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Object Area-based Method for Elliptic and Circular Fit of a Two-tone Image

D. Chaudhuri

Defence Electronics Applications Laboratory, Dehradun-248 001

ABSTRACT

Circular or elliptic fit of an object is important in target detection, shape analysis, and biomedical image analysis problems. Here, the problems of fitting circle or ellipse to an object in 2-D are considered. At first, the problems is converted to quadratic equation of single unknown by some constraints. Then its solutions for all the border points of the object are found and averaged. The major and minor axes of ellipse are presented by least sum perpendicular distance of all points of the object. The other unknowns are found using the equations of constraints. In the proposed method, the main constraint used for the circular and elliptic fit is that the area of the fitting circle or ellipse is equal to the area of the object to be fitted. The approach appears to be less sensitive to the object border noise and is computationally attractive. Some examples are presented to show the effectiveness of the approach. A measure of degree of circularity, ellipticity, etc in fuzzy set theoretic framework is also proposed.

Keywords: Circularity, ellipticity, morphometry, shape analysis, pattern recognition, target detection.

1. INTRODUCTION

Several geometric shape properties are measured for recognition and description of two-tone objects in digital grid. Of these properties, circularity and ellipticity play important roles in applications involving industrial parts inspection, biomedical image analysis[1-4] and target detection[5,6].

Generally, the approach of locating a parametric curve can be divided into two parts[7]. The first part detects the boundary of the curve. The second part estimates the parameters based on the boundary points found in the previous step. A measure for circularity is proposed by Haralick[8] where the centre of the fitting circle is assumed to coincide with the centroid of the object border. The radius r of the fitting circle is the average of magnitude of radius vectors from the centre to the border of the object. The standard deviation σ of these radii vectors wrt r is computed to find the degree of circularity represented in terms of r/σ.

Of all parametric curves, the elliptic shape has drawn the maximum attention[1,9-12] because circles are projected into ellipse in the perspective projection model. Various techniques are available in the literature about shape analysis problem using object border points. Some of these use maximum chord of the object and the width perpendicular to it as the major and minor axes of the fitting ellipse, respectively. A different approach is due to Proffit[13,14] who used two components of normalised polar Fourier expansion of border points to define the optimised fitting ellipse. His results are invariant under translation, rotation, and scaling. While this approach is convenient for ellipse, it cannot be readily extended for ellipsoidal fit. An optimal trial and error method to estimate the parameters of an ellipse and circle to an object in 2-D as well as to estimate the parameters of sphere, spheroid or ellipsoid to an object in 3-D using the border points of the object is proposed by Chaudhuri[15]. Almost all the techniques work mainly on border points of the object, which is susceptible to border noise resulting from object segmentation and spatial resolution of digital grid.

With the objective to find an alternative technique less dependent on the border, it is assumed that the centre of the ellipse coincides with the centroid of all points of the object and not with the centroid of the border points only. The major axis of the ellipse is assumed to pass through the line of best fit to all points belonging to the object. The minor axis passes through the centroid and is perpendicular to the major axis direction. Also, the area of fitting ellipse is assumed to be the same as that of the object to be fitted. Since all these computations are done on the area of the object, the results are less susceptible to noise. Next, an estimate of the major (or minor) axis length is found from the border points of the object.

2. ELLIPTIC AND CIRCULAR FIT

2.1 Elliptic fit

Consider an object A in R^2. Let $(x_i, y_i), i = 1, 2, ..., n$ be n pels belonging to A. Then the centroid $(\bar{x}, \bar{y})$ of A is defined by

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$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \; ; \quad \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \tag{1}$$

It is required to find a line so that the sum of the squared perpendicular distances of all $(x_i, y_i) \in A$ from the line is minimum. Let θ be the angle of the line with x-axis and the line passing through $(\bar{x}, \bar{y})$. Therefore, the equation of the line will be

$$\tan\theta\, x - y + \bar{y} - \tan\theta\, \bar{x} = 0 \tag{2}$$

From any point $(x_i, y_i), i = 1, 2, ..., n$ the perpendicular distance to Eqn (2) is

$$q_i = (x_i - \bar{x})\sin\theta - (y_i - \bar{y})\cos\theta$$

Therefore, the sum of the squared perpendicular distance will be

$$Q = \sum_{i=1}^{n}\left[\left(x_i - \bar{x}\right)\sin\theta - \left(y_i - \bar{y}\right)\cos\theta\right]^2$$

Now θ is chosen in such a way so that Q will be minimum. Therefore, $\partial Q/\partial\theta = 0$ gives[2,15]

$$\tan 2\theta = \frac{2\sum_{i=1}^{n}\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)}{\sum_{i=1}^{n}\left[\left(x_i - \bar{x}\right)^2 - \left(y_i - \bar{y}\right)^2\right]} \quad \text{and}$$

$$\cos 2\theta \left\{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2 - \sum_{i=1}^{n}\left(y_i - \bar{y}\right)^2\right\} + 2\sin 2\theta \sum_{i=1}^{n}\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right) > 0 \tag{3}$$

Let this line be considered as best fitting line to A. It can be shown that if A is translated, then $(\bar{x}, \bar{y})$ and the line are translated by the same amount. Also, the best fitting line is invariant under scaling of A. In addition if A is rotated by an angle φ, the best fitting line is also rotated by an angle φ. It is assumed that

(i) The centre of the fitting ellipse is $(\bar{x}, \bar{y})$. (It ensures translation invariance of fitting ellipse.)

(ii) The major axis of the fitting ellipse to A coincides with the best fitting line of A. Also the minor axis of the fitting ellipse is a line perpendicular to the best fitting line and passes through $(\bar{x}, \bar{y})$. (The assumptions ensure rotation invariance of the fitting ellipse. Also, it converts ellipse fitting problem to that of solving two parameters a and b in Eqn. (4) below.)

(iii) Let S be the area of A. Then the area of the fitting ellipse is also S. (This assumption ensures scale invariance of the fitting ellipse. Also, as shown below, it converts ellipse fitting problem that of solving a quadratic equation.)

If the centre of the ellipse is the origin (0,0) while the major and minor axes coincide with the x and y axes, respectively. Then it is well known that the equation of the ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{4}$$

where $2a$ and $2b$ are the intercepts of the ellipse with the x and y axes, respectively. The area of the ellipse is πab while $a > b$.

For fitting object A to the ellipse, A is translated and rotated so that (x, y) coincides with the origin and the line of best fit to A coincides with the x-axis of the coordinate system. Now according to assumption (iii), one has

$$S = \pi\, ab \tag{5}$$

Using Eqn (5) in Eqn (4), one gets the quadratic equation in b^2 given by

$$\pi^2 x^2 b^4 - S^2 b^2 + S^2 y^2 = 0 \tag{6}$$

For each pel on the border of A one gets a pair of solutions of b^2 from the solutions obtained from all border pels and use it as an estimate of b^2 for the fitting ellipse. From this estimate, the major axis length a can be found using Eqn (5). However, the solutions should be examined before these are accounted for averaging. Several situations may occur in solving Eqn (6) as below.

(a) The solution may contain imaginary parts, which happens when $S^2 < \left(2\pi\, xy\right)^2$. It signifies that an ellipse of area S and the specified major and minor axis cannot pass through the current border pel P. One would like to find the ellipse that is nearest to the current border pel and yet satisfies the constraints. In particular one likes to find P' on the line OP so that the ellipse passes through P' and $P'P$ is smallest (Fig. 1). Let the slope of OP be $m = y/x$.
If one wants to find $P'(x', y')$ so that $y/x = y'/x'$ and $\left(2\pi x' y'\right)^2 = S^2$, i.e,

$$x'^2 = \left(\frac{S^2}{4\pi^2 m^2}\right)^{1/2}$$

Then one has $b^2 = \dfrac{S^2}{2\pi^2 x'^2}$

while a is found using Eqn. (5).

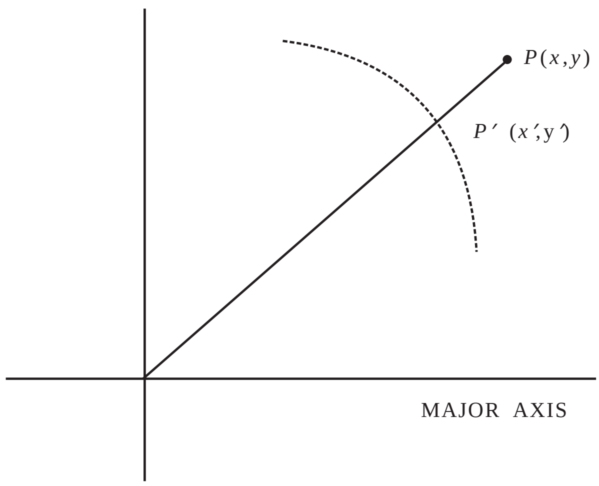

Figure 1. The case with complex solution to Eqn (6).

If one has $a < b$, then this pair of solution is not acceptable. One wants to find an ellipse so that $a \ge b$ and yet it is nearest to $P(x, y)$.

Actually under these constraints the ellipse is a circle with $a^2 = b^2 = S/\pi$. In brief, therefore, these circumstances may be taken care of using the following steps.

- Check if the current border is such that $S^2 < (2\pi\, xy)^2$.
- If yes, compute

$$x'^2 = \left(\frac{S^2}{4\pi^2 m^2}\right)^{1/2};\, b^2 = \frac{S^2}{2\pi^2 x'^2} \quad \text{and} \quad a^2 = (S/\pi\, b)^2$$

If $a^2 \ge b^2$, then accept the solutions. Otherwise accept $a^2 = b^2 = S/\pi$.

(b) The solutions may be real. For the two solutions of b^2, say b_1^2 and b_2^2 one gets two a^2, say a_1^2 and a_2^2 using Eqn. (5). Since one should get $b^2 \le a^2$, one disregards the solution, if any, for which $b^2 > a^2$. If however, both solutions satisfy $b^2 \le a^2$ one disregards the one that corresponds to $\max \{a_1^2, a_2^2\}$. (In this case, one of the solutions tries to make major axis length unnecessary lengthy to meet the area constraint of assumption (iii). Such situation may occur near to or on the major axis. Then, one shall get one of the b_i's equal to zero and its corresponding a_i equal to infinity, which must be disregarded).

When *a* and *b* are found from the averaged results with due considerations of (a) and (b), the fitting ellipse can be drawn. Choice of border pels should be done carefully. Equispaced points on the border should not be taken. This is because, if *A* has a thorn-like protrusion or intrusion, then points on those portions will be more heavily weighed. A better approach is to use equal area sweeping points. If the area swept out by a line from the origin as it moves from a border point *j* to *j*+1 is equal to that while moving from point *j*+1 to *j*+2, then one says that *j*, *j*+1 and *j*+2 are equal area sweeping points. If one want to consider *n* border points, then the area swept between *j* and *j*+1 should be π/n ab.

2.2 Circular fit

The circular fit to *A* can be obtained at once if one assumes that the area of *A* is equal to the fitting circle area *pr*^{2}, where *r* is the radius of the circle. Then the circle can be drawn with $(\bar{x}, \bar{y})$ as the centre and radius $r = \sqrt{S/\pi}$. The estimate of *r* obtained by this method is less sensitive to border noise and it is computationally attractive. Also, this method avoids the problem of choosing the appropriate border points encountered in other methods[8,16]. A comparative study of this method with that of Haralick's method[8] is given.

3. RESULTS AND DISCUSSION

To test the effectiveness of the proposed techniques, at first, a set of 2-D object borders was taken to test for the circular fit. Three types of circular fit were considered.

The first type is the proposed method where the centroid of all pels within the object is taken as the centre of circle while its area is equal to the area of the fitting circle.

- In the second type, the centre was chosen as above but the radius of the circle is taken equal to the average of radius vector lengths from this centre to the border of the object.
- In the third type, the centre was found as the centroid of the border of the object while the radius is found by second method.
- The error of fit is measured by two approaches: (a) the area of mismatch, and (b) the absolute sum of difference between radius vector and radius of all border points.

For both kinds of error of fit, it was found that the third type of fit is always inferior to the first and second types. Of the first and second types, the differences in error is very small and for one figure the first type of fit is slightly better while for the other figure, it is slightly inferior. A typical result with two circles is shown in Fig. 2 and is presented in Table 1. From these results we can draw the conclusions that the first and second type of fit yield identical results. Also, the centroid of all of the pels in the object is a stable and better measure for the centre of the fitting circle.

The last observation appears to be true in case of elliptic fit as well. When the centroid of all pels in the object is used for the centre of the fitting ellipse, the errors of fit of both types are less than the errors if the centroid of border is the centre of the ellipse.

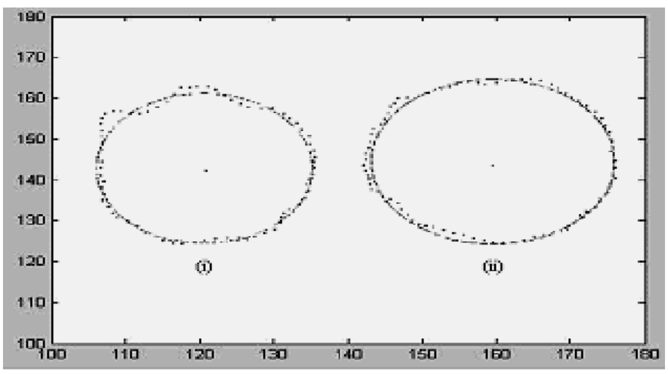

Figure 2. Two objects and their circular fits.

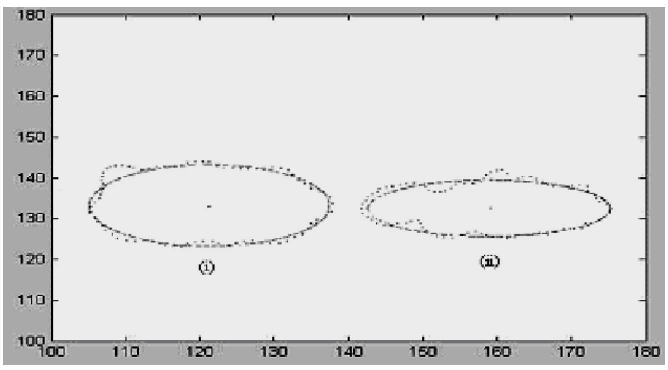

Figure 3. Two objects and their elliptic fits.

Table 1. Comparative study between the proposed method and the Haralick's method

Fig. No.	Error for			
	Type of error of fit	**First type fit**	**Second type fit**	**Third type fit**
(i)	Approach (a) error of fit	255	259	285
	Approach (b) error of fit	67.3	69.1	75.06
(ii)	Approach (a) error of fit	124	120	144
	Approach (b) error of fit	68.9	37.8	43.95

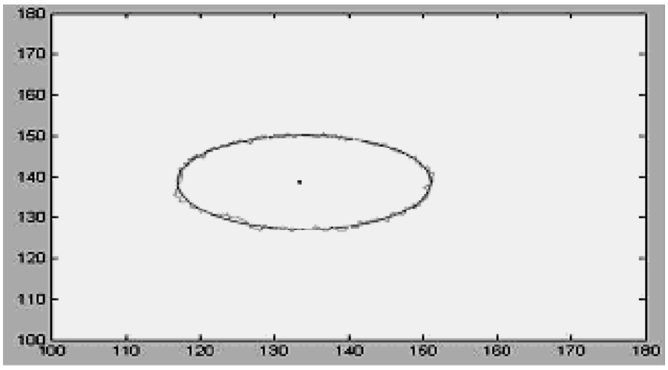

Figure 4. An ellipse with distorted border and its fitting ellipse by proposed method.

Two typical object borders with their elliptic fits according to the present approach are shown in Fig. 3. Also, in Fig. 4, an ideal ellipse is computed by noise and then the present method is applied to find the fitting ellipse. The computational efforts necessary is of the order of N for the major and minor axes direction where N is the number of pels in the object. To find major and minor axes lengths, an order on n real computation is necessary, where n is the number of chosen border points.

The degree of circularity and ellipticity may be modelled under fuzzy set theoretic framework[17]. Considering error of fit of type (a), let S and S_e be the area of the object A and its area of mismatch with the fitting circle or ellipse. Then, the fuzzy degree of circularity or ellipticity of A may be represented as

$$\mu_X(A) = \left[1 - \frac{S_e}{2S}\right]^{\beta} \tag{7}$$

where X denotes *circularity or ellipticity* and β is an exponential fuzzifier to control the shape of $\mu_X(A)$. Note that, $\mu_X(A) = 1$ when $S_e = 0$ and $\mu_X(A) = 0$ when $S_e = 2S$.

4. CONCLUSION

An approach of fitting closed conic sections in 2-D is proposed. The approach is based on converting the problem into quadratic equation of one variable by some constraints. The centroid of the object is constrained to be the centre of the fitting conic section, while the axes of the fitting conic are constrained to be the lines of best fit to the object. For elliptic fit, the area of the fitting ellipse is constrained to be equal to the area of the object. The approach appears to be less sensitive to the object border noise and is computationally attractive. Experimental results present effectiveness of the approach. A measure of degree of circularity, ellipticity, etc in fuzzy set theoretic framework is also proposed.

REFERENCES

1. Nakagawa, Y. & Rosenfeld, A. A note on polygonal and elliptical approximation of mechanical parts. *Pattern Recognition*, 1979, **11**,133-42.
2. Sychra, J.J.; Bartels, P.H.; Taylor, J.; Bibbo, M.& Weld, G.L. Cytoplasmic and nuclear shape analysis for computerized cell recognition. *Acta Cytologica*, 1976, **20**, 68-78.
3. Paton, K.A. Conic sections in chromosome analysis. *Pattern Recognition*, 1970, **2**, 39-51.
4. Belson, M.; Dudley, A.W. & Ledley, R.S. Automatic computer measurements of neurons. *Pattern Recognition*, 1968, **1**, 119-28.
5. Zhu, C. & Wang, R. A fast automatic extraction algorithm of elliptic object groups from remote sensing images. *Pattern Recognition Letters*, 2004, **25**(13),1471-478.
6. Zhang, S.C. & Liu, Z.Q. A robust real-time ellipse detector. *Pattern Recognition*, 2005,**38**(2), 273-87.
7. Chaudhuri, D. & Samal, A. A simple method for fitting of bounding rectangle to closed regions. *Pattern Recognition*, 2007, **40**, 1981-989.
8. Haralick, R.M. A measure of circularity of digital figures. IEEE Trans. System, *Man and Cybernetics*, 1974, SMC-4, 1974, 394-96.
9. Ellis, T.J.; Abbood, A.& Brillault, B. Ellipse detection and matching with uncertainty. *Image Vision Comput.*, 1992,**10**, 271-76.
10. Rosin, P.L. A note on the least squares fitting of ellipse. *Pattern Recognition Lett.*, 1993, **14**, 799-08.
11. Safaee-Rad, R.; Tchoukanov, I.; Benhabib, B.; & Smith, K. C.; Accurate parameter estimation of quadratic curves from gray-level images. CVGIP: *Image Understanding*, 1991, **54**, 259-74.
12. Wu, W. & Wang, M. Elliptic object detection by using its geometric properties. *Pattern Recognition*, 1993, **26**, 1499-509.
13. Proffit, D.; The measurement of circularity and ellipticity on a digital grid. *Pattern Recognition*, 1982, **15**, 383-87.
14. Proffit, D. Normalisation of discrete planer objects.

Pattern Recognition, 1982, **15**, 137-45.
15. Chaudhuri, D. Elliptic and circular fit of a two-tine image using optimal trail and error method and their 3-D extensions. *In* International Conference on Optics and Optoelectronics Theory, Devices, and Applications, (ICOL), Narosa Publishing House, 1998, **2**, pp. 817-22.
16. Albano, A. Representation of digitised contours in terms of conic arcs and straight-line segments. *Comp. Graphics Image Process.*, 1974, **3**, 23-33.
17. Zadeh, L.A. Fuzzy sets, *Information and Control*, 1965, **8**, 338-53.

Contributors

Dr D. Chaudhuri obtained his MSc in Applied Mathematics from Jadavpur University, Kolkata. He received his PhD in image processing and pattern recognition from Indian Statistical Institute, Kolkata. He has worked at Indian Statistical Institute; Institute of Armament Technology, Pune; and Integrated Test Range, Chandipur. He was a visiting Assistant Professor at University of Nebraska, USA in the Dept of Computer Science and Engineering for 2003-2004. Presently, he is working as a Scientist at Defence Electronics Applications Laboratory, Dehradun. He has more than 12 years experience in the field of image processing, pattern recognition, computer vision and remote sensing with extensive systems development andtheir implementation. He has extensive experience in the field of automatic target detection from satellite imagery. He has published more than 28 papers in reputed international journals.