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# 3-D Modified Proportional Navigation Guidance Law based on a Total Demand Vector Concept

# P.K. Tiwari, Prashant Vora, S. Srinivasan and R.N. Bhattacharjee Defence Research & Development Laboratory, Hyderabad-500 058

#### ABSTRACT

Different proportional navigation (PN)-based guidance laws-pure proportional navigation (PPN), true proportional navigation (TPN), and proportional navigation with boost acceleration compensation generally used cannot maintain fundamental parameter of proportional navigation, viz., Navigation constant to the desired value in the presence of significantly high lead angles and missile longitudinal accelerations/decelerations. In a real-life situation with sensor noises and hardware constraints, this navigation constant should be maintained tightly at the selected value, which is generally between 3 and 4, for optimum performance. In this paper; a new 3-D modified PN guidance law based on a total demand vector concept is presented, which can maintain the navigation constant to the designer-selected value for any 3-D engagement scenario with associated lead angles and any velocity profile with missile longitudinal accelerations/ decelerations. Generality of this guidance law is brought out and superiority of this guidance law over the commonly used proportional navigation-based laws like PPN, TPN and PN with boost acceleration compensation has been demonstrated by applying it to the real-life 3-D engagement scenarios of different hypothetical missiles.

Keywords: Proportional navigation, navigation guidance laws, pure proportional navigation, true proportional navigation, modified PN-based guidance

#### NOMENCLATURE

- θ Lead angle, angle between missile longitudinal axis and line-of-sight direction
- $V_m$  Missile velocity vector
- V, Target velocity vector
- $\alpha$  Missile angle of attack
- $f_{xd}$  Lateral acceleration demand in line-of-sight frame X-direction
- $f_{yd}$  Lateral acceleration demand in line-of-sight frame Y-direction

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- $f_{zd}$  Lateral acceleration demand in light-of-sight frame Z-direction
- $f_{xbd}$  Lateral acceleration demand in missile body frame X-direction
- $f_{ybd}$  Lateral acceleration demand in missile body frame Y-direction
- $f_{zbd}$  Lateral acceleration demand in missile body frame Z-direction
- N' Navigation constant
- *a*, Missile longitudinal acceleration

- V. Missile-target closing velocity
- $\lambda_a$  Azimuth sight-line angle
- $\lambda_{a}$  Elevation sight-line angle
- $N_{eff}$  Realised navigation constant

# **1. INTRODUCTION**

For commonly used guidance laws based on proportional navigation theory like pure proportional navigation (PPN), true proportional navigation (TPN), etc, in the presence of lead angles (defined here as angle between the missile velocity vector and line-of-sight vector) and missile longitudinal acceleration/ deceleration generally present; the realised acceleration is different from the one demanded by the proportional navigation law, in a plane perpendicular to line-ofsight vector. In other words, the effective or realised navigation constant is not the same as the desired one. For a practical system with sensor noises and hardware constraints, this navigation constant should be tightly maintained at the chosen value, which is generally between 3 and 4. However, in the presence of significant lead angles and missile longitudinal acceleration/deceleration, effective navigation constant can go up to fully unacceptable values, leading to fully unacceptable performance. It is, therefore, required to develop a guidance concept and law, which ensures that the realised navigation constant is the same as the one desired, even in the presence of real-life constraints, viz., in the presence of significant lead angles typical of 3-D interceptions and missile longitudinal acceleration/ deceleration. Such a concept is formulated and is visualised in 2-D scenario for the sake of understanding, and later extended to cater for a general 3-D interception scenario. Ultimately, it has been evolved as a 3-D guidance law in polar coordinates. The guidance law has been evaluated on a 6-DOF platform developed for a hypothetical surface-to-air missile with a 3-D interception scenario and also for an air-toair missile. The performance of the new guidance law is compared with the existing laws, viz., TPN, PPN and PN with boost acceleration compensation (BAC). It has been found that the performance of the new guidance law is appreciably better than the other guidancelaws listed above. The paper brings out the new guidance formulation and the significant performance improvement that can be achieved with the new guidance law. It is analytically shown that this new guidance law can tightly control the navigation constant to the desired value, whereas for TPN and PN with BAC; navigation constant can change considerably wrt the desired value set.

# 2. PROPORTIONAL NAVIGATION GUIDANCE

The problems associated with the existing PN guidance laws-TPN, PN with BAC, are highlighted. Also, the objective of the modified PN guidance, how these problems can be overcome in the new method, are discussed.

To orient the missile flight path towards constant bearing collision course, the PN guidance generates commands perpendicular to the line-of-sight direction and an angle  $\phi m$  is generated by the guidance depending on  $V_m/V_t$  speed ratio, target manoeuvres and errors/disturbances (Fig. 1). Guidance designer chooses an optimum navigation constant N depending on the target manoeuvres, noises in the system (ground radar, seeker, etc), errors and stability of missile guidance loop, including parasitic effects, etc. This generally vary between 3 to 4 for a reallife system. However, guidance commands can be applied only along body Y- and Z-axes (lateral plane) and those commands  $f_{vbd}$ ,  $f_{zbd}$  can be considerably reduced wrt the required PN demands generated along the LOS frame Y- and Z-directions, ie,  $f_{yd}/f_{zd}$ , depending on lead angle  $\approx \phi_m + \alpha$  (angle between the LOS vector and the missile longitudinal axis) which is generally time varying. Thus, effective navigation constant,  $N_{eff}$  varies due to the above resolution error and deviates from the desired value. This deviation can be considerable in many cases, affecting performance appreciably.

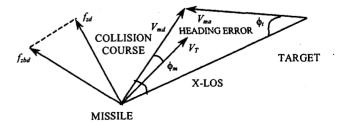


Figure 1. Collision triangle

# 2.1 Proportional Navigation with Boost Acceleration Compensation

In the process of transforming PN demands from the LOS frame to the body frame, a component of demand comes in body  $X_{h}$  direction (longitudinal axis) also, which can be much different from the missile longitudinal acceleration,  $a_x$ . Thus, effective lateral acceleration demands sent to the autopilot are either high or low in magnitude than the required acceleration and generated by the PN. In PN with BAC, the PN guidance commands have been improved through missile longitudinal acceleration compensation in the LOS frame by transforming acceleration  $a_{1}$ along the directions perpendicular to LOS, ie, Y-LOS and Z-LOS and then reducing/increasing the PN demands suitably by the transformed longitudinal acceleration commands. However, in the presence of significant lead angle, this compensation needs to be improved by correcting for the resolution error.

#### 2.2 Modified PN Guidance

To alleviate both resolution error and missile longitudinal acceleration constraints together, a unified PN guidance design approach based on a total demand vector concept is formulated, which ensures full guidance demands as per PN guidance law, and is actually implemented through body plane commands in the presence of any propulsion and drag profile. However, in this implementation, in the case of accelerating thrust profile, closing velocity  $V_c$  would considerably increase and for deceleration phase,  $V_c$  would decrease, while satisfying the full guidance requirement. With higher/lower closing velocity, flight time and system coverage would change. In general, for any missile with both accelerating and decelerating velocity profiles, average closing velocity,  $V_c$  would be higher for the new method, leading to lower flight time and higher system coverage.

# 3. NAVIGATION GUIDANCE DEMANDS FORMULATION

# **3.1 PN Guidance Demands Formulation**

The proportional navigation guidance demands lateral accelerations,  $f_{yd}$ ,  $f_{zd}$  in the LOS frame, ie,

along  $Y_{LOS}$ ,  $Z_{LOS}$ . The LOS frame  $X_{LOS}$ ,  $Y_{LOS}$ ,  $Z_{LOS}$ can be obtained from the launcher-fixed frame  $X_i$ ,  $Y_i$ ,  $Z_i$  by two successive rotations; first by an angle  $\lambda_a$  (azimuth sight-line angle) about  $X_i$  and then by an angle 90° –  $\lambda_e$  ( $\lambda_e$  = elevation sight-line angle) about newly obtained Y-axis, ie, Y-LOS (Fig. 2).

The LOS rates about Y-LOS direction,  $\lambda_e$  and same about Z-LOS direction,  $\dot{\lambda}_a \cos \lambda_e$  are used by PN guidance to generate latax demands  $f_{yd}$ ,  $f_{zd}$ in LOS frame as per:  $f_{zd} = N'V_c \dot{\lambda}_e$ ,  $f_{yd} = N'V_c \dot{\lambda}_a \cos \lambda_e$ . These are transformed to body frame with the help of either lead angles or gimbal angles, depending on measurements available to

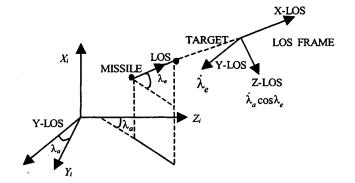


Figure 2. Launcher-fixed frame

the guidance system. For a homing missile system during mid-course, radar supplied target data and missile INS data can be used for the transformation, while in the terminal phase seeker, gimbal angles can be used for the transformation.

# 3.2 Modified PN Guidance Demands Formulation

To realise PN components  $f_{zd}$  and  $f_{yd}$  perpendicular to the LOS fully in the presence of missile longitudinal acceleration/deceleration  $a_x$ , consider a demand vector (Fig. 3) defined such that its component along body  $X_b$  axis is  $a_x$  and components perpendicular to LOS is  $f_{zd}$ . This demand vector has a component along the LOS which is  $f_{xd}$ .

From the definition, the demand vector in the LOS frame  $\begin{bmatrix} f_{xd} & f_{zd} \end{bmatrix}^r$  has a component  $a_x$  in the  $X_b$  direction. Therefore

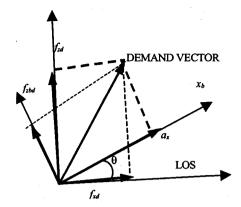


Figure 3. Modified PN formulation

$$f_{rd}\cos\theta + f_{rd}\sin\theta = a_r \tag{1}$$

or

$$f_{xd} = \frac{a_x - f_{zd}\sin\theta}{\cos\theta}$$
(2)

Again,

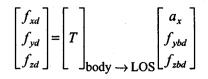
$$f_{zbd} = f_{zd} \cos\theta - f_{xd} \sin\theta \tag{3}$$

which is the command sent to the autopilot Thus, the acceleration realised in the Z-LOS direction is (assuming an ideal autopilot):

 $f_{zbd}\cos\theta + a_x\sin\theta$ 

It is easy to see by substitution from Eqns (2) and (3) that the above expression equals  $f_{zd}$ , implying that full latax demand as per the PN law is realised.

The above concept can be extended to 3-D engagement scenario by considering the demand vector in the LOS frame as  $[f_{xd} \ f_{yd} \ f_{zd}]^T$  where  $f_{yd}$  and  $f_{zd}$  are the demands calculated as per PN law. Demands along  $Y_{body}$ ,  $Z_{body}$  directions:  $f_{ybd}$ ,  $f_{zbd}$  are calculated such that those realized  $f_{ybd}$ ,  $f_{zbd}$ along with actual missile longitudinal acceleration  $a_x$ , when transformed from body frame to the LOS frame equals to the actual PN demands  $f_{zd}$ ,  $f_{yd}$ perpendicular to the LOS and also a new demand along the LOS  $f_{xd}$ . Thus,  $f_{ybd}$ ,  $f_{zbd}$  are obtained from



Therefore

$$\begin{bmatrix} a_{x} \\ f_{ybd} \\ f_{zbd} \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{bmatrix}_{\text{LOS} \to \text{body}} \begin{bmatrix} f_{xd} \\ f_{yd} \\ f_{zd} \end{bmatrix}$$
(4)

From the above Eqn (4),

$$f_{xd} = \frac{a_x - T_{12}f_{yd} - T_{13}f_{zd}}{T_{11}}$$

is obtained. This demand along LOS  $f_{xd}$  along the with normal PN demands,  $f_{yd}$ ,  $f_{zd}$  constitute the complete demand vector in the LOS frame, which need to be transformed to body frame to obtain the required  $f_{ybd}$ ,  $f_{zbd}$  satisfying the forward acceleration  $a_x$  constraint. Therefore, putting the above  $f_{xd}$  along with normal LOS frame, PN demands  $f_{yd}$ ,  $f_{zd}$  in Eqn (4); body plane demands  $f_{ybd}$  and  $f_{zbd}$  are obtained as

$$\begin{bmatrix} f_{ybd} \\ f_{zbd} \end{bmatrix} = \begin{bmatrix} T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{34} \end{bmatrix} \begin{bmatrix} f_{xd} \\ f_{yd} \\ f_{zd} \end{bmatrix}$$
(5)

The transformations used for conversion between the LOS frame (seeker inner gimbal frame while homing) and the body frame can be obtained using the seeker gimbal angles in the homing phase. In the mid-course phase, above transformation matrix can be obtained as a product of rotation matrix from the LOS frame to launcher-fixed frame using the LOS angles and launcher-fixed body frame using INS Euler angles.

Those body plane demands,  $f_{ybd}$ ,  $f_{zbd}$  applied to autopilot would ensure full PN demands  $f_{yd}$ ,  $f_{zd}$ realised in the LOS frame, ie, perpendicular to X-LOS in the presence of any  $a_x$  profile. Thus, desired navigation constant N' would be always maintained during the entire flight. TIWARI, et al.: 3-D MODIFIED PROPORTIONAL NAVIGATION GUIDANCE LAW BASED ON A TOTAL DEMAND VECTOR CONCEPT

#### 4. REALISED NAVIGATION CONSTANT

It is shown above that for the modified PN, the guidance demand is met fully. The navigation constant actually realised in other guidance schemes can be obtained by taking the component of total achieved acceleration in body frame into the plane perpendicular to the LOS vector, and thereafter, taking the ratio of that component with the product of sight-line rate vector magnitude and closing velocity  $V_c$ . In other words, the components  $f'_{zd}$ ,  $f'_{yd}$ in the plane perpendicular to LOS, of the realised acceleration vector  $[a_x, f_{ybd}, f_{zbd}]^T$  in the body plane are used for effective or realised navigation constant  $N_{eff}$  calculation. The corresponding LOS rates are  $\dot{\lambda}_a \cos \lambda_e$  and  $\dot{\lambda}_e$  (Fig. 2). Thus, total or resultant realised navigation constant,  $N_{eff}$  is obtained as

$$N_{eff} = \frac{\sqrt{f_{zd}^{\prime 2} + f_{yd}^{\prime 2}}}{V_C \sqrt{\lambda_e^2 + \lambda_a^2 \cos^2 \lambda_e}}$$

It may be noted that the numerator term is the total acceleration realised in the plane perpendicular to the LOS and  $V_c$  is the closing velocity. To show how the above realised N' can go to a much different value from the desired value set in TPN and also in TPN with BAC, the above relation is expanded.

In Fig. 4, a resultant latax demand vector  $f_{zr}$  perpendicular to the LOS is defined based on latax demands along Z-LOS  $f_{zd}$  and Y-LOS  $f_{yd}$ .  $F_{zrb}$  is the total or resultant latax demand achieved in the body frame lateral plane.  $\theta_2$  is the resolution error, between the PN demand vector in the LOS frame and acceleration achieved in the body lateral plane (which has been corrected in the modified guidance).

# 4.1 Realised Navigation Constant for PN with BAC

The acceleration demanded in the LOS lateral plane after missile longitudinal acceleration compensation =  $f_{zr} - a_x \cos \theta_1$ , assuming ideal autopilot-realised acceleration perpendicular to the LOS

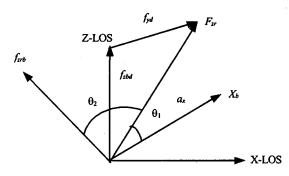


Figure 4. Realised navigation constant  $(N_{rr})$  calculation

$$f_{zrLOS} = (f_{zr} - a_x \cos \theta_1) \cos^2 \theta_2 + a_x \cos \theta_1$$
  
$$f_{zrLOS} \text{ (PN with BAC)}$$
  
$$= f_{zr} \cos^2 \theta_2 + a_x \cos \theta_1 \sin^2 \theta_2$$
  
$$= f_{zr} \left[ \cos^2 \theta_2 + \frac{a_x}{f_{zr}} \times \cos \theta_1 \sin^2 \theta_2 \right]$$

Thus, realised navigation constant,  $N_{eff}$  (PN with BAC) can be given as

$$\frac{f_{zrLOS}}{V_c \sqrt{\dot{\lambda}_e^2 + \dot{\lambda}_a^2 \cos^2 \lambda_e}}$$
$$= N' \left[ \cos^2 \theta_2 + \frac{a_x}{f_{zr}} \times \cos \theta_1 \sin^2 \theta_2 \right].$$

For planer case engagement;  $\theta_1 = 90^\circ - \theta_2$ , so the above expression reduces to

$$N_{eff}$$
 (PN with BAC) =  $N' \left[ \cos^2 \theta_2 + \frac{a_x}{f_{zr}} \times \sin^3 \theta_2 \right]$ 

#### 4.2 Realised Navigation Constant for TPN

For TPN, realised acceleration in a plane perpendicular to the LOS (assuming ideal autopilot)

$$f'_{zrLOS} = f_{zr}\cos^2\theta_2 + a_x\cos\theta_1$$

Thus

$$N_{eff}$$
 (TPN) =  $N' \left[ \cos^2 \theta_2 + \frac{a_x}{f_{zr}} \cos \theta_1 \right]$ 

Again for the planer case engagement,

 $\theta_1 = 90^\circ - \theta_2$ , thus the realised navigation constant for planar engagement is

$$N_{eff} = N' \left[ \cos^2 \theta_2 + \frac{a_x}{f_{zr}} \times \sin \theta_2 \right]$$

where N' is the desired navigation constant set by the designer. It can be shown based on the above relations that effective navigation constant can go to a much different value from the desired value chosen N' even for resolution error of 20° and above.  $N_{eff}$  variation from desired N' depends on both the resolution error,  $\theta_2$  and the forward acceleration to resultant latax demand,  $f_{zr}$  ratio along with their signs. The effect of resolution error is more on the realised navigation constant,  $N_{eff}$  in PN with BAC whereas  $N_{eff}$  in TPN can get affected equally appreciably, both due to resolution error and  $a_x/f_{zr}$  ratio. For example, with  $\theta_2 = 30^\circ$  and  $a_x = f_{rr}$ ; for planar case

$$N_{eff} (\text{PN With BAC}) = N' \left[ \cos^2 \theta_2 + \frac{a_x}{f_{zr}} \times \sin^3 \theta \right]$$
$$= N' \left[ 0.75 + \frac{1}{8} \right] = 0.875N'$$

and

$$N_{eff}$$
 (TPN) =  $N \left[ \cos^2 \theta_2 + \frac{a_x}{f_{zr}} \times \sin \theta_2 \right]$ 

With the same  $\theta_2 = 30^\circ$  and  $a_x = -f_{zr}$  for planar case

 $N_{eff}$  (PN with BAC) = 0.625 N', which is low, and

$$N_{eff}$$
 (TPN) =  $N' [0.75 - 0.5] = 0.25 N'$ 

which is unacceptably low, even for original N' = 4 choice of designer. Thus, simulation results presented in Section 5.1 for varying engagement scenarios, lead angles, and with accelerating/decelerating profile/phase of missile show wide variation in effective navigation ratios from the start of guidance to the end for both TPN and PN with BAC, whereas for modified scheme, it is held constant at the desired value.

#### 5. PERFORMANCE OF GUIDANCE LAWS THROUGH 6-DOFS SIMULATION

The performance improvement achievable with the new guidance law,  $vis-\dot{a}-vis$  other PN laws can be best shown in the cases involving high lead angles and high missile longitudinal acceleration/ deceleration levels.

For performance evaluation, modified PN and other PN-based guidance laws are applied to two different classes of missile systems. The TPN, PN with BAC, and modified PN were applied to a hypothetical surface-to-air missile with a boostcoast velocity profile, here  $V_m/V_t < 1$ . The TPN, PPN, and the modified PN were also applied to an air-to-air missile which was also having a boostcoast velocity profile but with a velocity advantage, ie  $V_{\perp}/V_{\perp} > 1$ . For a surface-to-air missile, different 3-D engagement scenarios were simulated to generate varying lead angle requirements. Tables 1 and 2 give comparison of performance of guidance laws for a surface-to-air missile. Results were generated by carrying out simulations on a 6-DOF platform for the hypothetical missile. Before sending the commands to autopilot, a time-varying latax limit was applied based on the minimum of latax limit coming due to  $(\alpha, \delta)$  deflection limit.

In the mid-course guidance phase, the radar errors on target position and velocity were introduced in a way to give certain heading error at the start of homing phase. Here, the heading error is defined as an angle between the collision course velocity vector and the actual missile velocity vector (Fig. 1). In Table 1, the heading error settling characteristics of different guidance laws and the miss-distance are compared. In Table 2 for a different engagement scenario, the time for which the latax was saturated

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was compared, and was found to have a direct implication on the miss-distance. The variation of effective navigation ratio is also tabulated. In Tables 3 and 4, the miss-distance comparison is done for MPN and PPN and MPN and TPN, respectively for different missile-target heading angles. By analysing Tables 1 and 2 and the Figs 5 to 9, the following general conclusions can be drawn.

• It is seen that maximum heading error at the end-phase is always less in modified PN compared to other guidance laws (Tables 1 and 2).

Table 1. Comparison of performance of TPN, PN with BAC, and modified PN guidance for a surface-to-air missile, N' = 4

Case	Guidance type	Height of kill (km)	$T_F$	Miss- distance	Heading error (end-phase)	
		KIII (KIII)	(s)	(m)	Elevation	Azimuth
I	TPN	10.0	35.42	5.44	0.03	0.33
	PN + BAC	10.0	32.05	5.91	0.09	1.00
	Mod. PN	10.0	32.22	0.87	0.09	0.25
II	TPN	12.5	37.81	9.87	0.05	0.50
	PN + BAC	12.5	34.85	14.09	0	0.88
	Mod. PN	12.5	35.04	3.02	0.01	0.18
III	TPN	15.5	40.21	23.70	0.59	0.55
	PN + BAC	15.5	39.00	17.25	0.08	0.62
	Mod. PN	15.5	39.01	11.62	0.32	0.17

Table 2. Comparison of MAPN and TPN performance for a surface-to-air missile, N' = 3

Case	Guidance type	Height of kill (km)	Time of flight (s)	Miss- distance (m)	Time of latax saturation (end-phase)	N' variation
1	MAPN	14.61	33.00	10.73	0.25	3.0
1	TPN	14.50	32.80	45.10	1.80	25.0 - 0.5
	MAPN	13.00	31.31	3.59	0.20	3.0
II	TPN	12.80	31.00	19.34	0.40	30.0 - 0.4
	MAPN	10.00	28.10	3.48	0.10	3.0
III	TPN	9.80	27.80	9.78	0.40	27.0 - 0.7

Table 3. Comparison of performance of PPN and MPN for an air-to-air missile,  $V_m = 520$  m/s,  $V_r = 520$  m/s, h = 15 km, target manoeuvre  $n_{sich} = -4$  g (const) at 10 km range to go

Guidance Heading angle (d)	Modified PN					Pure PN			
	Time of flight (s)	Miss- distance (m)	n <sub>max</sub> (g)	V <sub>m</sub> /V <sub>t</sub> impact	Range (km)	Time of flight (s)	Miss- distance (m)	n <sub>max</sub> (g)	V <sub>m</sub> /V <sub>t</sub> impact
180	34.50	14	12	1.57	50	34.50	22	12	1.58
150	22.05	16	9	1.93	30	22.0	70	9	1.90
120	25.10	9	12	1.72	25	25.05	62	12	1.78
90	25.06	22	15	1.71	18	24.98	120	15	1.80
70	28.84	30	14	1.72	18	28.26	110	14	1.80
50	32.25	10	. 12	1.73	18	31.95	93	12	1.77
30	34.11	8	10	1.81	18	33.91	47	10	1.83
0	34.64	7	8	1.90	18	34.38	71	8	1.92

Guidance			MPN			TPN			
Heading angle (d)	Time of flight (s)	Miss- distance (m)	n <sub>max</sub> (g)	V <sub>m</sub> /V <sub>t</sub> Impact	Range (km)	Time of flight (s)	Miss- distance (m)	n <sub>max</sub> (g)	V <sub>m</sub> /V <sub>t</sub> Impact
180	34.43	16	12	1.57	50	34.49	21	12	1.58
150	38.87	5	18	1.90	30	38.92	12	18	1.92
120	24.13	21	18	1.73	25	24.53	111	18	1.88
90	23.52	36	18	1.98	18	23.46	132	18	1.98
70	26.78	10	13	1.79	18	26.79	32	13	1.88
50	32.31	13	13	1.72	18	32.25	51	13	1.75
30	34.12	8	18	1.81	18	34.05	61	18	1.83
0	34.64	7	16	1.90	18	34.38	71	16	1.92

Table 4. Comparison of performance of TPN and MPN for an air-to-air missile,  $V_m = 520$  m/s,  $V_T = 520$  m/s, h = 15 km, target manoeuvre  $n_{mich} = -4$  g (const) at 10 km range to go

- The heading error always settles to a low value at the end-phase for MPN (Figs 7 and 8).
- The phenomenon can be attributed to meeting the guidance demand completely.
- With low heading error at the end-phase, missile flight path is driven close to ideal collision course in MPN, which is reflected in low missdistance achieved in all cases of simulations (Tables 1 and 2).

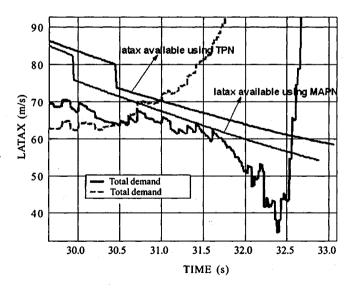


Figure 5. Duration of saturation comparison, (Table 2, Case 1)

- With appreciably higher heading error, the missdistance in other guidance laws is higher.
- It is also seen from Table 2 and Fig. 9 that the effective navigation constant in MPN is always maintained at the desired value set, whereas it can vary widely from the desired value in other guidance laws like TPN, etc. This is highly undesirable for any practical system where navigation constant is to be maintained generally between 3 and 4.

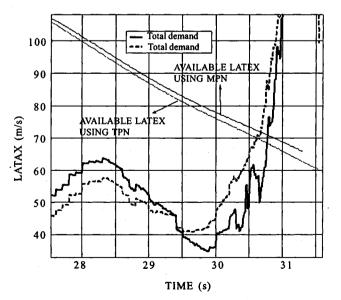


Figure 6. Duration of saturation comparison, (Table 2, Case 2)

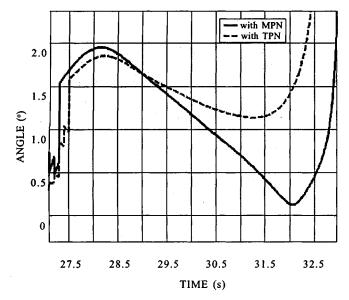


Figure 7. Heading error settling comparison, (Table 2, Case 1)

• For air-to-air missile, several engagement simulations carried out with different heading angles (angle between the missile and the target velocity vector) varying from 180° to 0° and performance comparison between MPN, TPN, and PPN carried out in terms of miss-distance flight time, and  $V_m/V_i$  ratio (Tables 3 and 4). again shows that miss-distance is greatly reduced for MPN wrt both TPN and PPN, where missdistance goes to unacceptably high values in many cases.

Summarising, it can be concluded that the new guidance law called modified PN, by maintaining the effective navigation constant to the desired value, gives better quality of guidance performance compared to the existing PN-based guidance laws in a general 3-D engagement scenario with both accelerating and decelerating velocity profiles of the missile.

#### 6. CONCLUSION

A new 3-D modified PN guidance law based on a total demand vector concept, has been presented. It ensures full guidance demands as per PN law implemented through body plane commands in the presence of any propulsion and drag profile, thereby maintaining the navigation constant to the desired value set. Derivation has been carried out to show

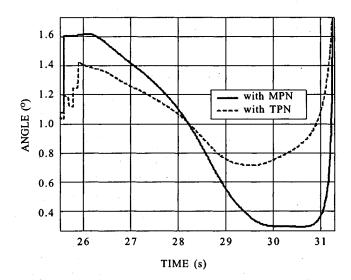


Figure 8. Heading error settling comparison, (Table 2, Case 2)

that this new law can tightly control the navigation constant to the desired optimum value, whereas for the other PN-based laws commonly used, navigation constant can change appreciably wrt the desired value set. Superiority of the new modified PN guidance wrt other PN-based laws has been demonstrated by applying these to widely varying 3-D engagement scenarios for different types of missile systems with boost-coast velocity profiles. In all the cases simulated, appreciable performance benefits have been obtained. Thus, this modified

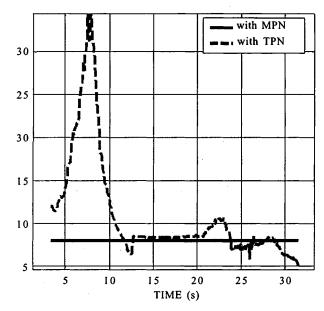


Figure 9. Realised navigation constant  $N_{eff}$ , (Table 2, Case 3)

PN guidance law is established as a general PN guidance law.

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#### Contributors



**Mr P.K. Tiwari** obtained his MTech (Control & Guidance) from the Indian Institute Technology Bombay, Mumbai in 2000. Since then, he is working at the Defence Research & Development Laboratory (DRDL), Hyderabad. His areas of interest include: System simulation, missile guidance, and nonlinear control.



**Mr Prashant Vora** obtained his MTech (Systems & Control) from the IIT Bombay, Mumbai in 2001. Since then, he is working at the DRDL, Hyderabad. His areas of interest include: System simulation, missile guidance, and nonlinear control.