Defence Science Journal, Vol. 55, No. 3, July 2005, pp. 253-264

Guidance Scheme for Solid Propelled Vehicle during Atmospheric Phase

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ABSTRACT

Strategic ballistic missiles are the long-range missiles designed to intercept stationary, defined targets. Guidance is in the boost phase which represents only a small fraction of the total time of flight of the missile. A missile has to be guided and steered to the required position and velocity states at the end of the boost phase. The problem of guiding a vehicle in atmospheric phase in the presence of drag and lift, poses a challenge for the designer. This paper describes an energy management guidance scheme for guiding the vehicle in atmospheric phase of the flight. The estimated drag is taken care of by the guidance scheme. Uncertainty in velocity due to drag or impulse variation at the end of the boost phase are corrected by an augmented velocity package. A closed-loop guidance scheme has been used during this phase also. A ballistic re-entry has been considered for this design study.

Keywords: Long-range missiles, missile guidance schemes, launch vehicle, ballistic missiles, energy management guidance scheme, atmospheric phase

1. INTRODUCTION

A large number of guidance schemes have been proposed for the launch vehicles and ballistic missiles. These are explicit, implicit and perturbation guidance schemes. The implicit guidance schemes ride a predefined nominal trajectory. However, these are not advocated for long-range ballistic flights where a tight control on trajectory parameter is difficult.

The perturbation guidance schemes are the most widely used schemes. Two main variations are δ-guidance, which defines the trajectory about a prefixed injection point, and the Q-guidance, which defines a set of influence coefficient matrix to update the trajectory about a nominal. The Q-guidance schemes have been used in the launch vehicles as well as in the ballistic missiles. These schemes simplify the onboard computational loads by trading these with an extensive simulation load on ground. These schemes have relatively lower tolerances to parameter variations compared to explicit guidance schemes but were preferred because of ease of implementation. The explicit guidance schemes solve the two point boundary value problems in flight to compute the velocity vector required to reach the target. These schemes compute the required velocity and a velocity to be gained (V_g) onboard and generate a steering command to drive the V_g to zero. These schemes offer maximum freedom to the designer with an inherent robustness at the cost of high computation loads. With the improvements in computing capability, it is possible to use this scheme without any computational load problems.

Received 15 November 2004

In cases where thrust termination is not possible, all guidance schemes lead to certain errors caused by off nominal propulsion behaviour. This has to be corrected by a velocity trimming package. The size of the velocity trimming package is governed by the guidance uncertainties and is a critical parameter in the mission design.

This guidance scheme offers the following advantages:

(a) It minimises the velocity uncertainties, thereby minimising the velocity trimming package requirements.

(b) No thrust termination, thereby the operational complexities of thrust termination requirements are eliminated.

2. BALLISTIC MISSILE FLIGHT PROFILE

A ballistic missile has three phases of flight, ie, powered phase, free-flight phase and the re-entry phase. Guidance can be done only during the powered phase. The powered phase is again divided into two parts, the pre-programmed phase and the guided phase. This paper concentrates on the powered phase of the flight.

2.1 Pre-programmed Flight Phase

The vehicle follows a pre-programmed pitch profile from the lift-off till start of the closed-loop guidance (Fig. 1) Pitch profile is designed with initial vertical rise, followed by pre-determined pitch rate to meet the range and to keep the trajectory α close to zero during high dynamic pressure region.

2.2 Closed-loop Guidance Phase

The closed-loop explicit guidance (energy management scheme) starts when the vehicle dynamic pressure is built up to provide adequate controllability. This scheme continues almost up to the end of the boost phase (Fig. 1).

2.3 Free-flight Phase & Reentry Phase

At the end of the guidance phase, vehicle follows a ballistic trajectory with controlled attitude up to the impact.

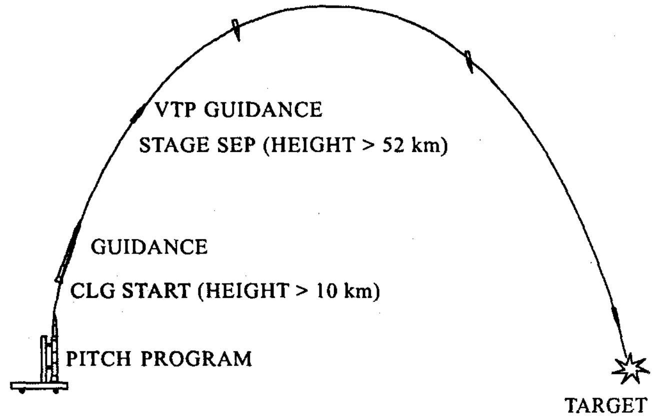


Figure 1. Pre-programmed flight phase

3. REFERENCE COORDINATE FRAMES

An explicit guidance follows Keplerian trajectory requiring knowledge of position and velocity in inertial frame. Initially, a local vertical frame is used to generate data for control during pre-programmed attitude phase. Body frame is used for rate of turn and acceleration measurement along the body axis. Earth-centred inertial frame is used for computation of the required velocity vector based on the inertial position. Instantaneous trajectory frame is used for computation of the steering commands.

These coordinate frames are body frame, earth-centred inertial frame, instantaneous trajectory frame, and the local frame.

3.1 Body Frame

This frame is defined by its X-axis along the roll axis of the vehicle, Y-axis along the pitch, and Z-axis forming a right-handed triad as in Fig. 2.

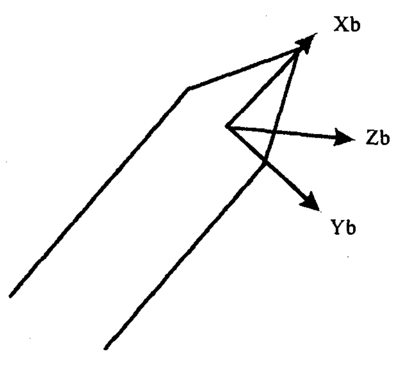


Figure 2. Body frame

3.2 Earth-centred Inertial Frame (ECI)

This frame has its origin at the centre of the earth with X-axis lying along zero meridian at t_0, Z-axis along spin axis of the earth, and Y-axis forming a right-handed triad as in Fig. 3.

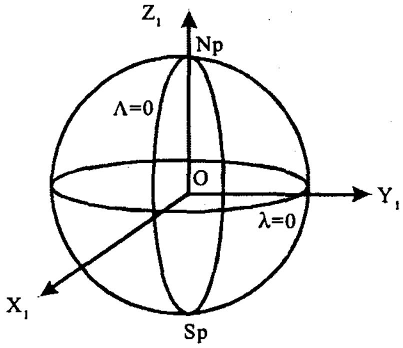


Figure 3. Earth-centred inertial frame

3.3 Instantaneous Trajectory Frame

This frame has its Z-axis along the radius vector of the earth, X-axis normal to the instantaneous trajectory plane, and Y-axis forming a right-handed triad as in Fig. 4.

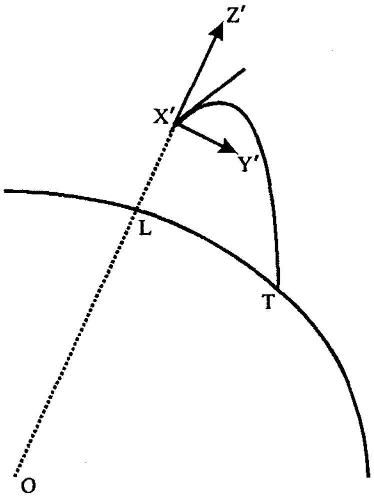


Figure 4. Instantaneous trajectory frame

3.4 Local Frame

This frame is a north-slaved local vertical system with X-axis along north, Y-axis along west, and Z-axis along the local vertical as in Fig. 5.

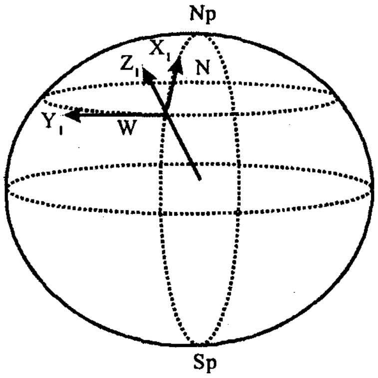


Figure 5. Local frame

4. ENERGY MANAGEMENT GUIDANCE

The guidance design has to consider the requirement of a variable range options for the mission with variable thrust-time profile of the solid propulsion performance bounds. The guidance scheme design has to meet the different launch point and target point azimuth variations. Well-known problems like rotating oblate earth and gravity anomalies also exist. Energy management guidance scheme designed is a mix of a prior information of data along with in-flight assessment of propulsion system performance.

4.1 Problems of Fully Burning Boost Vehicle Guidance

The guidance scheme developed was intended for a solid-stage propulsion without thrust termination, where there is no control on the energy imparted. For a single-stage vehicle having the larger part of the powered phase in the atmosphere, there is only a short duration for guidance corrections. There is no closed-form guidance solution for this class of vehicles. The guidance scheme has to take care of drag uncertainty, ensure structural integrity of the vehicle, and consider the lift forces. An integrated approach to control and guidance is required so that full controllability is possible during this guided phase which is also the high dynamic pressure region. Special design features are included to counter the coupling due to aero-latax loads and angle of attack limited to ensure controllability. The guidance scheme proposed has the unique features to take care of these problems.

5. GUIDANCE FLOW DIAGRAM

For vehicles without thrust termination, entire energy is imparted. This energy has to be managed to reach the target. Maximum energy capability of the vehicle, including drag and performance of propulsion system estimated onboard, is used for predicting burnout state vector. Present state vector module consists of the current velocity, current position, and corresponding direction cosine matrix and quaternions transformations obtained from the navigation system. Burnout state vector module consists of estimation of burnout velocity and burnout position. Range angle wrt to burnout point is computed with the estimated burn out state vectors.

Knowing the range angle, the downrange hit equation is solved by Newton-Raphson iterative technique, to find the flight-path angle. The flight-path angle module is iterated till the solution to the guidance equations coverages to a correct value. This module finds the γ required at burnout. The time of flight is estimated and the target position vector is updated for this time of flight. The inner loop consisting of range angle, flight-path angle, time of flight, and target update are executed thrice to get a convergence. The crossrange hit equation is solved for finding the out of plane velocity and the required velocity vector (V_R) is computed.

The difference of V_R and current velocity gives the velocity to be gained (V_g) vector. Steering law generates the steering command to drive V_g to zero. With the steering rotation vector defined by the quaternion $q\varepsilon$, the guidance loop is executed in an iterative manner till a convergent solution is obtained. A guidance flow diagram indicating the process is given in Fig. 6.

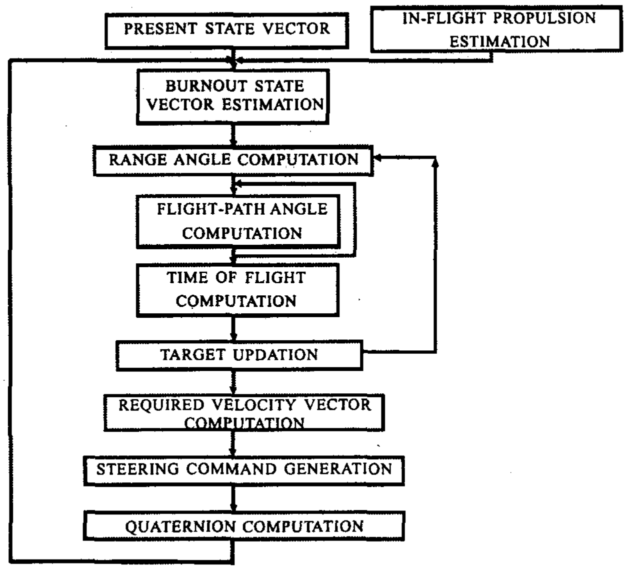


Figure 6. Guidance flow diagram

5.1 Burnout State Vector Estimation

An onboard estimation of propulsion performance is obtained using the onboard sensed instantaneous velocity (V_s) and generating a correction for the acceleration time profile based on a scaling factor

$$k = f\,(V_s / V_n)$$

where V_n is the nominal velocity at the current instant. This enables the estimation of stage performance onboard for any thrust profile thereby limiting the errors to the uncertainties of stage weight and specific impulse errors alone.

Inertial velocity at burnout without g loss is expressed in instantaneous required frame as,

$$\begin{pmatrix} U_{BO} \\ V_{BO} \\ W_{BO} \end{pmatrix} = \begin{pmatrix} U_{NEW} + V_S \\ V_{NEW} \\ W_{NEW} \end{pmatrix}$$

where

$\begin{pmatrix} U_{NEW} \\ V_{NEW} \\ W_{NEW} \end{pmatrix}$ Current velocity vector in the instantaneous required frame

V_s Stored velocity impulse-current velocity impulse at any instant.

5.2 Drag & Aerodynamic Lift Forces

The atmospheric effects are considered through the stored impulse which contains total impulse due to propulsion and loss due to drag. In-flight performance of the vehicle is determined by the actual measurement of velocity impulse achieved at any time instant onboard. This also accounts the contribution of lift forces to the actual vehicle velocity. Iterative process for estimation of burnout state vectors, ensures that there is no accumulation of errors.

Similarly, change in position at burnout is given as

$$\begin{pmatrix} \Delta R_X \\ \Delta R_Y \\ \Delta R_{ZO} \end{pmatrix} = \begin{pmatrix} U_{NEW} * t_{go} + P_S \\ V_{NEW} * t_{go} \\ W_{NEW} * t_{go} \end{pmatrix}$$

$$\Delta R_Z = W_{NEW} * t_{go}$$

where t_{go} is the time-to-go and is the difference in stage-burn time and the elapsed time. P_s is computed as integral of residual position to be added at any time instant.

5.3 Gravity Contribution

The change in gravity vector magnitude and direction along the flight path causes changes in velocity vector at burnout and needs to be computed. However, the gravity vector itself is a function of position, causing difficulties in solving. The gravity effects are, therefore, modelled around the present position. $\delta\phi$ is the instantaneous angle of present position $R(t)$ to burnout $R_{B.O}$. position. Vertical and forward components of gravity are computed using this angle and is shown in Fig. 7.

Since $\delta\phi$ variation is primarily due to forward velocity, it is not a strong function of gravity itself

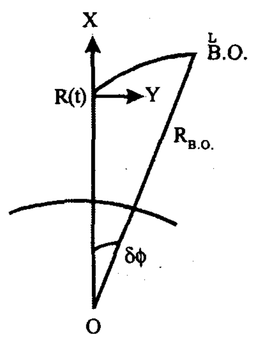


Figure 7. Computation of vertical and forward components of gravity using δϕ.

and a mid-value of $\delta\phi$ provides a reasonable approximation for small value of $\delta\phi$.

6. GUIDANCE FORMULATION & EQUATIONS

Equations relating trajectory parameters for a spherical, rotating earth are taken from the hit equation solved by Wheelon[1] as

$$r_0/a = 1-\cos\phi/\lambda \sin^2\gamma + \sin(\gamma-\phi)/\sin\gamma$$

where

$r_0 = (a + h)$

h = Height of the missile at burnout position

a = Radius of the earth

$\lambda = r_0 V^2 / GM$

ϕ = Range angle

γ = Flight-path angle at burnout

The burnout velocity V_R, required to impact at a range ϕ, for a fixed burnout angle γ, and height can be computed by solving the hit equations as

$$V_R^2 = GM/r_o [(1-\cos\phi) / (r_0/a) \sin^2\gamma + \sin(\phi-\gamma) \sin\gamma)]$$

For the solutions of guidance equations, a rotating oblate earth model has been considered. Gravitational potential for an oblate earth is given by

$$U(r,\sigma) = -GM/R_0 [(R_0/r) + (JR_0^3/r^3)(1/3 - \cos^2\sigma) + (D/35)(R_0^5/r^5) + (35\cos^4\sigma - 30\cos^2\sigma + 3) + \ldots]$$

and where σ is the co-latitude of the vehicle and J and D are the oblate constants. Detailed equations for oblate earth model are listed by Nautial[2]. A geometrical representation of the guidance problem is shown in Fig. 8.

Having obtained position and velocity corresponding to the burnout position and velocity, a solution for γ required is obtained using the rotating oblate earth model.

The γ solution is obtained by solving the following equations by N-R method[3], where $p = \sin\gamma$.

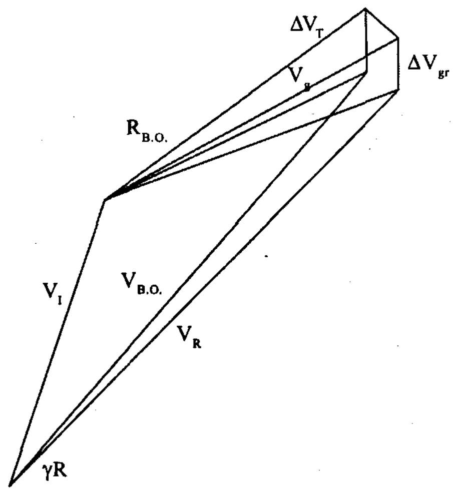


Figure 8. Geometrical representation of guidance problem

$$f(p) = [\cos\phi / Pmag] - \left[\sqrt{(1-p^2)}\sin\phi / p\,Pmag\right] + \left[X(\phi)/p^2\right] + \left[Y(\phi)/p^4\right] - \left[1/R_T\right]$$

$$f'(p) = \left[\sin\phi / p^2\; Pmag\sqrt{(1-p^2)}\right] - \left[2X(\phi)/p^3\right] - \left[4Y(\phi)/p^5\right]$$

$$p_{new} = p_{old} - f(p) \,/\, f'(p)$$

An iterative solution for γ required is obtained. The equation shows an excellent convergence.

7. GUIDANCE & STEERING LOGIC

7.1 Steering Logic

The required γ computation gives the desired velocity vector. Achieving this γ requires steering of the vehicle which in-turn changes the burnout state estimation. An iterative steering loop is proposed to find the desired solution for steering.

A V_g alignment steering law is used for iteration. The velocity to be gained (V_g) is given by

$$V_g = V_{R\,b.o.} - V_{Ib} - \Delta V_{gIb}$$

The velocity to be gained vector (V_g) is controlled by both $V_{R\,b.o.}$ and V_{Ib}, where $V_{R\,b.o.}$ is the required velocity, which is updated and iterated within every guidance cycle, V_{Ib} is the current velocity and is updated at every navigation cycle. ΔV_{gIb} is the velocity contribution due to gravity.

The theoretical steering angle is given by the angle θst and is computed from the quaternion $q\varepsilon$ based on the estimated parameters and the velocity to be gained vectors. From the actual steering angle achieved, a new $V_{R\,b.o.}$ required is estimated and a new steering rotation vector is defined as a quaternion obtained from the cross product of V_g with the thrust axis aT and is given by, $V_g \times aT$. A geometric representation of the steering process is given below in Fig. 9.

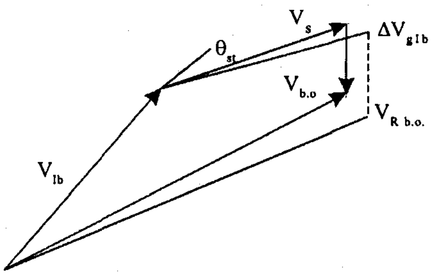


Figure 9. Geometric representation of steering process

A fresh estimate of burnout state vector is obtained for the desired thrust axis, which is used for an iterative computation of γ required and the steering rotation vector. The rotation is assumed to have occurred instantaneously. Once the steering convergence condition is achieved, the steering commands are given for execution by the control system.

7.2 Limiting of Guidance Demand

In case of sudden disturbance at closed loop, guidance starts steering command computed above can cause heavy loads on the vehicle. This has to be limited and angle of attack (α) is monitored on-board and α clamping is done to take care of aero-latax loading. Figure 10 shows the guidance loop and steering loop for control execution.

8. VELOCITY TRIMMING PACKAGE GUIDANCE

Based on the stage motor performance, the shortage in impulse or the velocity to be gained

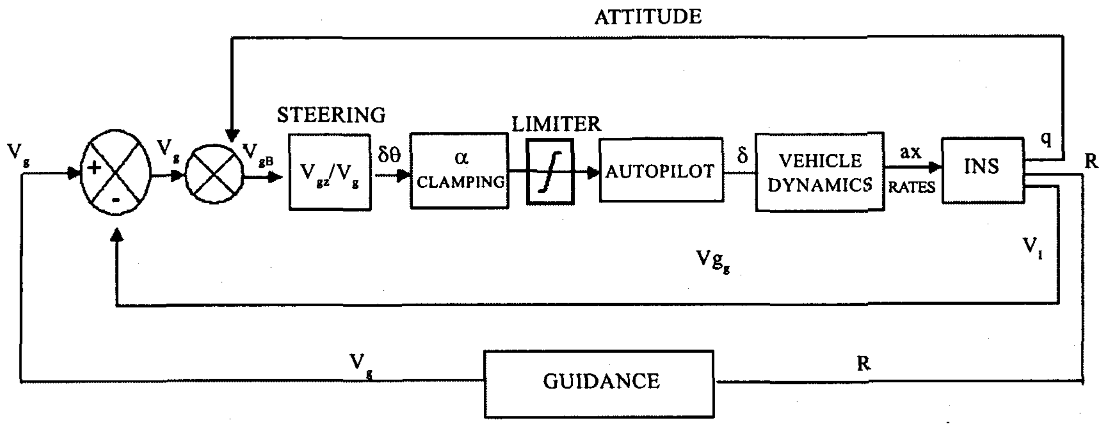


Figure 10. Guidance loop and steering loop for control execution

(V_g) is computed by the guidance system. The residual impulse after stage burnout is indicative of the difference in stored value of impulse vis-à-vis actual value of the impulse obtained in-flight. This V_g is corrected by the payload velocity trimming package.

The velocity trimming package thrusters are located in payload and this guidance is executed after S1 separation.

8.1 Velocity Trimming Package Guidance Scheme

There are two types of guidance schemes: (i) open-loop guidance and (ii) closed-loop guidance. In the open loop guidance, the velocity trimming package is fired for a pre-computed duration, based on the time-to-go. Disadvantage of this scheme, is that due to thrust variation and variation required in the velocity, the V_g will not be corrected fully. Therefore a closed-loop guidance scheme is proposed for the velocity trimming package region.

With the stage-estimated burnout conditions ($\vec{R}_{B.O}, \vec{V}_{B.O}$), the semi-major axis of the free-flight ellipse is computed. The free-flight ellipse with this semi-major axis is then projected to the actual burnout position and the vehicle is constrained to follow this ellipse from the actual burnout position as in Fig. 11. The instantaneous velocity required to follow this ellipse is computed using the equations below and the effects of oblateness are computed and included in the guidance equations.

$$V_{\mathrm{Req}} = \sqrt{\mu[(2/R_I) - (1/a)]}$$

where

$\mu = GM = 3.9861 \text{ E } 14 \text{ m}^3/\text{s}^2$

The semi-major axis a at stage

$V_{B.O.} = \mu * R_{B.O.} / (2 * \mu - R_{B.O.} * V_{B.O.}^2)$

and R_I is the present position magnitude.

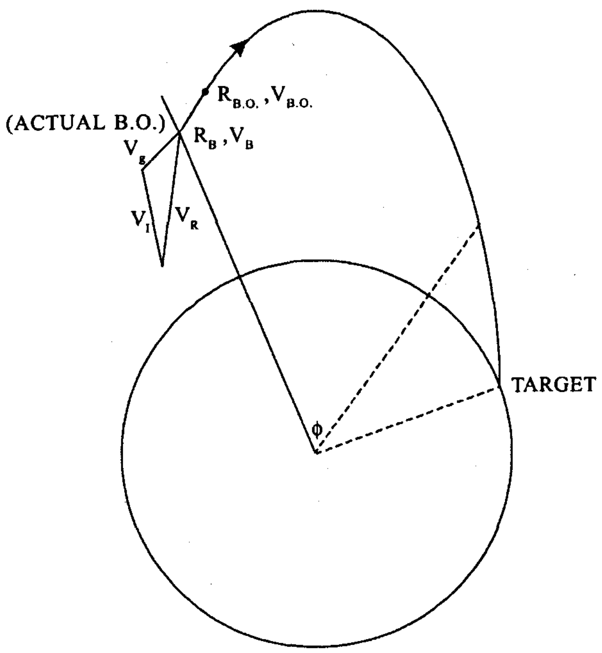


Figure 11. Free-flight ellipse with semi-major axis

After calculating the required velocity magnitude from the guidance equations, the range angle, flight-path angle, and the time of flight, are computed for the oblate earth. Then, the target is updated for earth rotation and the required velocity vector is computed. Based on the velocity to be gained, steering commands are generated and executed, till thrust termination conditions are met.

9. SIMULATION STUDIES

The flight trajectories of a missile system can be simulated by solving the 6-DOF equations of motion, three translational equations, and three rotational equations. A multi-step integration method was used to solve the equations of motion.

Multi-step update frames were used to represent the missile dynamics keeping in view the real-time implementation. The following update frames were used. Minor cycle update time of 6 ms for interrupt service and kinematics routines. Attitude and control gets updated in every 3rd minor cycle. Similarly, once in every 12th minor cycle, velocity, position, steering, aerodynamic parameters, system parameters, atmosphere, thrust model, moment of inertia and centre of gravity model were updated. Since, the guidance update cycle is not very critical, is done once in every 720 ms.

9.1 System Modelling

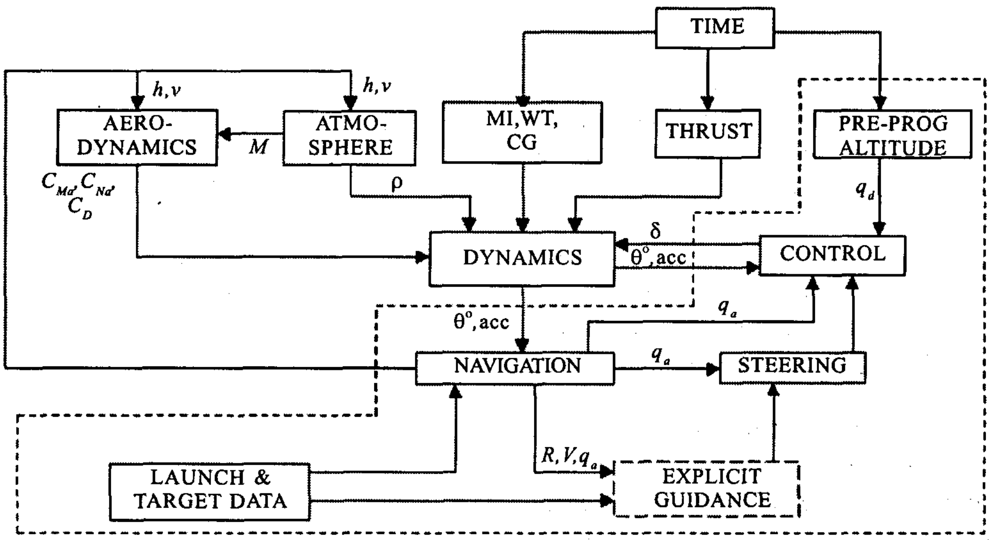
The flight vehicle simulation study has been carried out to predict the possible vehicle behaviour, to validate the design philosophy, and to work out the control and propulsion margins. In computer simulation, the first step is the system modelling by mathematical models. The systems are divided into subsystems and the mathematical models of these subsystems are modelled in the following manner.

Vehicle Configuration: Stage 1 + Payload

Dynamics: Rotational + Translational

Control: SITVC, HFTC, RCS

Guidance: Pre-programmed attitude, ascent explicit, steering, VTP guidance and steering, reentry orientation.



MULTI-STEP UPDATE			
CONTROL	18 ms	GUIDANCE	720 ms 360 ms during VTP
ATTITUDE	18 ms		
VELOCITY & POSITION	72 ms	STEERING	72 ms

Figure 12. Schematic representation of the model

Navigation: Attitude update, velocity update, position update

Environment: Atmosphere, winds, gravity, oblate earth.

A block schematic representation of the model is given in Fig. 12.

9.2 Navigation System

An inertial navigation system (INS) consists of a two 2-axis dynamically turned gyros and three force-balanced accelerometers. These sensors are incorporated on a strap-down element, which inturn is mounted onto the body of the missile. Incremental angles are available every 6 ms. Attitude represented by the quaternion needs three samples of incremental angles, and hence the attitude updation is done in every 18 ms by rotation vector estimation concept. The incremental velocity information is available every 72 ms along the body frame. The incremental velocities are then transformed into the reference frame by a mean attitude quaternion. Position is also updated every 72 ms employing trapezoidal integration on the velocity. Thus, the navigation system provides the information of position, velocity, and orientation of the vehicle wrt a reference frame, which in turn forms the input state vector for the guidance and control system.

9.3 Flight Control System

The flight control system uses attitude control for all the three planes using a digital autopilot controller. The control system stabilises the movements of the vehicle and executes the guidance demands to manoeuvre the vehicle to follow a trajectory to reach the destination. The various types of control used in different phases are:

9.3.1 *First-stage (boost) Control*

Secondary injection thrust vector control (SITVC) is used for controlling the vehicle during the first stage operation. In the nozzle expansion area of the rocket motor, injection valves are provided to inject the secondary fuel. These valves, when operated, are capable of giving pitch and yaw moments, respectively. When the secondary fuel is injected into the main thrust, a disturbance in the exhaust causes the main thrust to deflect, and hence, a side force is created which is used to control the vehicle movements.

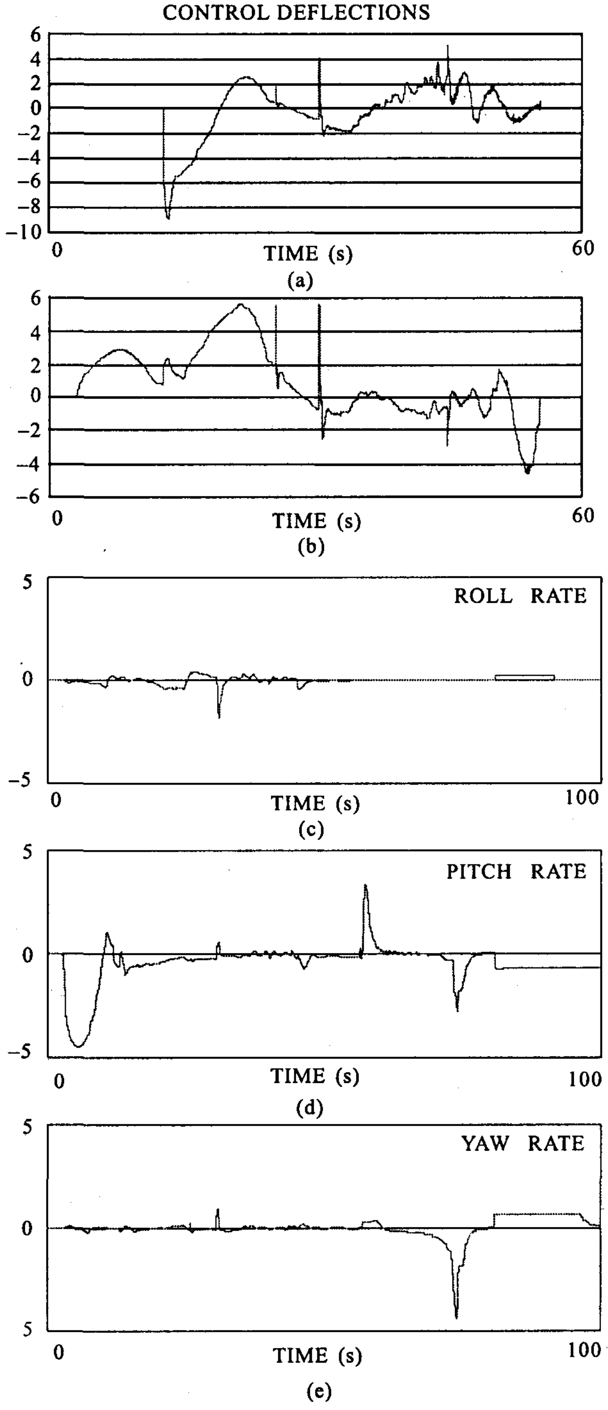
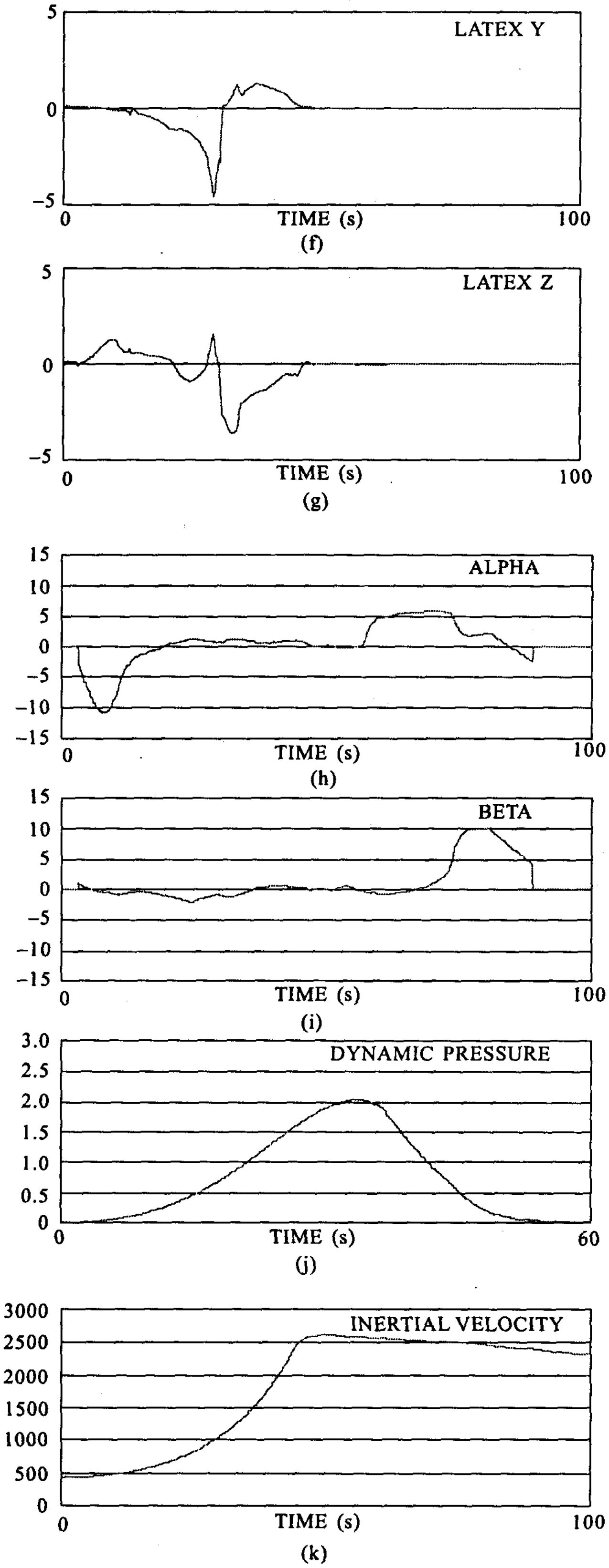


Figure 13 (a-e). Typical test results of the vehicle



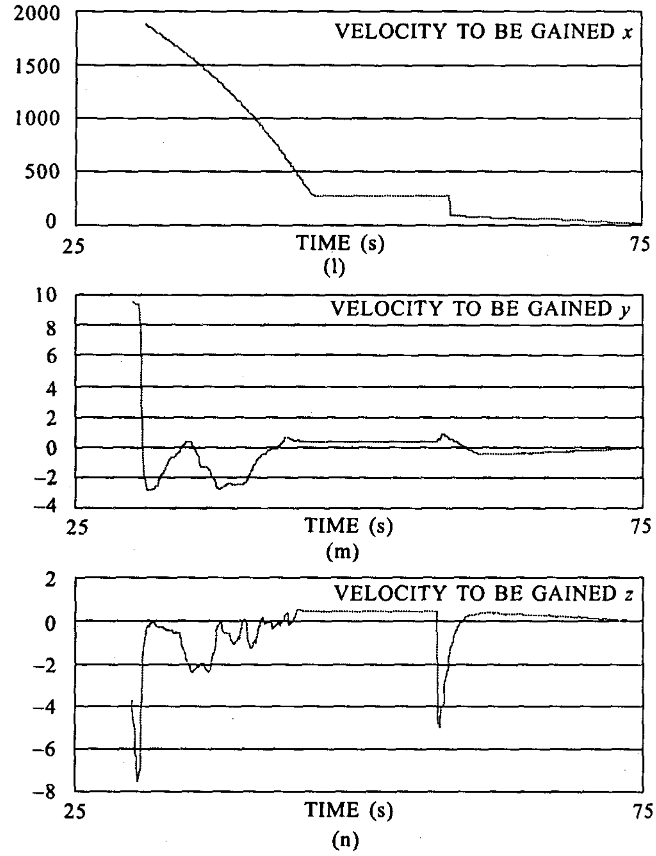


Figure 13 (f-n). Typical test results of the vehicle

Hydraulic fin tip control system (HFTC), where fins are mounted at the bottom of the missile. The tips of the fins are movable, by which aerodynamic forces normal to the vehicle longitudinal can be generated. The fins are mounted in such a configuration so that these fins are capable of producing pitch yaw and roll moments and are used to control the vehicle.

9.3.2 *Velocity Trimming Package Guidance Phase & Free-flight Phase*

Reaction control system is used to control the vehicle in the exoatmospheric free flight. During this phase, the vehicle is oriented to achieve zero angle of attack during reentry. Initial body rates generated due to stage separation are counter balanced by reaction control system. Control forces are generated by the reaction motors operating in Bang_Bang mode.

10. SIMULATION MODELS

10.1 Atmospheric Model

An Indian standard atmosphere as proposed by Ananthasayanam and Narasimha, called ISTA-8 has a seal-level temperature of 30 °C and a constant lapse rate of 6.5 °C/km up to 16 km in the troposphere, leading to a tropopause temperature of –74 °C. In the stratosphere, the temperature increases at the lapse rate of 2.3 °C/km reaching a value of –5 °C at the stratopause, which extends from 46 km to 52 km. Beyond this altitude, in the mesosphere, the temperature decreases through a lapse rate of 3 °C/km, reaching once again a temperature of – 74 °C (as in tropopause) at 75 km, followed by a constant temperature mesopause up to 80 km. This model is based on the assumption that the atmosphere above 80 km has negligible effect. The various atmospheric parameters are temperature, density, pressure, and velocity of sound.

10.2 Wind Model

Winds are the local disturbances of the free stream through which when the vehicle travels, it is subjected to additional aero-loads. A mean wind profile up to 60 km altitude observed over various months are used in the development of the program. Winds above 60 km altitude are neglected as their effect is very low due to atmosphere density being low. The effect of wind is on the angle of attack and the side-slip angles. Wind data consists of a southward and eastward components.

10.3 Earth's Gravity Model

Earth is oblate in shape, due to this reason gravitational force varies not only with height but also with latitude. Considering the oblateness, the potential at any point P at a distance R from the centre of earth is given in guidance equations.

11. SIMULATION RESULTS

Real-time 6-DOF simulation studies have been carried out to validate guidance model on a PC. A pre-defined attitude profile is used prior to the guidance. With the closed-loop guidance, a range variation of 400 km has been achieved with target definition.

For the in-flight estimation of the propulsion stage performance, 3 σ variations of thrust profile has been used for the simulation studies.

From these simulation studies, it is seen that an upper thrust profile be stored as the reference since the shortage in impulse can be corrected by the velocity trimming package. The velocity trimming package gives the provision of adding velocity, thereby correcting the shortage in forward velocity. The velocity contribution due to gravity and remaining velocity due to thrust are treated as a geometric vector addition, and steering angle is computed.

A close iterative was obtained for the burnout velocity, burnout position and guidance parameters like flight-path angle (γ), time of flight, etc. Control studies has been carried out with the lateral / roll autopilot designed along with perturbation cases to evaluate the guidance system. Under the worst case performance of the vehicle, the velocity trimming package impulse requirements and guidance performance evaluation is carried out.

Typical test results of the vehicle are given below Fig. 13 (a) to (n).

CONCLUSIONS

An explicit iterative, energy management guidance scheme has been designed for a single-stage solid propulsion vehicle. Simulation results confirmed the convergence of the guidance equations and high-impact accuracy. This guidance scheme minimises the velocity uncertainties, thereby minimising the velocity trimming package requirements. The operational complexities of the thrust termination requirements are eliminated for the solid system, since the fuel is burned to the end. Initial validation tests of the guidance model indicate that the guidance errors at the burnout are limited to the uncertainties of the stage weight and specific impulse (Isp) errors only.

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Contributor

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