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Gravity Modulation Effects on Transient-free Convection Flow Past an Infinite Vertical Isothermal Plate

R.K. Deka[1] and V.M. Soundalgekar[2]

[1]*Gauhati University, Guwahati*-781 014
[2]31A-12, *Brindavan Society, Thane*-400 601

ABSTRACT

An exact solution to the free convection flow influenced by gravity modulation has been derived by the Laplace-transform technique. Velocity profiles, penetration distance from the leading edge, and the skin-friction expressions are derived. Velocity profiles and penetration distance are shown graphically, and the numerical values of the skin friction are tabulated. It was observed that the transient velocity decreased with increasing the frequency ω of gravity-modulation or the Prandtl number but increased with increasing time t. Also, transition from conduction to convection was delayed with increasing ωt, but it occured nearer the leading edge as the frequency ω or the Prandtl number increased.

Keywords: Gravity modulation, free convection flow, penetration distance, transient-free convection

NOMENCLATURE

C_p	Specific heat at constant pressure
g_o	Acceleration due to gravity
k	Thermal conductivity
t'	Time
T	Dimensionless time
T'	Temperature of fluid
T'_w	Temperature of the plate
T'_∞	Temperature of the fluid away from the plate
u'	Velocity component
u	Non-dimensional velocity component
u_o	Reference velocity
X_p	Penetration distance
ε	Amplitude
β	Coefficient of volume expansion
ν	Kinematic viscosity
μ	Dynamic viscosity
Pr	Prandtl number
ρ	Fluid density
θ	Dimensionless temperature
τ	Dimensionless wall shear stress
ω'	Frequency
ω	Non-dimensional frequency
L, t_o	Reference length, time
y'	Coordinate normal to the vertical plate
y	Non-dimensional coordinate normal to the plate

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p, q, X, Y	Dummy variables
f, f_1, f_2 f_3, H	Functions of dummy variables
η,	$y/(2\sqrt{t})$
a, b	Constants

1. INTRODUCTION

From convection flows of an incompressible viscous fluid past, a semi-infinite or infinite vertical plate were studied during the past many years. Both steady and unsteady cases were studied because of their important applications in atmospheric and oceanic circulation, power transformer, nuclear reactor, vortex chambers, etc.

Transient-free convection flows past an infinite vertical plate were studied extensively in the 1960's by Gebhart[1], Chung and Anderson[2], Schetz and Eichhorn[3], Goldstein and Briggs[4], etc, who presented exact solutions under different physical situations. Siegel[5] studied unsteady-free convection flow near a semi-infinite vertical plate due to step-change in wall temperature or surface heat flux employing the momentum integral method, and established that the initial behaviour of the temperature and velocity fields for the semi-infinite vertical plate is the same as for the doubly infinite vertical flat plate.

Under this condition, the temperature field is the solution of an unsteady one-dimensional heat conducting equation. The transition from conduction to convection begins only when some effect from the leading edge has propagated up the plate to the particular point in question. This phenomenon was first presented by Siegel[5] and before this, it was not known that the plate has a leading edge. These studies were devoted to the physical situation where all physical properties of the fluids and all the forces acting were constant.

However, oscillating buoyancy forces also exist in nature and in microgravity, these flows are induced by a modulating gravity field or g-jitter. Compared with earth-based systems, the flow intensity is found to be drastically reduced as a result of a significant reduction in gravitational forces. A large number of papers have been published on the well-known topic of Benard convection under modulated gravity field in the case of both linear and nonlinear phenomena. Noted are those by Clever[6,7], *et al.*, and Fu and Shieh[8], etc.

In space technology, in space vehicle, there are transient or time-varying perturbations to the gravity field at a point. This g-jitter phenomenon arises from spacecraft manoeuvres and mechanical vibrations. An excellent account of this physical feature has been described by Ostrach[9]. Li[10], has studied the effect of g-jitter and the magnetic field on the natural convection phenomenon in a channel bounded by long vertical plate. He has represented the g-jitter as a sum of Fourier harmonic components with distinct frequencies and amplitudes, and studied different types of natural convection in magneto-hydrodynamic (MHD) channel flow.

However, the effect of oscillating gravitational field on the transient-free convection has not been studied in the past as literature study reveals. Because of good applications of this phenomenon, both in space and on the earth, it is necessary to study this phenomenon, and hence, it is now proposed to study the effect of oscillating gravity field on the transient-free convection flow past an infinite vertical isothermal plate.

2. MATHEMATICAL ANALYSIS

Consider unsteady-free convection flow of an incompressible viscous fluid past an infinite vertical plate. The x'-axis is taken along the plate in the vertical direction and the y'-axis is taken normal to the plate. Initially, the plate and the fluid are assumed to be at a uniform temperature T_∞' and stationary. At time $t' > 0$, the plate temperature is instantaneously raised to T_w' and stationary. At time $t' > 0$, the plate temperature is instantaneously raised to T_w' causing a temperature difference $T_w' - T_\infty' > 0$ which then causes convective motion of the fluid near the plate, and at the same time, the gravitational acceleration also starts oscillating with frequency ω' and amplitude ε, which is assumed constant. Then under usual Boussinesq's approximation, the flow can be shown to be governed by the following equations:

$$\frac{\partial u'}{\partial t'} = \beta g_0 (1 + \varepsilon \sin \omega' t')(T' - T'_\infty) + \nu \frac{\partial^2 u'}{\partial y'^2} \quad (1)$$

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y'^2} \quad (2)$$

with the following initial and boundary conditions:

$$\left.\begin{array}{l} u' = 0,\ T' = T'_\infty \ \text{for all}\ y',\ t' \le 0 \\ \left.\begin{array}{l} u' = 0,\ T' = T'_w \ \text{at}\ y' = 0 \\ u' = 0,\ T' = T'_\infty \ \text{as}\ y' \to \infty \end{array}\right\} t' > 0 \end{array}\right\} \quad (3)$$

As the plate is assumed to be infinite in length, the physical variables are functions of y' and t' only. Also, all the physical variables are defined in the nomenclature. The following scaling factor may be chosen for the velocity, length, and time.

$$\begin{aligned} u_o &= \left\{ \nu g_o \beta \left(T'_w - T'_\infty \right) \right\}^{1/3} \\ L &= \left\{ g_o \beta \left(T'_w - T'_\infty \right) / \nu^2 \right\}^{-1/3} \\ t_o &= \left\{ g_o \beta \left(T'_w - T'_\infty \right) \right\}^{-2/3} / \nu^{-1/3} \end{aligned} \quad (4)$$

and then the non-dimensional quantities are defined as follows:

$$\begin{aligned} & u = u'/u_0,\ y = y'/L,\ t = t'/t_0,\ \omega = \omega' t_0 \\ & \theta = (T' - T'_\infty)/T'_w - T'_\infty),\ Pr = \mu C_p / k \end{aligned} \quad (5)$$

Introducing Eqn (5) into Eqns (1)-(3), one has

$$\frac{\partial u}{\partial t} = \theta (1 + \varepsilon \sin \omega t) + \frac{\partial^2 u}{\partial y^2} \quad (6)$$

$$\Pr \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} \quad (7)$$

with the following initial and boundary conditions:

$$\left.\begin{array}{l} u = 0,\ \theta = 0 \ \text{for all}\ y, t \le 0 \\ \left.\begin{array}{l} u = 0,\ \theta = 1 \ \text{at}\ y = 0 \\ u = 0,\ \theta = 0 \ \text{as}\ y \to \infty \end{array}\right\} t > 0 \end{array}\right\} \quad (8)$$

These are coupled linear system of equations, which can be solved by the usual Laplace-transform technique. Then the solutions are:

$$\theta = erfc\left(\eta \sqrt{\Pr} \right) \quad (9)$$

$$\begin{aligned} u = & \frac{1}{Pr - 1} \left[H(\eta) - H(\eta \sqrt{Pr}) \right] \\ & + \frac{\varepsilon}{\omega} Rl \left\{ \begin{array}{l} e^{iat} f(\eta, a) - e^{bt} f(\eta, b) \\ + e^{bt} f(\eta \sqrt{Pr}, b - a) \end{array} \right\} \\ & + \frac{\varepsilon \cos \omega t}{\omega} erfc(\eta \sqrt{Pr}) \end{aligned} \quad (10)$$

where

$$\eta = y / 2\sqrt{t},\ a = i\omega,\ b = aPr/(Pr - 1)$$

$$f(p, q) = \frac{1}{2} \left\{ \begin{array}{l} e^{2p\sqrt{qt}} erfc(p + \sqrt{qt}) + \\ e^{-2p\sqrt{qt}} erfc(p - \sqrt{qt}) \end{array} \right\}$$

$$H(p) = t \left\{ (1 + 2p^2) erfc(p) - 2pe^{-p^2} / \sqrt{\pi} \right\}$$

Here $R1\{\ \}$ denotes the real part of the quantity enclosed and p, q are dummy variables. Moreover, the temperature profile given by Eqn (9) is well known.

To understand the effect of different parameters like Pr, ω, etc. the numerical values of u have been computed by taking the real part of Eqn (10) and these are plotted in Figs 1 and 2. It has been observed from these figures that the transient velocity increases with increasing ωt for both air and water, but the transient velocity decreases with increasing ω for both air and water. The transient velocity also decreases with increasing the Pr.

Now, it is interesting to understand the effect of gravity-modulation on the transition from conduction to convection, which is studied by deriving the

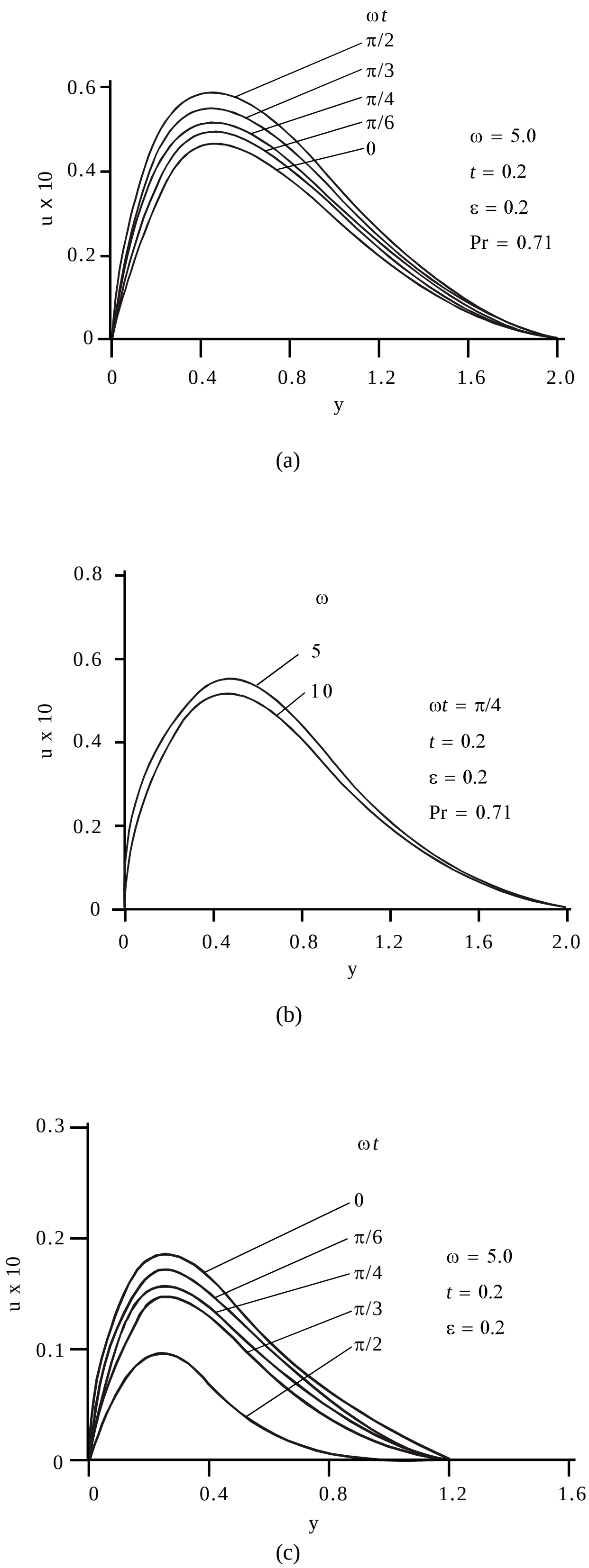

Figure 1. Velocity profiles.

penetration distance of the point from the leading edge. Here, the leading edge is defined as the distance from any point x' = 0 taken along the infinite plate in the upward direction.

It is given in non-dimensional form as and in terms of the Laplace transform and inverse transform, one has

$$X_p = \int_0^t u(y,t)dt \tag{11}$$

Substituting for $\overline{u}(y,s) = L\{u(y,t)\}$ from Eqn (10) in Eqn (11) and carrying out the algebra, one has

$$X_p = L^{-1}\left\{\frac{1}{s}\overline{u}(y,s)\right\} \tag{12}$$

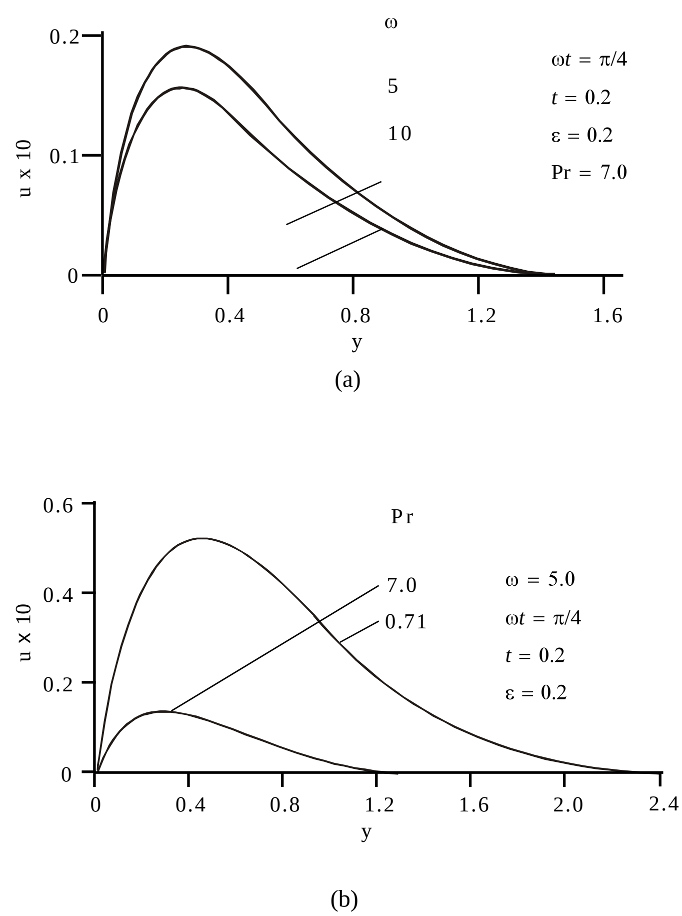

Figure 2. Velocity profiles.

$$
\begin{aligned}
X_p = {} & \frac{1}{Pr-1}\left[f_3(\eta) - f_3(\eta\sqrt{Pr})\right] \\
& + \frac{\varepsilon}{\omega} Rl \left[\begin{array}{l} \frac{1}{a}\left\{ e^{iat} f(\eta, a) - erfc(\eta) + f(\eta\sqrt{Pr}, -a)\right\} \\ + \frac{1}{b}\left\{ erfc(\eta) - e^{bt} f(\eta, b) + e^{bt} f(\eta\sqrt{Pr}, b-a) \right. \\ \left. - f(\eta\sqrt{Pr}, -a) \right\} \end{array} \right] \\
& - \frac{\varepsilon \cos \omega t}{a\omega} erfc(\eta\sqrt{Pr})
\end{aligned} \tag{13}
$$

where

$$
f(X, Y) = \frac{1}{2}\left\{ \begin{array}{l} e^{2X\sqrt{Y_1 1}} erfc(X + \sqrt{Yt}) \\ -e^{2X\sqrt{Yt}} erfc(X - \sqrt{Yt}) \end{array} \right\}
$$

$$
f_1(X) = 2\sqrt{t}\left\{ \frac{e^{-X^2}}{\sqrt{\pi}} - X erfc(X) \right\}
$$

$$
f_2(X) = \frac{2t}{3} f_1(X) - \frac{2X\sqrt{t}}{3} H(X)
$$

$$
f_3(X) = \frac{t}{2} H(X) - \frac{X\sqrt{t}}{2} f_2(X)
$$

where X, Y are dummy variables. Xp has been evaluated numerically and the variations in penetration distance are shown in Figs 3 and 4. It has been observed from these figures that the transition from conduction to convection is delayed as ωt increases for both air and water, whereas transition from conduction to convection occurs nearer the leading edge as ω increases or with increasing Pr.

From the velocity field, it is now proposed to study the skin friction. It is given in non-dimensional form by

$$
\tau = -\left.\frac{du}{dy}\right|_{y=0} \tag{14}
$$

Then from Eqns (10) and (14), one has

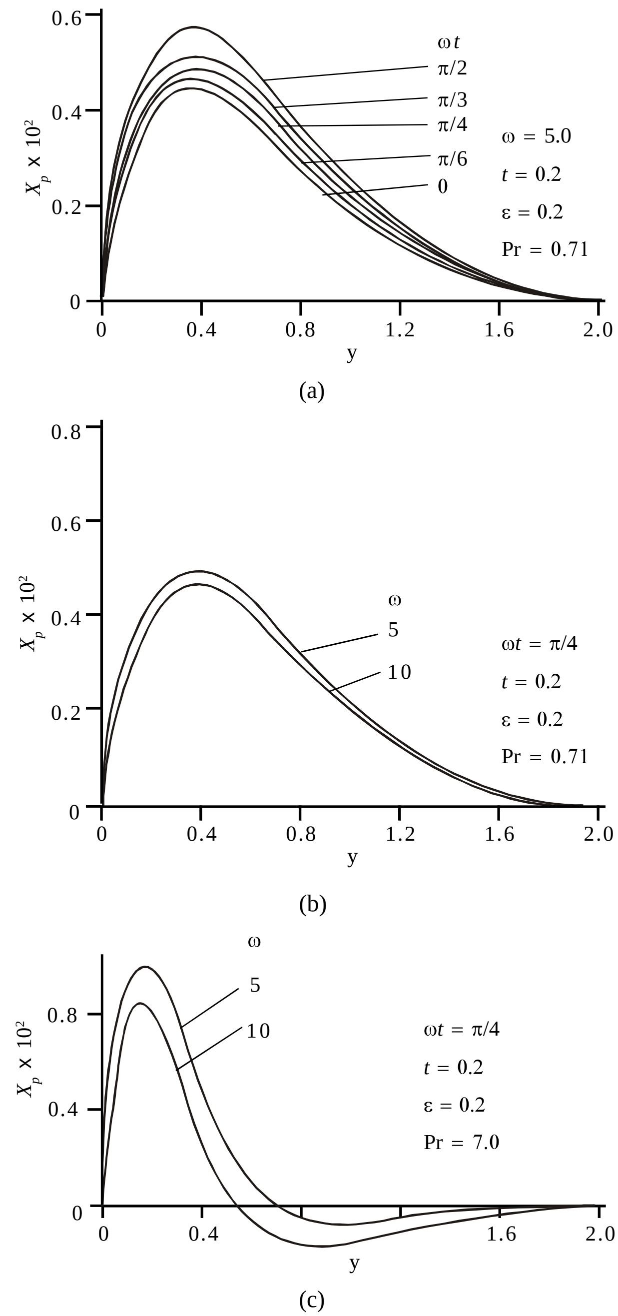

Figure 3. Penetration distance.

$$
\begin{aligned}
\tau = {} & \frac{1}{Pr-1}\left[\frac{2\sqrt{t}\left(1-\sqrt{Pr}\right)}{\sqrt{\pi}} \right] - \\
& \frac{\varepsilon}{\omega} Rl \left\{ \begin{array}{l} \sqrt{a} e^{at} erf(\sqrt{at}) - e^{bt}\sqrt{b} erf\sqrt{bt} + \\ \sqrt{b} e^{bT} erf(\sqrt{(b-a)t}) \end{array} \right\} \\
& + \frac{2\varepsilon}{\omega}\frac{\sqrt{Pr}}{\sqrt{\pi t}} \cos \omega t
\end{aligned} \tag{15}
$$

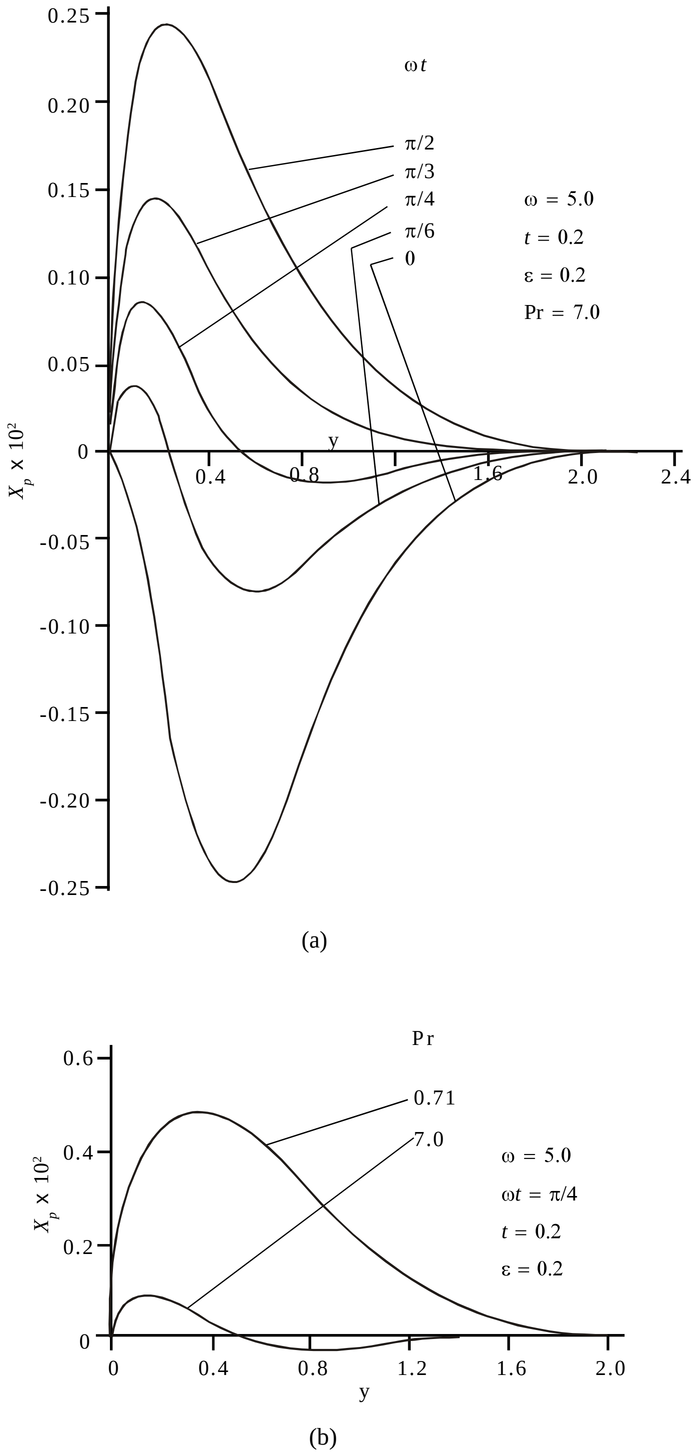

Figure 4. Penetration distance.

The numerical values of (τ) for ε = 0.2 and t = 2 have been computed and are listed in Table 1.

It has been concluded from Table 1 that with increasing ω or Pr, the skin friction decreases.

Table 1. Values of τ for ε = 0.2, *t* = 2

ω	ω*t*	Pr = 0.71	Pr = 7.0
5	0	0.5605	0.4211
5	π/6	1.9936	1.8359
5	π/4	1.4204	1.2692
5	π/3	1.1337	0.9865
5	π/2	0.8471	0.7038
10	π/4	0.8539	0.7027
15	π/4	0.6492	0.4979

3. CONCLUSIONS

The following conclusions have been drawn:

(a) The transient velocity increases with increasing the frequency ω or Pr but increases with increasing ω*t*.

(b) The transition from conduction to convection is delayed due to increasing ω but occurs nearer the leading edge as ω or Pr increases.

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Contributors

Dr R.K. Deka is Senior Lecturer of Mathematics at the Gauhati University. He visited Taiwan as Resesrcch Associate for few months. He has published more than 50 papers.

Dr V.M. Soundalgekar has published more than 500 papers, 90 of these are referred in the books published in USA/Russia/India. He has delivered lectures in many universities in Canada/USA/UK/Belgium/Germany/Czechoslovakia and presented papers at many international conferences. He has guided 40 research scholars for their PhDs.