Defence Science Journal, Vol. 58, No. 3, May 2008, pp. 377-389

Estimation of Drag Coefficient from Radar-tracked Flight Data of a Cargo Shell

G.G. Dutta, Ankur Singhal, A. Kushari, and A.K. Ghosh

Indian Institute of Technology, Kanpur-208 016

ABSTRACT

Two schemes to extract drag coefficient (C_D) by processing the radar-tracked trajectories of artillery shell in motion, have been proposed. Flight trajectories of artillery shell are considered. The proposed schemes are applied on the radar-tracked trajectory data of artillery shell to estimate the C_D. The C_D is strong function of Mach number. To capture the functional relationship between the C_D and the Mach number, the C_D of the ammunition was assumed to be a polynomial function of Mach number (separately for subsonic and supersonic). The coefficients of the assumed polynomial were estimated by minimising the error between measured and estimated trajectories. In the second scheme, whole trajectory was split into different sets containing 50 or 100 data points. Each data set was processed using the proposed schemes to estimate numerical values of C_D corresponding to the average Mach number of the chosen data set. The estimated values of the C_D (at different Mach numbers) have been presented along with its standard deviations. The difficulties encountered in processing the real trajectory data using the proposed schemes are also highlighted. It is observed that the proposed schemes could advantageously be applied to quickly estimate the numerical values of the C_D at corresponding Mach numbers, by processing the trajectory data of an artillery shell in motion.

Keywords: Cargo shell, radar-tracked trajectory, artillery shell, drag coefficient

NOMENCLATURE

a_0, a_1, a_2	Constants of series approximation
C_D	Drag coefficient
C_L, C_y	Lift and side force coefficients
C_l, C_m, C_n	Rolling moment, pitching moment, and yawing moment coefficients
$C_{L\alpha}$, $C_{m\alpha}$	Static longitudinal derivatives
C_{Lq}, C_{mq}	Longitudinal damping derivative
C_{lp}	Roll damping derivative
C_{yb}, C_{yr}	Lateral-directional force derivative
d	Diameter of the shell
$\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}$	Time derivatives of trajectory co-ordinates, x, y, z
GG^{-1}	Inverse of covariance matrix
g	Acceleration due to gravity
$J(\theta)$	Cost function
M	Mach number
m	Mass of projectile

Received 16 March 2007, revised 11 September 2007

p, q, r	Components of the angular velocity vector of the projectile
U_{X0}, U_{Z0}	Initial values of velocities in x- and z-directions
V	Total velocity of the shell
W_x, W_y, W_z	Wind velocity components in x-, y-, z-directions
u, v, w	Components of the velocity vector of the mass centre of the composite body in the body reference frame
X, Y, Z	Trajectory co-ordinates of mass centre in x-, y-, z-directions
X_0, Z_0	Initial values trajectory coordinates in x- and z-directions
Z_1, Z_2	Matrices of measured and estimated trajectory variables
α	Angle of attack
β	Side-slip angle
ϕ, θ, ψ	Roll, pitch, and yaw angle of the projectile
ρ	Density of air

1. INTRODUCTION

Artillery forms an important wing of the army in providing firepower, during both war and cross-border skirmishes with the enemy. Artillery shells are a class of projectiles around which much of aeroballistics theory was originally developed, and it continues to form a significant part of the aeroballistician's interest. The effectiveness of artillery shells is largely judged by its accuracy in hitting the target. The conventional approach used for predicting the behaviour and flight performance of a projectile such as an artillery shell was via mathematical models[1].

Beginning with the simplest, but relatively inaccurate, in-vacuuo trajectory model, more and more sophisticated models of increasing accuracy such as the point-mass model, the modified point-mass model, and the six-degrees of-freedom (6-DOFs) model, have been developed[1]. Numerical integration of the 6-DOFs equations of motion gives the most accurate solution possible, for the trajectory and flight dynamics behaviour of a rotationally symmetric, spinning or non-spinning projectile provided that all the aerodynamic forces and moments, and the initial conditions, are known to a high degree of accuracy[1,2].

At the preliminary design stage, theoretical methods[3,4] to estimate aerodynamic parameters are useful in spite of their limited accuracy. Nevertheless, computational fluid dynamics has in recent years positively influenced the analytical scenario by providing numerical solutions of total configuration via sophisticated and advanced Euler, and Navier-Stokes flow solver[5,6]. Experimental methods are essential to corroborate the analytical predictions. The wind tunnel testing improves the accuracy of the estimates but is often difficult to simulate the exact flight conditions[7,8]. Moreover, the model tested in the tunnel is generally slightly different from the actual model used in the flight due to last minute changes. It is, therefore, desirable that the wind tunnel estimates be updated using the estimates obtained from actual flight test data[9,10]. Estimation activities in aerospace science started in late 1950s. Aircraft parameter estimation using flight data is done on routine basis[9,10]. Flight data of an aircraft generated through special manoeuvers contain measurements of a large number of motion and control variables, like accelerations, speed, angle of attack, side-slip angle, etc. Acquisition of these flight variables is carried out using a large number of dedicated sensors[11,12]. Artillery shells generally do not have sufficient space to install dedicated sensors to acquire time histories of large number of motion/control variables. Further, considering the cost-effectiveness, it may not be feasible to house many sensors inside an artillery ammunition going through many development flight trials[13].

During early phase of development of an artillery ammunition, large number of these ammunitions are fired at different elevations (launch angle, θ) for subsystem validation. It is a routine practice to capture the trajectories of these ammunitions in motion, using ground-based tracking radar system. Generally, a typical trajectory data of a projectile in motion, acquired by tracking radar contain measurements of trajectory variables, namely range, (X), height (Z), drift (Y), and total velocity (V). Such flight data contain very limited dynamics for the parameter estimation using conventional methods. However, these trajectory data could effectively be processed using a proper estimation algorithm to estimate a few aerodynamic parameters of the artillery ammunition in motion.

The maximum likelihood estimator in its several forms has been the most widely used means of estimating the stability and control derivatives (parameters) from flight data of a flight vehicle[9,10]. Application of maximum likelihood method requires an *a priori* postulation of the flight dynamic model. As stated earlier, given the forces and moments, along with initial conditions, the complete in-flight motion of the projectile can be simulated using a 6-DOFs trajectory model. However, this model requires aerodynamic coefficients as input (e.g., drag coefficient (C_D), damping in roll derivative, (C_{l_p}), etc.) and the estimates available for these coefficients are not so reliable. In contrast, the point-mass trajectory model, which includes the aerodynamic drag force in addition to gravity, is a very practical and accurate approximation to the actual trajectory of any projectile that flies with predominantly small yaw. Accordingly, the point-mass trajectory model could be used as an *a priori* model while applying maximum likelihood method to estimate C_D by processing the radar tracked-trajectory data of the artillery shell in motion[13].

In this paper, schemes using maximum likelihood[8] method have been proposed to estimate C_D, by processing trajectories (acquired by tracking radar) of artillery shells in motion. The aerodynamic parameters are strong function of Mach number. To accurately estimate drag characteristics of projectiles in motion, it is necessary to estimate the numerical values of C_D as a function of flight Mach number. Application of maximum likelihood method requires an *a priori* postulation of the aerodynamic model to be used in estimation algorithm. The point-mass trajectory model uses C_D to model the aerodynamic force responsible for projectile's deceleration[1]. Since the artillery shells belong to a class of projectiles which encounter very small angle of attack (<1-2 degree) in-flight, the use of point-mass model in estimation algorithm for this purpose is fairly justified[1,2].

Two schemes using maximum likelihood method to extract C_D by processing the trajectory data of an artillery shell have been proposed. Both the schemes were initially validated using simulated trajectory data. On satisfactory performance of the schemes with simulated trajectory data, attempts were made to validate both the schemes using real trajectory data of an artillery shell in motion. In the first scheme, the C_D was assumed to be a polynomial function of Mach number. Maximum likelihood estimator was used to estimate the coefficients of the polynomial by minimising the error between the measured and estimated trajectory variables. The numerical value of the C_D was computed by plugging the values of the estimated coefficients in the chosen model of the C_D used in the estimation algorithm. In the second scheme, whole trajectory was split into a number of successive sets containing 50 or 100 time histories of trajectory variables. Each data set was processed using maximum likelihood method to estimate a constant numerical value of C_D corresponding to the average Mach number of the chosen data set. For this scheme, the initial conditions (velocity and elevation) required for generation of the estimated responses, for different successive sets, were obtained by differentiating (wrt time) the time histories of measured trajectory variables (X, Z, Y) of the ammunition in motion.

For this study, trajectory data corresponding to a cargo ammunition was chosen from the database for estimation of C_D. The estimated values of the C_D (at different Mach numbers) have been presented along with their standard deviations. The estimated values of C_D were then compared with the values of C_D (true value) used for preparing range table (firing table). Range tables (firing tables) are routinely used to estimate required values of elevation (θ) and bearing (ψ) of the barrel to engage a target at predefined location. The difficulties encountered in processing the real-trajectory data using the proposed schemes are also highlighted. It is observed that the proposed schemes could advantageously be applied to quickly estimate the numerical values of the C_D at corresponding Mach numbers, by processing the real-trajectory data of an artillery shell in motion.

2. GENERATION OF SIMULATED AND REAL TRAJECTORY DATA

Simulated trajectories of a routinely used artillery shell in motion were generated using 6-DOFs trajectory model[1]. The 6-DOFs include, three-position components of the mass centre of the artillery shell, as well as three Euler orientation angles of the body. Due to symmetry, the aerodynamic model was simplified to have the following form:

$$C_L = C_{L_\alpha}\alpha + C_{L_q}\frac{qd}{2V} \quad (1)$$

$$C_m = C_{m_\alpha}\alpha + C_{m_q}\frac{qd}{2V} \quad (2)$$

$$C_n = -C_{m_\alpha}\beta + C_{n_r}\frac{rd}{2V} \quad (3)$$

$$C_l = C_{l_p}\frac{pd}{2V} \quad (4)$$

$$C_y = C_{y_\beta}\beta + C_{y_r}\frac{rd}{2V} \quad (5)$$

Due to symmetry of artillery shell, $C_{mq} = C_{nr}$, $C_{lq} = C_{yr}$, the following are the 6-DOFs equations of motion:

$$\dot{u} = (\bar{q}s/m)C_x - qw + rv - g\sin\theta + (Th/m) \quad (6)$$

$$\dot{v} = (\bar{q}s/m)C_y - ru + pw + g\sin\phi\cos\theta \quad (7)$$

$$\dot{w} = (\bar{q}s/m)C_z - pv + qu + g\cos\phi\cos\theta \quad (8)$$

$$\dot{p} = \left\{\bar{q}sdC_l + qr(I_y - I_z)\right\}/I_x \quad (9)$$

$$\dot{q} = \left\{\bar{q}sdC_m + rp(I_z - I_x)\right\}/I_y \quad (10)$$

$$\dot{r} = \left\{\bar{q}sdC_n + pq(I_x - I_y)\right\}/I_z \quad (11)$$

$$\dot{\phi} = p + q\tan\theta\sin\phi + r\tan\theta\cos\phi \quad (12)$$

$$\dot{\theta} = q\cos\phi - r\sin\phi \quad (13)$$

$$\dot{\psi} = r\cos\phi\sec\theta + q\sin\phi\sec\theta \quad (14)$$

To derive the spatial position equations, above equations were used to transform the body-axis velocity (u, v, w) into earth-fixed axis. The equations are:

$$\begin{aligned}\dot{X} = {} & u\cos\psi\cos\theta \\ & + v(\cos\psi\sin\theta\sin\phi - \sin\psi\cos\phi) \\ & + w(\cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi) + W_x\end{aligned} \quad (15)$$

$$\begin{aligned}\dot{Y} = {} & u\sin\psi\cos\theta \\ & + v(\sin\psi\sin\theta\sin\phi + \cos\psi\cos\phi) \\ & + w(\sin\psi\sin\theta\cos\phi - \cos\psi\sin\phi) + W_y\end{aligned} \quad (16)$$

$$\dot{Z} = u\sin\theta - v\cos\theta\sin\phi - w\cos\theta\cos\phi + W_z \quad (17)$$

To generate the simulated trajectories, the 6-DOFs equations of motion [Eqns (6)-(17)] were numerically integrated using a fourth-order Runge-Kutta algorithm. The artillery shell configuration used in this simulation study is a representative of a direct fire artillery shell that is 0.8 m long, spin-stabilised shell with ranging capability up to 23 km. Table 1 gives the inertial, geometrical characteristics along with the initial condition used for generating the simulated data. Time histories of simulated trajectory variables, X, Z, Y, and V for both the trajectories (V = 818 m/s and 360 m/s) are presented in Fig. 1. The trajectory variables corresponding to V = 818 m/s and θ = 7.0° was processed to estimate variation of C_D with Mach number in the supersonic regime. The shell with initial velocity of 360 m/s maintained subsonic flight throughout the trajectory. This flight data was processed to estimate variation of C_D with Mach number in the subsonic flight regime only.

Table 1. Inertial and geometrical characteristics with initial conditions for example shell

Mass (m) = 42.6 kg		CG = 0.533 m
Roll inertia (I_{xx}) = 0.146 kg-m^2		Pitch inertia (I_{yy}) = 1.709 kg-m^2
u_0 = 818 and 360 m/s	v_0 = 0.0 m/s	w_0 = 0.0 m/s
ϕ_0 = 0.0°	θ_0 = 7.0°	ψ_0 = 0.0 deg
p_0 = 1657 rad/s	q_0 = 0.0 rad/s	r_0 = 0.0 rad/s
X_0 = 0.0 m	Y_0 = 0.0 m	Z_0 = 0.0 m

Real trajectory data (X, Z, Y, V) were selected from the trajectory database of artillery ammunitions available with Armament Research & Development Establishment (ARDE), Pune. Trajectory data generated using 130 mm cargo shell was considered for parameter estimation. Trajectories of seven rounds of the cargo shell (artillery shell) corresponding to launch angle of 8° and initial velocity of 810 m/s were used to estimate C_D of the cargo shell. It may be mentioned here again that, the trajectories of this cargo shell in motion were tracked and acquired by the Doppler radar, DR6700, located at the firing site. The radar-tracked trajectories of these ammunitions were processed to compute range (X), height, (Z) drift (Y) and total velocity (V) of the ammunition in motion. Figure 2 presents the real trajectories of the cargo shell used to estimate C_D. From Fig. 2, it is observed, that the real flight data for all the seven rounds fired on two different occasions, show good consistency in trajectory variables, namely, X, Z and V except for Y. Although, there seems to be large scatter in the values of drift (Y) among all the seven rounds, however, the absolute values of the drift for majority of cases are small when compared to height or range. Thus for estimation purpose, the numerical values of drift (Y) was not considered.

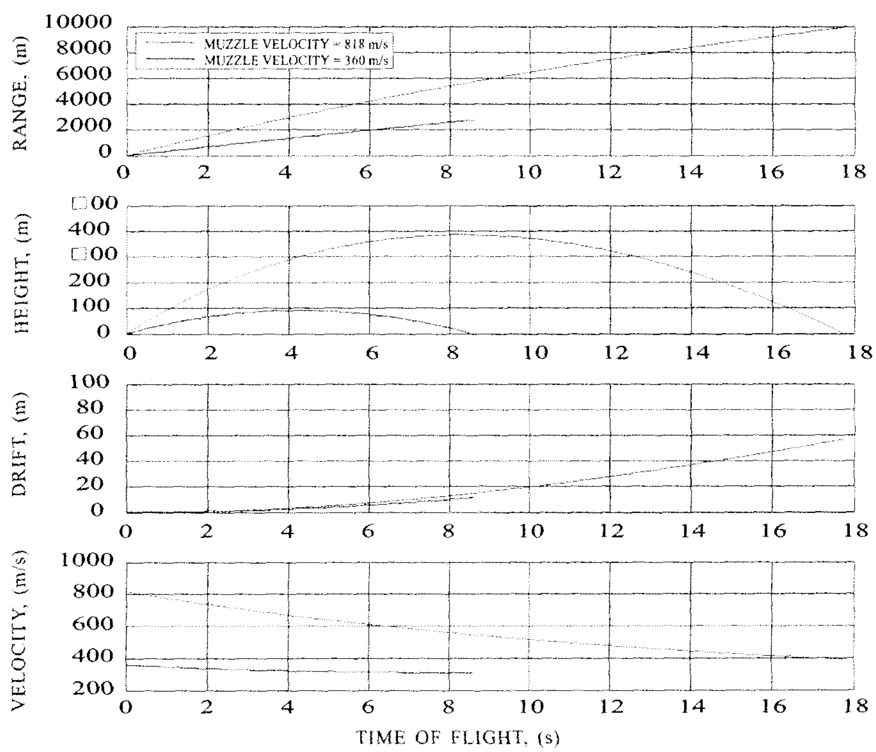


Figure 1. Time histories of trajectory parameters for the chosen artillery shell.

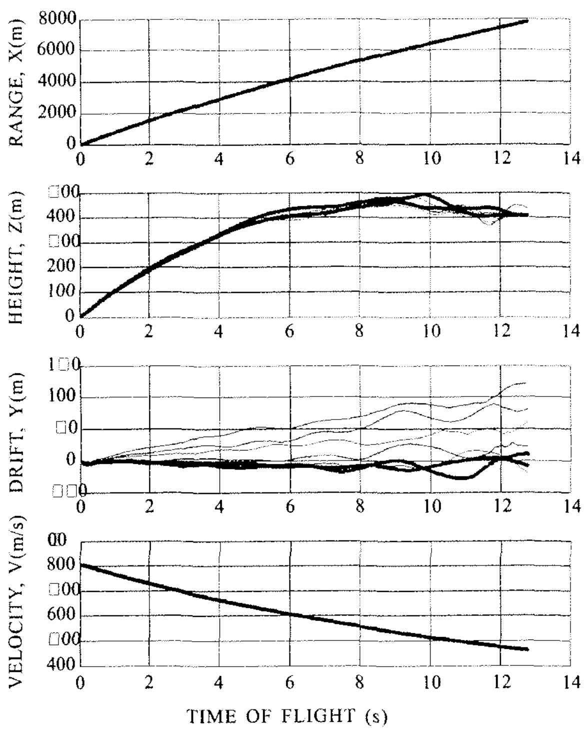


Figure 2. Time histories of real flight trajectory variables of cargo shell.

3. PROPOSED SCHEMES FOR PARAMETER ESTIMATION

Two schemes using maximum likelihood method to extract the C_D by processing trajectory data have been proposed. In Scheme 1, the C_D was assumed to have the following form in the aerodynamic model used in the estimation algorithm[14,15]:

$$C_D = a_0 + a_1 M + a_2 M^2 \tag{18}$$

Maximum likelihood estimator was used to estimate the coefficient of the assumed polynomial by minimising the cost function $J(\theta)$. The cost function $J(\theta)$ used for the purpose of estimation can be given as:

$$J(\theta) = \sum_{i=1}^{N} (Z_1 - Z_2)^T \times GG^{-1} (Z_1 - Z_2) \tag{19}$$

where Z_1 and Z_2 contain the measured and estimated trajectory variables (X, Z, Y, V) as their elements, respectively. Numerical values of the C_D were computed by substituting the estimated values of a_0, a_1, and a_2 into the Eqn (18).

In Scheme 2, the whole trajectory was split into different sets containing 50 or 100 time histories of trajectory variables. Each data set was processed using maximum likelihood method to estimate a constant numerical value of C_D, corresponding to the average Mach number of the chosen data set. The initial conditions required for the generation of the estimated responses for different successive sets were evaluated by numerically differentiating the time histories of the measured trajectory variables of the ammunition in motion. The measured trajectory variables (X, Z, Y) are expected to have measurement noise associated with it. The presence of measurement noise is likely to cause numerical difficulty in getting reliable initial conditions through numerical differentiation. While applying Scheme 2 on real-trajectory data, this aspect has been given more attention to evaluate the suitability of the Scheme 2 for actual field application.

4. RESULTS AND DISCUSSION

The numerical values of C_D obtained by validating the proposed schemes using simulated and real trajectory data have been presented. The difficulties faced in implementation of these schemes using real trajectory data have also been discussed. Before presenting the detail results of the estimated C_D and discussing its accuracy, a brief discussion is given about the initial values, number of data points, sampling rate, etc. Different combinations of the initial guess values of the polynomial coefficients (a_0, a_1, a_2) were used to start the iterative algorithm. In general, results found not much affected by the choice of the initial values except for a few cases. It is therefore decided to present the result for a typical set of the initial values. The sampling rate has been fixed at $\Delta t = 0.027$ s for all the cases. Maximum likelihood estimator was used to estimate the numerical values of C_D, by minimising the error between the measured and estimated trajectory variables namely X, Z, Y, and V. The estimation algorithm used the point-mass model, as represented by the following equations of motion[1] is

$$\frac{d^2x}{dt^2} = -\frac{\pi \rho d^2 C_D}{8m} V\left(\frac{dx}{dt} - W_x\right) \tag{20}$$

$$\frac{d^2z}{dt^2} = -g - \frac{\pi \rho d^2 C_D}{8m} V\left(\frac{dz}{dt} - W_z\right) \tag{21}$$

$$\frac{d^2y}{dt^2} = -\frac{\pi \rho d^2 C_D}{8m} V\left(\frac{dy}{dt} - W_y\right) \tag{22}$$

The results obtained by implementing the proposed schemes using simulated and real trajectory data are presented here:

4.1 Scheme 1: Study using Simulated Data

To start with, firstly, the proposed scheme 1 was validated using simulated data only. The simulated trajectory data as shown in Fig. 1 were treated as measured trajectory data. The trajectory data contained information about X, Z, Y, and V of the artillery shell in motion. The trajectory data corresponding to initial muzzle velocity of 818 m/s and 360 m/s were used. The launch angle, q for both the cases were fixed at 7°. For estimation purpose, a point-mass model [(Eqns (20)–(22)] was considered in the estimation algorithm. The C_D was expressed as a polynomial function of Mach number as given in Eqn (18). The numerical values of a_0, a_1, a_2 were estimated using maximum likelihood method by minimising the cost function $J(\theta)$ given in Eqn (19). Table 2 presents values of a_0, a_1, and a_2 along with estimates of Cramer-Rao bound obtained using trajectories corresponding to initial muzzle velocity of 818 m/s and 360 m/s. Using the values of a_0, a_1, and a_2, numerical values of estimated C_D were obtained through Eqn (18). The estimated values of C_D were compared graphically with true values of the chosen artillery shell in Fig. 3(a). It can be observed that

Table 2. Estimated values of polynomial coefficients (a_0, a_1 and a_2)

Constant	V = 818 m/s	V = 360 m/s
a_0	– 0.549 (0.0028)*	– 5.449 (0.6611)
a_1	– 0.2137 (0.0026)	9.737 (1.3507)
a_2	0.0346 (0.0007)	-3.983 (0.6887)

* Cramer Rao bound

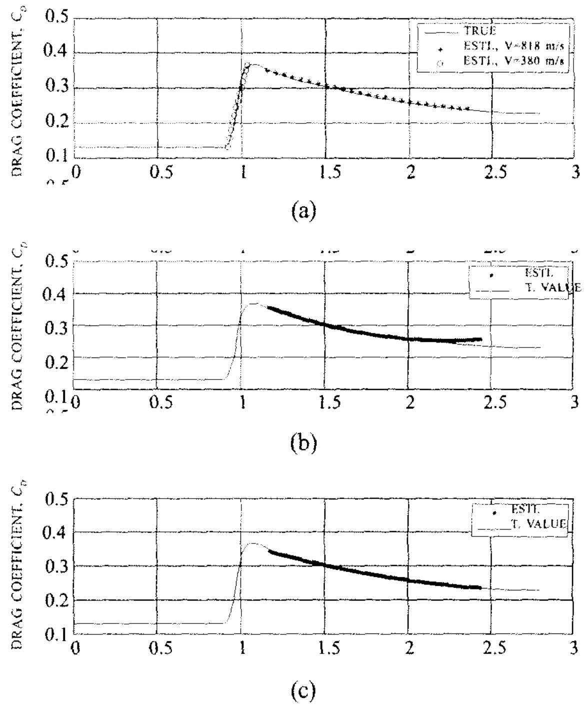


Figure 3. Comparison of estimated values and true values: Scheme 1: Simulated trajectory data: (a) noise=0 %; (b) noise=5 % (without initial conditions estimation), and (c) noise=5 % (with initial conditions estimation).

the estimated values of C_D lie in close agreement with the true values of C_D used for simulation. Further, low value of Cramer Rao bound in Table 2 reflects higher confidence level in the estimation.

It was also of interest to see how the accuracy of estimates is affected by the presence of measurement noise in the radar-tracked trajectory variables. To this purpose, pseudo noise were added to the trajectory data corresponding to V = 818 m/s. The noise was simulated by generating successively uncorrelated pseudo random numbers having normal distribution with zero mean and assigned standard deviation, the standard deviation corresponding approximately to the designated percentage (1 %, 5 %) of maximum error of 20 m in X and Z measurements. It was observed that 1 per cent noise had almost no effect on the accuracy of the estimate (C_D), a higher noise level of 5 per cent affected the parameter, C_D, appreciably. A comparison between estimated C_D obtained by processing flight data with 5 per cent noise and true value of C_D used for simulation is presented in Fig. 3(b). Referring Fig. 3(b), it could be appreciated that the estimated values of C_D with Mach number did not follow the expected trend. A careful look into the application of scheme 1 revealed that while adding pseudo noise in X and Z, the numerical values of X and Z during the initial phase of the trajectory got altered appreciably, however the effect of noise diminished as the trajectory progressed. To overcome this difficulty, it was decided to evaluate the initial conditions X_0, Z_0, U_{X0}, and U_{Z0} along with the polynomial coefficients a_0, a_1, a_2. The estimated values of a_0, a_1, a_2 were then plugged in Eqn (18) to compute variation of C_D with Mach number. A comparison between estimated C_D with true value of C_D is presented in Fig. 3(c). Appreciable improvement in the estimation of C_D could be observed for this case. This is expected as estimation of initial conditions helped in reconstructing the trajectory implicitly.

4.2 Scheme 1: Study using radar-tracked Trajectory Data

After validating the applicability of the Scheme 1 using simulated data, it was then applied on real-trajectory data of cargo shell (Fig. 2). Trajectory data of seven rounds of cargo shell were considered. From Fig. 2, it could be seen that the cargo shell maintained the supersonic flight from launch to terminal phase. Therefore, the variation of C_D during supersonic regime could only be estimated. The trajectory data contained information about X, Z and V. The C_D was expressed as a polynomial function of Mach number as given in Eqn (18). The numerical values of X_0, Z_0, U_{X0}, U_{Z0}, a_0, a_1, and a_2 were estimated using maximum likelihood method by minimising the cost function $J(\theta)$. Table 3 presents the numerical value of X_0, Z_0, U_{X0}, U_{Z0}, a_0, a_1, and a_2 along with its Cramer Rao bounds. Using the values of a_0, a_1, and a_2, the numerical values of C_D at a corresponding Mach number were evaluated using Eqn (18).

Figure 4 shows the variation of C_D with Mach number for all the real flight data of seven rounds of firing. It may be noted from Fig. 4, that the numerical values of C_D estimated using all the seven trajectories lie in proximity to each other. Further, these values of C_D were compared with the true C_D of the cargo shell used for developing firing tables for actual field applications. A close match in

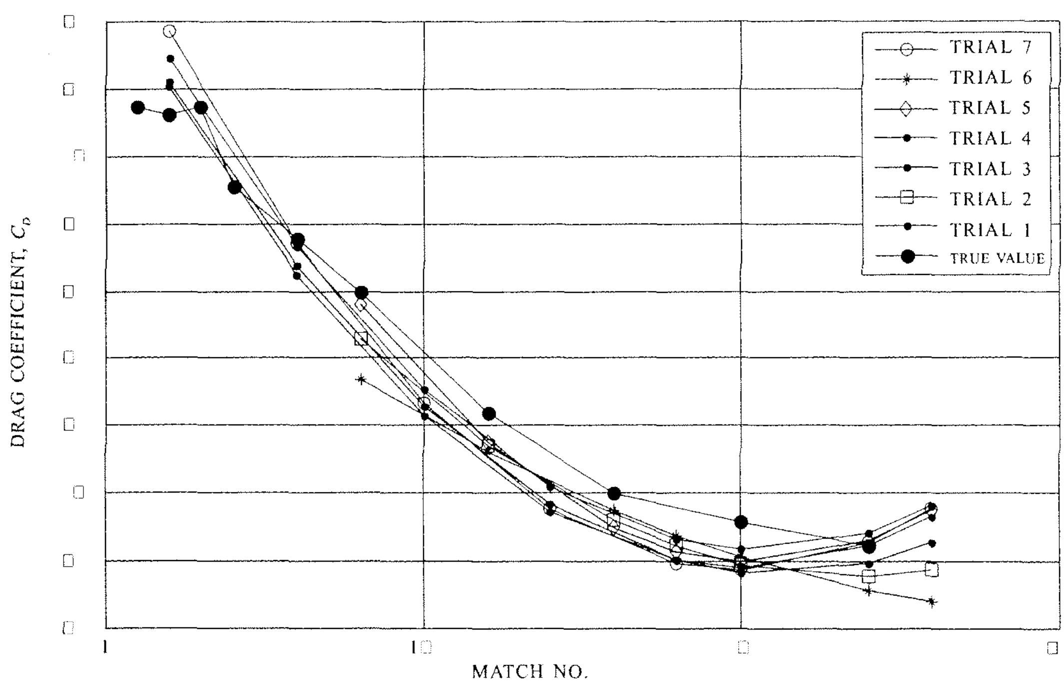


Figure 4. Comparison of estimated values and true values: scheme 1: with initial conditions estimation; Real-flight data.

Table 3. Estimated values of X_0, Z_0, U_{X0}, U_{Z0}, a_o, a_1 and a_2

Parameters	Estimated values for flight data of trail no. 5
X_0	105.79 (0.0269)*
Z_0	4.85 (0.0154)
U_{X0}	799.3 (0.02765)
U_{Z0}	108.61 (0.00411)
a_o	1.2156 (0.001)
a_1	–0.94607 (0.00116)
a_2	0.24168 (0.00033)*

* Cramer Rao bound

Fig. 4 suggests positively on the utility of the Scheme 1 to extract C_D by processing the radar-tacked trajectory data of artillery shell in motion.

4.3 Scheme 2: Study using Simulated Data

While applying Scheme 2, complete simulated (Fig. 1) time histories of trajectory variables X, Z, Y, and V were broken into finite number of smaller sets containing information of trajectory variables of successive 50 data points having total time duration of 1.35 s. Each of these sets was considered separately to estimate a constant value of C_D. The initial conditions required for solving the equations of motion were obtained by numerically differentiating the trajectory variables X and Z. Figure 5(a) graphically shows a comparison between estimated and true value of C_D of the chosen example artillery shell. The estimated values of C_D almost exactly follow the true values of C_D used in the simulation.

To evaluate the robustness of the Scheme 2, in relation to the presence of measurement noise in the trajectory data, it was decided to apply scheme 2 on the trajectory data having 5 per cent noise. Fig. 5(b) presents graphically a comparison between estimated and true C_D used for simulation. The poor matching between the estimated and true values of C_D was attributed to the presence of noise in the trajectory data. The Scheme 2, requires numerical differentiation of motion variables X, Z to evaluate successive initial conditions of the chosen set to estimate the trajectory variables. The presence of measurement noise in the trajectory data resulted in erratic value of the initial condition (X_0, Z_0, U_{X0}, U_{Z0}). To avoid this difficulty, it was decided to smoothen the noisy data by filtering it by a polynomial of 5th order. The initial condition of the trajectory variables corresponding to a particular set was obtained by numerically differentiating the smoothen trajectory data. A comparison between true values and estimated values of C_D after preprocessing (filtering) the noisy trajectory data is presented in Fig. 5(c). It could be observed that, by smoothing the noise trajectory data with a polynomial of 5th order, there was appreciable improvement in the estimated value of C_D.

4.4 Scheme 2: Study using Radar-tracked Trajectory Data

Accordingly, before applying Scheme 2, on real trajectory data, a polynomial fit was employed to smoothen the motion variable X, Z, Y. As shown in Fig. 6, the application of Scheme 2, on the real trajectory data, resulted in numerical values of C_D with large scatter. The average value of the estimated C_D did lie a little closer to the true value of C_D of the shell. The application of Scheme 2, seems to be highly sensitive to the presence of measurement noise in the trajectory data. Before application of Scheme 2, it is necessary to smoothen the trajectory data by filtering out the noise content present in the trajectory data. Thus, the applicability of the Scheme 2 on the real trajectory data would largely depend on the quality of trajectory data acquired by radar tracking system. The sensitivity of this Scheme 2 with respect to presence of measurement noise may restrict the use of it on real trajectory data for the purpose of estimation of aerodynamic parameters.

Figure 7 represents a comparison among the average values of the estimates obtained using Scheme 1 and Scheme 2. These are then compared with the true values of C_D at corresponding Mach Nos. Despite the poor quality of the trajectory data, both the schemes yields estimates in close proximity with the true values of C_D. Although few estimates obtained by Scheme 2 show large scatter, this scatter can be reduced by properly pre-filtering the trajectory data.

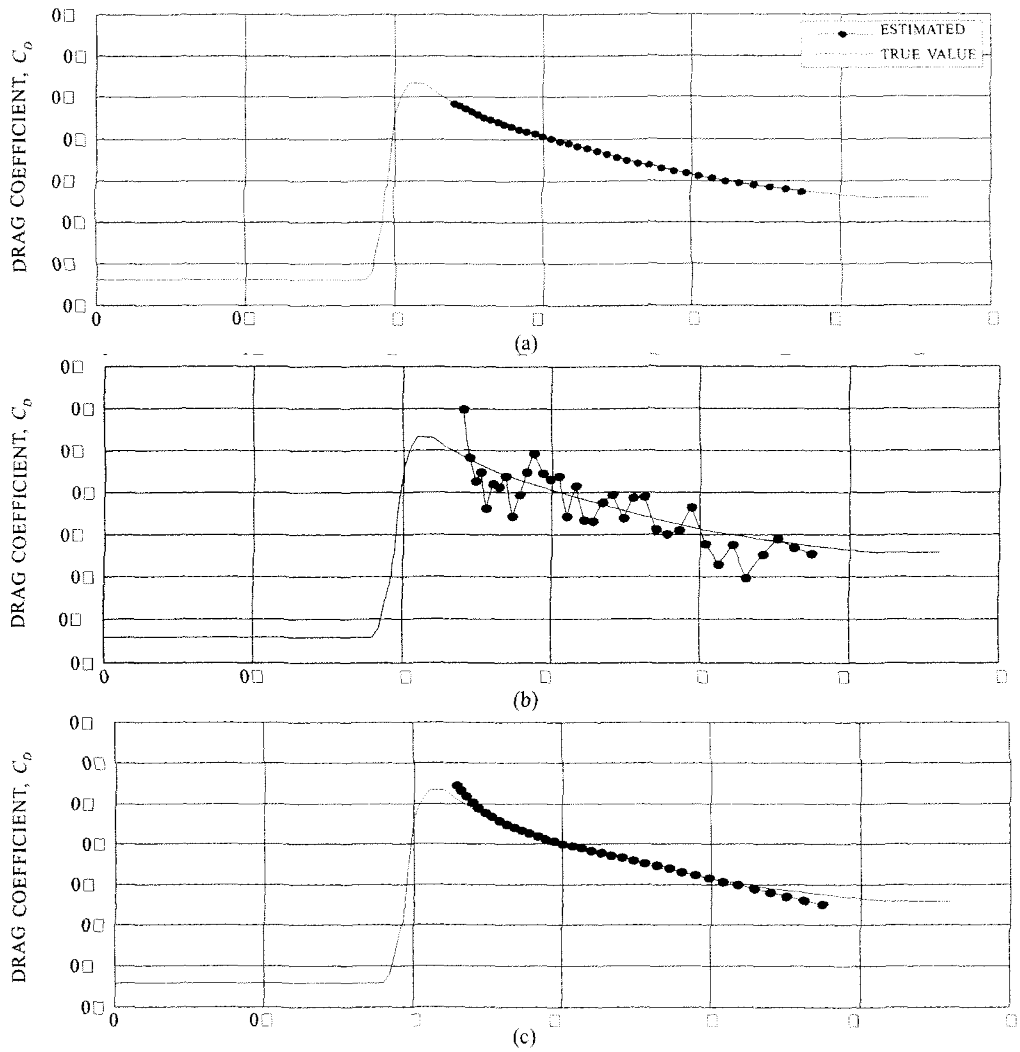


Figure 5. Comparison of estimated values and true values: Scheme 2: Simulated trajectory data: (a) noise = 0 %; (b) noise = 5 % (without initial conditions estimation); and (c) noise = 5 % (with initial conditions estimation).

5. CONCLUSIONS

In this paper, two schemes have been proposed to estimate the C_D (as function of Mach number) by applying maximum likelihood estimation algorithm on radar-tracked trajectory data of an artillery shell. Scheme 1 assumes series approximation of the C_D as function of Mach number in estimation algorithm. In Scheme 2, the whole trajectory was split into different sets of points and average values corresponding to each set of points were estimated.

Based on the results obtained via these schemes, it can be concluded that both the schemes can advantageously be used to estimate C_D by processing radar-tracked trajectory data of an artillery shell in motion. However, the applicability of Scheme 2 on the real-trajectory data would largely depend on the quality of trajectory data acquired by radar-tracking system. Thus, the sensitivity of this scheme wrt presence of measurement noise may restrict the use of it on real-trajectory data for estimation of aerodynamic parameters

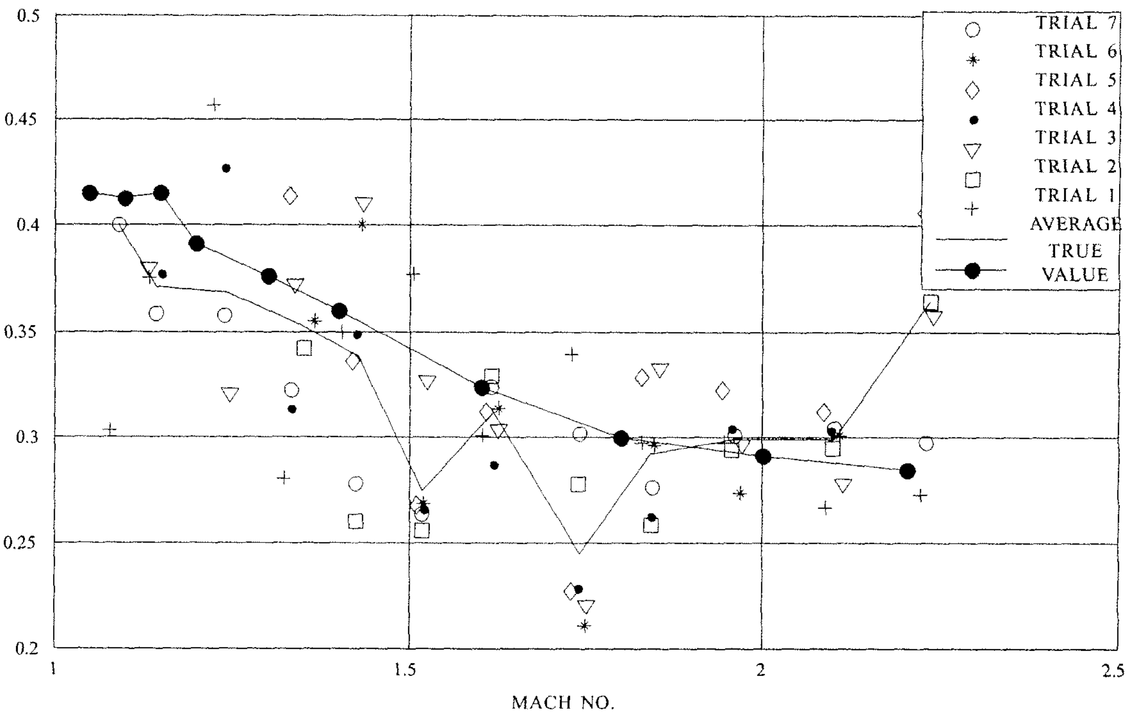


Figure 6. Comparison of estimated and true values of drag coefficient: Scheme 2 (with initial conditions estimation): Real-flight data.

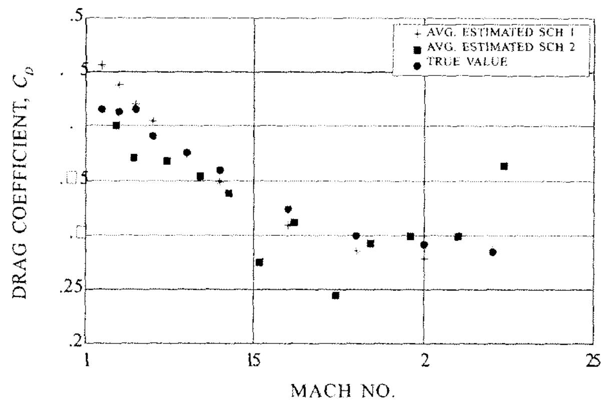


Figure 7. Comparison of average values of estimates through Scheme 1 and Scheme 2 with the true values of drag coefficient.

REFERENCES

1. Anonymous. External ballistics: Text book of ballistics and gunnery, Vol. 2. Her Majesty of Stationary Office (HMSO), London, 1987. pp. 443-97.

2. Mc Coy, Robert L. Modern exterior ballistics. Schiffer Publishing Ltd, Atglen, PA, 1999.

3. Moore, F.G. Approximate methods for weapon aerodynamics, Chaps. 7-14, Progress in Astronautics and Aeronautics, April 2000, .

4. Roskam, J. Methods for estimating stability and control derivatives for conventional subsonic airplanes. Roskam Aviation and Engineering Corporation, 1973.

5. Taylor, L.W.; Iliff, K.W. & Powers, B.G. A comparison of Newton-Raphson and other methods for determining stability derivatives from flight data. AIAA Paper No. 69-315, May 1969.

6. Hamel, Peter G. & Jategaonkar, R.V. The evolution of flight vehicle system identification. *Journal of Aircraft*, 1996, **33**(1), 09-28.

7. Klein, V. Estimation of aircraft aerodynamic parameter from flight data. Prog. Aerospace Sci., 1989, **26**, 1-77.

8. Seckel, E., & Morris, J.J. The stability derivatives of the Navion aircraft estimated by various methods and derived from flight test data, FAA-RD-71-6.

9. Iliff, K.W. Parameter estimation for flight vehicle. *J. Guid. Control, Dyn.*, 1989, **12**(5), 609-22.

10. Jategaonkar, R.V. Identification of the aerodynamic model of the DLR research aircraft ATTAS from flight test data. *DLR, DLR-FB 90-40*, Braunschweig, Germany, 1990.

11. Wincheback, G.L.; Randy, S. Buff; White, R.H. & Hathway, W.H. Subsonic and transonic aerodynamics of a wrap around fin configuration. *J. Guid. Control, Dyn.*, 1986, **19**(6).

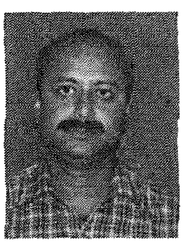
12. Dupis, Alan D. High spin effect on the dynamics of a high l/d finned projectiles from free-flight tests. *J. Guid. Control Dyn.*, 1989, **12**(2), 129-34.

13. Ghosh, A.K.; Sridhar, I.V.S.; Singhal, A. & Om Prakash. Estimation of drag coefficient from radar tracked trajectory data of an artillery shell. *J. Aerospace Sci. Technol.*, *2004*, ***56****(1)*, 62-65.

14. Mendel, J.M. Post flight data analysis by means of Adaptive Iterated EKF. *IEEE Trans. Automatic Control*, 1974, **AC-19**(5), 467-74.

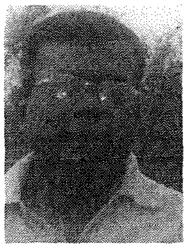
15. Iliff, K.W. & Taylor, L.W. Determination of stability derivatives from flight data using Newton Rapson minimization technique. NASA Report No.TND-8209, April 1976.

Contributors

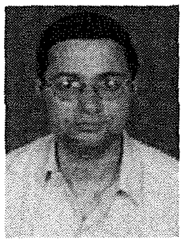


Mr G.G. Dutta is Scientist F at the Armament Research & Development Establishment, Pune, with 22 years experience in the field of design and devolvement of missile warheads, fuzes, and safe arming mechanism. Currently, he is pursuing his doctoral research in the field of aerodynamic parameter estimation from real flight data of free-flight rockets and shells from the Department of Aerospace Engineering, Indian Institute of Technology (IIT), Kanpur.

Mr Ankur Singhal is working as Research Associate in the Department of Aerospace Engineering at IIT, Kanpur. He is working in the area of flight mechanics, flight testing, parameter estimation from real flight data, and neural networks.



Dr A.K. Ghosh is Faculty Incharge, Flight Laboratory, IIT, Kanpur. He obtained his BTech, MTech and PhD from IIT, Kanpur. His area of interest are flight mechanics, flight testing, parameter estimation from real-flight data, and neural networks.



Dr Abhijit Kushari is Faculty Incharge, High Speed Wind Tunnel at IIT, Kanpur. He obtained his PhD from the Georgia Tech, USA. His areas of interest are rocket and gas turbine propulsion, instrumentation in combustion and fluid mechanics, liquid atomisation and liquid combustion, active flow control, combustion instability, experimental fluid mechanics, and high speed flows.