing policy, a single interceptor or a salvo is fired and then kill assessment is carried out after each round of fire. Second or subsequent rounds are fired if the target is not killed. In SS policy over-killing of a target is possible, whereas S-L-S firing policy tries to avoid it and thus saves interceptors from excess firing[11].

2.1 To Increase $p_{k|d}$ with Multiple Shots: Shoot-Shoot Firing Policy

In a shoot-shoot firing policy that fires multiple interceptors n against a threat almost continuously without assessing outcome of the interceptors fired, the kill probability $p_{k|d}$, becomes

$$p_{k|d} = \left(1-\left(1-p_k\right)^n\right) \tag{2}$$

It is assumed that the firing of interceptors is statistically independent with identical kill probability, p_k. The lethality of the ASMD thus can be expressed as

$$p = p_d\left(1-\left(1-p_k\right)^n\right) \tag{3}$$

ASMD lethality, is therefore, depends on detection probability (p_d), kill probability of an interceptor (p_k) and number of interceptors (n) fired. Table 1 indicates ASMD lethality values with number of interceptors n fired corresponding to a set of values of p_d and p_k. The results depict that the detection probability is a crucial parameter and it constraints the ASMD effectiveness as the upper limit.

2.1.1 *Determination of the Number of Interceptors to be Fired*

Total number of interceptors, n that ASMD can fire depends upon total available engagement duration (T_D), inter firing time (t_L), i.e the time between two successive interceptor launches and number of available launchers (α). Total engagement time

Table 1. ASMD lethality values with varying n, p_d and p_k

$P_d\rightarrow$	0.80	0.85	0.90	0.95	0.99	0.80	0.85	0.90	0.95	0.99
$P_k\downarrow$	n=1					n=2				
0.50	0.4000	0.4250	0.4500	0.4750	0.4950	0.6000	0.6375	0.6750	0.7125	0.7425
0.60	0.4800	0.5100	0.5400	0.5700	0.5940	0.6720	0.7140	0.7560	0.7980	0.8316
0.70	0.5600	0.5950	0.6300	0.6650	0.6930	0.7280	0.7735	0.8190	0.8645	0.9009
0.80	0.6400	0.6800	0.7200	0.7600	0.7920	0.7680	0.8160	0.8640	0.9120	0.9504
0.90	0.7200	0.7650	0.8100	0.8550	0.8910	0.7920	0.8415	0.8910	0.9405	0.9801
	n=3					n=4				
0.50	0.7000	0.7438	0.7875	0.8312	0.8662	0.7500	0.7969	0.8438	0.8906	0.9281
0.60	0.7488	0.7956	0.8424	0.8892	0.9266	0.7795	0.8282	0.8770	0.9257	0.9647
0.70	0.7784	0.8270	0.8757	0.9243	0.9633	0.7935	0.8431	0.8927	0.9423	0.9820
0.80	0.7936	0.8432	0.8928	0.9424	0.9821	0.7987	0.8486	0.8986	0.9485	0.9884
0.90	0.7992	0.8491	0.8991	0.9490	0.9890	0.7999	0.8499	0.8999	0.9499	0.9899
	n=5					n=6				
0.50	0.7750	0.8234	0.8719	0.9203	0.9591	0.7875	0.8367	0.8859	0.9352	0.9745
0.60	0.7918	0.8413	0.8908	0.9403	0.9799	0.7967	0.8465	0.8963	0.9461	0.9859
0.70	0.7981	0.8479	0.8978	0.9477	0.9876	0.7994	0.8494	0.8993	0.9493	0.9893
0.80	0.7997	0.8497	0.8997	0.9497	0.9897	0.7999	0.8499	0.8999	0.9499	0.9899
0.90	0.8000	0.8500	0.9000	0.9500	0.9900	0.8000	0.8500	0.9000	0.9500	0.9900

depends upon factors like, maximum/minimum effective firing range in the operational situation, ASCM velocity (V_{ASCM}) and the target detection range as follows

$$T_D = \frac{r_{L1} - r_{\min}}{V_{ASCM}} \tag{4}$$

Here r_{L1} is the initial interceptor launch range and $r_{min} = max(D_r, nr_e)$. D_r is considered as the denial range that is the range at which the target missile ASCM would not be allowed to cross and nr_e is the minimum effective interception range.

The number n can be derived as:

$$n = \text{int}\left(\frac{\alpha T_D}{t_L}\right) = \text{int}\left[\left(\frac{\alpha}{t_L}\right)\left(\frac{r_{L1} - r_{\min}}{V_{ASCM}}\right)\right] \tag{5}$$

where $int(x)$ represents the integral part of x. It is obvious from Eqn. (5) that, n decreases with the increase of V_{ASCM} or t_L or r_{min} or with the decrease of α or r_{L1}.

(a) Determination of initial interceptor launch range: r_{L1}

In case of early detection of the ASCM, the ASMD may engage the missile with a sequence of engagements starting from range r_{L1}, provided that the target is within the effective firing zone [nr_e, xr_e]. The initial launch range r_{L1} is determined as shown in *Appendix A* as

$$\begin{aligned} r_{L1} &= r_d\,,\ if\ \ r_d \le xr_e \\ &= xr_e\left(1 + \frac{V_{ASCM}}{V_{\text{int}}}\right) + (\tau + t_L)V_{ASCM}\,,\ if\ \ r_d > xr_e \end{aligned} \tag{6}$$

Here, V_{int} is the interceptor velocity (taken as constant) and τ is the initial reaction time of ASMD and xr_e is the maximum effective interception range.

(b) ASCM detection range (r_d)

Probability of detection of an attacking ASCM depends upon ASMD radar performance and also upon availability of the target line of sight. The target detection range r_d in a tactical situation is the minimum distance between the maximum radar detection range, R_{max}, and the radar horizon range, R_{hor}, i.e.

$$r_d = \min(R_{\max}, R_{hor}) \tag{7}$$

R_{max} depends upon radar characteristics, target characteristics and environment factors and can be determined using radar equation[9]. It is found from the radar equation that, R_{max} is directly proportional to radar cross section as

$$R_{\max} \propto \left[\frac{\sigma}{SNR}\right]^{1/4} \tag{8}$$

where SNR is the signal to noise ratio. For a small radar cross section(σ) the radar detection range R_{max} becomes significantly short.

The radar horizon range, R_{hor} can be calculated using the following equation[9]

$$R_{hor} = 4.124\left(\sqrt{h_{radar}} + \sqrt{h_{ASCM}}\right) \tag{9}$$

where h_{radar} and h_{ASCM} are considered as height of the radar and flight altitude of the ASCM. If ASCM flies at a low altitude then the radar horizon becomes short reducing R_{hor} that further reduces the detection range. Thus, small sized low flying missiles are difficult to detect.

2.1.2 Defence Requirement to Meet Performance Criterion

It seems reasonable to characterize mission requirements by establishing a 'denial area' at 'acceptable risk' that refers to that area of coverage by ASMD within which it has a desirable level of defence against the ASCM attack. As the denial area is expanded, the ASMD cannot provide effective defence to the entire area with the available resources which increases the possibility of the ASCM to survive ASMD and thus leak or penetrate the defensive area and becomes a threat to ship's survivability. The acceptable level of leakage is dependent on the mission objective.

Survival probability, p_s of the target missile can be derived from Eqn. (3), as

$$p_s = 1 - p = 1 - p_d\left(1-\left(1-p_k\right)^n\right) \tag{10}$$

If p_a is considered as the probability associated with acceptable risk of leakage then we should have $p_s \le p_a$. Keeping in mind the possible attack density of missiles, duration and time of attack, ASMD system has to fire the interceptors judiciously against each of the threats. Therefore, it requires to fire optimum number of interceptors against a single threat for a desired level of lethality. If μ is the minimum number of interceptors to be fired by the ASMD against the threat ASCM under an acceptable risk of leakage, then from Eqn. (10) we get,

$$1 - p_a = p_d\left(1-\left(1-p_k\right)^\mu\right) \tag{11}$$

Therefore, one can derive the minimum number of interceptors μ to be fired under a desired level of risk, p_a as:

$$\mu = \frac{\log(1-(1-p_a)/p_d)}{\log(1-p_k)} \tag{12}$$

Equation (12) can be used to determine minimum interceptor requirements against an ASCM threat with given values of p_a, p_d and p_k. Figure 1 below indicates expected number of minimum interceptors to be fired by the ASMD against an ASCM to achieve lethality more than 0.80 (i.e. less than 0.20 leakage probability). To achieve minimum μ, it is required to increase p_k or p_d or both p_k and p_d. It is further to be noted that the minimum detection probability to achieve the lethality must be greater than 0.80.

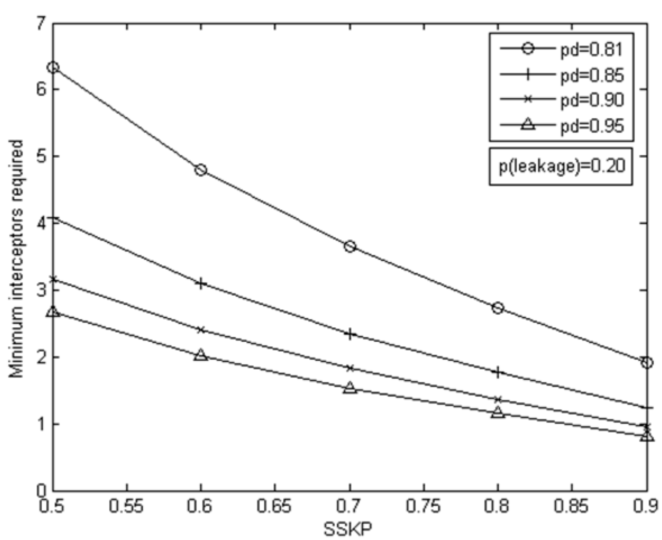

Figure 1. Expected number of minimum interceptors required.

2.2 To Increase $p_{k|d}$ with Multiple Shots: Shoot-Look-Shoot (S-L-S) Firing Policy

In S-L-S firing policy, a sequence of engagements are made against the target and kill assessment of the target is carried out after each engagement. Engagements continue till the target is killed. S-L-S firing policy restricts over-killing of the target and thus controls high interceptor firing cost.

If there are m engagements and $r_1, r_2, \ldots, r_m$ are the ranges of successive engagements, then $r_{L1} \ge r_1, r_2, \ldots, r_m \ge nr_e$, where r_{L1} is defined in Eqn. (6). Also, if p_{kj} is considered as the kill probability at the j^{th} engagement ($j=1,2,...,m$), then the cumulative kill probability of the ASMD in case of S-L-S firing policy is given as

$$\begin{aligned} p_{k|d}(m) &= p_{k1} + (1-p_{k1})p_{k2} + (1-p_{k1})(1-p_{k2})p_{k3} + \ldots\ldots \\ &\quad + (1-p_{k1})(1-p_{k2})\ldots\ldots(1-p_{k(m-1)})p_{km} \\ &= 1-(1-p_{k1})(1-p_{k2})\ldots\ldots(1-p_{km}) \\ &= 1-\prod_{j=1}^{m}\left(1-p_{kj}\right) \end{aligned} \tag{13}$$

Here, we consider, $0<p_{kj}<1$, if $nr_e \le r_j \le xr_e$ else $p_{kj} = 0$; for $j=1,2..,m$. We can express the overall kill probability under S-L-S policy with m sequence of engagements as

$$p = p_d\, p_{k|d}(m) = p_d\left(1-\prod_{j=1}^{m}\left(1-p_{kj}\right)\right) \tag{14}$$

If the number of engagements is not adequate to kill the target, then a salvo of interceptors needs to be fired to increase the lethality to the desired level at each engagement. This would change S-L-S firing policy to salvo-look-salvo firing policy. The Eqn. (14) becomes

$$p = p_d\left(1-\prod_{j=1}^{m}\left(1-p_{kj}\right)^{w_j}\right) \tag{15}$$

where w_j is the size of the salvo at the j^{th} engagement.

Assuming, p_k as the constant kill probability for all the engagements i.e. $p_k = p_{kj}$, *for* $j = 1,2,..m$ Then the lethality under salvo-look-salvo firing policy becomes

$$p = p_d\left(1-\left(1-p_k\right)^{\sum_{j=1}^{m} w_j}\right) \tag{16}$$

2.2.1 To Compute Number of Possible Sequence of Engagements (m)

To compute the number of possible engagements, m, we proceed to calculate successive interception time points T_j and interception ranges, R_j ($j=1,2,..,m$) as follows:

Time to first interception after detection, T_1 is expressed[6] as

$$T_1 = \frac{r_{L1} - \left(\tau + t_L\right)V_{ASCM}}{V_{\text{int}} + V_{ASCM}} \tag{17}$$

and the interception range is calculated as

$$R_1 = V_{\text{int}} T_1 \tag{18}$$

The second engagement may take place after time $(T_1+t_L+t_k)$, if the first one is a miss. Here t_k is considered as the kill assessment time of the target.

For the second engagement, the interception time and interception range are calculated as

$$T_2 = \frac{R_1 - \left(t_L + t_k\right)V_{ASCM}}{V_{\text{int}} + V_{ASCM}} \tag{19}$$

Using Eqn. (17) and Eqn. (18), we get

$$T_2 = \left(\frac{r_{L1} - (\tau + t_L)V_{ASCM}}{V_{\text{int}} + V_{ASCM}}\right)\left(\frac{V_{\text{int}}}{V_{\text{int}} + V_{ASCM}}\right) - \left(\frac{(t_L + t_k)V_{ASCM}}{V_{\text{int}} + V_{ASCM}}\right) \quad (20)$$

and

$$R_2 = V_{\text{int}} T_2 \quad (21)$$

The second engagement is feasible, if $R_2 > r_{min}$.

For determining third and subsequent sequence of engagements, the values of T and R can be computed iteratively. Finally, time to intercept the target at the m^{th} engagement, T_m, can be determined as

$$T_m = \left(\frac{r_{L1} - (\tau + t_L)V_{ASCM}}{V_{\text{int}} + V_{ASCM}}\right)\left(\frac{V_{\text{int}}}{V_{\text{int}} + V_{ASCM}}\right)^{m-1} - \left(\frac{(t_L + t_k)V_{ASCM}}{V_{\text{int}} + V_{ASCM}}\right)\sum_{i=0}^{m-1}\left(\frac{V_{\text{int}}}{V_{\text{int}} + V_{ASCM}}\right)^{i} \quad (22)$$

The m^{th} engagement is feasible, if

$$\underset{m}{Max}\ T_m \geq \frac{r_{\min}}{V_{\text{int}}} \quad (23)$$

Using Eqn. (12) above, one can determine analytically the number of interceptors to be fired with known values of p_a, p_d, and p_k. However, in case of S-L-S firing policy, expected number of interceptors to be fired is obtained through iterative process only.

2.2.2 To Determine Expected Number of Effective Interceptors

The expected number of interceptors, required to fire in the case of salvo-look-salvo firing policy with m engagement sequences can be determined as

$$\bar{n} = w_1 + \sum_{i=2}^{m} w_i (1 - p_k)^{\sum_{j=1}^{i-1} w_j} \quad (24)$$

where w_i $(i=1,2,..m)$ is the salvo size of the i^{th} engagement. For a salvo of fixed size w, for all the engagements, Eqn. (24) can be expressed as[11]

$$\bar{n} = w\left[1 + \sum_{i=2}^{m} (1 - p_k)^{(i-1)w}\right] \quad (25)$$

Table 2 shows expected number of interceptors required in salvo-look-salvo firing with varying salvo sizes and kill probabilities (p_k = 0.5, 0.7, 0.8 and 0.9). In a salvo firing (of size 2 shots) of a single sequence, both the rounds are fired almost simultaneously. In the S-L-S firing policy (one round each in two sequences), the second round is fired if the first round is a miss and therefore with kill probability of 0.5, the expected number of interceptors required is 1.50. The potential savings in interceptors is therefore observed as 0.5 whereas with kill probability of 0.7, the expected saving becomes 0.7 interceptors. The expected number of interceptors depends upon number of engagements, kill probability and firing policy.

3. COLLABORATIVE DEFENCE AGAINST ASCM ATTACK

The new breed of ASCMs with low signature, supersonic speed and maneuverability designed to be capable of delivering more precise effects against a wide spectrum of targets at sea and ashore have made the ASMD increasingly difficult. By taking advantage of GPS-aided precision guidance and navigation allied with improved ship-borne mission planning facilities brought the abilities to execute complex multiple waypoint flight profiles in confined littoral environments and pick out the intended target from clutter, countermeasures and other shipping contracts have added further complexities[12]. Therefore, to defeat improved ASCMs with complex attack scenarios must be multi-layered and network-centric rather platform-centric. The conceptual model for cruise missile defence is the combined use of early warning airborne system, fighters and in many instances, airborne surveillance radars to detect, track and engage both launch aircraft and cruise missiles[13]. The future development to upgraded ASMD with increased radar scan rates, improved fire control mechanism with fast processing, low flying target detection and maneuvering target tracking capability will increase ability of ASMD to engage of the ASCMs. The collaborative engagement of ships networked with other platforms and airborne sensors along with upgraded engagement capability of individual ships further increase interception probability of incoming missiles. With this capability, an interceptor with active seeker can engage an incoming missile that is not detected and controlled by the interceptor firing platform but has rather been informed by other platforms in the network[14].

Table 2. Expected number of interceptors required in salvo-look-salvo firing

Firing sequence	P_k= 0.5	P_k= 0.7	P_k= 0.8	P_k= 0.9
SS	2	2	2	2
S/S	1.50	1.30	1.20	1.10
SS/S	2.25	2.09	2.04	2.01
S/SS	2.00	1.60	1.40	1.20
SS/SS	2.50	2.18	2.08	2.02
S/S/S	1.75	1.39	1.24	1.11
SS/SS/SS	2.63	2.20	2.08	2.02
SS/S/SS	2.50	2.14	2.06	2.01
SS/SS/S	2.56	2.19	2.08	2.02

Firing policy-
SS: a salvo of 2 shoots
S/S: shoot-look-shoot
S/S/S: shoot-look-shoot-look-shoot
SS/S: salvo(2)-look-shoot
SS/SS: salvo(2)-look-salvo(2)
$S^i/S^j/\ldots(n)$.: salvo(i)-look-salvo(j)-look-…n sequence

3.1 Effect of Sensor Network

In case of multi-platform sensor network model, the detection probability p_d in Eqn. (3) can be modified as

$$p_d = (1 - (1 - \delta_i)^S) \quad (26)$$

where S is the number of sensors and δ_i the detection probability of the i^{th} sensor. Effect of sensor networking increases the detection range r_d [Eqn. (7)] by influencing the limitation of radar horizon and hence the radar detection range. The attacking missile can thus be engaged at the maximum effective range of the interceptor launched from the ASMD system under reference.

3.2 Effect of Layered Defence

Assuming p'_{kl} as the kill probability of the l^{th} layer of ASMD having L non-overlapping layers, the cumulative kill probability of the ASCM can be expressed as

$$p_k = 1 - \prod_{l=1}^{L} \left(1 - p'_{kl}\right) \quad (27)$$

The layered defence may be capable to engage not only any long range missile but also the missile launch platform.

4. CONCLUSIONS

This paper analyses the anti-ship missile defence system effectiveness by considering a scenario of a single ship defence with multiple interceptors firing against a single missile attack. Effect of different firing policies undertaken by the defender to achieve maximum ship survivability and expected number of interceptors to be fired to achieve desired outcome have been considered. The probabilities considered here are taken as constants; however, these probabilities can be evaluated using detailed models separately. The model can further be used for analysing impact of new technology, upgradation and tactics on the effectiveness of both anti-ship missile system and anti-ship missile defence system after suitable modifications. The model can be extended by incorporating multiple missile engagements from single or multiple directions at same or different time sequences, maneuvering missile threats, multi-layered defence and multi-platform defence scenarios. However, these would make the model complex and therefore difficult to get analytical solutions for probability computations and in such cases, simulation may be preferable to determine the solution of the model.

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Appendix A

1. CALCULATION OF LAUNCH INITIATION RANGE : R_{L1}

Anti-ship missile defence (ASMD) requires time to react and launch the first interceptor after the initial detection. If the detection range, r_d is greater than the maximum interceptor effective range, xr_e, then ASMD waits for some time to launch the intercept or so as to intercept the target at the range xr_e. In case of $r_d < xr_e$, the interceptor will be launched as soon as ASMD detects the threat. Figure A1 below illustrates the sequence of events.

If ASMD initiates launching the first interceptor after the target detection and firing decision at time $t=0$, then after $(\tau + t_L)$ time the first interceptor is launched. During this time the ASCM covers a distance $(\tau + t_L)V_{ASCM}$. If t_f is the time taken by the interceptor to intercept the target at the maximum effective range xr_e, then we get,

$$t_f = \frac{xr_e}{V_{\text{int}}} \tag{A1}$$

And the range to launch the first interceptor is:

$$r_{L1} = V_{\text{int}} t_f + V_{ASCM}\left(t_f + \tau + t_L\right) \tag{A2}$$

Thus, we get

$$\begin{aligned} r_{L1} &= r_d\,,\ if\ \ r_d \le xr_e \\ &= xr_e\left(1 + \frac{V_{ASCM}}{V_{\text{int}}}\right) + \left(\tau + t_L\right)V_{ASCM}\,,\ if\ \ r_d > xr_e \end{aligned} \tag{A3}$$

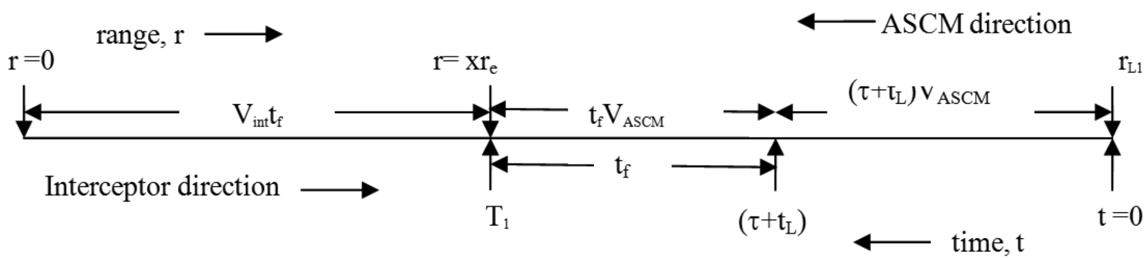

Figure A1. Sequence of events diagram for initial launch of the interceptor.