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SHORT COMMUNICATION

Maximisation of Expected Target Damage Value

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ABSTRACT

The weapon assignment problem has been modelled as a nonlinear integer programming problem[1]. The problem is to assign weapons to the targets to maximise the optimum-target damage value. There are constraints on various types of weapons available and on minimum number of weapons by types to be assigned to various targets. The objective function is nonlinear, the constraints are linear in nature, and the decision variables are restricted to be integers. The results obtained by Bracken and McCormick[1] should not be applied to solve the problem of weapon assignment to target to maximise the optimum target damage value, because firstly, the results violate the constraints, and secondly, instead of using the integer programming techniques, the crude method of rounding off has been used to obtain the solution. In this study, the I-GRST algorithm developed by Deep and Pant[2,3] has been used to solve the weapon assignment problem. The results obtained are better than the results obtained by Bracken and McCormick[1] and also do not violate any constraints.

Keywords: Weapon assignment, optimisation, integer programming problems, genetic algorithm, target damage, I-GRST algorithm

1. INTRODUCTION

Operations research has wide applications in defence-related problems. One such area is the weapon-target assignment problem, which is a fundamental problem. In such types of problems, the objective is to assign the weapons to targets such that the target damage value is maximum. Such problems have integer restrictions imposed upon. These can be categorised as integer programming problem. The objective function of the problem is nonlinear and the constraints are linear in nature and the variables are subjected to integer restrictions.

The I-GRST algorithm[2,3] for obtaining the global optimal solution of the integer and the mixed integer programming problem has been presented and well-tested on benchmark problems. In the present study, the I-GRST algorithm[2,3] is used to solve weapon assignment problem. A hypothetical data has been taken to show the working of I-GRST algorithm. The numerical and graphical results obtained are compared with the results obtained by Bracken and McCormick[1]. It is concluded that the results obtained are better than the results obtained by Bracken and McCormick[1].

2. FORMULATION OF THE PROBLEM

The mathematical model of the weapon assignment problem has been presented.

Let x_{ij} be the variables to be determined, where x_{ij} is the number of weapons of type i assigned to

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target j, $i = 1,...p$ and $j = 1,...q$.

Limitations on the weapons assigned are specified in terms of the following:

a_i = Total number of weapons of type i available

b_j = Minimum number of weapons of all types assigned to the target j

The constraints on total number of weapons and on minimum weapons assigned to targets are:

$$\sum_{j=1}^{q} x_{ij} \leq a_i \text{ , } i=1,...p \tag{1}$$

$$\sum_{i=1}^{p} x_{ij} \leq b_j \text{ , } j=1...q \tag{2}$$

The objective function is calculated in terms of probability of damage of various targets weighted by the military value.

Let α_{ij} be the probability the target j will be undamaged by an attack using one unit of weapon i, and u_i be the military value of the target j

The expected damage to the target j by an assignment of x_{ij} weapons of type i is:

$$\left[1-\alpha_{ij}^{x_{ij}}\right]$$

and the expected damage to the target j by the overall assignment of weapons of all types is:

$$\sum_{i=1}^{p} x_{ij} \text{ is} \left[1-\prod_{i=1}^{p} \alpha_{ij}^{x_{ij}}\right]$$

The total expected target damage value is the sum of the expected damages to targets weighted by the military value of the targets,

$$\sum_{j=1}^{q} u_j \left[1-\prod_{i=1}^{p} \alpha_{ij}^{x_{ij}}\right] \tag{3}$$

The objective of the problem is to determine x_{ij}'s to maximise the objective function defined above subjected to the linear constraints define in Eqns (1) and (2).

As an example, suppose that weapons of types are to be assigned to 20 different target: Let the weapon of 5 types be:

(a) Intercontinental ballistic missiles

(b) Medium-range ballistic missiles from the fir: firing area

(c) Long-range bombers

(d) Fighter-bombers

(e) Medium-range ballistic missile from the secon firing area.

The parameters needed for the model lik probabilities that the targets will be undamaged b the weapons, total number of weapons availabl minimum number of weapons to be assigned, an military value of targets, are given by Bracken an McCormick. Thus, the objective function become:

$$Max\ z = 60\left[1.00-\left(1.00^{x_{11}}.84^{x_{21}}.96^{x_{31}}.1.00^{x_{41}}.92^{x_{51}}\right)\right]$$

$$+...$$

$$+...$$

$$+...$$

$$+150\left[1.00-\left(1.00^{x_{1,20}}.85^{x_{2,20}}.92^{x_{3,20}}.1.00^{x_{4,20}}1.00^{x_{5,20}}\right)\right]$$

Subjected to the constraints on the total numbe of weapons of 5 types:

$$x_{11} + ... + x_{1,20} \leq 200$$

$$...$$

$$...$$

$$...$$

$$x_{5,1} + ... + x_{5,20} \leq 250$$

and the constraints on the minimum number c assignments of weapons to the 7 specified targe that must be attacked are:

$x_{11} + \ldots + x_{5,1} \geq 30$

...

...

...

$x_{1,20} + \ldots + x_{5,20} \geq 10.$

3. I-GRST ALGORITHM

The I-GRST algorithm developed by Deep and Pant[2,3] has been used to solve the nonlinear integer optimisation problem described above. This algorithm is used for solving the linear as well as nonlinear integer programming problems of the type:

$Min\ f(X),\ X = (x_1, x_2, \ldots x_n),\ s.\ t.\ g_j(X) \leq 0;$ $j = 1, 2 \ldots k.$

$g_j(X) = 0;\ j = k+1,\ k+2, \ldots p.$ and $a_i \leq x_i \leq b_i;$ $i = 1, 2 \ldots n.\ \forall i \in I$

where x_i is integer $\forall\ i \in I \subseteq N = \{1, 2 \ldots n\}$; a_i *and* b_i are assumed to be integers $\forall i \in I$.

The I-GRST algorithm is based on the GRST algorithm[2], which in turn is the merger of the genetic algorithm approach with the random search technique (RST2) of Mohan and Shanker[4].

The I-GRST algorithm works in three phases. In the first phase, the objective function is evaluated at a number of randomly generated feasible solutions. If each of these is an integer solution, then these are adopted as these are, whereas if these are non integer, then these are modified to satisfy the integer restriction. In the second phase, at each iteration these solutions are manipulated by local searches (using quadratic approximation) to yield possible candidate for global optima. This is also checked to satisfy the integer requirements.

In an iteration, in case no satisfactory approximation is obtained, then instead of abandoning the iteration, the algorithm enters the third phase where the genetic algorithm operators, namely reproduction and crossover are activated with the hope of finding a better approximation.

The computational steps of the I-GRST are as follows:

Phase I

Step 1.

Choose a suitably large number NBIG say NBIG=10×(n +1) n-dimensional random feasible solutions (say x_i) and evaluate the objective function at each of these. The integer restrictions are checked as

(a) $x_i = x_i$, if x_i is the integer

and

(b) If x_i is not an integer, then generate a random number (say r) between 0 and 1.

If $r < 0.5$, then $x_i = [x_i]$ else $x_i = [x_i] + 1$

Store these solutions in an NBIG by (n+1) array A.

Phase II

Step 2.

Out of these feasible solutions, determine M and L as the feasible solutions with the greatest and the least function values $f(M)$ and $f(L)$ respectively. If stopping criteria,

$$\left|\frac{f(M) - f(L)}{f(L)}\right| < \varepsilon$$

is satisfied, stop with the message that L is the global minimum solution. Otherwise goϕ to *Step* 3.

Step 3.

From the current array A, choose three distinct feasible solutions $R_1 = L$, R_2 and R_3 randomly and determine the next feasible solution P as the point of minima of the quadratic curve passing through R_1, R_2 and R_3 by the formula:

$$P = .5\left(\frac{\begin{array}{c}(R_2^2 - R_3^2)f(R_1) + \\ (R_3^2 - R_1^2)f(R_2) + (R_1^2 - R_2^2)f(R_3)\end{array}}{\begin{array}{c}(R_2 - R_3)f(R_1) + \\ (R_3 - R_1)f(R_2) + (R_1 - R_2)f(R_3)\end{array}}\right) \tag{4}$$

If $P = (p_1, p_2, p_3, \text{---}, p_n)$ satisfies the integer restrictions given in Eqn (4), P is feasible and $a_i \leq p_i \leq b_i$, $i = 1, 2, \text{---}n$, then go to *Step* 4, otherwise set $p_i = b_i$, if $p_i \geq b_i$ and $p_i = a_i$ if $p_i \leq a_i$ and go to *Step* 4.

Step 4.

Find $f(P)$ .If $f(P) < f(m)$ go to *Step* 5 otherwise go to *Step* 6.

Step 5.

Replace M by P in the array A and go to *Step* 2.

Phase III

Step 6.

Convert the n-dimensional elements of array A into their binary equivalent array B of fixed binary length (10 here). Perform crossover for each element of A by randomly selecting its mate from the elements of array A. Convert these back to real numbers and evaluate function value at each of these, thus obtaining a 2NBIG $\times$ $(n+1)$ array. If these are not integers, then these are modified to satisfy the integer restrictions using Eqn (4). Let Q be the element with least function valve $f(Q)$. If $f(Q) < f(m)$, replace M by Q in array A and go to *Step* 2, otherwise go to *Step* 3.

The above algorithm has been well-tested by Deep and Pant[2,3] on benchmark test problems and a comparison of the algorithm with the other algorithms in this category has been presented. It has been concluded that the I-GRST algorithm outperforms the other algorithms of this category in terms of percentage of success in obtaining the global optimal solution of the integer and the mixed integer programming problems.

4. NUMERICAL RESULTS & CONCLUSIONS

The results obtained by Bracken and McCormick[1] are discussed and the results of using the I-GRST algorithm for solving the weapon assignment problem is presented and compared with other algorithms.

In the study by Bracken and McCormick, the quoted results have been obtained by rounding off the solution to the nearest integer. It displays the objective function value as 1001. It may be noted that while using the rounding off technique, the assignment of 101 weapons of type 2 exceeds the availability. Hence the constraints is violated.

Since I-GRST is a probabilistic technique, the problem given in Section 2 has been executed five times and the numerical results are presented in Tables 1-5. The objective function values obtained are 998, 1000, 999, 996, and 998. The best solution obtained is 1000. Since I-GRST is designed in such a manner that integer conditions are checked in each phase, all the runs are obtained without violating any constraint.

The graphical interpretation of the results are displayed in Figs 1(a) to 1(e). In each case, the X-axis represents the target number from 1 to 20, and the Y-axis represents the number of weapons of type i assigned to target j. Different symbols are used to show the assignment of five weapons to different targets. The figures indicate the pattern of allotting the weapons to the targets.

Table 1. Solution obtained in the first run of I-GRST algorithm

	Targets																				
Weapons	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	α_i
1		16				99	33	29	21			2									200
2			1	20	19										28	30					98
3													1	3	46		70	60	60	60	300
4											1	38	48	58			5				150
5	48	50	46							48	58										250
Total weapons assigned to targets	48	66	47	20	19	99	33	29	21	48	59	45	49	61	74	30	75	60	60	60	998 Obj Fun

Table 2. Solution obtained in the second run of I-GRST algorithm

	Targets																				
Weapons	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
1		16				98	32	30	24												200
2				20	20										27	33					100
3													2	2	44		70	64	56	59	300
4												40	46	56			5				150
5	46	52	50							44	56										250
Total weapons	46	68	50	20	20	98	32	30	24	44	56	40	48	58	71	33	75	64	62	59	1000
assigned to targets																					Obj fun

Table 3. Solution obtained in the third run of I-GRST algorithm

	Targets																				
Weapons	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
1		16				98	32	32	22												200
2				20	19										26	34					99
3														4	46		70	64	62	58	300
4												40	50	54		6					150
5	50	50	50							48	52										250
Total weapons	50	66	50	20	19	98	32	32	22	48	52	40	50	58	72	40	70	64	62	58	999
Assigned to targets																					Obj fun

Table 4. Solution obtained in the fourth run of I-GRST algorithm

	Targets																				
Weapons	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
1		16				96	34	30	24												200
2				21	19										27	32					99
3													2	2	44		70	67	56	59	300
4												40	46	56			5				147
5	46	54	50							44	56										250
Total weapons	46	70	50	21	19	96	34	30	24	44	56	40	48	58	71	32	75	67	62	59	006
assigned to targets																					Obj fun

Table 5. Solution obtained in the fifth run of I-GRST algorithm

	Targets																				
Weapons	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
1		16				96	32	30	24			2									200
2				21	20										26	32					99
3															48		70	62	60	60	300
4												40	46	56			8				150
5	46	52	50							44	56										250
Total weapons	46	68	50	21	20	98	32	30	24	44	56	42	48	58	71	33	75	60	62	59	998
assigned to targets																					Obj fun

Based on these numerical and graphical results, it has been concluded that as compared to the value of the objective function in the source[1], the value of the objective function obtained is better in all the five runs of the algorithm. Moreover, the solution given in the source[1] violates the constraints,

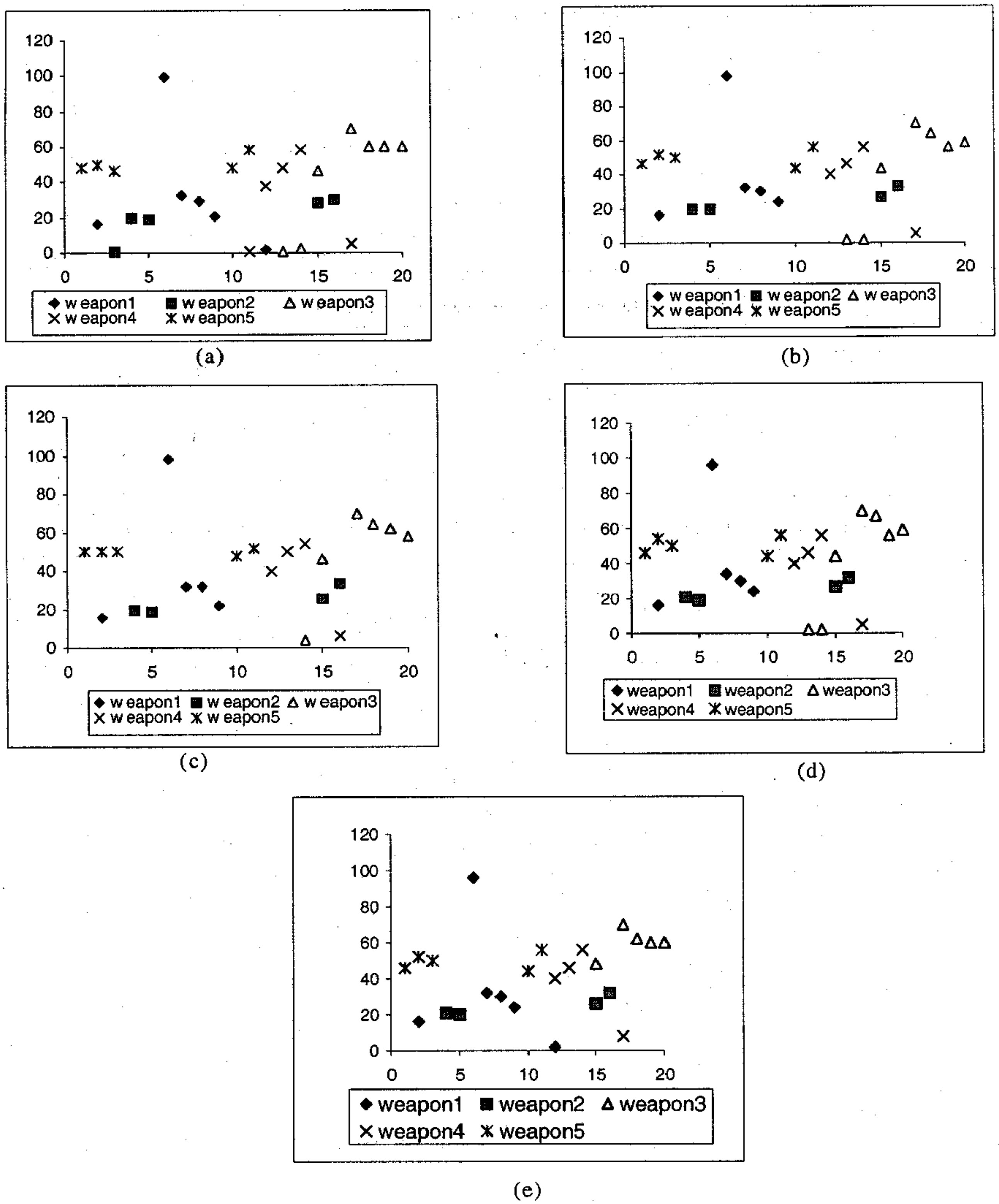

Fig. 1 Graphical representation of optimum assignment using five runs of I-GRST

whereas all the five runs obtained by our approach satisfy all the constraints. Since the results obtained are superior in quality to the ones presented in the source[1], the present approach is more reliable and trustworthy.

From the results, it is observed that instead of using the crude rounding off technique for solving the integer programming problems, the systematic I-GRST algorithm should be used to solve the weapon assignment problem. This algorithm gives better results, and that too, without violating the constraints.

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