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Estimation of Rheological Properties of Snow Subjected to Creep

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ABSTRACT

Creep is one of the most important phenomenons to determine the settlement of snow. Snow, in natural conditions, exists at temperature quite close to its melting point and deforms very fast. The settlement of snow is the result of creep phenomenon under the action of overburden pressure as well as due to metamorphic processes going on within the snowpack. In this communication, creep behaviour of snow is simulated with four-parameter viscoelastic fluid model. This viscoelastic character is basically controlled or monitored by various rheological constants. Estimation of all the rheological constants in the four-parameter viscoelastic fluid model appropriate for the creep properties of snow is done. Total 91 uniaxial unconfined constant stress experiments on sieved snow were conducted at controlled temperature conditions. The effect of density and varying temperature on these constants is found to be remarkable.

Keywords: Snow, snowpack, creep, rheological model creep behaviour, visco-elastic model, rheological properties, snow mechanics, creep deformation behaviour, Maxwell model, Burger's model

NOMENCLATURE

$\varepsilon, \varepsilon_1, \varepsilon_2, \varepsilon_3$	Strains
σ	Stress
E_1, E_2	Spring stiffness or elasticity modulus
$\dot{\varepsilon}, \dot{\varepsilon}_1, \dot{\varepsilon}_2, \dot{\varepsilon}_3$	Strain rates
t	Time
η_1, η_2	Coefficients of viscosity
α	Initial slope of creep curve
β	Slope of secondary creep curve
χ^2	Chi square function
σ_1	Standard deviation

1. INTRODUCTION

Snow is considered as viscoelastic material with highly variable morphological properties. However, no demarcation exists to discern the bounds of linearity in terms of either stress or time. Most of the descriptive and experimental studies were done between 1930 to 1980 and were reviewed by Bader[1], Mellor[2,3], Salm[4], and Shapiro[5], *et al*. The objective was to determine the parameters required for application of linear elasticity, viscosity, and viscoelasticity to problems of snow mechanics. The effort followed the recognition that some creep deformation behavior in snow samples in a laboratory or field setting could be described by linear relationships. Bader[6], *et al*. discussed the creep of snow for snow settlement. Experiments on samples in both uniaxial confined and unconfined compression were conducted, but

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no attempt was made to formulate a constitutive relationship to describe the process. Yosida[7], *et al*. used the data from Bader[6], *et al*. to calculate values for the coefficient of Newtonian viscosity of snow. The most general constitutive relationship used for snow prior to about 1970 was the equation for a four-parameter viscoelastic fluid with linear elements. (Eqn). De Quervain[8], to interpret the results of torsion experiments, first used it in snow mechanics. Bucher[9] included a sketch of a Maxwell model (a spring and dashpot in series, as shown in Fig. 1 and used the constitutive relationship for a linear viscous fluid to find the coefficient of Newtonian viscosity for compacted snow as a function of temperature, duration of loading, and a variety of snow, grain sizes, and ages. Interestingly, although the Maxwell model includes a spring element, Bucher made no mention of the elastic properties of snow. Yosida[10], *et al*. measured Young's modulus of snow in static uniaxial compression tests. Bader1 also suggested that the one-dimensional hyperbolic sine relationship:

$$\frac{d\varepsilon}{dt} = \varepsilon_o \sinh(A\sigma) \quad (1)$$

Where ε is the strain, σ is the stress and t is time, and ε_o and A are constants might be used to describe creep behaviour of snow; and could replace the linear relationship for the dashpot of the Maxwell element of the four-parameter viscoelastic fluid model. Mellor[2] introduced an additional term into Eqn (1) by dividing the coefficient of the hyperbolic sine by a viscosity coefficient, η. He also discussed the use of exponential and power relationships to represent compactive viscosity (i.e., the viscosity determined from the compaction of natural snowpacks or from confined compression experiments in the laboratory) in terms of the snow density as derived from data sets collected by various investigators.

Constitutive relationship is the equation that mathematically represents the stress-strain behaviour of any material. The relationship is unable to fulfill its objective unless all the constants used in the relation for the material are known. Various researchers have analysed elastic and viscous behaviours of snow in detail, but the transient creep (i.e., primary creep) remained untouched. In this communication, mainly the transient phase of creep is dealt. Here, the creep phenomenon is implemented by simple rheological model rather than complicated constitutive relations. The model used is the well-known four-parameter viscoelastic fluid model and simulates the creep behaviour of snow with a good correlation. Constants of the four-parameter viscoelastic fluid model are determined by the Levenberg-Marquardt method. The dependence of rheological constants on snow density and temperature is studied in the present work.

2. THEORY

The idea of using the law, for what is now called the four-parameter viscoelastic fluid to represent the deformation of a material, was apparently first proposed by Nadai[11] based on observations of experimental creep curves. Any material that responds to loads by three distinct types of strains $\varepsilon_1, \varepsilon_2, \varepsilon_3$ and two types of stresses, σ_1, σ_2. The three types of strains are: (i) an ideal elastic strain that respond instantly to changes in stress, (ii) a component of permanent strain that changes as a function of time and load, and (iii) a semi-permanent, recoverable strain which represents a time-dependent elastic response to the applied load. The familiar spring-dashpot model for this material [shown in Fig. 1(a) with linear springs and dashpots] illustrates how the total strain results from the summation of the three distinct types of strains listed above. The first two are from the Maxwell model, while the third is from the Kelvin model. Summing the strain components in this manner implies the assumption that these are independent of each other. This effect has never been studied experimentally. The two types of stresses referred to by Nadai[11] are simply the stresses across the arms of the Kelvin model (Fig. 1). Equilibrium requires that these sum to the magnitude of the stress, σ, applied across the model. The summation and integration of the strains for a constant stress so applied at time t = 0 to the model with linear elements, leads to equation (Findley[12]).

$$\varepsilon(t) = \sigma\left\{\frac{1}{E_1} + \frac{t}{\eta_1} + \frac{1}{E_2}\left(1 - e^{-\frac{E_2 t}{\eta_2}}\right)\right\} \quad (2)$$

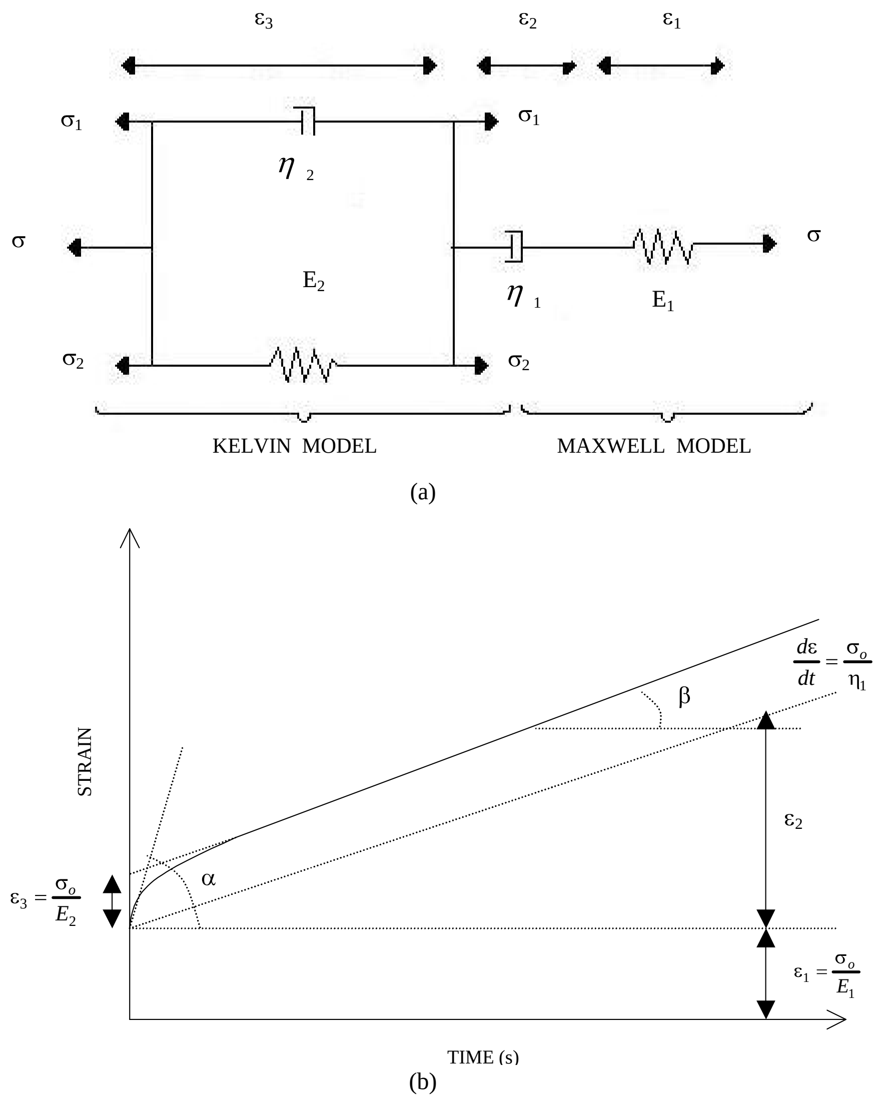

Figure 1. (a) Four-parameter spring-dashpot model of a visco-elastic fluid showing nomenclature, and (b) stress-strain laws and relationships for determining values of the parameters from a creep curve.

A plot of this equation approximates a creep curve as shown in Fig. 1(b), the parameters in Eqn (2) can be determined from a single experimentally-derived creep curve. The four-parameter viscoelastic fluid model has four parameters (material constants), which describe the response of material under constant stress. These four parameters are spring stiffness of Maxwell model, coefficient of viscosity of Maxwell model dashpot, spring stiffness of Kelvin model, and coefficient of viscosity of Kelvin model dashpot.

Instantaneous strain simulation is taken care of by the spring in series; this strain is directly proportional to the stress applied, so E_1 represents the Young's modulus of elasticity. Similarly η_1 represents the coefficient of viscosity of secondary creep. E_2 and η_2 take care of the transient creep (primary creep) or delayed elasticity. Evaluation of these parameters is done with the help of experimental creep curve. From experimental creep curve, the values of parameters are estimated as follows:

Creep rate $\dot{\varepsilon}$ at any time t is

$$\dot{\varepsilon} = \frac{\sigma}{\eta_1} + \frac{\sigma}{\eta_2} e^{-E_2 t/\eta_2} \tag{3}$$

Thus, the creep rate at time t = 0+ with a finite value

$$\dot{\varepsilon}(0+) = \frac{\sigma}{\eta_1} + \frac{\sigma}{\eta_2} = \tan(\alpha) \tag{4}$$

and approaches asymptotically to the value

$$\dot{\varepsilon}(\infty) = \frac{\sigma}{\eta_1} = \tan(\beta) \tag{5}$$

It may be observed from Figs 1(a) and 1(b) that $\varepsilon_1=\sigma/E_1$ and $\varepsilon_3=\sigma/E_2$. Thus in theory, the material constants E_1, E_2, η_1,η_2 may be estimated from a creep experiment by measuring α, β, ε_1 and ε_3 from the creep curve. There is a practical problem to measure the values of α, β, ε_1 and ε_3; as initial creep rate is so rapid that a value of α other than 90^o is hard to justify. For the same reason, ε_1 and ε_3 are almost impossible to determine directly. Thus, these values of material constants estimated from experimental creep curve may further be optimised by employing statistical methods and by minimising the errors.

3. OPTIMISATION OF PARAMETERS

The parameters in four-parameter rheological model (i.e., Burger's model) are estimated manually from creep data and then optimized in a regression analysis. Here, a standard nonlinear regression method the Levenberg-Marquardt method is used to perform the data fitting and optimisation of parameters. This method combines the steepest-descent method and a Taylor series-based method to obtain a fast, reliable technique for nonlinear optimisation. Neither of the above optimisation methods are ideal at all the time; the steepest descent method works best far away from the minimum and the Taylor series method works best close to the minimum. The Levenberg-Marquardt (L-M) algorithm allows for a smooth transition between these two methods as the iteration proceeds.

The four-parameter modelling equation (with one independent variable) can be written as follows:

$$\varepsilon = \varepsilon(t, E_1, \eta_1, E_2, \eta_2) \tag{6}$$

The expression simply states that the dependent variable ε can be expressed as a function of the independent variable t and parameters E_1,η_1,E_2,η_2. Then, the merit function (Chi-square function) to be minimise is

$$\chi^2\left(E_1,\eta_1,E_2,\eta_2\right) = \sum_{i=1}^{N}\left(\frac{\varepsilon_i - \varepsilon\left(t_i;E_1,\eta_1,E_2,\eta_2\right)}{\sigma_i}\right)^2 \tag{7}$$

where N is the number of data points, t_i denotes the t data points, ε_i denotes the ε data points, σ_i is the standard deviation (uncertainty) at point i, and $\varepsilon(t_i, E_1, \eta_1, E_2, \eta_2)$ is an arbitrary nonlinear model evaluated at the i^{th} data point. This merit function simply measures the agreement between the data points and the parametric model; a smaller value for the merit function denotes better agreement.

4. EXPERIMENTATION

To study the creep behaviour of sieved snow, experiments were conducted in unconfined compression mode of loading. All experiments were carried out in cold room Manali (Himanchal Pradesh, India) under controlled conditions.

4.1 Sample Preparation

For sieved snow experiments, samples were prepared using snow collected from Patseo. Snow was sieved using mechanical sieve shaker in cold laboratory to get adequate snow grain size in the range 0.5 mm to 1.0 mm. Sieved snow grains were filled up into the aluminum samplers using a brush. Silicon grease was applied along the inner periphery of sampler before filling, for easy and safe removal of sample. After the sample preparation, it was allowed to undergo aging for seven days at –20 ^{o}C. Unconfined compression tests were conducted on cylindrical snow samples of 150 mm height and 65 mm dia. Density was measured by weighing the sampler and snow. Dimensions of samplers used were the same for all the experiments.

4.2 Experimental Setup

The experiments were conducted at different temperature levels of –3 °C, –6 °C and –10 °C. Density range of snow for all experiments was 180 kg/m^3 to 450 kg/m^3. Samples were extracted from the sampler using plunger (after seven days of aging) and maintained at the experimentation temperature for 2-3 h. The sample was mounted on a hydro-mechanical universal-testing machine (UTM) with electronic controls. Stresses applied to the samples were in the range of 4 KPa-50 KPa. All inputs and outputs were controlled and recorded using an electronic console and Dartec software.

5. RESULTS AND DISCUSSION

The four parameters (namely E_1, E_2, η_1, η_2) used in four-parameter viscoelastic fluid model creep compliance have been calculated for 91 unconfined compression creep tests, in the density range 180-450 kg/m^3. The parameters are first estimated

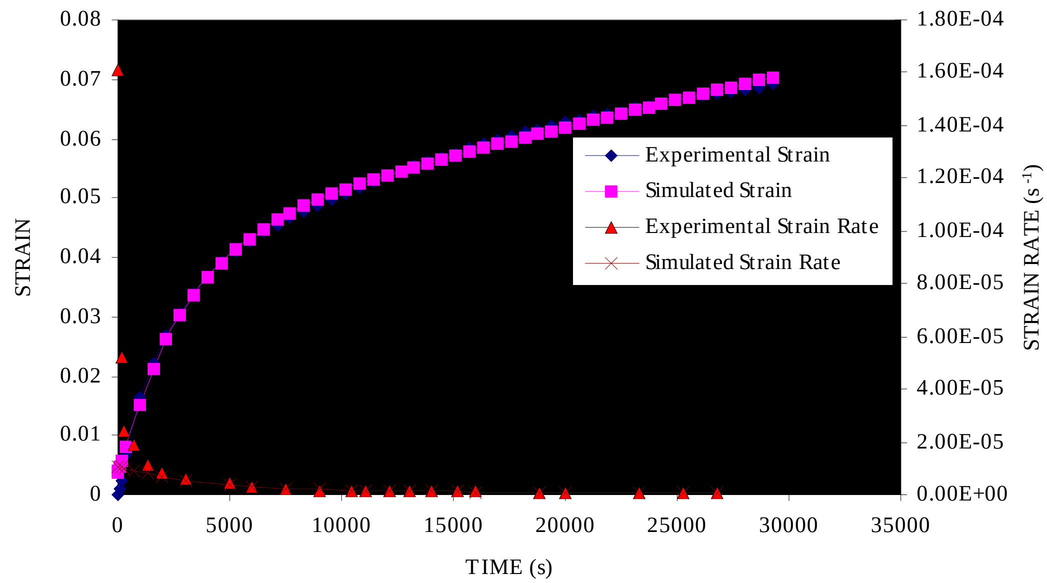

Figure 2. Experimental and simulated strain and strain rate curves at –3 °C.

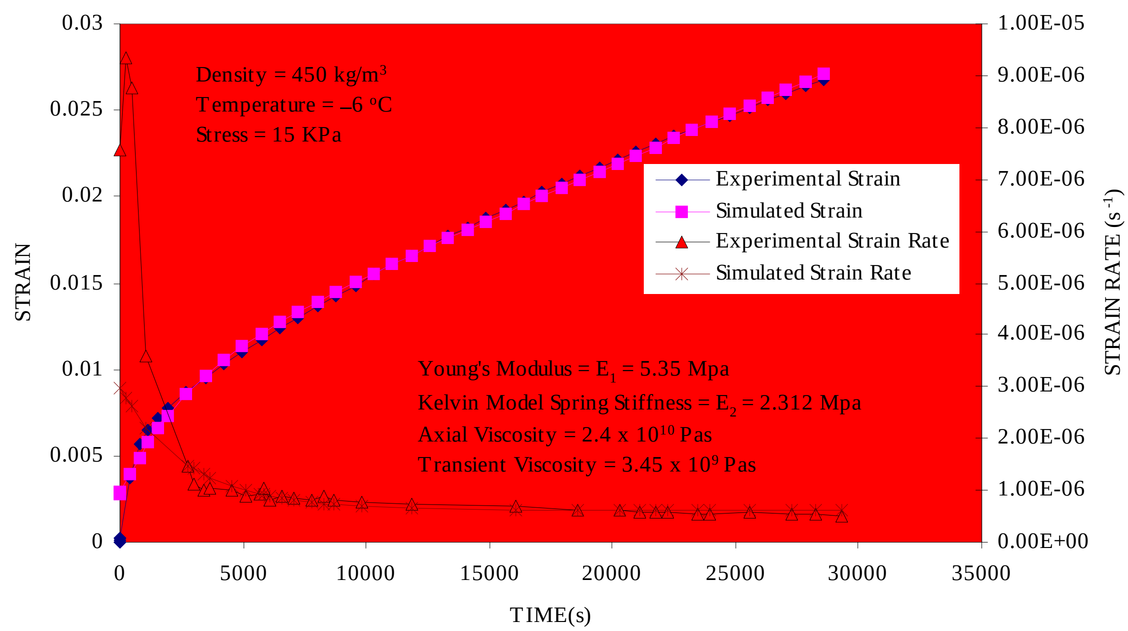

Figure 3. Experimental and simulated strain and strain rate curves at –6 °C.

and then optimised using numerical method, i.e., the Levenberg-Marquardt method. Mellor[3] described the range 0.11 - 20 MPa for static Young's modulus and 10^2-10^6 MPa-s for axial viscosity in the density range 100 - 400 kg/m^3, while in this work, the values of spring stiffness of Maxwell model (Young's modulus) are found to be in the range 1.08 MPa to 13.4 MPa (Fig. 5). The spring stiffness of Kelvin's model (modulus responsible for delayed elasticity) is found to be in the range 0.0634 MPa to 12.9 Mpa (Fig. 6), axial viscosity is found to be in the range 3.21 x 10^9 Pas to 4.37 x 10^{10} Pas (Fig. 7) and the viscosity coefficient of dashpot (in parallel) responsible for transient creep, varies from 3 x 10^8 Pas to 1.92 x 10^{10} Pas (Fig. 8). It is clear from Fig 2, 3 and 4 that the values calculated from experiments simulates the creep curve with a good correlation. Coefficient of correlation between the experimental strain and simulated creep curve is around 0.98 for all the experiments.

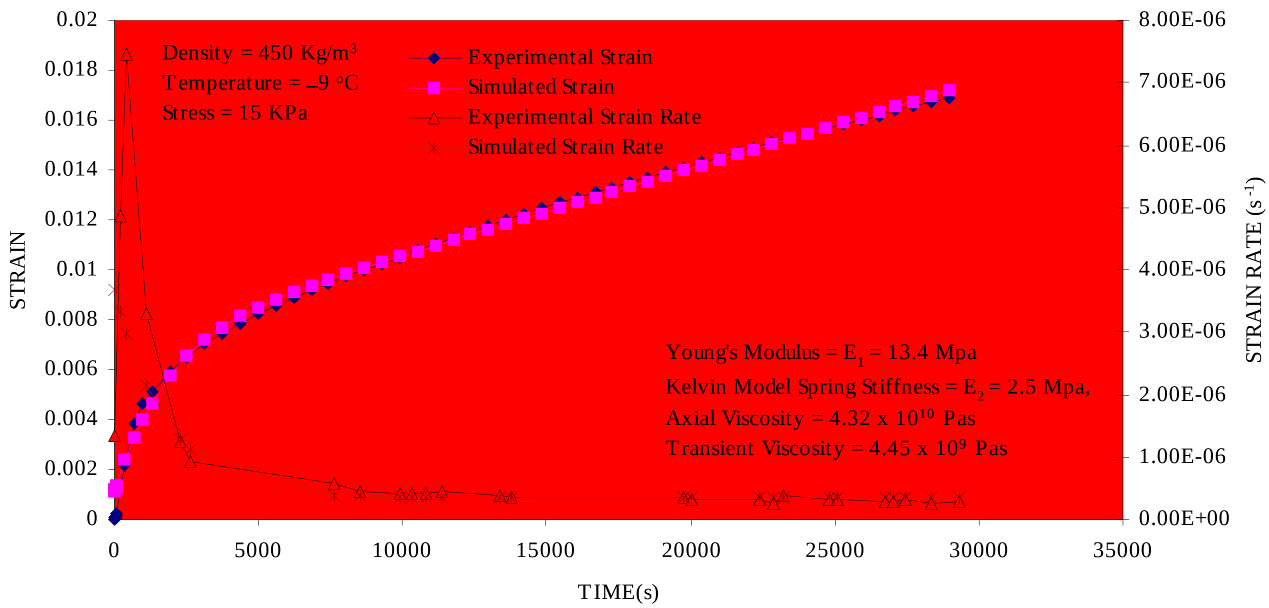

Figure 4. Experimental and simulated strain and strain rate curves at –9 °C.

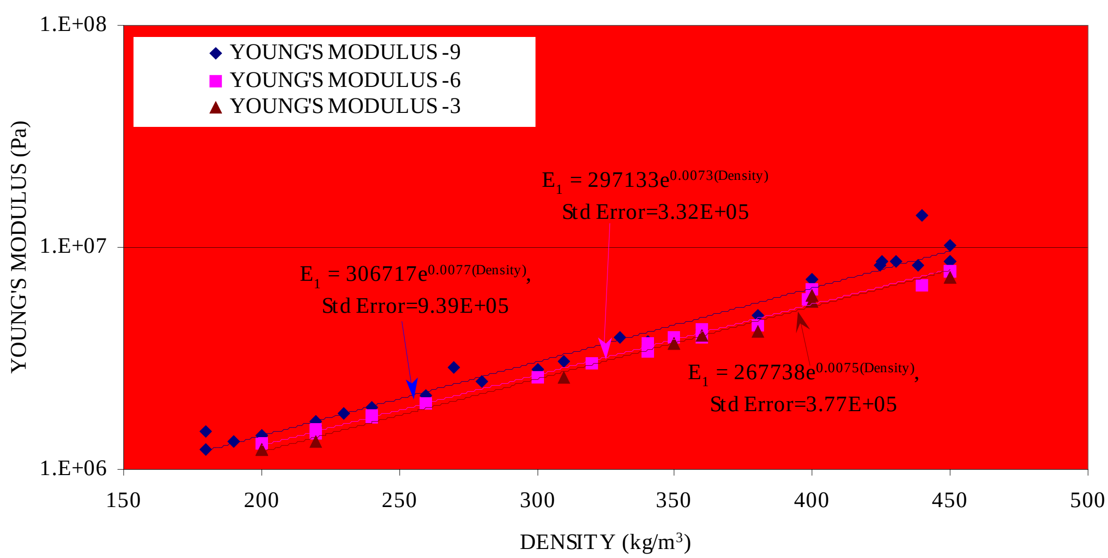

Figure 5. Effect of density and temperature on Young's modulus.

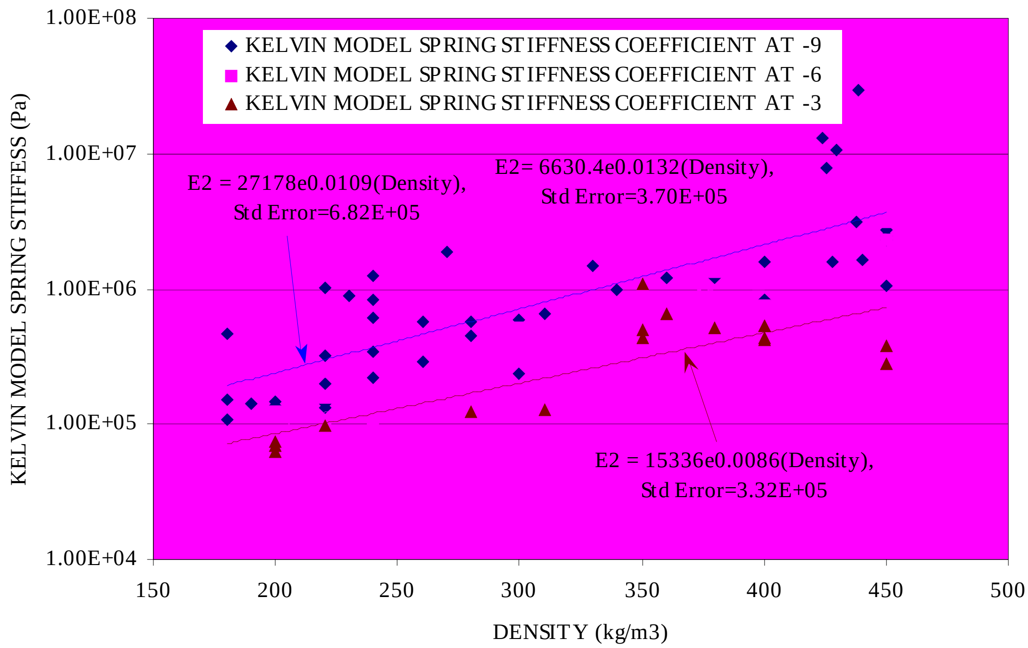

Figure 6. Effect of density and temperature on Kelvin model spring stiffness coefficient.

It can also be seen that the difference in the strain rate curves is large initially, but after some time, both strain rates are almost the same. The difference in the strain rates is because of the stress rate applied to achieve the required constant stress (stress rate = required constant stress/time to achieve the stress), as on UTM machine, stress cannot be applied all of a sudden. The successive stresses correspond to greater strains every time (because of elastic properties), which give rise to higher strain rate in experimental strain rate curve.

The effect of density and temperature on all the four parameters is prominent. The exponential increase in all the four parameters is observed wrt density. This is shown in the respective graphs by fitting exponential regression equations for all the parameters. The deviation in the measured and the

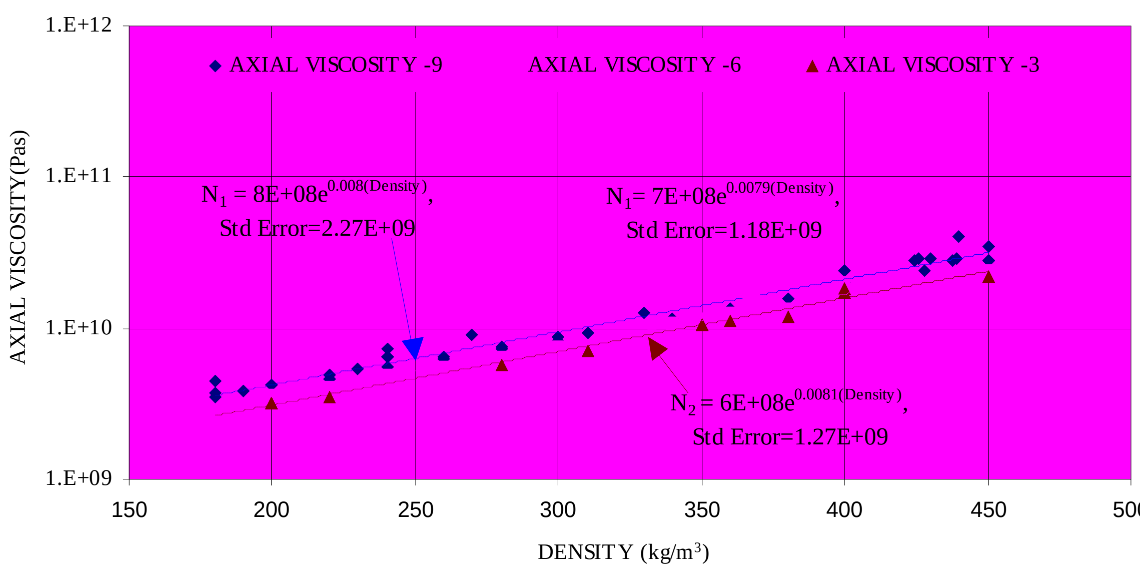

Figure 7. Effect of density and temperature on axial viscosity.

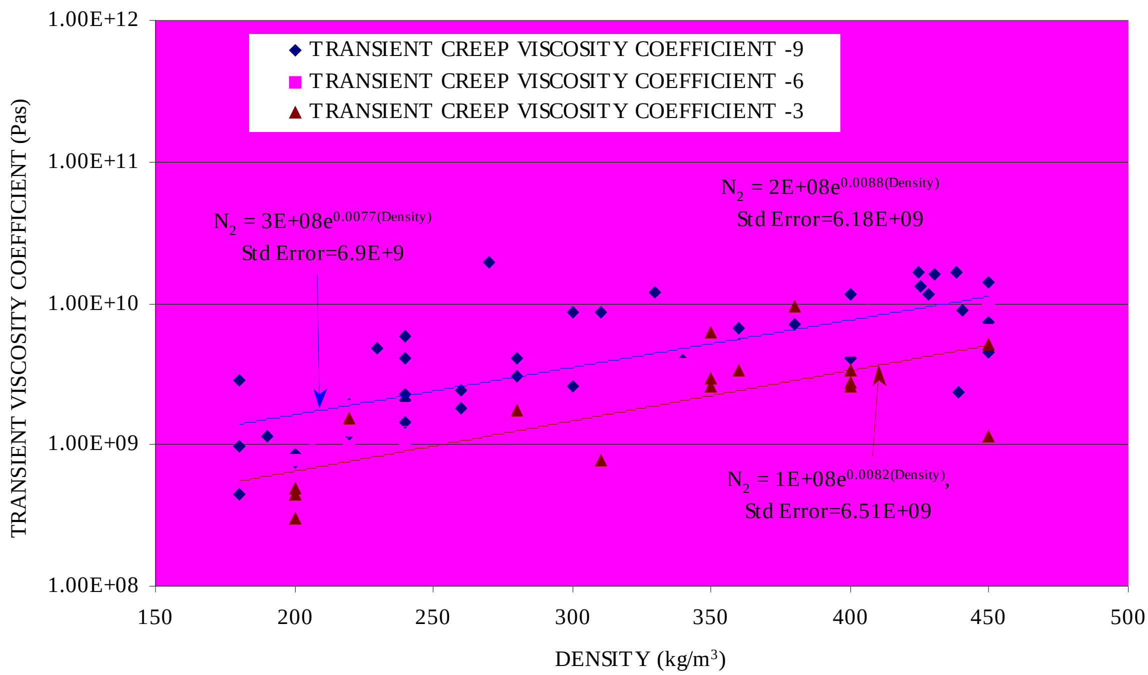

Figure 8. Effect of density and temperature on transient viscosity coefficient.

calculated values of the parameters is calculated in terms of standard error and this is shown in the respective graphs. The values of all the four parameters are increasing wrt decrease in temperature but this trend is not the same in all the parameters. To say something concrete with confidence about the effect of temperature on these parameters, the difference between different temperature levels at which experiments were conducted should be more and not so close.

6. CONCLUSIONS

The four-parameter rheological model seems to simulate the creep behaviour of snow clearly. From the 91 experiments conducted in the laboratory, it is observed that the values of all the four parameters are clustered in different ranges as specified in the results and discussion. It was also observed that that the creep behaviour of snow with a density range of 180-450 kg/m^3, can be idealised using four parameter constitutive relation, using the parameters value falling in the above said range. Effect of density and temperature on all the four parameters is remarkable. Microstructure study can further improve the accuracy of the parameters thus calculated and scatter in the values can be reduced.

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