Defence Science Journal, Vol. 63, No. 5, September 2013, pp. 456-461, DOI : 10.14429/dsj.63.2575

A Predictive Explicit Guidance Scheme for Ballistic Missiles

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ABSTRACT

A new approach to the design of ballistic missile guidance is presented in this paper. The proposed method uses the missile model to predict the likely impact point at every guidance cycle and apply course corrections based on the predicted impact point (PIP) deviations. The algorithm also estimates the in-flight thrust variation from nominal and accordingly updates the model to reduce the uncertainty in the prediction of the impact point. The performance of the algorithm is tested through 6-DOF simulation. The simulation results show excellent performance of the proposed guidance scheme in nominal & off nominal cases.

Keywords: Ballistic, point mass, 6-DOF, down range, cross range

NOMENCLATURE

r	Radial position vector
V	Missile velocity
γ_p	Flight path angle in elevation plane
γ_y	Flight path angle in azimuth plane
μ	Geocentric longitude
λ	Geocentric latitude
m	Vehicle mass
$t_{burnout}$	Propulsion burnout time
T	Missile thrust
D	Drag
C_d, $C_{N\alpha}$	Aerodynamic coefficients
ρ	Density
S	Effective surface area
ω	Earth angular speed
Ψ, Φ, θ	Euler angles
α, β	Angle of attack in elevation and azimuth planes
ΔDR	Impact point down range error
ΔCR	Impact point cross range error

1. INTRODUCTION

The role of a ballistic missile is to deliver one or more warheads to a predetermined stationary target.

The design and development of ballistic missile and its guidance has a long history. The problem of ballistic missile guidance has been discussed in numerous literatures such by Siouris[1] and Zarchan[2].

A conventional guidance algorithm aiming for required height, velocity and flight path angles combination at the burnout to reach the desired impact point was presented by White[3]. A nominal trajectory following implicit guidance scheme popular in early days of ballistic missile guidance development was discussed by Schultz[4], *et al.* In this scheme, the missile is guided to follow the nominal trajectory generated on ground and loaded to on-board computer (OBC) before missile lift-off. The guidance algorithm computes the deviation of the current missile position from the nominal position at each instant and commands the required course correction. The disadvantage in this method is that it requires high lateral accelerations pull to correct guidance errors in presence of disturbances such as wind, gust, etc.

In the present scenario, most of the ballistic missiles use explicit guidance scheme in which complete set of trajectory equations are solved on-board and the desired burnout conditions are obtained to hit the impact point. In[5] an optimal explicit guidance scheme for a satellite launch vehicle was presented.

In this paper, a new approach has been presented which uses a missile prediction model residing in OBC to predict the likely impact point. The guidance works on the predicted impact point dispersions and brings the missile to optimum burnout states so that the subsequent ballistic phase ensures the missile impact at the desired impact point.

Most of the guidance schemes for ballistic missiles have been developed for the missiles having majority of flight duration out of sensible atmosphere where the effect of drag is negligible. The proposed guidance scheme is advantageous as it works well for short-range ballistic missiles as well where the missile spends significant duration within sensible atmosphere. This is possible since the missile prediction model provides flexibility to incorporate all the necessary data affecting vehicle dynamics. The high computational requirement of the proposed guidance scheme is met by the present day high speed processors. The details of the algorithm, missile model, guidance command computations and simulation results are discussed in subsequent sections.

Received 31 July 2012, revised 22 August 2013, online published 25 September 2013

2. REFERENCE FRAMES

The guidance scheme assumes the input state vectors of the missile in the earth centered earth fixed frame (ECEF). The north-east-down (NED) frame acts as the guidance reference frame as the key guidance variables are defined in this frame. The diagram of ECEF co-ordinate frame along with NED frame is shown in Fig.1. The ECEF frame has its origin at the center of the earth and rotates with the earth. The X-axis lies in the equatorial plane going through the intersection of Greenwich meridian and the equator, the Z-axis goes through the North Pole and the Y-axis completes the right handed triad. The Local NED frame has its center at the missile's center of gravity with its X-axis pointing towards local north, Y-axis pointing towards local east and the Z-axis pointing in the local vertical direction. The velocity vector and the flight path angles used as variables in missile model are shown in Fig. 2.

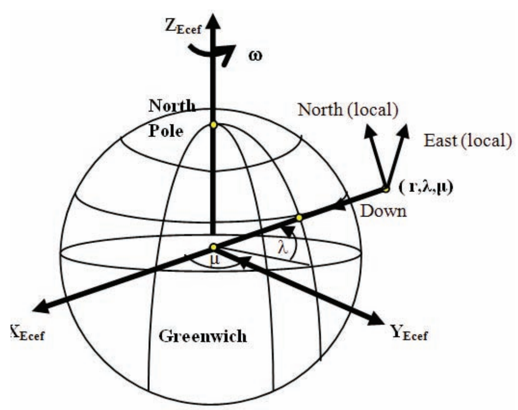

Figure 1. Reference frames- ECEF and local NED.

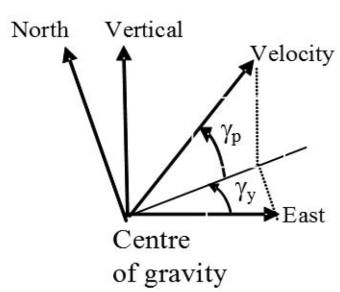

Figure 2. Flight path angles in elevation and azimuth plane.

3. EXPLICIT GUIDANCE ALGORITHM

The objective of the guidance algorithm is to guide the missile to reach the specified impact point on the ground. The guidance works on the predicted impact point (PIP) and gives the guidance corrections as a function of predicted terminal error.

Let

$$\dot{X}_m = f_1(X_m, u(t), t) \tag{1}$$

$$\dot{X}_m = f_2(X_m, u(t), h) \tag{2}$$

be the equations of motion of the missile with time and altitude as the independent variables respectively. If the initial conditions, the independent guidance vector 'u' and the missile state vector X_m (from Onboard INS) are known at every guidance cycle, the equations of motion can be integrated forward to find the probable impact point. In other words,

$$X_{mf} = \int_{t_c}^{t} f_1(X_m, u(t), t)\,dt + \int_{h_{\min}}^{0} f_2(X_m, u(t), h)\,dh \tag{3}$$

The time-based integration is done from current time until the predicted height becomes less than prefixed altitude in the descent phase of the trajectory. The integration of trajectory equations continue with *h* as the independent variable from the prefixed altitude in the descent phase until the predicted height becomes zero. This is to ensure that the prediction continues until impact on the ground. The propagation using both 'time' and 'height' as independent variables is advantageous in terms of computation time as propagation using 'time' alone requires very small integration step size at the end.

The desired impact point X_t is known before the launch of the missile. Therefore, the predicted impact point deviation can be calculated by taking the difference between predicted missile impact point and the desired impact point i.e., X_t - X_{mf}. The predicted impact point error is resolved to get the down range and cross range errors (ΔDR, ΔCR). Finally, the guidance variables are updated based on predicted errors to minimize the impact point deviation. The detailed block diagram of the guidance algorithm is shown below in Fig. 3 for better understanding of the flow of the algorithm.

The success of the algorithm mainly depends on the missile model, choice of the guidance variables and the law for updating the guidance variables in minimizing the impact point deviations.

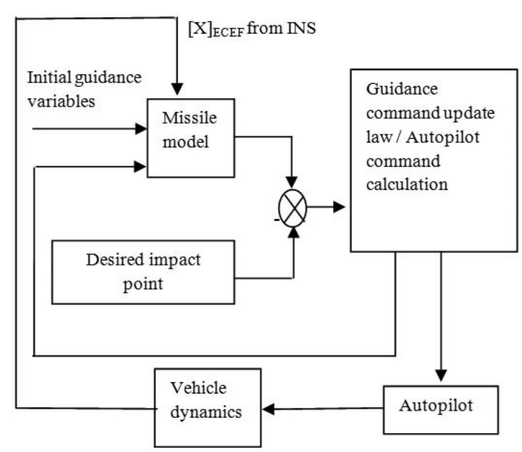

Figure 3. Block diagram of guidance algorithm.

3.1 Guidance Variables for Minimising Impact Point Deviations

The guidance variables chosen are $u = (\dot{\gamma}_p, \dot{\gamma}_y)'$, where $\dot{\gamma}_p$, $\dot{\gamma}_y$ are the flight path angle rates of the missile in elevation and azimuth plane respectively. The flight path rates are the basic trajectory parameters, which can be altered for lateral correction by the guidance system to achieve the desired impact point.

For missiles with liquid propulsion, thrust shut off time can be used as another guidance variable whereas the same is not possible for missiles with solid propulsion and hence the thrust shut off time is not used as the guidance variable. However, the burnout time is essential for the proposed predictive guidance as the missile model requires it for propagation. The burnout time is initialized with nominal burnout time and is updated in-flight based on the propulsion performance variation and frozen before the start of closed loop guidance.

The in-flight variation of thrust is accounted in the prediction model by re-scaling the thrust ~ time curve, computed based on the sensed acceleration measured over the nominal acceleration assuming total impulse of the solid motor constant. The problem of ballistic missile guidance assuming constant total impulse of the solid rocket motor is solved in[6]. In the realistic scenario, the constant impulse of the solid propulsion system is not ensured therefore a velocity trimming guidance scheme is required as presented[7].

3.2 Guidance Command Calculation

The guidance assumes the proper initialization of the variables $(\dot{\gamma}_p, \dot{\gamma}_p)$. This initialization is done on ground before the launch of the missile. The missile model for the first time uses these initialized values and subsequently uses the previous guidance cycle commanded values to calculate PIP. The following procedure is carried out at every guidance cycle for minimizing the PIP deviations.

From the Eqn. (3) and the missile prediction model, it is clear that the final state is in polar form (r, λ, μ). The final state is computed in ECEF Cartesian form, which is the PIP in ECEF frame. This PIP then transformed into launch point east-north-vertical (ENV) frame and subsequently into down range-cross range-altitude (DCH) frame. The down range axis is the line joining launch point (LP) and target point (TP). The cross range axis is the line perpendicular to down range axis. Similar sequence of transformations is applied on the desired ECEF state of the target point to get the desired impact point in DCH frame i.e.,

$$\left[X_{mf}\right]_{DRCR} = [T]_{lpenv}^{drcr} . [T]_{ecef}^{lpenv} . [X_{mf}]_{ecef}$$

$$\left[X_t\right]_{DRCR} = [T]_{lpenv}^{drcr} . [T]_{ecef}^{lpenv} . [X_t]_{ecef}$$

$$[E]_{DRCR} = [X_t]_{drcr} - [X_{mf}]_{drcr}$$

ΔDR = E(1) ΔCR = E(2)

The down range error (ΔDR) and the cross range error (ΔCR) are the functions of the missile burnout parameters in terms of its position, velocity and the flight path angles in elevation and azimuth planes. The missile prediction model estimates the burnout states such as position, velocity and flight path angles. The flight path angle change required at burnout to hit the desired impact point taking estimated burnout position and velocity are as follows

$$\Delta\gamma_p = (\partial\gamma_p/\partial DR) * \Delta DR \tag{4}$$

$$\Delta\gamma_y = (\partial\gamma_y/\partial CR) * \Delta CR \tag{5}$$

where $(\partial\gamma_p/\partial DR)$ and $(\partial\gamma_y/\partial CR)$ are the sensitivity coefficients determining the per unit change requirement in flight path angle for unit change in down range and cross range respectively. ΔDR and ΔCR are the impact point down range and cross range errors as computed by the missile prediction model.

The sensitivity coefficients are computed at every guidance cycle by perturbing flight path angles at burnout and evaluating its effect on impact down range and cross range. The angular flight path angle change requirements $\Delta\gamma_p$ and $\Delta\gamma_y$ are distributed from current time to estimated propulsion burnout time ($t_{burnout}$). The guidance demanded flight path rates are computed as,

$$\dot{\gamma}_p(i+1) = \dot{\gamma}_p(i) + \frac{\Delta\gamma_p}{(t_{burnout} - t)} \tag{6}$$

$$\dot{\gamma}_y(i+1) = \dot{\gamma}_y(i) + \frac{\Delta\gamma_y}{(t_{burnout} - t)} \tag{7}$$

The guidance demanded flight path rate is to be achieved by axial thrust and aerodynamics. The flight path angle rates are translated into an equivalent attitude command and communicated to the attitude autopilot for execution. It can be seen that lateral acceleration, flight path angle rate and the angle of attack are all closely related. The relation among them is given below.

$$\alpha = \frac{mV\dot{\gamma}_p + g\cos\gamma_p}{T + 0.5\rho V^2 SC_{N\alpha}} \tag{8}$$

$$\beta = \frac{mV\cos\gamma_p\,\dot{\gamma}_y}{T + 0.5\rho V^2 SC_{N\alpha}} \tag{9}$$

Now, the attitude command to the autopilot to achieve $\dot{\gamma}_p, \dot{\gamma}_y$ (or equivalently α, β) can be communicated. Attitude command in terms of euler angles from NED to body is,

$$\psi = \frac{\pi}{2} - \beta - \gamma_y; \varphi = \pi; \theta = -(\gamma_p + \alpha) \tag{10}$$

4.0 MISSILE MODEL FOR IMPACT POINT PREDICTION

The missile is modeled as a point mass traveling over a spherical rotating earth. The propulsive, aerodynamic and gravity forces are modeled. The detailed derivation of trajectory equations is discussed by Vinh[8]. The trajectory equations are presented by Song & Tahk[9] as a function of time is given below

$$\dot{v} = k.\frac{(T\cos\alpha\cos\beta - D)}{m} - g\sin\gamma_p \tag{11}$$

$$\dot{\gamma}_p = (\dot{\gamma}_p)_{demand} \qquad \text{if } (t < t_{burnout}) \tag{12}$$

$$\dot{\gamma}_p = \frac{-g\cos\gamma_p}{v} + \frac{v}{r}\cos\gamma_p + \frac{\omega^2 r}{v}\cos\lambda(\cos\gamma_p\cos\lambda + \sin\gamma_p\sin\gamma_y\sin\lambda) + 2\omega\cos\gamma_y\cos\lambda \qquad (t >= t_{burnout}) \tag{13}$$

$$\dot{\gamma}_y = (\dot{\gamma}_y)_{demand} \qquad \text{if } (t < t_{burnout}) \tag{14}$$

$$\dot{\gamma}_y = -\frac{v\cos\gamma_p\cos\gamma_y\tan\lambda}{r} + 2\omega(\tan\gamma_p\sin\gamma_y\cos\lambda - \sin\lambda) - \frac{\omega^2 r}{V\cos\gamma_p}\cos\gamma_y\sin\lambda\cos\lambda \qquad \text{if}(t >= t_{burnout}) \tag{15}$$

$$\dot{r} = v \sin\gamma_p \quad (16)$$

$$\dot{\mu} = \frac{v\cos\gamma_p \cos\gamma_y}{r\cos\lambda} \quad (17)$$

$$\dot{\lambda} = \frac{v\cos\gamma_p \sin\gamma_y}{r} \quad (18)$$

where

$$D = \frac{1}{2}\rho v^2 s C_d$$

The aerodynamic drag coefficient C_d is a function of mach no. and height. The air density (ρ) and gravitational acceleration (g) are functions of height.

The equations of motion need to be integrated numerically to obtain the final missile states. A fourth-order Runge-kutta (RK4) method has been chosen for solving the trajectory equations. The RK4 method is discussed in details by Sastry[10].

4.1 Estimation of Net Acceleration Variation

The proposed predictive guidance is sensitive to modeling and data inaccuracies as it will lead to higher predicted impact errors hence higher control effort. Therefore, it is important to identify all the necessary parameters affecting missile in-flight dynamics. The main parameters that can vary in real time are the vehicle propulsion and aerodynamics. The variation in these two parameters is accounted by introducing a scale factor (k) in the Eqn. (11). The scale factor is estimated during missile flight at very guidance cycle by taking the ratio of sensed acceleration obtained from onboard INS and the nominal acceleration. The nominal acceleration is calculated using stored nominal thrust and drag profiles, resolved in the current velocity direction. For instance, a scale factor of 1.07 would indicate a net acceleration (i.e., (Thrust – Drag)/Mass) variation of 7 % over that of the nominal.

Therefore, the equation for the net acceleration along the flight direction will be

$$\dot{V} = k.\left(\frac{T-D}{M}\right)$$

here k is the scale factor.

5. SIMULATION STUDIES

The proposed guidance algorithm is thoroughly tested in 6-DOF simulation under nominal and off nominal cases. The 6-DOF simulation test bed is written in 'FORTRAN' programming language. The nominal missile data used for evaluating the guidance algorithm is shown in Table 1. Off-nominal cases involve perturbation on the vehicle thrust, weight and aerodynamics. The algorithm is also tested under wind conditions. The perturbation cases used for testing the algorithm is mentioned in Table 2. The simulation results show an excellent performance of the algorithm and the impact error is observed well below 50 m in all the cases.

The simulation results of nominal and perturbation cases are discussed below. The guidance predicted errors (down range and cross range errors) in nominal, thrust-up and thrust down cases are shown in Figs. 4 and 5 and are observed well within a dead zone of 20 m. It takes few guidance cycles (8-10 guidance cycles) to converge into guidance tunnel of specified value. The simulation study is carried out taking guidance cycle of 100 ms. The trajectories in azimuth and elevation plane are shown in Figs. 6 and 7.

Table 1. Missile data

m_0 = 4000kg, g_0 = 9.81 m/s^2, Isp = 250 s
Thrust = 6.0 ton at sea level
Nominal burn time = 100.0 s

Table 2. Vehicle data perturbation cases

S. No.	Thrust	H/W Wt.(kg)	Cd0
1. Nominal	nom	nom	nom
2. Thrust up	+7%	-1.5 %	-3%
3. Thrust down	-7%	+1.5%	+3%

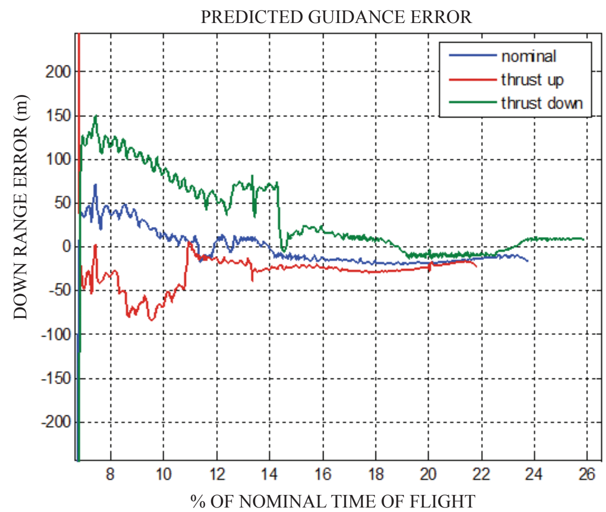

Figure 4. Predicted down range error at impact.

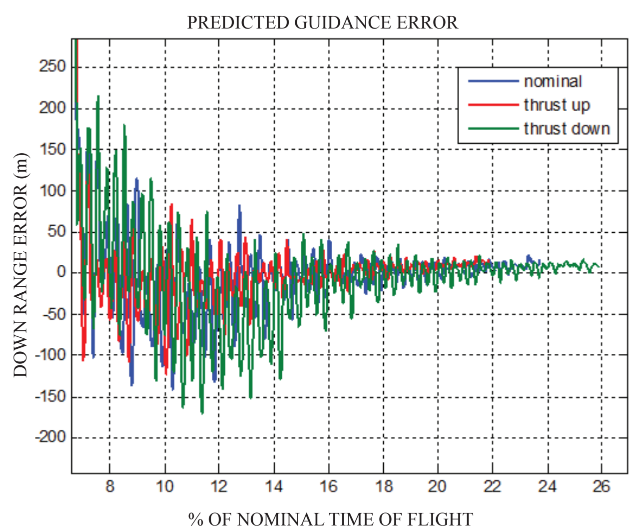

Figure 5. Predicted cross range error at impact.

5.1 Hardware-in-loop Simulation Results

Hardware-in-loop simulation (HILS) test bed consists of 6-dof model, On board computer (OBC), Missile interface unit (MIU) and Inertial navigation system (INS). The test bed

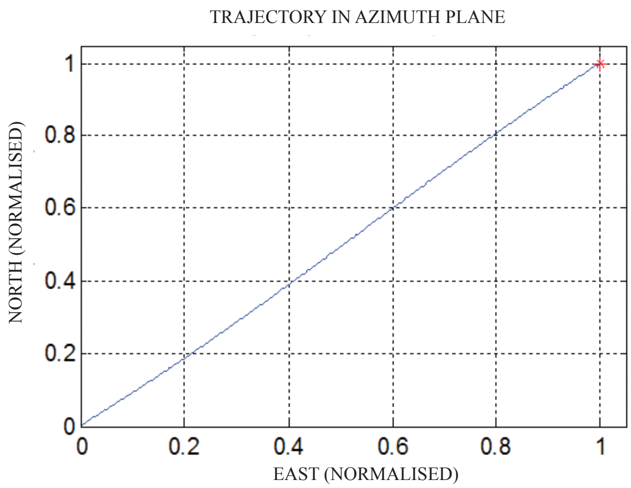

Figure 6. Trajectory in azimuth plane.

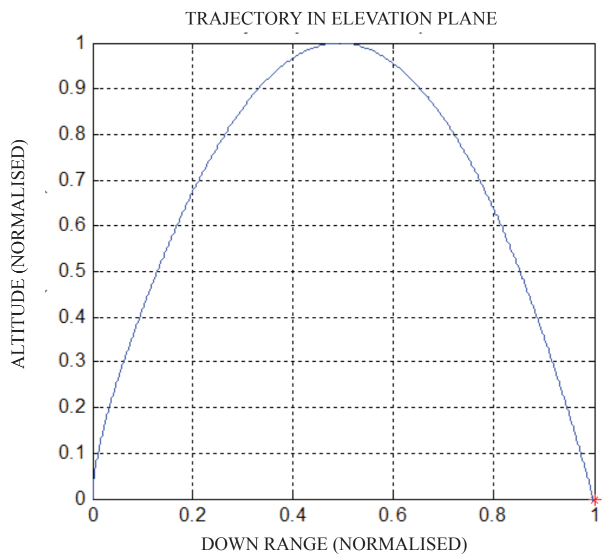

Figure 7. Trajectory in elevation plane.

was based on 1553 bus communication protocol. The mission software consisting of guidance algorithm is validated in HILS environment using OBC in loop configuration. It is also validated through using sensor in loop, actuator in loop and sensor-actuator in loop. A typical actuator in loop simulation results are shown below in Figs. 8 and 9.

6. CONCLUSION

A novel explicit guidance scheme has been developed for the ballistic missiles. A single stage flight vehicle with solid propulsion system was used to evaluate the performance of the guidance algorithm in this paper. The guidance algorithm can be used for multiple stage flight vehicles by incorporating necessary data of different stages in the prediction model. The guidance algorithm developed is advantageous especially for short-range ballistic missiles having significant flight within atmosphere. The guidance algorithm has been tested and validated through HILS test bed and all the simulation results were satisfactory.

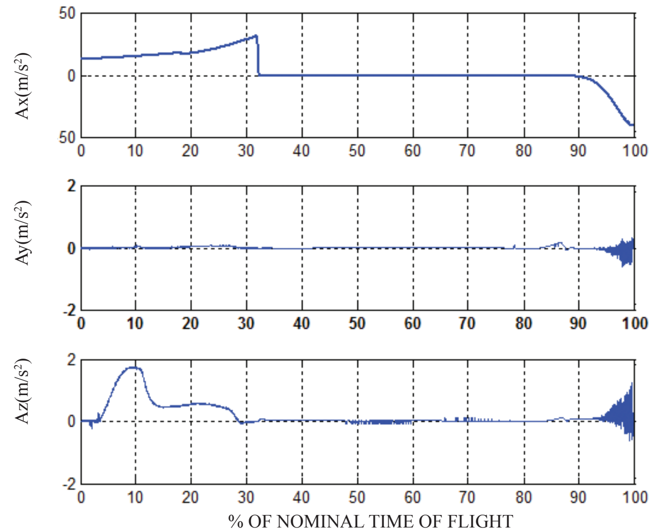

Figure 8. Missile axial (Ax) and accelerations (Ay and Az).

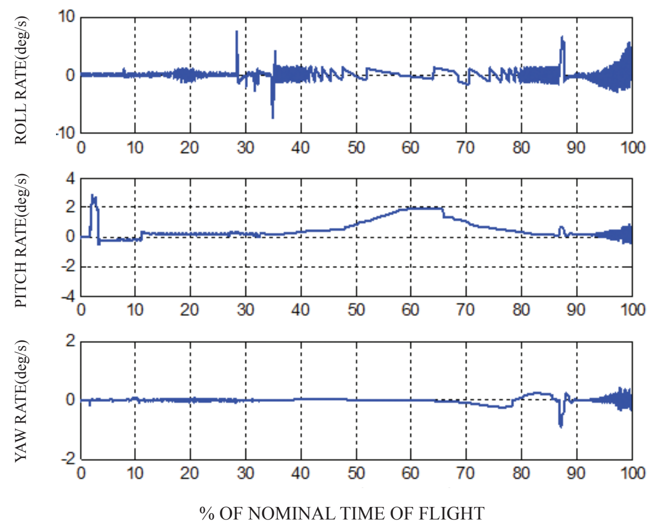

Figure 9. Missile body rates- roll, pitch and Yaw.

ACKNOWLEDGEMENTS

The authors acknowledge Shri N.V. Kadam for his motivation and support during the course of the work. The authors also acknowledge the immense involvement and guidance of Dr S.K. Choudhury during the testing and validation of the algorithms through HILS test bed.

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