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SHORT COMMUNICATION

Punctured Turbo Codes for Bandwidth-efficient Transmission

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ABSTRACT

Turbo codes are the error-coding schemes applied nowadays in wireless networks. In naval applications, the information is mostly sent through wireless networks and the data is more prone to noise. Since very important data has to be communicated, it is necessary to get back the original data in the receiver. In military applications also, the soldiers wear electronic jackets which are connected by wireless networks. In such applications, the data loss is not affordable and there is also a need to utilise the bandwidth efficiently through puncturing by means of which certain bits are deleted before transmission from the output of encoder. By means of this punctured turbo codes, bandwidth-efficient coding is achieved. Hence, it is necessary to design turbo codes with an efficient puncturing pattern so that the performance of the punctured code is also improved in spite of deletion of few bits before transmission. This paper deals in choosing the puncturing patterns that lead to systematic rate-compatible punctured turbo codes (RCPTCs) which also give a reduction in bit-error rate. The design criterion for choosing the best puncturing patterns is based on the minimum weight of code words and their multiplicities. The best puncturing pattern chosen is tested for its performance by simulating turbo codes for an additive white Gaussian noise (AWGN) channel. Compared with the existing puncturing pattern, the pattern proposed is able to achieve a gain of 0.5 dB at a bit-error rate of 10^{-3}.

Keywords: Wireless transmission, turbo codes, error-coding schemes, rate-compatible punctured turbo codes, additive white Gaussian noise, MAP algorithm, soft output viterbi algorithm, RCPC, rate-compatible punctured codes, recursive systematic convolutional encoders

1. INTRODUCTION

The introduction of turbo codes has increased the interest in the coding area since these codes give most of the gain promised by the channel-coding theorem. Turbo codes are able to achieve very low bit-error rate (BER) at relatively low signal-to-noise ratio (Eb/No). It is possible to get different code rates for the existing turbo codes to utilise the bandwidth efficiency. Puncturing is a means to achieve different code rates. Hence, it is necessary to find good puncturing patterns so that the bit-error rate is also less inspite of deleting few parity bits of the encoder.

2. TURBO CODES

Error-control codes have become a vital part of modern digital networks, enabling reliable transmission to be achieved over noisy and fading channels. Turbo codes have been widely considered to be the most powerful error-control codes of practical importance. Turbo codes emerged in 1993 and

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have since become a popular area of communications research[1,2]. The important characteristics of turbo codes are the small BER achieved even at low signal-to-noise ratio (SNR) and the flattening of the error-rate curve, ie, the error floor at moderate and high values of SNR. The performance of turbo codes is due to the randomness created by the interleaver and the iterative decoding.

2.1 Turbo Encoder

The basic structure of turbo code encoder is shown in Fig. 1. It consists of two binary recursive systematic convolutional (RSC) encoders with small constraint lengths usually set between 3 and 5 which are concatenated in a parallel fashion using a turbo code interleaver and a puncturing and multiplexing device. The binary input sequence u, which has finite duration, is fed into the first RSC encoder, yielding the redundancy sequence x with the same finite duration, as u. Sequence $u1$, which is an interleaved replica of u, is put into RSC encoder 2. The output of RSC encoder 2 yields redundancy sequence y. Both the redundancy sequences x and y, as well as u are punctured and multiplexed to form output sequence b.

2.2 Punctured Turbo Codes

Puncturing is the process of deleting some bits from the code word according to a puncturing matrix[2]. The puncturing matrix (P) consists of 0s and 1s where the 0 represents an omitted bit and the 1 represents an emitted bit. It is usually used to increase the rate of a given code. An example of the puncturing matrix to go from rate 1/2 to rate 2/3 is given by Eqn (1).

$$P = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \tag{1}$$

This matrix implies that the first bit is always transmitted while every other second bit is omitted. The puncturing is made at the output of the encoders on the symbols as shown in Fig. 1. In case of two encoders, every other symbol is selected from different encoder output. The symbol stream to different encoders is interleaved.

To achieve the optimum performance, while applying a certain degree of puncturing, following are some specific rules recommended:

- Systematic code symbols should not be punctured
- The number of the punctured parity symbols should be distributed evenly. If exact even distribution is not possible due to the frame size constraint, the puncturing will be distributed as even as possible.

2.3 Turbo Decoder

There are two main algorithms used in the decoding of turbo codes[3]. One is the maximum *a posteriori* (MAP) algorithm[4] and the other is soft output viterbi algorithm (SOVA). Both the algorithms can be used as an iterative algorithm. SOVA is less complex compared to MAP but the performance of MAP is better compared to SOVA in terms of

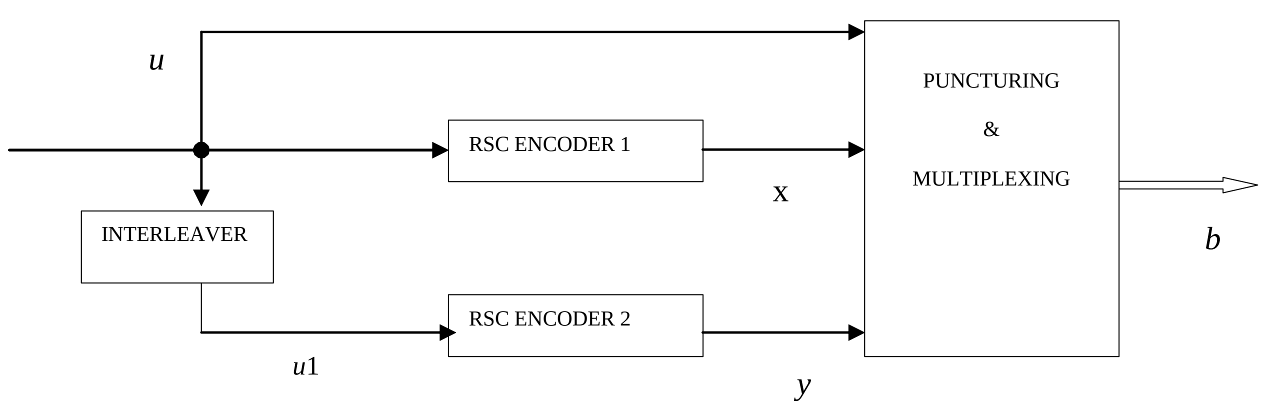

Figure 1. Structure of turbo encoder with puncturing.

the BER. In SOVA, a branch metric is assigned to each state transition for every state and for each decoding cycle. The path with the best path metric is selected. The path metrics are accumulated for different time instants. Based on the history of the best path, the values of the data are taken from a memory available in the decoder.

In the MAP decoder, for each transmitted symbol, an estimate is made on the received bit by means of *a posteriori* probability on the basis of the received sequence r. The decoder computes a log-likelihood ratio given by Eqn (2).

$$\Lambda(c_t) = \log \frac{P_r\{c_t = 1|r\}}{P_r\{c_t = 0|r\}} \quad (2)$$

for $1 \le t \le n$, where n is the received sequence length and compares this value to a zero threshold to determine the hard estimate The estimate of c_t is taken as 0 if the log-likelihood ratio is < 0, otherwise c_t is taken as 1. Based on the performance analysis of both the decoders, MAP algorithm proves to be better. In this study, iterative MAP algorithm is applied as its performance is better when the number of iterations are increased.

2.4 Rate-compatible Punctured Codes

Rate compatibility is a very important feature of rate-compatible punctured codes (RCPC). It makes these suitable for the implementation of variable-rate error control systems using a single encoder/decoder pair. The codes are obtained by puncturing of a single low-rate mother code. The codes form a hierarchy in the sense that the set of bit-positions that are punctured by a low-rate code is a subset of the set of bit-positions that are punctured by any code of strictly larger rate. The codes within a family of rate-compatible codes exhibit different code rates and error-correction capabilities, but can be encoded and decoded using the same encoder and decoder structures.

3. PUNCTURING PATTERN

For rate-compatible punctured turbo codes (RCPTC), the code choice consists essentially of finding the puncturing patterns satisfying the optimality criteria of code word weight and their multiplicities[5]. The structure of interleaver plays an important role in the performance of turbo codes. Random and block interleavers are the most commonly used interleavers[3]. Block interleaver formats the input sequence in a matrix of m rows and n columns, such that $N = m*n$. The input sequence is written into the matrix row-wise and read out column-wise. In the random interleaver, a block of N input bits is read into the interleaver and read out randomly. Since random interleaver gives better performance in terms of the BER, random interleaver is employed in this study.

The following design criteria are employed to find the best puncturing patterns.

- Define as d_w the minimum weight of code words generated by input words with weight w, and by N_w, the number of nearest neighbours (multiplicities) with weight d_w.
- Determine the pairs (d_w, N_w), for $w = 2,\ldots w_{max}$. Since w_{max} is infinite, the design algorithm can be applied with a defined value of w_{max}. In this paper, the algorithm has been applied for the weights taken from 2 to 8. Select the candidate yielding the optimum values for (d_w, N_w), ie, the one which sequentially optimises the pairs (d_w, N_w) (first d_w is maximised, and then N_w is minimised).

This paper proposes new puncturing patterns apart from the existing one, that is puncturing the bits alternatively from the two encoders. New puncturing patterns proposed are:

- Deletes two bits in the first component encoder and one in the second encoder and vice versa.
- Deletes three bits in the first component encoder and one in the second encoder and vice versa.
- Deletes four bits in the first component encoder and one in the second encoder.

3.1 Algorithm to Find the Best Puncturing Pattern

Step 1: Generate 2^N sets of input words and the two-component encoder encodes each input.

Input to the second encoder is given after interleaving through a random interleaver.

Step 2: The encoded outputs from the two encoders are then punctured using all the puncturing patterns, and the code word weights are calculated.

Step 3: Code weights are then separated wrt the different input word weight. Code word giving minimum weight d_w and their multiplicities N_w are chosen for all the corresponding information weight varying from weight 2.

Step 4: The puncturing pattern giving the maximum d_w and minimum N_w is considered to be the best puncturing pattern.

4. PERFORMANCE ANALYSIS

The following parameters were chosen for turbo codes for simulation:

- Generator polynomial: (111, 101)
- Decoder algorithm: MAP algorithm
- Interleaver structure: Random interleaver
- Channel: Additive white Gaussian noise
- Number of decoder iterations: 5

The algorithm has been applied for a frame size of 10 bits and the best puncturing patterns were found.

In Table 1 are listed corresponding code word weights d_w after puncturing for various input word weights and their multiplicities N_w. The results are taken for six puncturing patterns and in applying the optimal criteria leads to a $d_w = 15$ for puncturing pattern 3 and puncturing pattern 4. Since puncturing pattern 4 has lesser value of N_w, puncturing pattern 4 is taken as the optimal pattern. The simulations have been run for 500 frames of frame size 100 bits. The performance analysis of the chosen puncturing pattern is compared with the existing puncturing scheme and also with an unpunctured code. The simulation results are shown in Fig. 2. The proposed puncturing pattern gives a gain of 0.5 dB at a BER of 10^{-3} compared to the existing scheme.

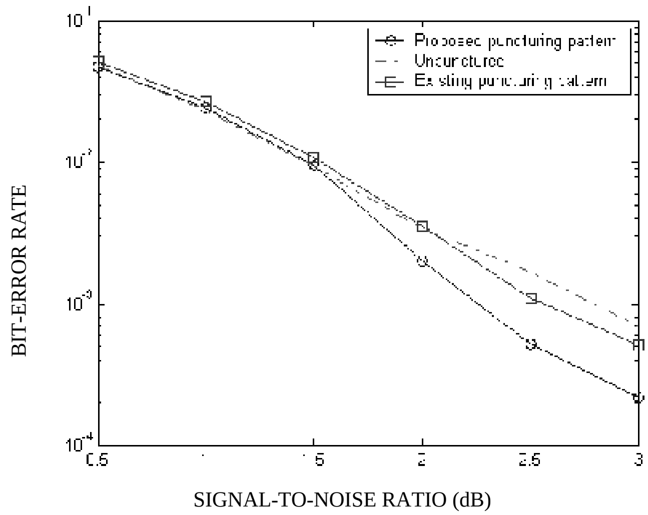

Figure 2. Performance analysis of proposed pattern with unpunctured code and the existing puncturing pattern.

Table 1. Code weight and their multiplicities (d_w, N_w) for puncturing patterns

Input word	Pattern 1		Pattern 2		Pattern 3		Pattern 4		Pattern 5		Pattern 6	
(d_w, N_w)	d_w	N_w	d_w	N_w	d_w	N_w	d_w	N_w	d_w	N_w	d_w	N_w
Weight 2	4	1	4	1	4	3	5	3	3	1	4	1
Weight 3	3	1	4	2	4	4	4	3	4	3	5	10
Weight 4	4	1	6	9	5	1	4	1	5	2	5	2
Weight 5	7	3	7	1	5	1	7	6	7	1	6	1
Weight 6	9	8	8	1	8	1	9	9	7	1	7	1
Weight 7	9	1	10	1	9	1	9	3	11	4	10	1
Weight 8	14	1	12	1	15	2	15	1	13	2	13	2
$\max_{(d_w)}$	14	1	12	1	15	2	15	1	13	2	13	2

5. CONCLUSION

Puncturing has been applied to turbo codes and the best puncturing patterns are chosen. The study is based on optimality criterion of code word weight and their multiplicities. These are tested for their performance in terms of the bit-error probability and it gives better results than the unpunctured code. The results show a lowering of the error floor. Hence, it is possible to achieve bandwidth-efficient transmission in defence applications where wireless transmission is desirable.

Future research should be to analyse the performance of these puncturing patterns for a fading channel. The algorithm can be applied for different interleaver structures.

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