# Tactical Trajectory Planning for Stealth Unmanned Aerial Vehicle to Win the Radar Game 

Hongfu Liu*, Shaofei Chen, Lincheng Shen, and Jing Chen<br>National University of Defense Technology, Changsha-410 073, China<br>*E-mail: liu_ho@163.com


#### Abstract

In this paper, problem of planning tactical trajectory for stealth unmanned aerial vehicle (UAV) to win the radar game is studied. Three principles of how to win the radar game are presented, and their utilizations for stealth UAV to evade radar tracking are analysed. The problem is formulated by integrating the model of stealth UAV, the constraints of radar detecting and the multi-objectives of the game. The pseudospectral multi-phase optimal control based trajectory planning algorithm is developed to solve the formulated problem. Pseudospectral method is employed to seek the optimal solution with satisfying convergence speed. The results of experiments show that the proposed method is feasible and effective. By following the planned trajectory with several times of switches between exposure and stealth, stealth UAV could win the radar game triumphantly.


Keywords: Radar game, unmanned aerial vehicle, stealth UAV, trajectory planning, pseudospectral method

## 1. INTRODUCTION

Winning the radar game has been and will remain central to future joint air operations ${ }^{1}$. Stealth unmanned aerial vehicles (UAVs), such as X-45, X-47A, X-47B, X-48B, and Europe Neuron, which have low radar cross section (RCS) of the majority circumferential curve and several narrow peaks ${ }^{2}$, are preferred candidates as the leading actor to win the radar game in an integrated air defense system. However, stealth UAVs still have several RCS peaks; these peaks are high observability aspects for radar detecting and tracking. Hence radar tracking avoidance is an important problem for winning the radar game. Tactical trajectory planning for stealth UAV must be elaborated to evade radar tracking in the whole penetration process.

Most of aircraft trajectory planning researches consider the radar threats as regular shapes for simplification ${ }^{3,4}$, such as hemisphere or cylinder. However, the detecting range of practical radar depends crucially on RCS of the target. RCS of an aircraft are nonisotropic, especially for the stealth UAVs. Hence the threat of radar is a transformable shape depends on relative azimuth between the aircraft and the radar. Grant introduces the game between the aircraft and the radar, gives the understanding of stealth and aircraft survivability ${ }^{1}$. He points out that mission planning of the aircraft enhances the effectiveness and flexibility in the radar game. Norsell ${ }^{5}$, et al. constructs the constraints of radar detecting system based on nonisotropic RCS model. Misovec ${ }^{6}$ and Inanc ${ }^{7}$ apply nonlinear trajectory generation method to trajectory planning, which considered the lock-loss feature of radar system. Kabamba ${ }^{8}$, et al. formulated aircraft low observable trajectory planning as a minimax optimal control problem. However, these researchers do not integrate the fuel consumption of aircraft and the features of radar tracking. In addition, previous researches have not
considered the optimization of the comprehensive efficiency of winning the radar game.

To address the problem mentioned above, we propose an elaborate framework of planning tactical trajectory for stealth UAV to win the radar game. We first analyze the principles of the game; three aspects of radar tracking avoidance are modeled. We then define the constraints and multi-objective of penetration and formulate the trajectory planning problem based on multi-phase optimal control, which can grasp the features of radar game well. Next, we propose a pseudospectral optimal control method to solve the problem. Finally, simulation experiments are presented to illustrate the feasibility and efficiency of our method.

## 2. PRINCIPLES OF STEALTH UAV TO WIN THE RADAR GAME

### 2.1 Principle of Ephemeral Exposure

During the radar game process, the radar network requires detecting and tracking the target in a continuous period. The whole tracking process includes three sub-periods. First, the guidance radar needs search targets that are designated by warning radar. The search time states as $T_{\text {search }}$. Second, for calculating the parameters of missile launch, it needs continuous track during a response time $T_{\text {resp }}$. After that, from the missile launch to grasp a target, the guidance radar is required to continually track the aircraft for missile guidance. The missile flyout time states as $T_{f o}$. The complete tracking time is defined as $T_{\text {track }}=T_{\text {search }}+T_{\text {resp }}+T_{f o}$. From the standpoint of the radar game, a stealth UAV does not have to keep stealth all the time. A conservative allowable exposure time states as $T_{\text {exposure }}=T_{\text {search }}$ $+T_{\text {resp }}$. So the aircraft just needs to evade continuous exposure to the radar system in interval $\left[t-T_{\text {exposure }} t\right]$.

### 2.2 Principle of Radar Tracking Lock-loss

The radar will lose track of the target after a continuous period of no detection, which is named as 'lock-loss' condition. The loss time interval is expressed as $T_{\text {loss }}$. Accordingly, from the perspective of the radar game, after the aircraft is exposed to radar, as long as it keeps stealth in the interval $\left[t, t+T_{\text {loss }}\right]$, it could ensure that the aircraft has thrown off the radar tracking at the time $t+T_{\text {loss }}$.

### 2.3 RCS of Stealth UAV

The RCS of stealth UAV is nonisotropic. Thus at the same distance between the radar and the aircraft, the detecting probability is varied on different azimuth. From the standpoint of the radar game, one means of radar tracking avoidance is to change the relative aspect to the radar into a low observable one; thereby a lower RCS value of the aircraft makes itself stealth to the radar. The aircraft changes the relative aspect by attitude and heading angle control.

Figure 1 shows a scenario of the radar game. The stealth UAV is allowed ephemerally exposes to radars during the radar responding time, afterward keeps stealth to drive the radar into the lock-loss condition. The whole process is separated into stealth and exposure phases.


Figure 1. Stealth UAV wins the radar game.

## 3. PROBLEM FORMULATION

### 3.1 Radar Detecting Probability Model

Stealth UAV is a kind of fluctuating target. As the target feature of stealth UAV is composed of several small scatterers, it can be considered as a Swerling I type target ${ }^{9}$. The probability of detection for the Swerling I target is

$$
\begin{equation*}
P_{d}=e^{-V_{T} /(1+S N R)}, \quad V_{T}=\sqrt{2 \psi^{2} \ln \left(\frac{1}{P_{f a}}\right)} \tag{1}
\end{equation*}
$$

where $V_{T}$ is threshold voltage, $S N R$ is signal to noise ratio, $\psi^{2}$ is the variance of signal, $P_{f a}$ is probability of false alarm. According to the basic radar equation

$$
\begin{equation*}
S N R=S N R_{0} \times\left(\frac{R_{0}}{R}\right)^{4} \times \frac{\delta}{\delta_{0}} \tag{2}
\end{equation*}
$$

where $\delta$ is RCS, $R$ is detection range, $S N R_{0}, R_{0}$, and $\delta_{0}$ are performance parameters of radar.

The radar network dynamic detecting model is set up based on integrated characters of radar network and each radar model. The detecting probability $P_{D}$ can be calculated as

$$
\begin{equation*}
P_{D}=1-\prod_{i=1}^{N}\left(1-P_{d i}\right) \tag{3}
\end{equation*}
$$

where $P_{d i}$ is the detecting probability of the $i^{\text {th }}$ radar, and $N$ is the total number of the radars.

### 3.2 Stealth UAV Model

### 3.2.1 Kinematic Dynamics Model

Tactical trajectory planning for the stealth UAV requires kinematic dynamics model, since the RCS of the aircraft may fluctuate dramatically even from small change of aspect. The dynamics according to a full-blown three degrees of freedom model is as follows ${ }^{10}$

$$
\left\{\begin{array}{lll}
\dot{x}=v \cos \gamma \cos \varphi, & \dot{y}=v \cos \gamma \sin \varphi, & \dot{h}=v \sin \gamma  \tag{4}\\
\dot{v}=g\left(n_{x}-\sin \gamma\right), & \dot{\varphi}=\frac{g n_{y}}{v \cos \gamma}, & \dot{\gamma}=\frac{g\left(n_{h}-\cos \gamma\right)}{v}
\end{array}\right.
$$

where $x, y, h$ are the east, north, and up components of the earth-fixed reference frame. $x, y, h$ is longitude, latitude, and altitude, respectively, $v$ is the speed of aircraft, $j$ is the heading angle, $g$ is the flight path angle, and $n_{x}, n_{y}, n_{h}$ are load factors for the three aspects.

### 3.2.2 Aircraft Fuel Consumption Model

The information for estimating fuel consumption (FC) is from practical flight experimentation. Sufficient recorded data is compiled as FC model in the flight manual. The model of Bell 407 aircraft is adopted ${ }^{11}$. The fuel consumption can be calculated as:

$$
\begin{equation*}
M_{\text {fuel }}=\int_{t} F F \times\left(1+f_{1} \times \dot{v}_{t}+f_{2} \times\left|\dot{\varphi}_{t}\right|\right) \tag{5}
\end{equation*}
$$

where $F F$ stands for fuel flow, $f_{1}$ is the FC factor for speed up, and $f_{2}$ is the FC factor for lateral turn. A second order polynomial can express relation between fuel flow and speed:

$$
\begin{equation*}
F F=k_{1} \times v^{2}+k_{2} \times v+k_{3} \tag{6}
\end{equation*}
$$

And $k_{1}, k_{2}, k_{3}$ is third order polynomial of altitude $h$, respectively.
$\int k_{1}=2.073 \times 10^{-14} \times h^{3}-2.772 \times 10^{-10} \times h^{2}+1.325 \times 10^{-6} \times h+2.682 \times 10^{-2}$
$\left\{k_{2}=-2.002 \times 10^{-12} \times h^{3}+2.142 \times 10^{-8} \times h^{2}-1.406 \times 10^{-4} \times h-3.411\right.$
$k_{3}=4.853 \times 10^{-11} \times h^{3}-1.296 \times 10^{-7} \times h^{2}-1.593 \times 10^{-3} \times h+344.045$

### 3.2.3 RCS Model

The practical RCS of the stealth UAV need minutely measurements from all azimuths with diverse detecting frequency. The metrical RCS is practical, but complex with many burrs. The aircraft RCS numerical simulation model with circumferential and curve peak characters is built up ${ }^{2}$. Here a curve with three peaks is adopted as the RCS model of the stealth UAV. The parameters of model for X bands: $\delta_{\text {ave }}$ $=-20 \mathrm{dBsm}, \phi_{0}=60^{\circ}, \phi_{1}=180^{\circ}, \phi_{2}=300^{\circ}, \delta_{1(\operatorname{Max})}=10 \mathrm{dBsm}$ ,$\delta_{0(\text { Max })}=\delta_{2(\text { Max })}=13.01 \mathrm{dBsm}$, and $\phi_{j(\text { Width })}=2^{\circ},(j=0,1,2)$. The model is described in Fig. 2.


Figure 2. RCS curve of stealth UAV.

### 3.3 MTTP Problem Formulation

Here, the problem is formulated as multi-objective tactical trajectory planning (MTTP). First temporal constraint is defined. Figure 3 describes the sequence of phases in trajectory, which includes short periods of exposure interspersed with periods of stealth.


Figure 3. Sequence of phases in trajectory.
Given $2 P$ phases, the $(2 p-1)^{\text {th }}$ phase is assumed as a process that the probability of detection keeping low level, while the probability of detection could be high in the $(2 p)^{\text {th }}$ phase (where $p=1, \ldots, P$ ), so that

$$
\begin{gather*}
0 \leq P_{t}^{(2 p-1)}(t) \leq P_{\mathrm{low}}, \quad t \in\left[t_{0}^{(2 p-1)}, t_{f}^{(2 p-1)}\right]  \tag{8}\\
0 \leq P_{t}^{(2 p)}(t) \leq P_{\mathrm{high}}, \quad t \in\left[t_{0}^{(2 p)}, t_{f}^{(2 p)}\right]
\end{gather*}
$$

where $P_{\text {high }}$ and $P_{\text {low }}$ stand for the high and low level of the detecting probability constraints, $t_{0}^{(q)}$ and $t_{f}^{(q)}$ stand for the start and end time respectively in the $q$ th phase (where $q=1,2, \ldots$, $2 P+1$ ). Considering the continuous track time and lock-loss condition of missile, the temporal constraints are expressed as follows

$$
\begin{align*}
t_{f}^{(2 p-1)}-t_{0}^{(2 p-1)} & \geq T_{\text {loss }}, t_{f}^{(2 p)}-t_{0}^{(2 p)} \leq T_{\text {exposure }}  \tag{9}\\
t_{f}^{(2 p-1)} & =t_{0}^{(2 p)}, t_{f}^{(2 p)}==t_{0}^{(2 p+1)}
\end{align*}
$$

Moreover, the goal of mission is optimising the comprehensive efficiency of the flight trajectory. The objective function includes three aspects:
(1) Minimize the total fight time $t_{f}^{(2 P)}$.
(2) Minimize the average detection probability of low observable phases $\left(1^{\text {th }}, 3^{\text {th }}, \ldots,(2 p-1)^{\text {th }}, \ldots,(2 P-1)^{\text {th }}\right.$ phase $)$,

$$
\begin{equation*}
P_{\Delta}=\sum_{p=1}^{P}\left(\frac{1}{t_{f}^{(2 p-1)}-t_{0}^{(2 p-1)}} \int_{t_{0}^{(2 p-1)}}^{t_{f}^{(2 p-1)}} P_{t}^{(2 p-1)}(\tau) d \tau\right) . \tag{10}
\end{equation*}
$$

(3) Minimize the total fight fuel consumption $M_{\text {fuel. }}$

Hence, it is a multiple objective problem. The integrated objective function is given as

$$
\begin{equation*}
J=w_{1} \times t_{f}^{(2 P)}+w_{2} \times P_{\Delta}+w_{3} \times M_{f u e l}, \tag{11}
\end{equation*}
$$

where $w_{1}, w_{2}, w_{3}$ are proportional factors.
To address the models mentioned above, MTTP problem becomes the optimization problem of integrated constraints defined by Eqns (1) to (9). It is a multi-objective multi-phase continuous-time optimal control with differential constraint, and temporal constraint.

## 4. TACTICAL TRAJECTORY PLANNING METHOD

### 4.1 Framework of Tactical Trajectory Planning Method

For stealth UAV tactical trajectory planning, the load factors $n_{x}, n_{y}, n_{h}$ are control variables, which determine the position, heading angle and speed of the aircraft through Eqn (4). The state variables are expressed as $\{x, y, h, \nu, \varphi, \gamma, t\}$. The tactical trajectory planning framework is described in Fig. 4. The first part gives the models of stealth UAV and radars. The second part is the formulation of MTTP. In the third part, a trajectory planning algorithm based on Gauss pseudospectral method (GPM) is developed to solve these problems efficiently with high convergence speed.


Figure 4. Framework of numerical procedure for tactical trajectory planning.

### 4.2 Trajectory Planning Algorithm-based on GPM

As mentioned above, the tactical trajectory planning is formulated as a multi-objective multi-phase optimal control
problem. First, the form of multi-phase optimal control problem is introduced. For a set of $K$ phases, minimize the cost functional.

$$
\begin{align*}
J & =\sum_{k=1}^{K} J^{(k)}=\sum_{k=1}^{K} \ddot{\mathrm{O}}^{(k}\left(\boldsymbol{x}^{(k)}\left(t_{0}\right), t_{0} \boldsymbol{x}^{(k)}\left(t_{f}\right), t_{\dot{f}} ; \boldsymbol{q}^{(k)}\right)  \tag{12}\\
& \left.+\int_{t_{0}^{(p)}}^{t_{f}^{(p)}} \ell\left(\boldsymbol{x}^{(k)}(t), \boldsymbol{u}^{(k)}(t), t ; \boldsymbol{q}^{(k)}\right) d t\right]
\end{align*}
$$

(where $k=1, \ldots, K$ ). Subject to the dynamic constraint

$$
\begin{equation*}
\dot{\boldsymbol{x}}^{(k)}=f^{(k)}\left(\boldsymbol{x}^{(k)}, \boldsymbol{u}^{(k)}, t ; \boldsymbol{q}^{(k)}\right) \tag{13}
\end{equation*}
$$

boundary conditions

$$
\begin{equation*}
\phi_{\min } \leq \phi^{(k)}\left(\boldsymbol{x}^{(k)}\left(t_{0}\right), t_{0}^{(k)}, \boldsymbol{x}^{(k)}\left(t_{f}\right), t_{f}^{(k)} ; \boldsymbol{q}^{(k)}\right) \leq \phi_{\max } \tag{14}
\end{equation*}
$$

inequality trajectory constraints

$$
\begin{equation*}
\boldsymbol{C}^{(k)}\left(\boldsymbol{x}^{(k)}(t), \boldsymbol{u}^{(k)}(t), t ; \boldsymbol{q}^{(k)}\right) \leq 0 \tag{15}
\end{equation*}
$$

phase continuity (linkage) constraints

$$
\begin{align*}
& \boldsymbol{P}^{(s)}\left(\boldsymbol{x}^{\left(k_{l}^{s}\right)}\left(t_{f}\right), t_{f}^{\left(k_{l}^{s}\right)} ; \boldsymbol{q}^{\left(k_{l}^{s}\right)}, \boldsymbol{x}^{\left(k_{u}^{s}\right)}\left(t_{0}\right), t_{0}^{\left(k_{u}^{s}\right)}\right.  \tag{16}\\
& \left.\boldsymbol{q}^{\left(k_{u}^{s}\right)}\right) \leq 0,\left(k_{l}, k_{u} \in[1, \ldots, K], \quad s=1, \ldots, L\right)
\end{align*}
$$

where $\boldsymbol{x}^{(k)}, \boldsymbol{u}^{(k)}, \boldsymbol{q}^{(k)}$, and $t$ are respectively the state, control, static parameters, and time in phase $k \in[1, \ldots, K], L$ is the number of phases to be linked, $k_{l}^{s} \in[1, \ldots, K],(s \in[1, \ldots, L])$ are the left phase numbers, and $k_{u}^{s} \in[1, \ldots, K],(s \in[1, \ldots, L])$ are the right phase numbers.

In this paper, the method selected to solve the multiphase optimal control problem is GPM, which is an orthogonal collection method where the collocation points are the Legendre-Guass points. An outline of the GPM for solving optimal control problem is provided here, and details is discussed by Huntington ${ }^{12}$.

The standard interval considered here is denoted as $\tau \in$ [-1,1]. By using a linear transformation, the actual time $t$ can be expressed as a function of $\tau$ via

$$
\begin{equation*}
t=\left[\left(t_{f}-t_{0}\right) \tau+\left(t_{f}+t_{0}\right)\right] / 2 \tag{17}
\end{equation*}
$$

where $t_{0}$ and $t_{f}$ stands for the initial and final time respectively.
The direct approach to solve optimal control problem is to discrete and transcribe optimal control problem to a nonlinear programming problem (NLP). The state is approximated using a basis of $N$ Lagrange interpolating polynomials, $L$

$$
\begin{equation*}
\boldsymbol{x}(\tau) \approx \boldsymbol{X}(\tau)=\sum_{i=0}^{N} \boldsymbol{X}\left(\tau_{i}\right) L_{i}(\tau) \tag{18}
\end{equation*}
$$

The control is approximated using a basis of $N$ Lagrange interpolating polynomials, $L^{*}$

$$
\begin{equation*}
\boldsymbol{u}(\tau) \approx \boldsymbol{U}(\tau)=\sum_{i=0}^{N-1} \boldsymbol{U}\left(\tau_{i}\right) L_{i}^{*}(\tau) \tag{19}
\end{equation*}
$$

The dynamics, boundary and trajectory constraints are transcribed into algebraic constraints. The discretized cost function and constraints are used to define an NLP whose solution is an approximate solution to the optimal control problem.

By using multi-phase optimal control method based on GPM above, the MTTP formulations could be transcribed into NLP problems, which could be solved by using some powerful
numerical methods. The framework of tactical trajectory planning algorithms is described in Fig. 4. GPM exhibits global convergence properties in many applications. Our experimental research of trajectory planning also shows that GPM generates high accuracy solution with satisfying convergence speed. A good initial trajectory will speed up the convergence process. An initial guess is generated by solving the problem integrating the dynamics and temporal constraints, but not considering the radar detection constraints.

Afterwards, the sequential quadratic programming (SQP) algorithm is employed to solve the generated NLP. TOMLAB/ SNOPT is a software toolbox of NLP algorithms, which is especially effective for nonlinear problems with the functions and gradients are expensive to evaluate. Thus, it is appropriate to solve the MTTP after the problem is transcribed into a large-scale NLP. The results given by our method consist of time sequence, state variables sequence and control variables sequence.

## 5. SIMULATIONS

The common parameters of models in our simulations are listed in Table 1. The simulation experiments carry out on a $2.4-\mathrm{GHz}$ Core 2 Duo, 2 G RAM computer with MATLAB R2009b. The following conventions are adopted in the result figures:
(1) Initial position, waypoints, and destination are alphabetically labeled with circles and triangle.
(2) Radar position is shown by a diamond and star.
(3) The range at which $P_{d}=0.5$, for a target with RCS $\sigma=-10 \mathrm{dBsm}$, is shown by a dashed arc of circle.
(4) The trajectory is shown by solid line: thicker red segments denote the exposure phases; thinner blue ones denote the stealth phases.
(5) The instantaneous $P_{d}$ is indicated by the darkness of a line of sight from the radar toward the aircraft as shown in the legends.

Table 1. Common parameters of UAV and radar

| Item | Parameter | Value |
| :--- | :---: | :---: |
| Maximum flight speed (m/s) | $v_{\text {max }}$ | 232 |
| Minimum flight speed (m/s) | $v_{\text {min }}$ | 165 |
| Maximum tangential load factor (G) | $n_{x}$ | 2 |
| Maximum lateral load factor (G) | $n_{y}$ | 4 |
| Maximum pitch load factor (G) | $n_{z}$ | 4 |
| Radar search time (s) | $T_{\text {search }}$ | 8 |
| Radar response time (s) | $T_{\text {resp }}$ | 7 |
| Radar loss track time (s) | $T_{\text {loss }}$ | 60 |
| High level of the probability of detection | $P_{\text {high }}$ | 1.0 |
| Low level of the probability of detection | $P_{\text {low }}$ | 0.2 |
| Fuel consumption factor for speedup | $f_{I}$ | 0.05 |
| Fuel consumption factor for lateral turn | $f_{2}$ | 3.18 |

### 5.1 Scenario 1: Crossing Trajectory between Two Radars for Various Multi-objective

Radar $\mathrm{R} 1=(0,40 \mathrm{~km})$ and $\mathrm{R} 2=(0,-40 \mathrm{~km})$. The initial position $\mathrm{A}=(-90 \mathrm{~km}, 0)$ and the destination $\mathrm{B}=(90 \mathrm{~km}, 0)$. According to the specified performance parameters of radar, for a target with RCS $\sigma=-20 \mathrm{dBsm}$, the detecting range is 35.82 km . So, even if at the lowest detectable azimuth, the stealth UAV just leaves from exposure less than 5 km .

Figure 5(a) illustrates the planned trajectory for winning the radar game and $P_{d}$ of each radar along the trajectory, which optimizes an integrated multi-objective. Figure 5(b) shows the trajectory of minimum total fuel consumption. Figure 5(c) shows the trajectory of minimum probability of detection. Figure 5(b) demonstrates the linear trajectory without stealth maneuver is dangerous. The maximal continuous exposure time of this trajectory is 222.1 s , which is much more than 15 s. Figure 5(c) demonstrates the trajectory is serpentine and fuel consuming, which results in difficult flight. Compared with the two other trajectories, the results of Fig. 5(a) reduce 16.3 per cent time, 3.9 per cent total flight distance and 16.2 per cent fuel consumption. At the same time, the continuous exposure time is less than allowable exposure time rigidly, and it keeps very low probability of detection in stealth phase. The planed trajectory utilizes the radar tracking features to win the game effectively.

### 5.2 Scenario 2: A Realistic Mission and Radar Placement Map

A realistic mission and radar placement map are created in this scenario. We study a stealth UAV performs the reconnaissance mission in a realistic combat environment. It is considered that several types of radars are encountered by the stealth UAV, which include one long range surveillance radar with L band, one long range surveillance radar with S band, two medium range fire control radars with Ku band, and two short range fire control radars with X band. The parameters of the radar placement map are listed in Table 2. Here are two reconnaissance targets $\mathrm{T} 1=(248,272) \mathrm{km}$, and $\mathrm{T} 2=(168,320)$ km . The stealth UAV takes off from blue base, flies in a stealthy trajectory, arrives at T1 and T2 orderly, executes reconnaissance mission, and then returns back to home territory. The start point and destination is blue base, which position is $(180,0) \mathrm{km}$. This is a complex game with multiple radars. Six radars compose a rigid air defense system. Long range surveillance radar R1 provides early warning for fire control radars R4 and R6.


Figure 5. Planning results of scenario 1 with three kinds of objectives.

Table 2. Parameters of the radars in scenario 2

| Radar type | Radar's <br> name | Band | Detecting range <br> $\boldsymbol{\sigma}=\mathbf{0} \mathbf{d B s m}(\mathbf{k m})$ | Position <br> $(\boldsymbol{x}, \boldsymbol{y})(\mathbf{k m})$ | Denoted range <br> $\boldsymbol{\sigma}=\mathbf{- 1 0} \mathbf{d B s m}(\mathbf{k m})$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Long range | R 1 | L | 268 | $(136,192)$ | 151 |
| surveillance radar | R 2 | S | 225 | $(280,160)$ | 126 |
|  | R 3 | Ku | 120 | $(240,120)$ | 67 |
| Medium range | R 3 | 120 | $(120,136)$ | 67 |  |
| fire control radar | R 4 | Ku | 52 | $(264,256)$ | 29 |
| Short range fire <br> control radar | R 5 | X | R | X | 52 |

Surveillance radar R2 provides early warning for fire control radars R3 and R5. Medium range fire control radar R4 and its missile system safeguard surveillance radar R1. Medium range fire control radar R3 safeguards surveillance radar R2. Short range fire control radar R5 and its missile system defense target T1. And short range fire control radar R6 defenses target T2.

Figure 6 shows the planned trajectory for the stealth UAV performs this reconnaissance mission in a realistic threat environment. Figure 7 shows the profile of the trajectory. Figure 8 displays a 3D view of stealth UAV flight: take off from blue base, toward reconnaissance target T1. The total flight distance is 915.7 km , the total fight time is 4076 s , and the total fuel consumption is 609.2 kg . The instantaneous $P_{d}$ of the flight trajectory is displayed by the pie charts. From these views of results, the planned trajectory keeps almost stealth through all the process of mission. During the trajectory crosses between R3 and R4, and between R1 and R2, the UAV operates evadable maneuver, throws off the continuous track of radars. When


Figure 6. Planned trajectory for stealth UAV in scenario 2.


Figure 7. Planned profile for stealth UAV flight in scenario 2.


Figure 8. 3D trajectory of stealth UAV flight, from take off, toward T1.
the UAV approaches the T1 nearby R5, it makes the head of the aircraft towards the radar firstly. The RCS of head part is little. As the same reason, the UAV operates fast maneuvers for target T2 reconnaissance mission nearby R6. The trajectory keeps very low probability of detection in stealth phase, and satisfies the temporal constraint. The maximal continuous exposure time is less than allowable exposure time rigidly, and it can utilize lock-loss condition to throw off radar tracking effectively. So stealth UAV achieves safety with a better comprehensive efficiency in the radar game.

## 6. CONCLUSIONS

A novel analytical result and tactical trajectory planning method for stealth UAV to win the radar game is presented. The principles and constraints of the game are modeled. Afterwards, the trajectory planning algorithm based on pseudospectral multi-phase optimal control is proposed. Moreover, compared with minimizing the total fuel consumption and minimizing the probability of detection, the defined multi-objective optimized the comprehensive efficiency. The validity of the proposed method is illustrated with some simulation result. By utilizing several times of switches between exposure and stealth, the stealth UAV could win the radar game effectively. For the future work, how to plan tactical trajectory for conquering the challenges brought by high dynamic threats is important.

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## Contributors


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Mr Shaofei Chen is pursuing his PhD from Laboratory of Mission Planning, College of Mechatronic Engineering and Automation, NUDT, China. His research interests include: Combat vehicle mission planning and artificial intelligence.
Mr Hongfu Liu is pursuing his PhD from Laboratory of Mission Planning, College of Mechatronic Engineering and Automation, National University of Defense Technology (NUDT), China. His research interests include: Combat vehicle mission planning and intelligence control.


Dr Lincheng Shen working as a Dean and Professor of College of Mechatronic Engineering and Automation in NUDT, China. He has published over 100 technical papers in refereed international journals and academic conferences. His research interests include : Mission planning, SAR image processing, biomimetic robotics, automation and control engineering.


Dr Jing Chen obtained his PhD in control science and engineering from NUDT in 1999. Presently working a Professor of College of Mechatronic Engineering and Automation in NUDT, China. His research interests include: Artificial intelligence and mission planning of aircraft.

