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Construction of Graceful Signed Graphs

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ABSTRACT

In this paper, the mathematical problem of automation of encoding a communication/transportation network is considered for enabling design of appropriate plans for ground-troop movement on a complex terrain that motivated to discuss a general method of constructing infinite families of graceful signed graphs from a given gracefully numbered signed graph. This method gives graceful numberings for signed graphs on $K_2 + K_t^c$, $t \geq 1$, where G^c denotes the complement of the graph G.

Keywords: Ground-troop movement, graceful signed graph

1. INTRODUCTION

Deployment and movement of ground troops on a war front depend much on the knowledge of roads and walk-ways, including their physical conditions for movement. Given a map of such road links and walk-ways (which, together are referred to as links) on the terrain of deployment, it is necessary to assign them weights indicative of their physical conditions. The simplest weighting scheme to begin with could be identifying good links or bad links which is done by assigning +1 to each good link and –1 to each bad link on the map. As a result, the map of the road/walk-way network on the frontier would look like a signed graph. As such, the network could be quite complicated to handle manually for the purposes of planning and, therefore, it is necessary to lodge the network on a computer and encode its elements (ie, vertices and edges of the network representing bunkers and links interconnecting them respectively) so as to enable devising appropriate troop-movement plans. For instance, in such a plan, it may be required to find a path with least resistance for movement, such as encountering least total length of bad roads to be traversed for reaching from one point to another on the war front.

Further, one may like to use least possible number of symbols to be assigned to the elements of the network to accomplish a given task on the network. For example, if one is able to assign to each vertex u in the network a distinct number $f(u)$ then one can assign $|f(u) - f(v)|$, the absolute value of the difference between $f(u)$ and $f(v)$, to each link uv in the network, thereby not requiring a separate numbering process for identifying the links. The basic question for identifying the links would then be to require the links to receive their labels (which are the numbers here) uniquely. Is it always possible to attain such a numbering for any arbitrarily given network? While it is not difficult to see that such a numbering, which is called a geodetic numbering scheme, is always possible for an arbitrary network, it could be quite formidable to obtain such an optimal numbering. The study of

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graceful signed graphs[1] could be one means by which optimal encoding of the road network could be used for automation of planning appropriate troop-movement strategies. Most interesting aspect of this process is that when a particular network does not admit a desired optimal encoding, it would be possible to expand the network to accommodate the required type of encoding[2], embedding an unyielding network into a facilitating network. Of course, one needs to optimise the size and complexity of so-augmented network links. While the latter problem would need separate consideration, the problem of extending an existing optimal geodetic numbering scheme to a larger integrated network will be solved in this paper.

2. PRELIMINARIES

For standard terminology and notation in graph theory, refer to F. Harary[3] and for signed graphs (*sigraph*, in short) refer to Chartrand[4] and Zaslavsky[5,6].

Most of graph labelling methods trace their origin to one introduced by Rosa[7] in 1966, who meant by a β-valuation of a graph G with q edges an injective function f from the vertices of G to the set $\{0,1,2,..., q\}$ such that when each edge uv is assigned the number $g_f(uv) = |f(u) - f(v)|$, the resulting edge numbers form the set $\{1,2,..., q\}$. Golomb[8] subsequently called such labellings graceful numberings of G and this is now the popular term (note here that terms labelling and numbering are distinguished in the sense that, while a labelling is an assignment of any type of entities as labels, a numbering is an assignment of numbers to the elements of a given graph).

A (p,m,n)-sigraph is an ordered pair $S = (G, s)$ where $G = (V, E)$ is a (p,q)-graph, called its underlying graph, and $s: E \rightarrow \{+, -\}$ is a function called its signing function. Let $E^+(S)$ and $E^-(S)$ denote the sets of positive and negative edges of S where $E^+(S) \cup E^-(S) = E(S) := E$. Then $|E^+(S)| = m$, $|E^-(S)| = n$ so that $m + n = q$. An all-positive sigraph S is one in which $E^+(S) = E(S)$ and an all-negative sigraph is one in which $E^-(S) = E(S)$. A sigraph S is said to be homogeneous if it is either all positive or all-negative and heterogeneous otherwise[9].

An indexer of a sigraph $S = (G, s)$ is an injection f: $V(S) \rightarrow N$, where $V(S)$ is the set of vertices of S, and N is the set of nonnegative integers. Let I_S denote the set of all indexers of S. $f \in I_S$ is called an encoder (or, equivalently, a numbering) of S if the induced function $g_f : E(S) \rightarrow Z$, where Z is the set of nonzero integers, defined by

$$g_f(uv) = s(uv)|f(u) - f(v)|, \forall\ uv \in E(S) \quad (1)$$

is also injective. The set of all encoders of S will be denoted by Γ_S. Clearly, if $f \in \Gamma_S$ then for any positive integer t, the function f_t: $V(S) \rightarrow \boldsymbol{N}$, defined by saying $f_t(u) = f(u) + t, \forall\ u \in V(S)$, is also an encoder of S such that $g_{f_t}(e) = g_f(e)$ for every $e \in E(S)$. Also, the complement f^c of f defined by saying $f^c(u) = M(f) - f(u)$ is an encoder of S, where

$$M(f) = \max\ \{f(u) : u \in V(S)\}.$$

Next, for any $f \in \Gamma_S$ let $f(S) = \{f(u): u \in V(S)\}$

$$g_f(S) = \{g_f(e): e \in E(S)\}.$$

Then the number

$$\theta(S) = \min\ \{M(f): f \in \Gamma_S\}$$

is called index of gracefulness of S. An encoder $f \in \Gamma_S$ for which $M(f) = \theta(S)$ is said to be optimal and, in particular, one calls f a graceful numbering (or, equivalently, graceful encoder) of S if f: $V(S) \rightarrow \{0, 1, . . . , q = m + n\}$ is such that in the induced edge function defined by Eqn (1) the set of numbers received on the positive edges of S is $\{1, 2, . . ., m\}$ and the set of numbers received on the negative edges of S is $\{-1, -2, . . . , -n\}$, respectively; if S admits such an encoder then S itself is called graceful. Note that if $n = 0$ this notion coincides with that of *a* β-valuation as in Rosa[7] and, equivalently, with that of gracefulness of graphs as in Golomb[8]. Some examples of graceful sigraphs are shown in Fig. 1.

Observation 1: For any sigraph S, if f is a graceful numbering of S then f is also a graceful numbering of the negation $\eta(S)$ of S, where $\eta(S)$ is obtained by reversing the sign of each edge in S.

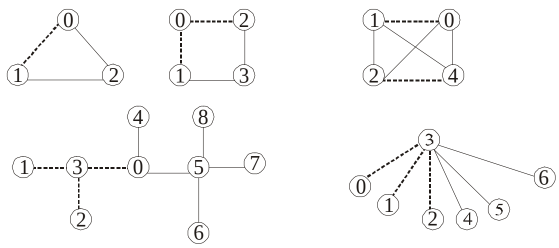

Figure 1. Some examples of graceful sigraphs.

No characterisation of graceful sigraphs has been discovered so far. The following two theorems are the only general results known on graceful sigraphs[1,2]:

Theorem 1[1]: Let $S = (G, s)$ be any (p, m, n)-sigraph with $G = (V,E)$ as its underlying graph. Then necessary condition for S to be graceful is that it is possible to partition $V(G) := V(S)$ into two subsets V_o and V_e such that the numbers $m^+(V_o, V_e)$, $m^-(V_o, V_e)$ of positive and negative edges of S, respectively, each of which joins a vertex of V_o with one of V_e are given by

and $m^+(V_o, V_e) = \lfloor (m+1)/2 \rfloor$

$m^-(V_o, V_e) = \lfloor (n+1)/2 \rfloor$

where $\lfloor x \rfloor$ denotes the greatest integer not greater than x.

Theorem 2[2]: Every numbered sigraph can be embedded as an induced subsigraph in a gracefully numbered sigraph.

While *Theorem* 1 gives a necessary condition for a sigraph S to be graceful, *Theorem* 2 claims the impossibility of having a characterisation of graceful sigraphs by means of forbidding a class of sigraphs to be the induced subsigraphs. To date, several families of graceful as well as non-graceful graphs (sigraphs) have been discovered[1,2,7,8,10-15] towards gaining better insight into their general properties.

Let $G = (V, E)$ be a (p, q)-graph with a graceful numbering f and let r be the number of integers which are not assigned as vertex labels in G. Let $H = G \cup K_r^c$ and assign the missing numbers to these r new vertices so augmented to G. It can be easily seen that H is a $(q+1, q)$ graceful graph. Then H is called a fully augmented graph on G in which G is an induced subgraph.

The following results are known in the theory of graceful graphs:

Theorem 3[10]: Let (G, f) be a gracefully numbered (p, q)-graph and let G_f be its full augmentation. Then for every $n \geq 1$, the graph $G_f + K_n^c$ is graceful.

Theorem 4[16]: If T is any graceful tree, then $T + K_t^c$, $t \geq 1$, is graceful.

Theorem 5[17]: Let G be a graceful graph with p vertices and q edges. Then any graceful numbering of G may be extended to a graceful numbering of $[G \cup (q-p+1)K_1] + K_t^c$, $t \geq 1$.

The aim here is to describe a method of constructing infinite families of connected graceful sigraphs embedding the given gracefully numbered sigraph as one of its induced subsigraphs. Such a construction is useful in expanding the facility of graceful addressing and identification systems to larger communication networks as elucidated in the introduction.

3. RESULTS

Every graceful (p, m, n)-sigraph S of size $q = m + n$ can be embedded in a graceful $(q+1, m, n)$-sigraph S . This may be achieved as follows: let f be a graceful numbering of S. Then $\{0, 1, 2, \ldots, q\} - f(S)$ has $\rho_f(S) = q - p + 1$ numbers each of which does not appear as a vertex number in (S, f), called missing vertex integers. Then adjoin $\rho_f(S)$ isolated vertices (or isolates) to S and assign them the numbers from $\{0, 1, \ldots, q\} - f(S)$. For example, two sigraphs on K_4 which can be gracefully numbered satisfy $\rho_f(K_4) = 3$ and they can be augmented by K_3^c as shown in Fig. 2.

Theorem 6: Let (S, f) be a gracefully numbered sigraph on a (p, q)-graph. Then for any integer $1 \leq r \leq q - p + 1$, $S \cup K_r^c$ is also graceful.

Proof: Let S be a graceful sigraph and f be a graceful numbering of S. Then, the vertices of S

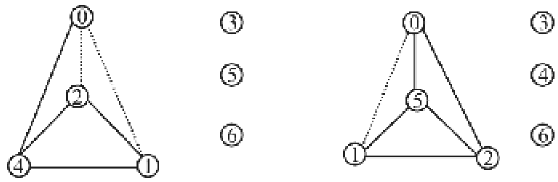

Figure 2. Full augmentation of graceful sigraphs.

can be labelled from $\{0, 1, 2, \ldots, q = m + n\}$ and from the definition of graceful sigraph the edge labels are given by $g_f(S) = \{1, 2, \ldots, m, -1, -2, \ldots, -n\}$. Therefore, the vertices of K_r^c can be labelled from $\{0, 1, 2, \ldots, q = m+ n\} - f(S)$ in a one-to-one manner. Let $f^*: V(S \cup K_r^c) \to \{0, 1, \ldots, q = m + n\}$ denote the so extended numbering of $S \cup K_r^c$. Clearly, adjoining of $r \leq q - p + 1$ isolated vertices do not change the original edge labels. So, f^* is also a graceful numbering of $S \cup K_r^c$. Hence, $S \cup K_r^c$ is graceful.

Let $S_1 = (V_1, E_1, \sigma_1)$ and $S_2 = (V_2, E_2, \sigma_2)$ be any two sigraphs. Their sum $S_1 + S_2$ has been defined[9] as a sigraph $S = (V, E, \sigma)$ in which $V = V_1 \cup V_2$, $E = E_1 \cup E_2 \cup (E_1 \otimes E_2)$, $E_1 \otimes E_2 = \{uv : u \in V_1 \textit{ and } v \in V_2\}$ and $\sigma(xy) = -1$ if and only if exactly one of the following three conditions is satisfied:

(a) $xy \in E_1$ and $\sigma_1(xy) = -1$

(b) $xy \in E_2$ and $\sigma_2(xy) = -1$

(c) $x \in V_1$, $y \in V_2$ and

$\prod (\sigma_1(e) : x \in e \in E_1(x)) = \prod (\sigma_2(f) : y \in f \in E_2(y)) = -1$

where $E_i(w) = \{g \in E_i : w \in g\}$, $i \in \{1, 2\}$, is the edge-neighbourhood of the vertex w in S_i.

Theorem 7: Let (S, f) be a gracefully numbered sigraph. Then the sigraph $S' = [S \cup (q - p + 1)K_1] + K_t^c$, $t \geq 1$, is graceful.

Proof: Let (S, f) be a gracefully numbered sigraph. Let $v_{p+1}, v_{p+2}, \ldots, v_{q+1}$ be the isolated vertices. Then, by *Theorem* 6, one can extend $f : V(S) \to \{0, 1, \ldots, q\}$ to $f^* : V(S \cup (q - p + 1)K_1) \to \{0, 1, \ldots, q = m+n\}$ so that f^* is bijective. Now, label the vertices $x_1, x_2, \ldots, x_t$ of K_t^c, $t \geq 1$, with t new numbers $i(q + 1) + m$, $1 \leq i \leq t$, and let f^{**} denote the so extended numbering of S'. It is easy to verify that f^{**} is a graceful numbering of S'.

In Fig. 3, the proof is illustrated by taking an example.

Theorem 8: All the sigraphs on $K_2 + K_t^c$, $t \geq 1$, are graceful.

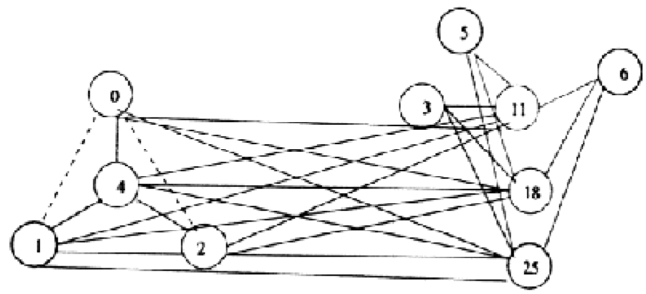

Figure 3. Example of graceful sigraph $[S \cup (q\text{-}p\text{+}1)K_1]+K_3^C$.

Proof: It is enough to provide a graceful numbering of the sigraph S on $K_2 + K_t^c$, $t \geq 1$, with m and n denoting the number of positive and the number of negative edges in the sigraph, respectively. Without loss of generality, one takes $m \geq n$. Let u, v denote the vertices of K_2 and let $t = a + b + c + d$, where a, b, c and d are the number of vertices of K_t^c of the following varieties: Each of the vertices $x_1, x_2, \ldots, x_a$ is joined to u by a negative edge and to v by a positive edge; each of the vertices $y_1, y_2, \ldots, y_b$ is joined to u by a positive edge and to v by a negative edge; each of the vertices $z_1, z_2, \ldots, z_c$ is joined to both u and v by negative edges; and each of the vertices $w_1, w_2, \ldots, w_d$ is joined to both u and v by positive edges. If any of a, b, c and d is zero then the corresponding subset of the vertex-set of K_t^c will be assumed to be empty. First, one assumes that K_2 is positive and without loss of generality, let $a \leq b$. Let f be a numbering of S defined as follows:

$f(u) = a$, $f(v) = a + 1$

$f(x_i) = i - 1$, for $1 \leq i \leq a$

$f(y_i) = 2a + 1 + i$, for $1 \leq i \leq b$

$f(z_i) = 2a + b + 2i$, for $1 \leq i \leq c$

$f(w_i) = 2a + b + (2i + 1)$, for $1 \leq i \leq d$.

Then the induced edge function g_f yields the numbers on the positive edges as

$g_f\ (uv) = 1$

$\{g_f\ (uy_i) = s(uy_i)|f(u) - f(y_i)| = (a + i + 1),\ 1 \le i \le b\} = \{a + 2,\ a + 3,\ \ldots,\ a + b + 1\}$;

$\{g_f\ (uw_i) = s(uw_i)|f(u) - f(w_i)| = (a + b + 1 + 2i),\ 1 \le i \le d\} = \{a + b + 3,\ a + b + 5,\ \ldots,\ a + b + 2d + 1\}$;

$\{g_f\ (vx_i) = s(vx_i)|f(v) - f(x_i)| = (a+2-i),\ 1 \le i \le a\} = \{a+1,\ a,\ a-1. \ldots,\ 2\}$;

$\{g_f\ (vw_i) = s(vw_i)|f(v) - f(w_i)| = (a+b+2i),\ 1 \le i \le d\} = \{a+b+2,\ a+ b + 4,\ \ldots,\ a + b + 2d\}$;

and the numbers on the negative edges as

$\{g_f\ (ux_i) = s(ux_i)|f(u) - f(x_i)| = -(a + 1 - i),\ 1 \le i \le a\} = \{-1,\ -2,\ \ldots,\ -a\}$;

$\{g_f\ (uz_i) = s(uz_i)|f(u) - f(z_i)| = -(a + b + 2i),\ 1 \le i \le c\} = \{-(a + b + 2),\ -(a + b + 4),\ \ldots,\ -(a + b + 2c)\}$;

$\{g_f\ (vy_i) = s(vy_i)|f(v) - f(y_i)| = -(a + i),\ 1 \le i \le b\} = \{-(a + 1),\ -(a + 2),\ \ldots,\ -(a + b)\}$;

$\{g_f\ (vz_i) = s(vz_i)|f(v) - f(z_i)| = -(a + b + 2i - 1),\ 1 \le i \le c\} = \{-(a + b + 1),\ -(a + b + 3),\ \ldots,\ -(a + b + 2c - 1)\}$.

Clearly, the numbers on positive edges and negative edges so defined are all distinct (Fig. 4).

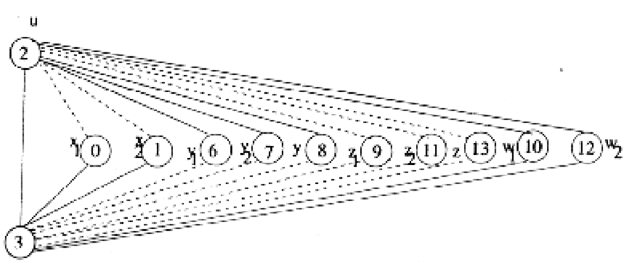

Figure 4. Example of graceful sigraphs on $K_2+K_{10}^c$.

If $a = 0$, then the graceful numbering of S is as follows:

$f(u) = 0,\ f(v) = 1$

$f(y_i) = i + 1$, for $1 \le i \le b$

$f(z_i) = b + 2i$, for $1 \le i \le c$

$f(w_i) = b + 1 + 2i$, for $1 \le i \le d$.

Then the induced edge function g_f yields the numbers on the positive edges as

$g_f\ (uv) = 1$

$\{g_f\ (uy_i) = s(uy_i)|f(u) - f(y_i)| = (i + 1),\ 1 \le i \le b\} = \{2,\ 3,\ \ldots,\ b + 1\}$;

$\{g_f\ (uw_i) = s(uw_i)|f(u) - f(w_i)| = (b + 1 + 2i),\ 1 \le i \le d\} = \{b + 3,\ b + 5,\ \ldots,b + 2d + 1\}$;

$\{g_f\ (\mathrm{v}w_i) = s(\mathrm{v}w_i)|f(\mathrm{v}) - f(w_i)| = (b+2i),\ 1 \le i \le d\} = \{b+2,\ b+4,\ \ldots,\ b+ 2d\}$;

and the numbers on the negative edges as

$\{g_f\ (uz_i) = s(uz_i)|f(u) - f(z_i)| = -(b + 2i),\ 1 \le i \le c\} = \{-(b + 2),\ -(b + 4),\ \ldots,\ -(b + 2c)\}$;

$\{g_f\ (vy_i) = s(vy_i)|f(v) - f(y_i)| = -i,\ 1 \le i \le b\} = \{-1,\ -2,\ \ldots,\ -b\}$;

$\{g_f\ (vz_i) = s(vz_i)|f(v) - f(z_i)| = -(b+2i-1),\ 1 \le i \le c\} = \{-(b+1),\ -(b+3),\ \ldots,\ -(b + 2c - 1)\}$.

Clearly, the numbers on positive edges and those on negative edges so defined are all distinct.

In case, when any one of the possibilities $b = 0$, $c = 0$ and $d = 0$ occurs, one can obtain a graceful numbering of S in the similar manner as stated above.

If $a = 0$ and $b = 0$, then one can see that the following numbering of S is graceful:

$f(u) = 0,\ f(v) = 1$

$f(z_i) = 2i$, for $1 \le i \le c$

$f(w_i) = 2i + 1$, for $1 \le i \le d$.

Then the induced edge function g_f yields the numbers on the positive edges as

$g_f\ (uv) = 1$

$\{g_f\,(uw_i) = s(uw_i)|f(u) - f(w_i)| = (1+2i),\ 1 \le i \le d\} = \{3, 5, ..., 2d+1\};$

$\{g_f\,(vw_i) = s(vw_i)|f(v) - f(w_i)| = (2i),\ 1 \le i \le d\} = \{2, 4, . . . , 2d\};$

and the numbers on the negative edges as

$\{g_f\,(uz_i) = s(uz_i)|f(u) - f(z_i)| = -2i,\ 1 \le i \le c\} = \{-2, -4, . . . , -2c\};$

$\{g_f\,(vz_i) = s(vz_i)|f(v) - f(z_i)| = -(2i - 1),\ 1 \le i \le c\} = \{-1, -3, . . . , -(2c - 1)\}.$

Clearly, the numbers on positive edges and those on negative edges so defined are all distinct.

In this case, when any two of the numbers *a*, *b*, *c* and *d* are zero, one can prove the result in the similar manner as stated above.

If $a = 0$, $b = 0$ and $c = 0$, then a required numbering *f* may be defined as follows:

$f(u) = 0,\ f(v) = 1$

$f(w_i) = 2i + 1$, for $1 \le i \le d$.

In this case, it is easy to verify that *f* is a graceful numbering of *S*.

In case any three of *a*, *b*, *c* and *d* are zero, one can prove the result in the similar manner as above.

Next, it is assumed that K_2 is negative and $a \le b$. Let *f* be defined as follows:

$f(u) = a + 1,\ f(v) = a$

$f(x_i) = i - 1$, for $1 \le i \le a$

$f(y_i) = 2a + 1 + i$, for $1 \le i \le b$

$f(z_i) = 2a + b + (2i + 1)$, for $1 \le i \le c$

$f(w_i) = 2a + b + 2i$, for $1 \le i \le d$.

It is easy to verify, as described above in each case, that the given numbering *f* of *S* is indeed a graceful numbering (Fig. 5).

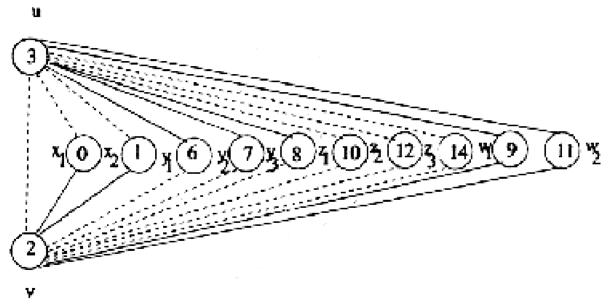

Figure 5. Example of graceful sigraphs on $K_2 + K_{10}^C$.

The injectivity of *f* can be seen straightforwardly by its very definition, in each of the above cases. Also, in each case, the induced edge labelling g_f has been verified to be injective. Thus, *f* in each case is a graceful labelling of the corresponding sigraph, completing the proof.

4. CONCLUSION

Given a gracefully numbered sigraph *S*, a method of constructing an infinite sequence ($S' = S_1, S_2, ...$) of gracefully numbered connected graceful sigraphs S_i has been given such that S_i is in S_{i+1} as an induced subsigraph for $i = 1, 2, ...$ If *S* is non-graceful, one can obtain an infinite sequence ($H = S_1, S_2, ...$) of gracefully numbered sigraphs S_i with the said property by the above mentioned method, where *H* contains *S* as an induced subsigraph; that such an imbedding exists has been shown[2]. Invoking this method, it has been shown that all sigraphs on $K_2 + K_t^c$ are graceful.

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Contributors

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