Defence Science Journal, Vol 50, No 3, July 2000, pp. 263-27

Transient Heat Transfer in Cylinders

M.G. Chopra, Ashwani Gupta and I.J. Marwah
Laser Science & Technology Centre, Delhi – 110 054

ABSTRACT

A numerical solution has been obtained for transient heat transfer in cylinders by appropriate choice of body conforming grid points. The physical domain is transformed to computational domain using elliptic partial differential equation technique, wherein the grid spacing becomes uniform. The advantage of this method is that the discretisation of transformed equations and accompanying boundary conditions become very simple. The applicability of this method is very broad, as it can be used for carrying out study of any complex domain in contrast to finite difference methods, which have limited applicability. Detailed calculations have been carried out to trace the evolution of temperature distribution from the initial stages to the steadystate for circular cylinder, elliptical cylinder and square block with circular hole. This paper is aimed for general-shaped bodies and it has been applied to study transient heat transfer in combustion-driven shock tube.

NOMENCLATURE

f	Any function
J	Jacobian of transformation
P, Q	Source functions
t	Time
T	Temperature
x,y	Coordinates in physical domain
α,β,γ	Coefficients in transformed partial differential equation
α_d	Coefficient of thermal diffusion
ξ,η	Coordinates in computational domain
i, j	Variable at point (i, j)
Δ	Increment in variable

Superscript

n	Time iteration number

Revised 23 December 1999

1. INTRODUCTION

Transient heat transfer is important in atmospheric, earth, biological and technological sciences. Structural technology depends very much on transient and steadystate heat transfer studies, particularly in material selection, for example in supersonic/hypersonic nozzles, re-entry shields, chemical and thermal reactor components and combustion devices.

This investigation deals with the study of transient heat transfer in different cylindrical geometry. The analytical study of unsteady heat transfer in circular cylinder has been carried out by Carslaw and Jaeger[1] and Meyer[2]. However, the analytical method involves grappling with Bessel functions because of circular cylindrical shapes. The analytical study becomes almost impossible for non-circular and non-rectangular cylinders, and physical situations which are not very uncommon phenomena. The method of grid generation has been widely used to analyse fluid flow and heat transfer problems in complicated domains[3,4]. The heat transfer problems involving irregular and complex cross-sections are solved by generation of body conforming grids which may not necessarily be uniformly spaced within the domain of investigation. Curvilinear grid of physical (x, y) plane is related to uniform grid of computational plane (ξ, η) through the solution of elliptic partial differential equation with Drichlet boundary conditions. The governing heat transfer equation for irregular-shaped bodies is transformed on to regular computational plane, wherein it is easy to setup uniformly spaced nodal points[5]. The finite difference analogue of the thermal equation and the accompanying boundary conditions are solved to trace steadystate evolution of temperature with time[6-9]. A thorough review of the body conforming grid generation technique was carried out during a refresher course[10] on computational fluid dynamics (CFD) known as 'CFD-update 1992' held at the Indian Institute of Technology, Madurai. This development is for general-shaped cylindrical bodies, and detailed calculations have been carried out to study the evolution of temperature to steadystate in elliptical cylinder and in a square block with a circular hole.

The study has also been applied to transient heating of the driver section of combustion-driven shock tube. It is observed that for stainless steel shock tube (internal diameter: 12 cm and thickness: 1 cm) subjected to 2500 K at the inner boundary, only a millimeter of the thickness senses a rise of 11 K in 5 ms.

2. FORMULATION & SOLUTION

The distribution of points in the interior of the domain is determined by solving

$$+\xi_{yy} = P(x,y) \quad (1)$$

$$\eta_{xx} + \eta_{yy} = Q(x,y) \quad (2)$$

where ξ, η represent coordinates in the computational domain and P and Q control the point spacing in the interior of the physical domain. Equations (1) and (2) can be transformed to computational space by interchanging the roles of independent and dependent variables.

It may be observed that for a function $f(\xi,\eta)$, such that

$$\xi = \xi(x,y) \quad (3)$$

$$\eta = \eta(x,y) \quad (4)$$

$$\nabla^2 f = (\nabla^2 \xi) f_\xi + (\xi_x^2 + \xi_y^2) f_{\xi\xi} + 2(\xi_x \eta_x + \xi_y \eta_y) f_{\xi\eta} + (\eta_x^2 + \eta_y^2) f + (\nabla^2 \eta) f_\eta \quad (5)$$

where

$$\nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \quad (6)$$

Interchanging the role of dependent and independent variables for $f(x,y)$ where

$$x = x(\xi,\eta)$$

$$y = y(\xi,\eta)$$

$$f_\xi = x_\xi f_x + y_\xi f_y \quad (7)$$

$$f_\eta = x_\eta f_x + y_\eta f_y \quad (8)$$

Using Eqns (7) and (8), one gets the values of f_x and f_y :

$$f_x = \frac{1}{J}(y_\eta f_\xi \quad y_\xi f_\eta) \quad (9)$$

$$f_y = \frac{1}{J}(x_\xi f_\eta - x_\eta f_\xi) \quad (10)$$

where

$$J = (x_\xi y_\eta - y_\xi x_\eta) \quad (11)$$

For

$$\xi_x = y_\eta / J, \qquad \xi_y = -x_\eta / J \quad (12)$$

For $f = \eta$

$$\eta_x \quad -y_\xi / J, \qquad \eta_y = x_\xi / J \quad (13)$$

Using Eqns (5), (11), (12) and (13), Eqns (1) and (2) transform to the following system of quasi-linear partial differential equations:

$$-2\beta x_{\xi\eta} + \gamma x_{\eta\eta} = -J^2 \left(P x_\xi + Q x_\eta\right) \qquad (14)$$

$$\alpha y_{\xi\xi} - 2\beta y_{\xi\eta} + \gamma y_{\eta\eta} = -J^2 \left(P y_\xi + Q y_\eta\right) \qquad (15)$$

where

$$\alpha = x_\eta^2 + y_\eta^2$$

$$\beta = x_\xi x_\eta + y_\xi y_\eta$$

$$+ y_\xi^2$$

$$J = x_\xi y_\eta - y_\xi x_\eta$$

Equations (14) and (15) can be finite differenced using difference formulae for

$$x_\xi,\ x_\eta,\ y_\xi,\ y_\eta,\ x_{\xi\xi},\ x_{\xi\eta},\ x_{\eta\eta},\ y_{\xi\xi},\ y_{\xi\eta} \text{ and } y_{\eta\eta}$$

which yield

$$x(i,j) = \frac{1}{2(\alpha+\gamma)} \left[\begin{array}{l} \alpha\{x(i+1,j)+x(i-1,j)\} \\ -2\beta\{x(i+1,j+1)-x(i+1,j-1) \\ -x(i-1,j+1)+x(i-1,j-1)\}/4 \\ +\gamma\{x(i,j+1)+x(i,j-1)\} \\ +J^2[P\{x(i+1,j)-x(i-1,j)\} \\ +Q\{x(i,j+1)-x(i,j-1)\}]/2 \end{array} \right] \qquad (16)$$

$$y(i,j) = \frac{1}{2(\alpha+\gamma)} \left[\begin{array}{l} \alpha\{y(i+1,j)+y(i-1,j)\} \\ -2\beta\{y(i+1,j+1)-y(i+1,j-1) \\ -y(i-1,j+1)+y(i-1,j-1)\}/4 \\ +\gamma\{y(i,j+1)+y(i,j-1)\} \\ +J^2[P\{y(i+1,j)-y(i-1,j)\} \\ +Q\{y(i,j+1)-y(i,j-1)\}]/2 \end{array} \right] \qquad (17)$$

New values of $x(i, j)$ and $y(i, j)$ are obtained from values of previous iteration and the iterative process is carried out to achieve third decimal accuracy.

Heat transfer equation governing the temperature profile in the physical domain is given by:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{1}{\alpha_d}\frac{\partial T}{\partial t} \qquad (18)$$

In the computational domain, using coordinate transformations [Eqns (3) and (4)], Eqn (18) becomes:

$$\frac{1}{J^2}\left[\alpha T_{\xi\xi} - 2\beta T_{\xi\eta} + \gamma T_{\eta\eta}\right] + PT_\xi + QT_\eta \quad \frac{1}{\alpha_d} T_t \qquad (19)$$

which governs the temperature profile in computational domain. Using central difference analogue for the derivatives of T with uniform grid spacing $\Delta\xi = \Delta\eta = 1$, Eqn (19) becomes:

$$T^{n+1}(i,j) = T^n(i,j) + \alpha_d \Delta t[1/J^2\{\alpha T_{\xi\xi} - 2\beta T_{\xi\eta} + \gamma T_{\eta\eta}\} + PT_\xi + QT_\eta] \qquad (20)$$

The equation is iteratively solved for second decimal accuracy which is achieved by grid consisting of 99 × 99 points. Computer software has been developed and used to study the evolution of temperature in the following cases:

(a) Stainless steel 304 cylindrical shock tube with

Inner diameter = 12 cm

Outer diameter = 14 cm

(b) Stainless steel elliptical cylinder of thickness 1 cm

Inner semi-minor axis = 5 cm

Inner semi-major axis = 6 cm

(c) Stainless steel square block with circular hole

Side of square block = 4 cm

Diameter of circular hole = 2 cm

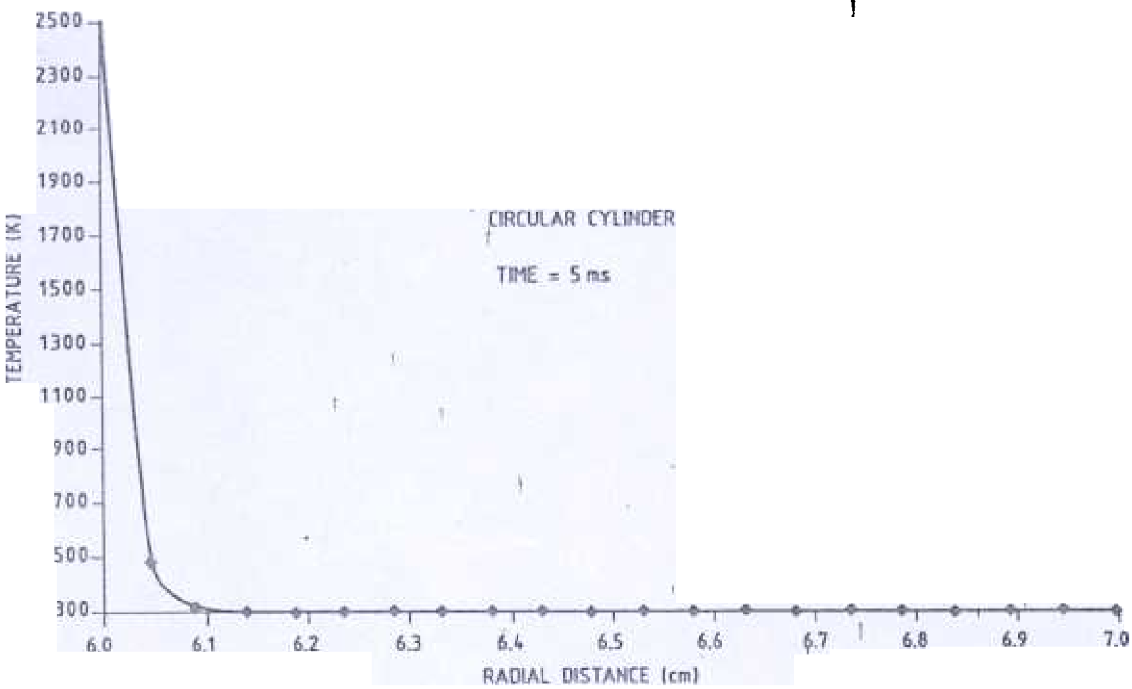

Figure 1. Temperature *vs* radial distance

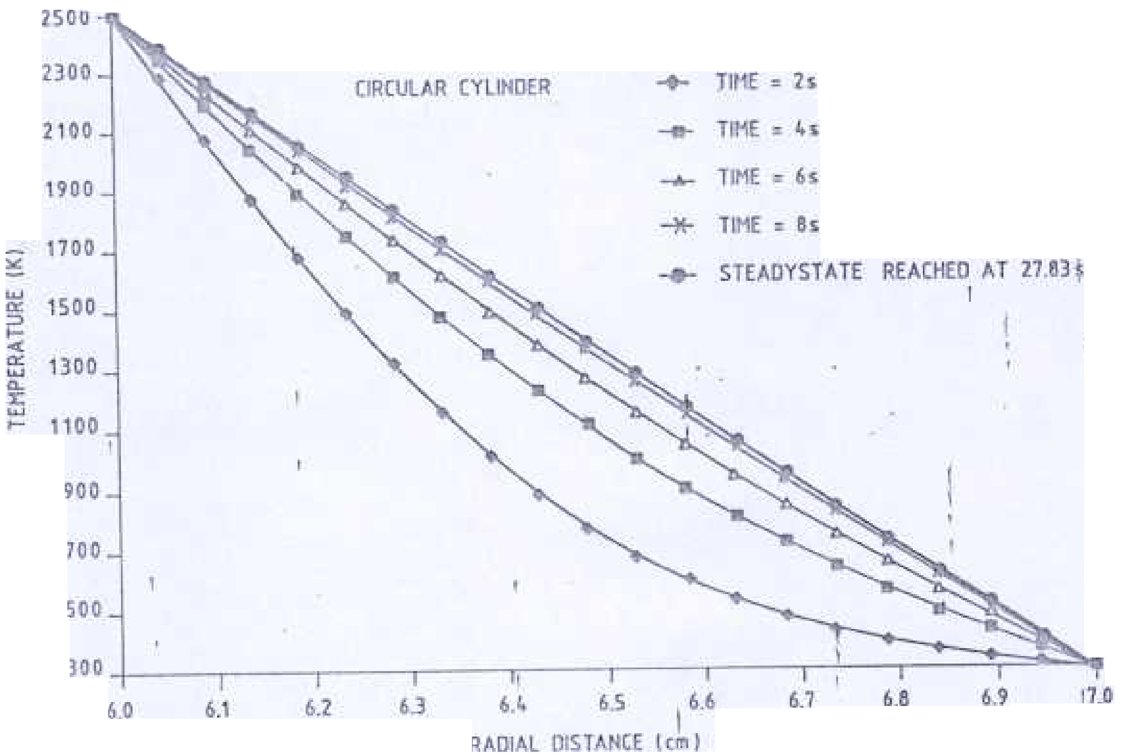

Figure 2. Temperature *vs* radial distance

3. RESULTS & DISCUSSION

The finite difference Eqn (20) has been solved iteratively beginning with initial ambient temperature (T_o). The outer surface of the tube is supposed to be kept at T_o and inner surface raised to T_i at time t = 0. Temperature profiles have been

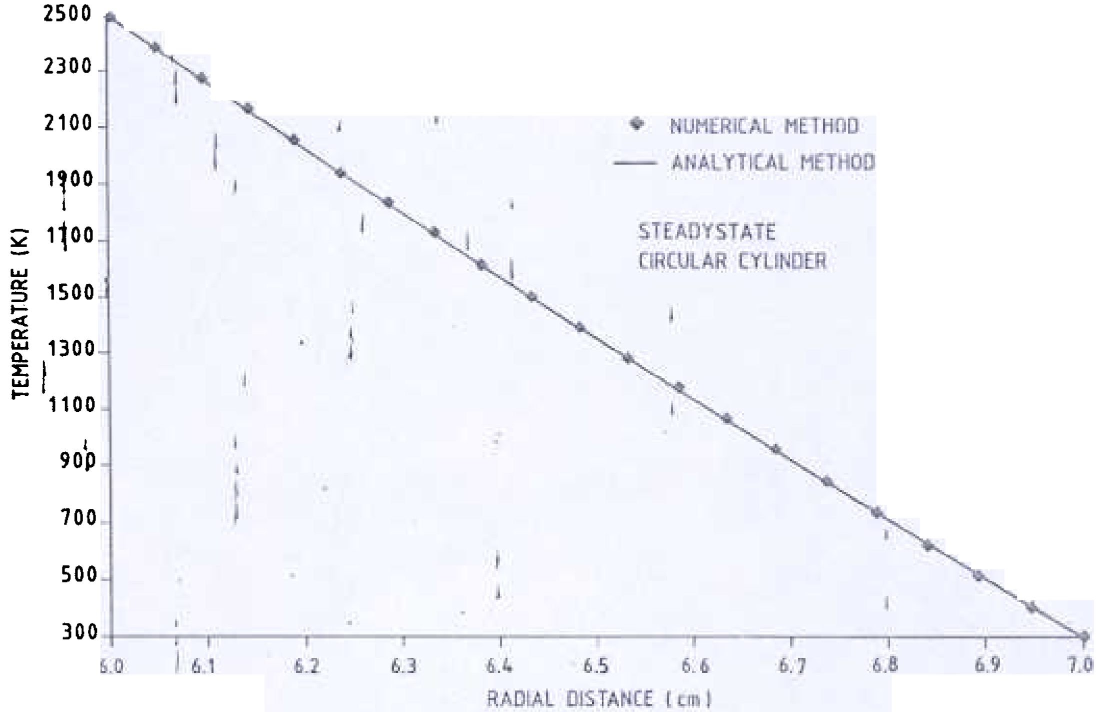

Figure 3. Comparison of analytical with numerical method

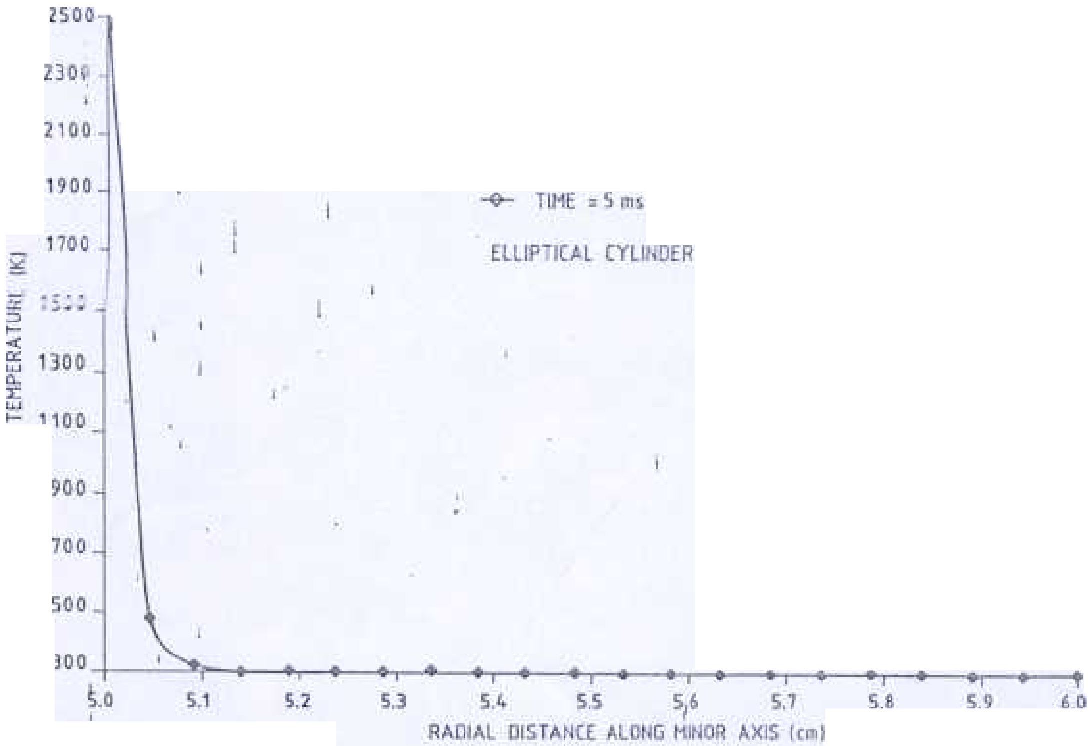

Figure 4. Temperature *vs* radial distance

obtained in different cases of this Drichlet boundary value problem with temperature at the inner boundary raised to 2500 K and that at the outer boundary kept at 300 K.

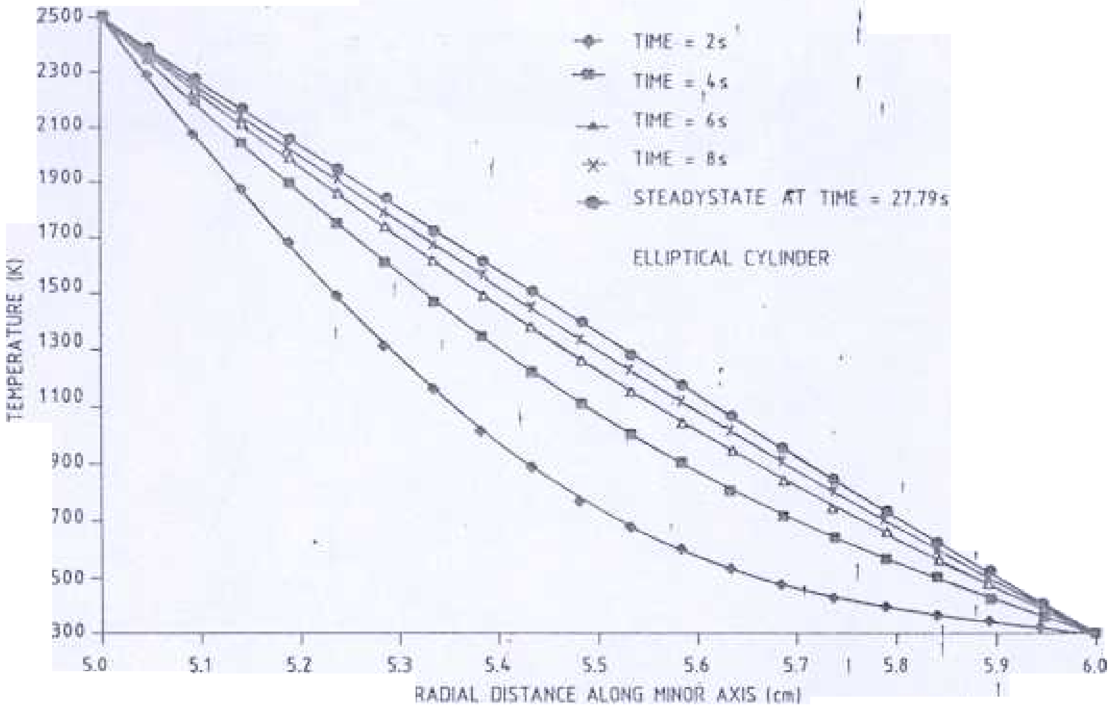

Figure 5. Temperature *vs* radial distance

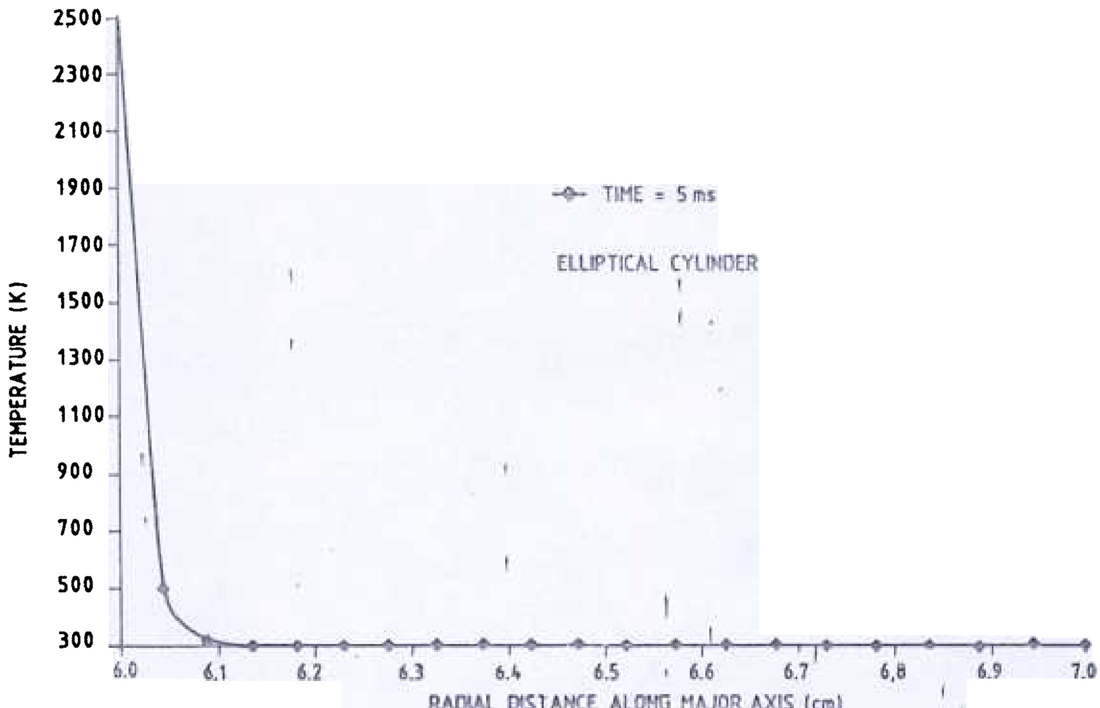

Figure 6. Temperature *vs* radial distance

Figure 1 gives the temperature profile in the driven section of shock tube after 5 ms which is more than the run time of Laser Science & Technology Centre (LASTEC) shock tube. It is observed that about a millimeter of the inner side of the tube senses a rise of 11 K. Figure 2 gives the variation of temperature profile within the cylindrical tube with time. Steadystate is reached after 27.83 s and the temperature profile agrees within 0.01 per cent of the available analytical solution for steadystate[1]. This result is plotted in Fig. 3. Morever, this adds to the confidence in the

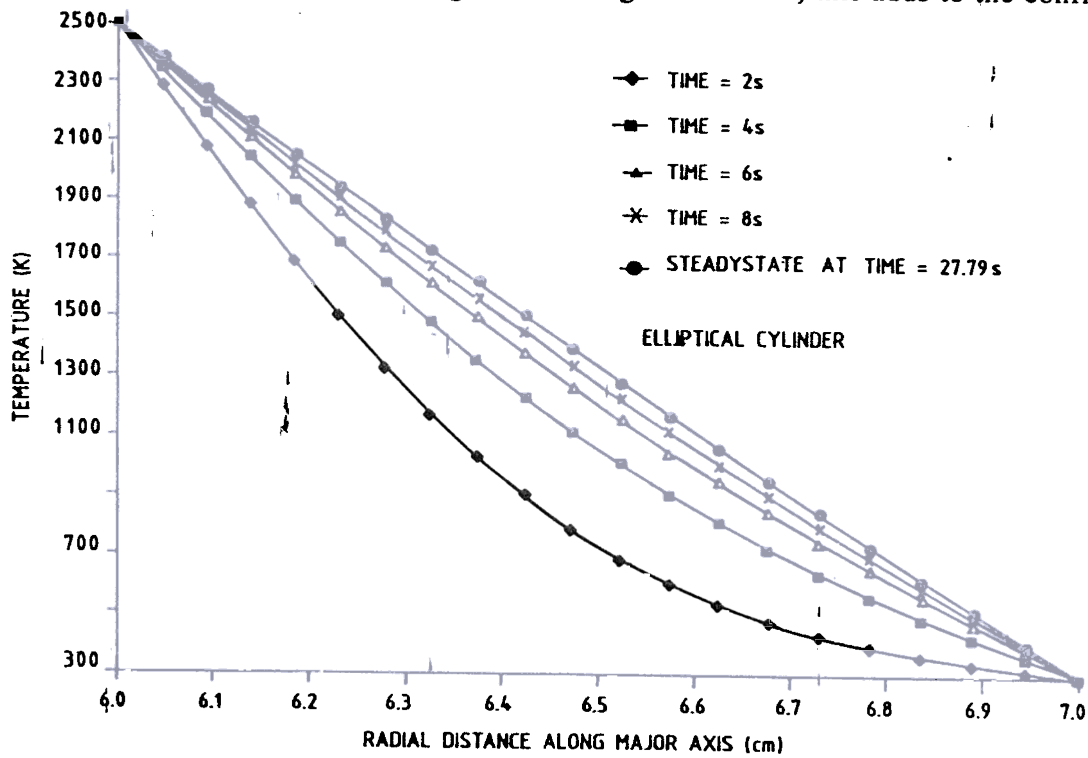

Figure 7. Temperature *vs* radial distance

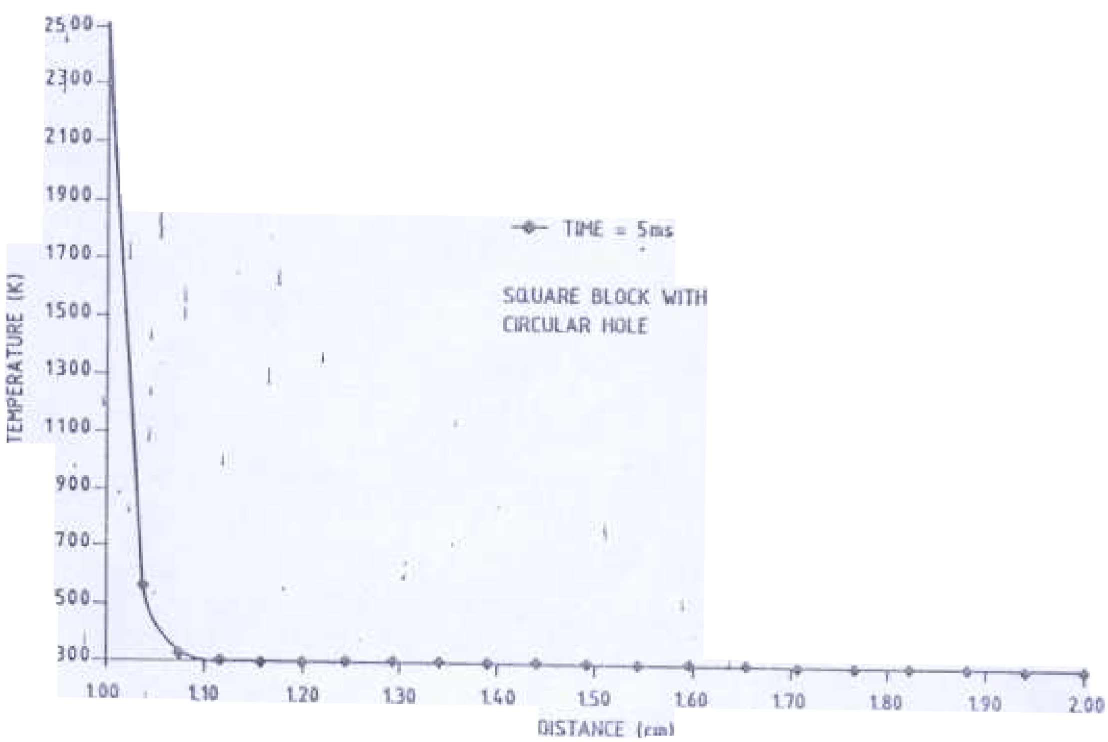

Figure 8. Temperature *vs* distance

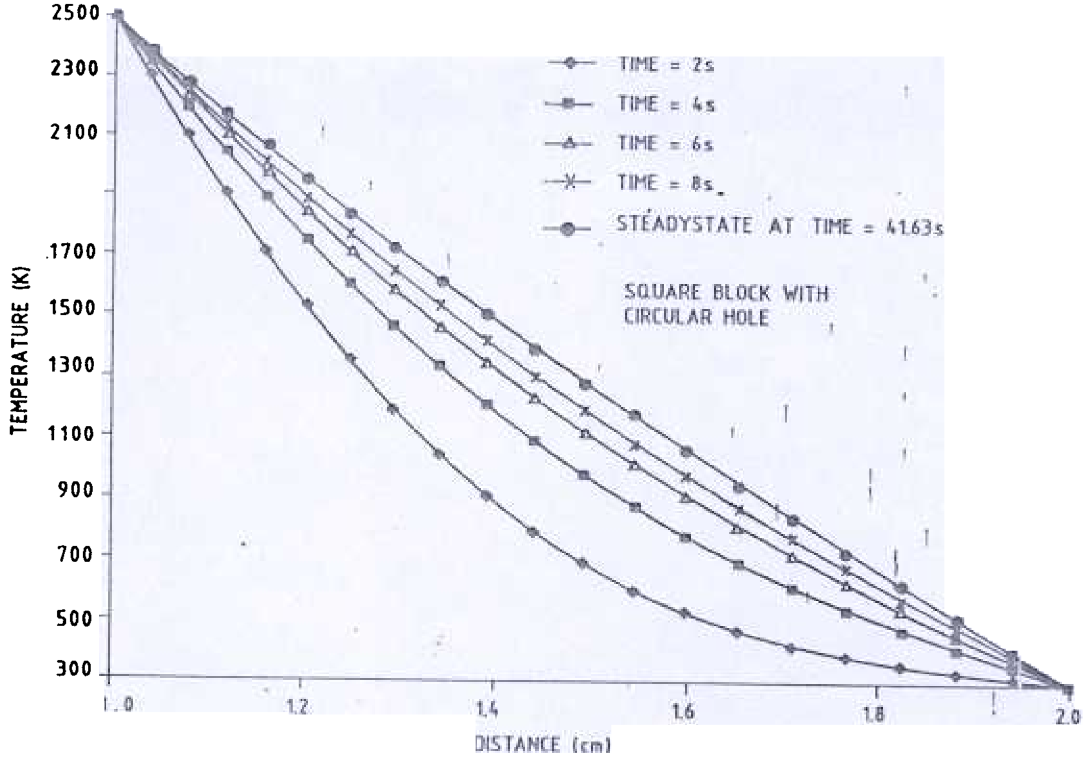

Figure 9. Temperature *vs* distance

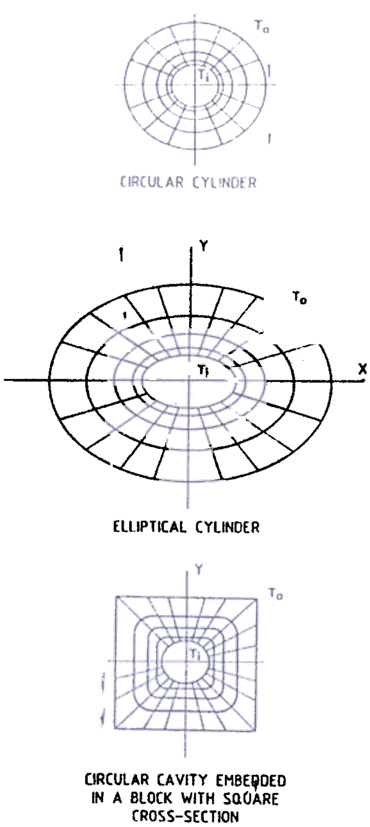

Figure 10. Schematic of grids in physical domain

analysis and the body conforming grid generation technique is applied to study transient heat transfer in elliptical cylinder and square block with circular hole. Figures 4 to 7 give the evolution of temperature in elliptical cylinder of thickness 1 cm with major and minor axes of 12 and 10 cm, respectively. Figure 4 gives temperature profile in the cylinder along minor axis after 5 ms and Fig. 5 gives the evolution of temperature to steadystate along minor axis. Similarly, Fig. 6 gives temperature profile in elliptical cylinder along major axis at 5 ms and Fig. 7 gives the evolution of temperature to steadystate along major axis. It is observed that the temperature profile gets very close to the steadystate profile within first 8 s and then it takes 20 s more to achieve steadystate. Figure 8 gives the temperature profile after 5 ms, whereas Fig. 9 gives the evolution of temperature in a square block with circular hole. The schematic of the grids in physical domain is depicted in Fig. 10.

It is observed from Figs 2,5,7 and 9 that the time taken to reach steadystate increases with the complexity of the domain. It takes 43 s to reach steadystate in square block with circular hole which is almost double the time required to reach steadystate in a circular cylinder.

ACKNOWLEDGEMENTS

The authors are grateful to Shri Amitav Mallik, Director, LASTEC, Delhi, for his keen interest and providing encouragement to carry out this work.

REFERENCES

1. Carslaw, H.C. & Jaeger, J.C. Conduction of heat in solids. Oxford University Press, 1959.
2. Meyer, G.H. A numerical method for heat transfer in an expanding rod. *Int. J. Heat Mass Transfer*, **21**, 1975, 824.
3. Sonar, T. Grid generation using elliptic partial differential equations. DFVLR, Institut fur Entwurfsaerodynamik, Braunschweig, Koln, 1989. DFVLR - FB 89-15.
4. Thompson, J.; Thomes, F.F. & Mastin, C.N. TOMCAT: A code for numerical generation of boundary fitted curvilinear coordinate system on fields containing any number of arbitrary two-dimensional bodies. *J. Comp. Phy.*, 1977, **15**, 299.
5. Anderson, D.A.; Tannehill, J.C. & Pletcher, R.M. Computational fluid mechanics and heat transfer. McGraw-Hill, New York, 1984.
6. Anderson, J.D, *et al.* Introduction to computational fluid dynamics. Von Karman Institute for Fluid Dynamics Lecture Series, Belgium, 1986.
7. Thompson, J.F. (Ed). Numerical grid generation. Elsevier Science Publishing Co., 1982.
8. Fletcher, C.A.J. Computational techniques for fluid dynamics, Vol. II. Springer Verlag, New York, 1987.
9. Bose, T.K. Numerical fluid dynamics. Narosa Publishing House, New Delhi, 1997.
10. Tulapurkara; RamaKrishna, M.; Swaminathan, V. & Adimurthy, V. C.F.D. update, Vols. I and II. IIT Madras, September1992.