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SHORT COMMUNICATION

Statistical Tolerance Limits for Process Capability

S. Rajagopalan
Vikram Sarabhai Space Centre,Thiruvananthapuram-695 022

ABSTRACT

In this paper, an attempt has been made to highlight the methodology of statistical tolerance limits and its applicability for estimating the process capability. The theory developed is applied to an actual case study and the results are discussed.

Keywords: Process capability, statistical tolerance limits, probability density function, PDF, cumulative distribution function, CDF

1. INTRODUCTION

Variations in the observed measurements are inevitable and no experiment is 100 per cent accurate in practice. Keeping this in view, it is often customary to set limits for allowable deviations which are inevitable. These are the limits which quantify the maximum acceptable deviations from the nominal and are taken care of during design and development[1,2]. A wrong notion of setting 3a limits for process capability is prevailing in industry, under the assumption that these limits will contain 99.73 per cent of the observation. This is true if and only if the mean and standard deviation are known to be stable, or otherwise these are population mean and standard deviation. In practice, the population parameters are unknown and are to be estimated from sample statistics, and thus, uncertainty due to sampling cannot be ignored. The method of statistical tolerance limits proposed here gives an appropriate alternative for estimating the process capability with proper accountability to sampling uncertainty. These limits are obtained in such a way that, at least, a given percentage *(p)* of the population can lie within these limits with a certain degree of confidence (v). The values that specify the computed intervals are called statistical tolerance limits, the minimum fraction of population, which the limits are intended to include, is denoted by *p* and the degree of certainty is referred to as confidence coefficient and is denoted by v.

This paper highlights the salient features of the statistical tolerance limits along with the generalised computer program for evaluating these limits. The study includes: (i) theory of statistical tolerance limits, (ii) formulae for one-sided and two-sided normal statistical tolerance limits, (iii) a generalised computer program for direct evaluation of the statistical tolerance limits from raw data, and (iv) case study to illustrate the methodologies discussed.

2. THEORY OF STATISTICAL TOLERANCE LIMITS

Statistically, for any continuous distribution, *f*(*), the tolerance limits can be defined[3-5].

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Let $[x_i\}$, $(i = 1,2,...AO$ be a sample from a population with probability density function (PDF) $/ (x)$ and cumulative distribution function (CDF) $F(x)$.

Let

$$L = L (x_1, x_2, ...x_N)$$

and

$$U = U (x, x_2 \quad X_N)$$

be the two sample statistics (L < U), such that

$$\text{Prob } [(F(U)-F(L)\} > p] > v$$

Then, the interval (L, U) is palled lOOp per cent statistical tolerance interval[6] for x at confidence level of v, in the sense that the lower limit (L) and the upper limit (U) will contain at least 100 p per cent of the population with PDF/(;c) with a degree of confidence 100 v per cent.

If only one-sided limit (either L or U), say L, is required, then if Prob [F(L) > p] > v, the limit L = L (;c, $x_2 \quad X_N$) is called 100 p per cent of lower limit at confidence level in the sense that at least p per cent of the population will be above the limit L with a degree of confidence v.

3. STATISTICAL TOLERANCE LIMITS

Since most of the physical quantities follow a normal distribution or at least can well be approximated by a normal frequency curve, the method for computing the statistical tolerance limits for normally distributed variables has been discussed here.

For a normal distribution with both mean and the variance being unknown and for a given sample (*, $x_2 \quad X_N$) of size N, the one-sided limits can be given[7] as follows:

Either L = Lower limit = ($\bar{X}$ - $K.s$), or U = Upper limit = ($\bar{X}$ + $K.s$), where $\bar{X}$ and s are the sample mean and standard deviation, and K is the statistical tolerance factor, which takes into account for the sampling errors in X and s as well as in the population fraction p and can be obtained using $K= N$ ~$^m t'$ \f, 8, 1-v], where, t' is the 100 v per cent point of the non-central ^-distribution with / $(=N$-\) DOFs and the non-centrality parameter 8 = Z t_ N ~$^{1/2}$, (Z,_ being the 100 p per cent point of standard normal deviate) and can be obtained using the probability distribution as given by Owen[8] and Resnikoff[9], for a set of values of p and v.

Table 1 gives K factors for two-sided tolerance intervals, ie, factors K such that the probability is 0.95 that at least a proportion p of the distribution will be included between ($\bar{X}$ ± $K.s$), where $\bar{X}$ and 5 are computed from a sample of size N. The values of K taken from Table 1 give a 95 per cent confidence that at least a population fraction p will be included[10-11] in the interval ($\bar{X}$ ± $K.s$).

Table 1. Two-sided tolerance factors

Sample size N	Proportion p			
	0.750	0.900	0.950	0.990
5	3.002	4.275	5.079	6.634
6	2.604	3.712	4.414	5.775
7	2.361	3.369	4.007	5.248
8	2.197	3.139	3.732	4.991
9	0.078	2.967	3.532	4.631
10	1.987	2.836	3.379	4.433
17	1.679	2.400	2.858	3.754
37	1.450	2.073	2.470	3.246
145	1.280	1.829	2.179	2.864
oo	1.150	1.645	1.960	2.576

Table 2 gives K values for one-sided tolerance intervals, ie, factors K such that the probability is 0.95 that at least a proportion p of the distribution will lie either above ($\bar{X}$ - $K.s$) or below ($\bar{X}$ + $K.s$), where $\bar{X}$ and s are computed from a sample of size N. The values of K taken from Table 2 give a 95 per cent confidence that at least a fraction p will be either above ($\bar{X}$ - $K.s$) or below ($\bar{X}$ + $K.s$).

Table 2. One-sided tolerance factors

Sample size *N*	Proportion *p*			
	0.750	0.900	0.950	0.990
5	2.150	3.412	4.212	5.751
6	1.895	3.008	3.711	5.065
7	1.733	2.756	3.400	4.644
8	1.618	2.582	3.188	4.356
9	1.532	2.454	3.032	4.144
10	1.465	2.355	2.911	3.981
17	1.220	2.002	2.486	3.414
37	1.014	h.717	2.149	2.972
145	0.834	1.481	1.874	2.617
oo	0.674	1.282	1.645	2.326

4. CASE STUDY

The theory explained above is applied to a typical case of quality check parameter of the pyrocharge used in the igniter of a launch vehicle rocket motor. Because of the classified nature of the problem, only the data is presented in Table 3. The details are available in the technical report[12].

Table 3. Data pertaining to a typical case study

Parameter of pyrocharge	Batch No.
26.83	5/94
28.75	”
29.68	”
29.30	”
26.77	”
24.44	2/98
25.44	”
23.92	”
23.36	”
24.52	”

Statistical tolerance limits can be determined in **the** form $((\bar{X} \pm K.s)$,

where

$\bar{X}$	Mean of the experimental data obtained
K	$r(N, p)$ X $u(f, v)$
$r(N, p)$	Factors of N and p
N	Number of test data
p	Percentage of data expected to be within the statistical tolerance limits
$u(f, v)$	Factors of f and v
f	Degrees of freedom
v	Confidence level
s	Standard deviation of the test data

The factors *r (N, p)* and «(/", v) are computed using the software STL.F, developed in FORTRAN.

The factor *K* for each value of *p* is calculated and the statistical tolerance limits are worked out. The results are given below:

Number of data *(N)*	10
Mean $(\bar{x})$	26.30
Standard deviation *(s)*	2.3239
Degrees of freedom (/)	9
Range of data	23.36 to 29.68

Based on the results shown in Table 4, it is seen that the given data follows the limits given by (26.3 ± 5.39) corresponding to 2a limits with 60 per cent confidence limits.

4. ADVANTAGES OF STATISTICAL TOLERANCE LIMITS

In cases, where a large number of data points are available, the classical method of fixing tolerance limits, viz., first computing mean { $\bar{X}$) and standard deviation *(s)* from the test data and then evaluating the values of $(\bar{X} \pm 2s)$ and considering these as tolerance limits is all right. However, in situations,

Table 4. Statistical tolerance limits for a typical case study

Limits	v = 0.60		v = 0.90		v=0.95	
	LTL*	UTL*	LTL	UTL	LTL	UTL
1 o limits (Prob.: 0.6827)	23.5909	29.0111	22.7002	29.9018	22.2690	30.3330
2 a limits (Prob.: 0.9546)	20.9117	31.6903	19.1404	33.4616	18.2829	34.3191
3o limits (Prob.: 0.9973)	18.2530	34.3490	15.6078	36.9942	14.3274	38.2746

* LTL = Lower tolerance limit and UTL = Upper tolerance limit

wherein the number of data points is very small, such an approach will give much closer tolerance limits under ideal conditions. In fact, for such cases, one has to compute the statistical tolerance limits by the above described procedure, viz., evaluate the values of $(\bar{X} \pm K.s)$ and consider these as tolerance limits. This will yield the tolerance limits with wider variation considering the uncertainty due to limited data. It can be seen that the statistical tolerance limits, as described in the paper, approach the classical limits as the sample size tends to infinity.

5. CONCLUSION

The concept of the statistical tolerance limits and its applicability to process capability studies have been dealt. Further, as an illustration, the theory developed is applied to a case study and the results have been presented.

REFERENCES

1. Bowker, A.H. Tolerance limits for normal distributions. *In* A technique of statistical analysis. McGraw-Hill, New York, 1947. pp. 95-110.

2. Bowker, A.H. Computation of factors for tolerance limits on a normal distribution when the sample is large. *In* Annals of Mathematical Statistics, Vol-17, pp. 238-40.

3. Faulkenburry, G.D. & Daly, J.C. Sample size for tolerance limits on a normal distribution. *Technometrics,* 1970, 12(4), 813-21.

4. Faulkenburry, G.D. A note on tolerance limits for a binomial distribution. *Technometrics,* 1970, 12(4), 920-22.

5. Faulkenburry, G.D. & Weeks, D.L. Sample size determination for tolerance limits. *Technometrics,* 1968, **10,** 343-48.

6. Prochan, F. Confidence and tolerance intervals for the normal distribution. *J. Amer. Stat. Assoc.,* 1953, **48,** 550-64.

7. Weissberg, A. & Beatty, G.H. Tables of tolerance limit factors for normal distributions. *Technometrics,* 1960, 2(4), 483-500.

8. Owen, D.B. Handbook of statistical tables. Addison Wesley Publishing Co. 273p.

9. Resnikoff, G. J. Tables to facilitate the computation of percentage points of the non-central ^-distribution. *Ann. Math. Stat.,* 1963, 33, 580-83.

10. Wald, A. & Wolfowitz, J. Tolerance limits for a normal distribution. *Ann. Math. Stat.,* 1944, **15,** 214-16.

11. Wilks, S.S. Determination of sample sizes for setting tolerance limits. *Ann. Math. Stat.,* 1941, 12, 91-96.

12. Rajagopalan, S. Statistical tolerance limits for maximum pressure output of BO1 pellets for RH 125 igniter. Vikram Sarabhai Space Centre, Thiruvananthapuram. Technical Report No. VSSC/SR/QDMS/QAR/TR/O/118. July 1998.