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Performance Improvisation of Cantilever-type Silicon Micro Acceleration Sensors Using Stress Concentration Regions Technique

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ABSTRACT

Acceleration sensors find applications in missile and competent munitions subsystems. Cantilever-type sensor's sensitivity and bandwidth are dependant on material properties of the cantilever and structure of proof mass. It is always desired to design a sensor as sensitive as possible but also maintaining higher bandwidth. In piezoresistive (cantilever-type) accelerometers, various techniques were employed by designers to enhance their sensitivity and bandwidth. Most of these techniques are usually focused on shape and size of either cantilever or proof mass. This paper presents a concept of creating stress concentration regions (SCRs) on the cantilever for enhancing its sensitivity. Five types of structures were simulated to study the behaviour of piezoresistive sensors with SCRs implementation. Use of SCRs results in substantial increase in the sensitivity, which is of the order of 1.85 times the nominal sensitivity. It was aimed at maximising sensor's performance factor, which is the product of sensor bandwidth and sensitivity. This study gives new dimension to the ways of improving performance of cantilever-type inertial piezoresistive sensor.

Keywords: Micro accelerometer, piezoresistive micro sensor, stress concentration regions, inertial sensor, acceleration sensor, cantilever-type sensor

NOMENCLATURE

β	Piezoresistance doping factor (max. 1)
σ	Nominal stress
σ_B	Stress at section B
ν	Poison's ratio
π_l	Longitudinal piezoresistive coefficient
$\pi_{11}, \pi_{12}, \pi_{44}$	Piezoresistive coefficients
π_L	Piezoresistive coefficient of silicon along <110> axis
π_t	Transverse piezoresistive coefficient
ΔI	Change in current
ΔR	Change in resistance
σ_l	Longitudinal stress
σ_t	Transverse stress
a	Major axis
b	Minor axis
A_r	Piezoresistor area
f	Applied acceleration
I	Current

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K_t	Stress concentration factor
R	Resistance
R_{PZR}	Resistance of the piezoresistor
S	Sensitivity
r_c	Radius of curvature

1. INTRODUCTION

Acceleration (i.e., sensing of setback force) is one of the important parameters in the safety arming mechanisms of missiles and rockets. Missile acceleration at boost phase vary from 5 g to 30 g, depending on the type of launching. Competent munitions have launch acceleration of the order of even 400 g. Cantilever-type of piezoresistive sensor, which has strong overload protection against shock, would be useful in this environment of setback force. Moreover, piezoresistive-type accelerometers have the ability to meet requirements of small size and mass, low cost, high sensitivity, high performance, reliability, large-scale production, and ease of integration with signal processing circuits.

A piezoresistive acceleration/inertial sensor basically consists of a proof mass attached to a micro cantilever (flexure), a structure made out of silicon[1-4]. The cantilever gets deflected when it is subjected to inertial force. This deflection causes the development of strain in the cantilever, which can be measured by an implanted set of piezoresistors (in bridge form). In cantilever-type sensors, its sensitivity is mostly dependent on structural factors such as length and thickness of cantilever and/or size and shape of proof mass. Bandwidth of a cantilever-type sensor is its first resonant frequency[5,20].

It is usually desirable to design a sensor as sensitive as possible, maintaining higher bandwidth. Various techniques have been employed by designers of accelerometers all over the world to enhance sensitivity and also maintain higher resonant frequency. These techniques are mostly concentrated either on various shapes of proof mass or on dual-beam (cantilevers)- type structures for sensors[6-7]. A deliberate introduction of stress concentration regions (SCRs) on the cantilever is technique of increasing sensitivity of piezoresistive-type sensors. The simplest type of SCR was used by Gupta[8], *et al.*, where cantilever thickness was reduced at a particular region to enhance stress. It was concluded that SCR should be placed nearer to the anchored-end. Rectangular and elliptical holes on the cantilever aligned in the longitudinal direction were used as SCRs by Kassegne[9], *et al*. It was established that SCR helps to increase longitudinal stress more as compared to transverse stress on the cantilever, thus increasing difference between ($\sigma_l - \sigma_t$) is desired to get maximise sensitivity. Yang[10], *et al.* employed techniques of elliptical holes and reduced cantilever width for the formation of SCRs. All these have evolved design principles from their studies on design of cantilever-type sensor with SCR aimed at designing cantilever-type sensor for either scanning probe microscopy application or as a means of detecting molecular adsorption.

It is for the first time that studies on SCR effect on cantilever used as an acceleration sensor have been taken up. This paper presents some new concepts of SCRs tried on cantilever. The introduction of various shapes of SCRs such as rectangular holes, long horizontal slits, combination of slits and circular holes, are explored. The product of structure's resonant frequency and Von-Mises stress (sensitivity) is defined as the performance factor of a sensor[11,12]. It is termed as *P*-factor. Variation in sensor's sensitivity vis-à-vis changes in its resonant frequency has been studied with an aim of optimising the *P*-factor. Piezoresistive analysis was also carried out to measure sensor sensitivity. The sensor models were created and analysed in Coventorware 2003 using Mem-Mech and Mem-PZR solvers. The study concludes with a type of SCR that gives optimised P-factor for an acceleration sensor.

2. SENSOR STRUCTURE

Coventorware 2003 tool was used for the creation of inertial sensor structure, which consists of a small cantilever, termed as flexure, a proof mass attached to this flexure. Simple cantilever (flexure), as shown in Fig. 1(a) was modelled with flexure having length of 100 μm, width of 50 μm and thickness of 8 μm. The proof-mass has a length of 2000 μm, width of 400 μm and thickness of 50 μm.

The basic sensor was modelled in X-Y plane. The Y-Z face of flexure was treated as fixed-end and the direction of application of inertial force was in Z-direction. The stress value thus obtained can be regarded as a response of the device or sensitivity of the device. Modal analysis was run on the model to find first resonant frequency of the structure, which can be regarded as the bandwidth of the device[1].

The flexure is the spring element, which gets deflected on application of inertial force. The deflection of the flexure generates stress in this element. The amount of deflection, and therefore, the stress developed depends on many factors such as mass of the proof mass, CG of the structure, material density, flexure dimensions, proof mass dimensions, etc. Mechanical analysis was run to obtain stress values developed in flexure after applying inertial force in Y direction. The Von-Mises theory was used to pick up maximum stress generated in the flexure. Figure 1(b), shows the variation in stress along the flexure length when subjected to 10 g acceleration, that maximum stress value of 950 MPa exists at anchored-end and goes on reducing towards free- end of the flexure.

3. STRESS CONCENTRATION REGIONS

Stress concentration regions (SCRs) is the result of discontinuities such as holes, grooves, keyways, cracks or sharp change in one of the dimensions of the structure. These structural discontinuities amplify the stress in their vicinity. Although the SCRs are not desired in engineering design of any structure, the discontinuities can be used to amplify the stress on the cantilever-type of sensor. Discontinuity shown in Fig. 2 is a simple circular hole, drilled through the depth of the flexure on its centreline.

The Fig. 2 shows the stress distribution at two sections, A and B of a cantilever beam, and illustrates the amplification of stress around the hole at section *B* as compared to stress at section *A*. The stress at section *B*, as shown in the adjoining enlarged view, as given by Inglis solution is given in Eqn (1).

$$\sigma_B = \sigma\left(1 + \frac{2b}{a}\right) \qquad (1)$$

Thus the stress at SCR increases by a factor of (1 + 2b/a). By definition, the stress concentration factor, K_t, [Eqn (2)] is the ratio of the maximum stress at the hole to the nominal stress at the same point[14]. Therefore, one gets

$$K_t = \frac{\sigma_B(\max)}{\sigma_B(nom)} = \left(1 + \frac{2b}{a}\right) \qquad (2)$$

When the hole is circular, b/a is 1 and maximum stress is 3-times the nominal value. Whereas, when b/a is large, the ellipse approaches a crack transverse

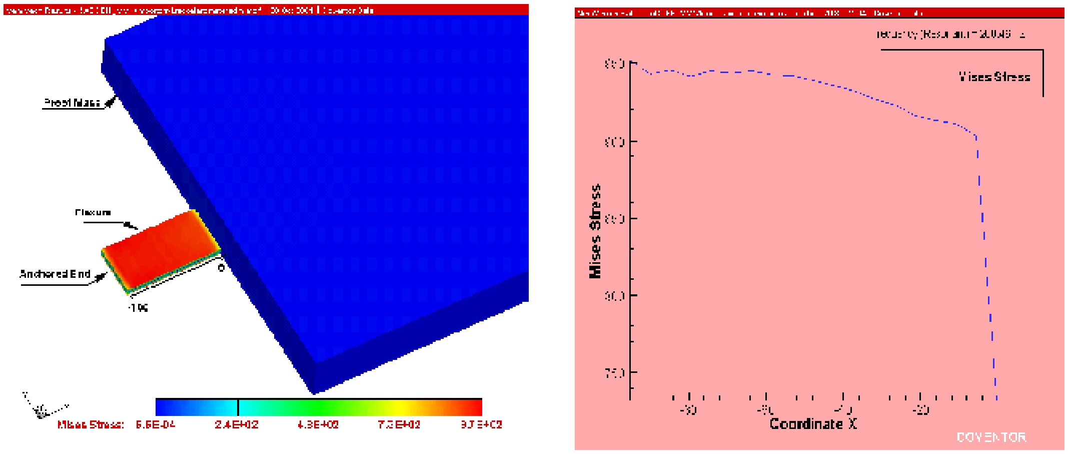

Figure 1. (a) Cantilever structure with flexure and proof-mass, (b) stress variation along longitudinal direction (X-axis) for simple flexure.

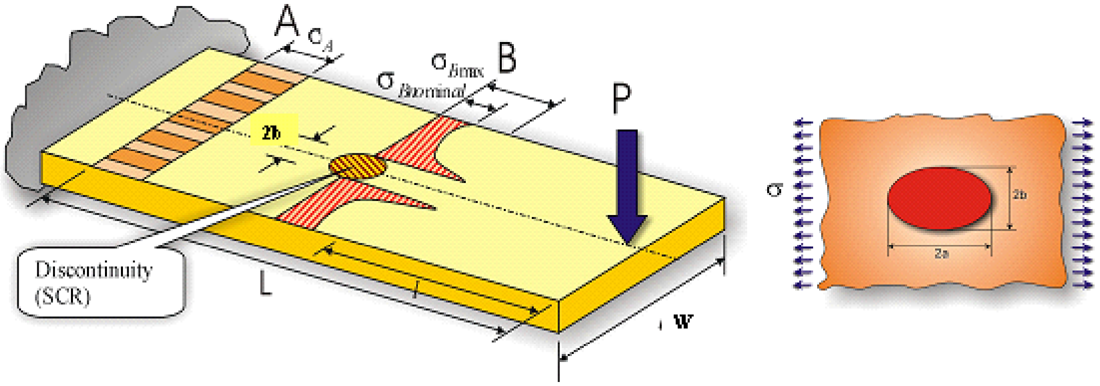

Figure 2. Schematic of stress distribution on cantilever and at discontinuity (SCR).

load and stress concentration increases. When b/a is small, the ellipse approaches a longitudinal slit and the increase in the stress is small. For ellipse with large aspect ratio, the value of K_t is expressed as equal to $2(b/r_c)^{½}$, where r_c is the radius of curvature[14].

4. PIEZORESISTIVE EFFECT IN SILICON

The piezoresistive effect in silicon results as a change in resistance R with applied stress, which is a function of crystal orientation, dopant type, and doping concentration. For a resistor with area A_r, the piezoresistive sensitivity[8] is given by Eqn (3) as

$$\left(\frac{\Delta R}{R}\right)\Big/ UnitLoad = \left(\int_0^{A_r} (\sigma_l \pi_l + \sigma_t \pi_t)\partial A\right)\Big/ A_r \tag{3}$$

The orientation of beam is determined by its anisotropic fabrication. The surface of the silicon wafer is usually a (100) plane and the edges of etched structures are intersections of (100) and (111) planes and are thus <110> directions. Therefore, the orientation of the piezoresistors wrt silicon crystal is (110) plane. The longitudinal piezoresistive coefficient in the <110> direction is $\pi_l = ½(\pi_{11} + \pi_{12} + \pi_{44})$ and the corresponding transverse coefficient[13,18] is $\pi_t = ½(\pi_{11} + \pi_{12} - \pi_{44})$. For P- and N- type resistors, the sensitivity is approximated by Eqns (4) and (5) respectively[18].

$$\frac{\Delta R}{R} = \frac{\pi_{44}}{2}.(\sigma_l - \sigma_t) \tag{4}$$

$$\frac{\Delta R}{R} = \frac{\pi_{11} + \pi_{12}}{2}(\sigma_l + \sigma_t) \tag{5}$$

The mechanical response of micro cantilever is due to the surface stress generated on the cantilever by acceleration. The sensitivity of the piezoresistive-type of sensor is given by the Eqn (6) for sensor with (110) orientation plane[9,15].

$$\frac{\Delta R}{R} = \beta.\frac{3.\pi_L}{t}(1-\nu).(\sigma_l - \sigma_t) \tag{6}$$

It was stated by Kassegne[9], *et al*. that sensitivity is proportional to differential stress distribution over a cantilever surface, which depends on the geometric factors of the layers and acceleration force applied on the cantilever beam. Therefore, by maximising differential stress (σ_l-σ_t), the sensitivity can be increased by changing the geometric parameters or by introducing SCRs on the cantilever. The piezoresistive analysis module of Coventorware 2003 was used to compute change in current due to applied stress. The piezoresistive analysis is a two-pass analysis. Mechanical analysis (Mem-Mech) of the structure was carried out and was used as an input to piezoresistive analysis. The Mem-Mech simulations were carried out for each type of SCR. The fixed-value potential was applied to piezo device, which gives the change in current as a result of induced stress. The sensitivity can be calculated from the percentage change in current obtained from the simulation as per the formula given below.

$$S = \left(I \times \frac{\Delta I}{100} \times R_{PZR} \right) / (V \times f) \tag{7}$$

5. SIMULATION RESULTS OF MECHANICAL SOLVER ON SCRs IMPLEMENTED STRUCTURES

Five types of discontinuities named *Type I* to *Type V*, were created on the flexure discussed earlier. [Figs 3(a) to 7(a) to see above defined types] These types are:

- *Type I*: Flexure with six rectangular holes as SCRs; (each hole size was 15 µm × 6 µm)
- *Type II*: Flexure with longitudinal slits as SCRs; (each slit size was 80 µm × 6 µm)
- *Type III*: Flexure with circular holes on longitudinal slits as SCRs
- *Type IV*: Flexure with staggered circular holes on longitudinal slits as SCRs
- *Type V*: Flexure with partially-etched staggered circular holes on longitudinal slits.

Mem-Mech simulations were run on each type of SCR to study stress development on flexure surface and to find resonant frequency of each structure. The results of simulations were obtained in Coventorware MEMSCAD by subjecting structures to acceleration of 100 g in the Z-direction. Refer to Figs 3(b) to 7(b) for variation stress along the sensor's longitudinal axis.

Table 1 gives average Von-Mises stress values and first modal (resonant) frequency obtained from

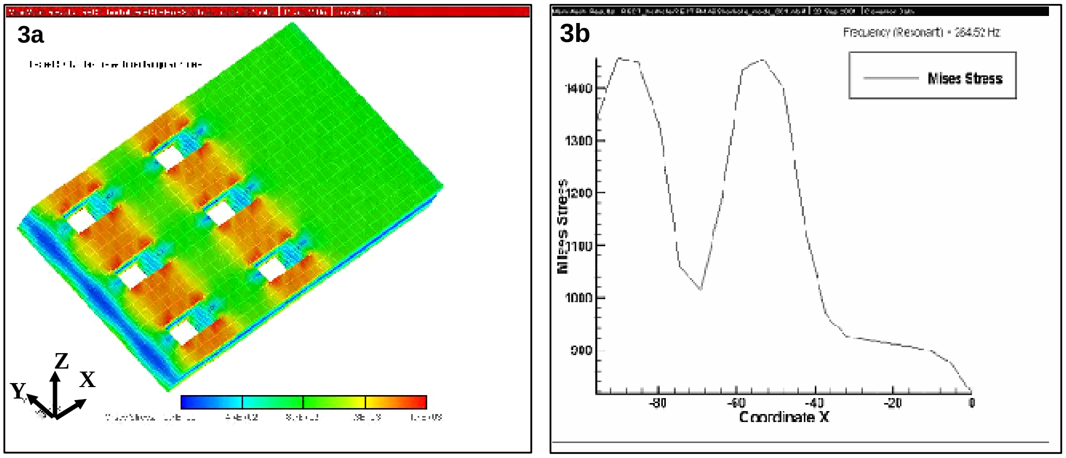

Figure 3. (a) *Type I* SCR: Rectangular holes on flexure, (b) stress variation along longitudinal direction of flexure with *Type I* SCR.

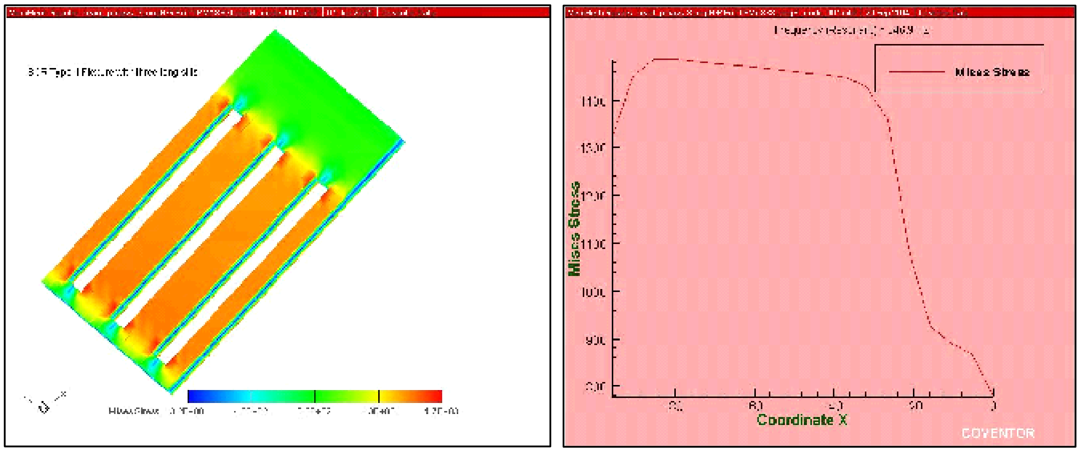

Figure 4. (a) *Type II* SCR: Long slit on flexure, (b) stress variation along longitudinal axis for *Type II* SCR.

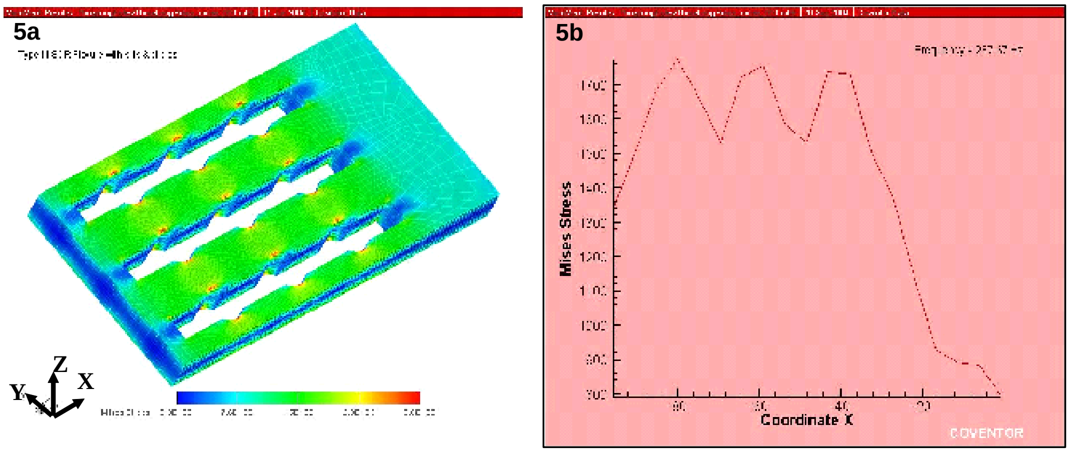

Figure 5. **(a)** ***Type III*** **SCR: Long slit with circular holes on flexure, (b) stress variation along longitudinal axis for** ***Type III*** **SCR.**

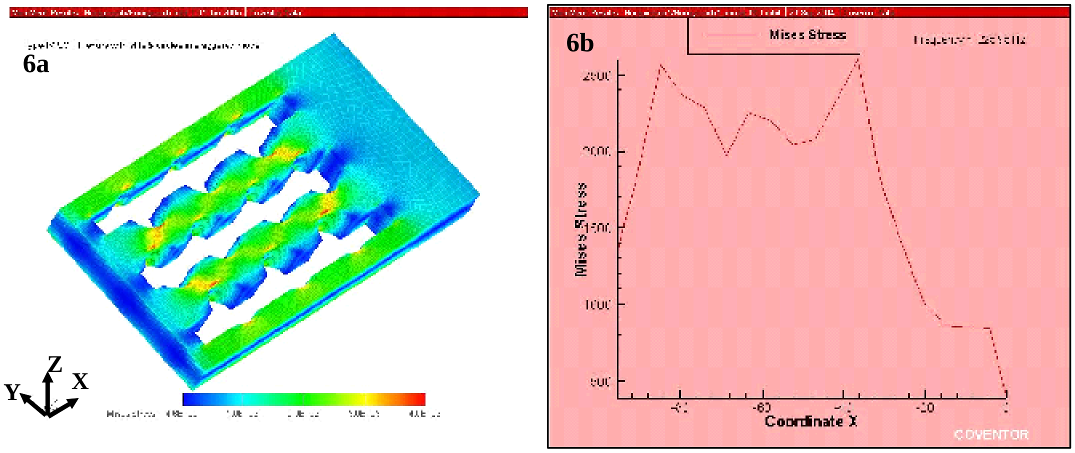

Figure 6. **(a)** ***Type IV*** **SCR: Long slit with staggered circular holes on flexure, (b) stress variation along longitudinal axis for** ***Type IV*** **SCR.**

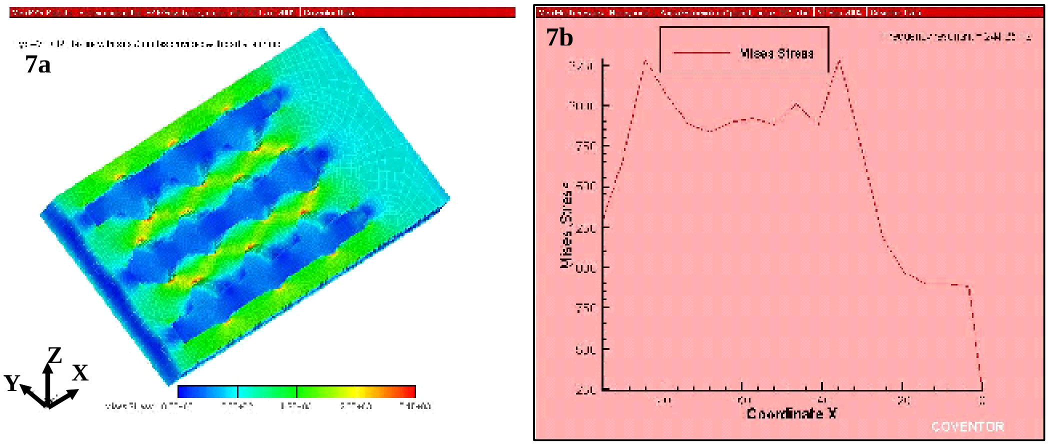

Figure 7. **(a)** ***Type V*** **SCR: Partially etched** ***Type IV*****, (b) stress variation along longitudinal axis for** ***Type V*** **SCR.**

simulations in Coventorware 2003 on simple cantilever and five types of structures and the computed performance factor (*P*-factor) for each type.

Results obtained by simulations are comparable with the results quoted by Gupta[8], *et al*. and Kassegne[9], *et al*. as far as magnification in sensitivity is concerned.

Table 1. Average Von Mises stress and modal frequency of various structures

	Type of SCR	Resonant frequency (Hz)	Average stress (MPa)	*P*-factor (X 1000)
	Simple cantilever	288	900	260
Type I	Six rectangular holes	264	1200	317
Type II	Long rectangular slit	247	1400	345
Type III	Long slit and circular holes	238	1600	380
Type IV	Long slit and staggered holes	226	2300	520
Type V	Partially-etched SCR	244	1900	464

Reference is also made to US patent (obtained by Ishikawa[19], *et al.*) where inventors claimed that sensor sensitivity improves almost by a factor of two by employing stress concentration holes in the form of holes on the cantilever. Moreover since silicon used was single crystal, it was assumed for all intents and purposes that the material does not yield until fracture occurs. The fracture strength of silicon given by Petersen[16] is of the order of 7000 MPa.

Type I SCR is a set of six rectangular holes. It can be observed in Fig. 3(b) that there is substantial increase in the stress value around the SCR locations. Here as per Eqn (2), stress concentration factor, K_t = 1.85. However, introduction of SCR results in reduction in resonant frequency because of increase in its flexibility.

Introduction of *Type II* SCR, which is a long slit on the flexure, further enhances stress development in flexure for the same excitation. Figure 4(b) shows the variation in stress value along the longitudinal direction of flexure. Here the advantage of long slit-type of SCR (*Type II*) is that there is uniform stress along the flexure unlike that in the case of holes on the flexure.

Type III SCR is formed by introduction of circular holes on the *Type II* SCR. *Type III* SCR increases the stress value by factor of another 22 per cent. However, resonant frequency reduces by a small factor only.

Table 2. Average stress and sensitivity for various structures

Type of SCR	Average stress σ_{av} (MPa)	Sensitivity (mV/V/g)
Simple flexure	900	5.82
Type I	1200	7.46
Type II	1400	9.18
Type III	1600	10.36
Type IV	2300	15.67
Type V	1900	13.17

Type IV SCR is staggered placement of holes on the slits, which creates even more powerful SCRs on the flexure. Figure 6(b) shows variation in stress along longitudinal axis of flexure for *Type IV*.

To increase bandwidth of the sensor, it is required to conserve the strength of the flexure. Hence SCR of *Type V* was constructed, which was similar to *Type IV* above, but with only partial etching up to half the thickness of flexure. Figure 7(b) shows variation in stress along longitudinal axis of *Type V*-type of SCR. It was observed that partial etching helps to maintain higher resonant frequency as compared to the case of through and through holes. This is due to improvement in strength of the flexure.

6. PIEZORESISTIVE ANALYSIS OF SCR STRUCTURES

Piezoresistive analysis was carried out on all the above structures. Mem-PZR was used for this analysis, which takes input from the respective Mem-Mech solver solutions of each of the simulated structures. The piezoresistive coefficients assumed were for crystal silicon material. These values[17,18] are π_{11} = 6.6 E-5 (MPa)$^{-1}$, π_{12} = –1.1 E-5 (MPa)$^{-1}$, π_{44} = 1.38 E-3 (MPa)$^{-1}$. Table 2 gives average stress and computed sensitivity for each of the simulated structures. It may be noted that sensitivity obtained for simple cantilever is 5.82 mV/V/g. The results of Mem-PZR analysis indicate that the sensor sensitivity increases in the same proportion of stress amplification as obtained in Mem-Mech simulations.

7. CONCLUSIONS

Table 3 summarises all five types of SCR structures giving performance comparison wrt simple cantilever obtained from Mem-Mech and Mem-PZR simulations. The table gives percentage change in each resonant frequency, average stress, P-factor, and sensitivity wrt simple flexure for all five types of SCR implemented structures. From the above simulations, it has been concluded that *Type II* SCR has uniform and higher value increase in stress as compared to *Type I*. *Type III*, which is the combination of *Type I* and *Type II*, gives even higher performance. Staggering of holes, i.e. *Type IV* further enhances the performance. However, *Type V*, which is partially-etched-*Type IV*, gives the optimised performance from both sensitivity and bandwidth point of view. This is because increase in its stress value is double that of *Type II* SCR but it maintains the same resonant frequency as *Type II* SCR (Table I). Therefore, it can be stated that long-slit, partially-etched staggered holes-type of SCR gives the optimum performance.

Table 3. Performance comparison of various types of SCR with respect to simple flexure

	Type of SCR	Resonant frequency (Hz) (% decrease)	Average stress (MPa) (% increase)	*P*-factor (% increase)	Sensitivity (mVV/g) (% increase)
Type I	Six rectangular holes	8.3	33	22	28
Type II	Long rectangular slit	14	55	33	57
Type III	Long slit and circular holes	17	77	49	77
Type IV	Long slit and staggered holes	21	155	100	169
Type V	Partially-etched long slit, staggered holes	15	111	80	126

The simulations results are agreeing with the theoretical formulas for the SCRs discussed above. For example, the stress concentration factor for the *Type I* structure is 1.85 and the stress amplification at the SCR location observed in this structure is also 1.85-times the average stress in the flexure. The results of piezoresistive analysis carried out on these structures are also in agreement with stress amplification in comparison to simple flexure obtained in each type of SCR. Sensitivity magnification obtained is in comparison with the results quoted by Gupta[8], *et al*., Kassegne[9], *et al*., and Yang[10], *et al*.

The simulation study concludes that stress amplification on the flexure surface can be achieved by the introduction of SCRs on the flexure of a sensor. Deliberate implementation of SCR increases stress but reduces resonant frequency only by a small margin, and thus results in higher performance factor of a sensor. Creation of this type of SCRs on cantilever-type of sensor should be possible with deep reaction ion etching (DRIE) process of fabrication.

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