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Aging Maxwell Constitutive Model for Concrete

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ABSTRACT

Fully-hydrated concrete not involved in any reaction has been observed to exhibit aging creep. Both the solidification theory and the dissolution-precipitation theory are incapable of predicting such a behaviour. The microprestress theory proposed for this purpose is based upon an ambiguous physical mechanism. In this paper, a constitutive model motivated by Drozdov's adaptive link mechanism has been proposed. The model is capable of predicting aging creep, recovery and relaxation for linear elastic concrete subjected to diverse load histories and temperatures. The theoretical significance of the proposed aging Maxwell model has been critically evaluated.

Keywords: Constitutive model, aging creep, viscoelasticity, mature concrete, microprestress, linear elastic concrete

1. INTRODUCTION

Concrete is a versatile material of construction for civil, industrial and defence structures. Under the action of working stresses, concrete is known to be a homogeneous isotropic linear aging viscoelastic solid exhibiting the creep, recovery, and stress relaxation. Relevant available experimental data reveals that concrete[1] obeys McHenry's superposition principle and exhibits the effect of age at loading on the creep of hydrating concrete[1].

Conventional rheological models have proved to be incapable of explaining the observed aging creep of hydrating concrete. As a physically motivated adaptation of these rheological models, the solidification theory has been quite successful in providing an explanatory mechanism and a prediction model for the same. As per this theory, creep is associated with the assumed viscoelastic nature of the hydration products while aging is sought to be explained in terms of concrete microstructure evolving with the progress of hydration[2,3].

In the later experimental investigations, mature concrete, otherwise exhibiting little creep, has been observed to do so while involved in some reactions, e.g., with sulphate and nitrate solutions[4-7]. Dissolution-precipitation theory has been proposed to explain and predict the time-dependent mechanical behaviour of reacting elastic concrete. Here, creep is associated with gradual transformation of the stressed reacting phase of concrete into stress-free solute on dissolution and the rate of creep is proportional to the rate of reaction. Time-dependent increase of material elasticity and viscosity parameter with progress of reaction explains the aging viscoelastic behaviour[8-10].

Mature concrete has also been observed to exhibit aging creep. For a fully-hydrated concrete, the solidification theory and the dissolution precipitation

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theory are not capable of explaining this phenomenon. To remove this lacuna, microprestress solidification theory has been proposed but the underlying physical mechanism for aging creep of mature concrete is not convincing[11,12]. In this paper, motivated by Drozdov's adaptive link mechanism[13], the required constitutive model for aging creep of mature concrete has been presented. The proposed model has been compared with the microprestress model.

2. PROPOSED AGING MAXWELL MODEL

Drozdov[13] has presented the theory of adaptive links to study the thermoviscoelastic behaviour of aging solid polymers in the rubbery and glassy states. Here, the viscoelastic solids are modelled as a network of long chains connected at discrete locations to each other by adaptive elastic physical cross-links and entanglements. These chains experience relative movement due to thermal micro-Brownian motion resulting in continual breaking and reformation of the adaptive links. The time-dependent behaviour of these stressed materials is sought to be explained by the process of annihilation and creation of the adaptive links. The instantaneous viscosity of the material at a certain temperature and in a certain state of strain and stress depends upon the rates of breakage and reformation of the adaptive links. A class of materials, called thermo-rheologically simple materials, is shown to obey time-temperature superposition principle in terms of thermal shift.

The proposed constitutive model for linear aging viscoelastic behaviour of concrete, has been motivated by Drozdov's adaptive link mechanism[13]. The thermal micro-Brownian motion of the atoms resulting in the continual breakage and restoration of inter-atomic bonds is a random phenomenon. The frequency of the atomic vibration varies from atom to atom. Consequently, the frequency of the bond breakage and restoration process is different for different pairs of atoms. At any instant, the fraction of bonds in the broken state about to be restored depends upon the probability distribution of vibration frequency of the atoms. Obviously, the appropriate approach to constitutive model has to be based on statistical mechanics.

However in this study, ignoring its discrete atomic/molecular constitution, the material has been assumed to a continuum. Also, the proposed constitutive model is stated in terms of deterministic variables and parameters. To be specific, the fraction of the bonds in the broken state at any instant has been assumed to denote the fraction of the area incapable of offering elastic resistance to deformation. Also, this fraction has been assumed to be a deterministic continuous function of time and ambient temperature. Here, concrete has been assumed to be isotropic linear elastic composite undergoing only small deformations. Its constituent phases are uniformly distributed and include aggregate, cement paste, air, water, etc. The process of bond breakage and restoration in all the phases is assumed to proceed at the same pace and does not result in any change in its phase composition.

Let the material point under consideration be under the action of a state of applied stress σ_{ij}. Let f denote the instantaneous fractional rate of increase of the area with broken bonds. The increase in such fractional area in infinitesimal time interval dt is fdt. Just after the bond breakage, this fractional area is incapable of resisting the applied stresses acting on it just an instant earlier. Since the applied state of stress does not change in this time interval, the incremental state of stress $d\sigma_{ij}$ considered to be imposed on the material point turns out to be $\sigma_{ij} fdt$. Consequently, material point experiences incremental strains given by

$$d\varepsilon_{ij} = D_{ijkl}\sigma_{kl}\, fdt \tag{1}$$

where D_{ijkl} is the fourth rank compliance tensor for the material. The above incremental constitutive equation can be restated in the rate form as

$$\dot{\varepsilon}_{ij} = f\; D_{ijkl}\sigma_{kl} \tag{2}$$

Here, the superposed dot implies time derivative of the variable. Thus, there is a continuous transfer of stress from the area with breaking bonds to the rest of the solid. The fractional rate f is assumed to be small enough to be ignored. This implies that the material compliances D_{ijkl} remain constant over time.

Thus, the process of breakage of bonds is followed by the incremental elastic deformations. Only then, these broken bonds are superposed to be restored and arranged in parallel to the remaining unbroken bonds in their current deformed sate. These newly restored bonds which are stress-free at the instant of restoration get stressed as and when their new natural state changes with further deformation of the material. Evidently, interatomic forces continue to be redistributed with newly formed bonds carrying lesser forces. In view of this fact, all the bonds can justifiably be assumed to be carrying equal interatomic force. This assumption implies that all the microconstituent phases of the material point can be considered to always be subjected to the same state of average applied stress. These microconstituent phases include areas with bonds broken and restored at different instant in the past. This is the justification for assuming the incremental stress components to be $\sigma_{ij} f dt$ in the above derivation.

These interatomic forces are randomly distributed amongst the bonds having the same probability of being in the broken or restored state.

As per Arrhenius principle, the fractional rate f of bond breakage and restoration depends upon the ambient temperature T and thermal activation energy E_0. Here, the activation energy has been assumed to be independent of the applied state of stress or strain. In this study, the temperature remaining constant, f is further assumed to decay exponentially with time. Thus

$$f = Ae^{-\lambda t} e^{-\frac{E_0}{RT}} = Ae^{-\left(\lambda t + \frac{E_0}{RT}\right)} \tag{3}$$

where A is a material constant. At constant temperature, f varies with time as

$$f = Be^{-\lambda t} \quad \text{and} \quad B = Ae^{-\frac{E_0}{RT}} \tag{4}$$

Since f keeps on decreasing with time and vanishes asymptotically, the realisable integrated fraction F has an upper limit F_0. The integrated fraction F realised till time t is obtained as

$$F = \int_0^t f dt = \frac{B}{\lambda}\left[1 - e^{-\lambda t}\right] \tag{5}$$

Let $\tau = \frac{1}{\lambda}$ be the characteristic material time, then

$$F = B\tau\left[1 - e^{-\frac{t}{\tau}}\right] \tag{6}$$

From above, one obtains

$$F_0 = \frac{B}{\lambda} = B\tau \tag{7}$$

The remaining unrealised integrated fraction at time t is given by

$$\overline{F} = F_0 - F = B\tau e^{-\frac{t}{\tau}} = f\tau \tag{8}$$

The Eqn (8) implies that, at any instant, if the fractional rate of bond breakage and restoration is kept constant, the remaining part of the integrated fraction can be realised in same duration τ.

The Eqn (2) can be interpreted to yield the instantaneous rate of creep under sustained state of stress. If the state of stress also varies with time, then additional elastic strain rate is also present and given by $D_{ijkl}\dot{\sigma}_{kl}$. Thus, the following constitutive equation for aging mature concrete is obtained:

$$\begin{aligned} \dot{\varepsilon}_{ij} &= f\,D_{ijkl}\sigma_{kl} + D_{ijkl}\dot{\sigma}_{kl} \\ &= D_{ijkl}\left(f\,\sigma_{kl} + \dot{\sigma}_{kl}\right) \end{aligned} \tag{9}$$

The constitutive Eqn (9) for creep rate can be restated as

$$\dot{\varepsilon}_{ij} = m_{ijkl}\sigma_{kl} + D_{ijkl}\dot{\sigma}_{kl}\,; \quad m_{ijkl} = f\,D_{ijkl} \tag{10}$$

where m_{ijkl} is the material mobility tensor as an inverse of the material viscosity tensor.

The viscosity tensor turns out to be inversely proportional to the fractional rate of bond breakage and restoration. Thus, material viscosity varies with progress of time. If f happens to be constant, the above constitutive equation can be recognised as corresponding to Maxwell rheological model for general state of stress and strain. Thus, material with varying viscosity can be said to obey an aging Maxwell rheological model.

Sudden application of the state of stress at any instant introduces the corresponding elastic strains given by

$$\varepsilon_{ij}^{e} = D_{ijkl}\sigma_{kl} \tag{11}$$

In view of this Eqn (2) for rate of creep can be restated as

$$\dot{\varepsilon}_{ij} = \varepsilon_{ij}^{e} f \tag{12}$$

Thus, the rate of creep for any strain component is proportional to the corresponding elastic strain component and to the instantaneous value of the parameter f. The total creep occurring under sustained stated stress between time t_1 and $t_2 \geq t_1$ is obtained as

$$\varepsilon_{ij}^{c} = \int_{t_1}^{t_2} \varepsilon_{ij}^{e} f dt = \varepsilon_{ij}^{e}\left(F_2 - F_1\right) \tag{13}$$

In case, $t_2 \to \infty,\ F_2 \to F_0$, the maximum creep depends upon age at loading t_1 as

$$\varepsilon_{ij}^{c} = \varepsilon_{ij}^{e}\left(F_0 - F_1\right) \tag{14}$$

In other words, the creep coefficients θ_{ij} for the strain ε_{ij} in the material loaded at age t_1 can be estimated as

$$\theta_{ij} = \frac{\varepsilon_{ij}^{c}}{\varepsilon_{ij}^{e}} = F_0 - F_1 \tag{15}$$

In case, $t_1 = 0$ and $F_1 = 0$,

$$\theta_{ij\max} = F_0 \tag{16}$$

The integrated fraction F_0 represents maximum value of the creep coefficient. In general, creep coefficient depends upon age at loading t_1. The expression $F_0 - F_1$ can be interpreted as unrealised creep potential for the concrete loaded at time t_1.

Multiaxial stress relaxation under constant state of strain is obtained as

$$f\ \sigma_{kl} + \dot{\sigma}_{kl} = 0 \quad \text{or} \quad \frac{\dot{\sigma}_{kl}}{\sigma_{kl}} = -f \tag{17}$$

Thus, the fractional rate of relaxation for all stress components turns out to be same. Similarly, if the strains are held constant after time t_1, the residual stress component σ_{kl} at time $t \geq t_1$ is obtained as

$$\frac{\sigma_{kl}}{\bar{\sigma}_{kl}} = e^{-(F-F_1)} \tag{18}$$

where $\bar{\sigma}_{kl}$ is the corresponding stress component at time t_1. Thus, the residual stresses required to keep maintain the same strain keep on decreasing with time, but complete stress relaxation never occurs. The fractional relaxation at any time is the same for all the stress components.

As f depends upon time as well as ambient temperature, the proposed constitutive Eqn (12) is applicable to linear aging thermoviscoelastic concrete. As the exponent in Eqn (3) is $\left(\frac{t}{\tau} + \frac{E_0}{RT}\right)$, the ambient temperature affects the origin of the time scale. A change in temperature is equivalent to Drozdov's thermal shift in the time scale.

Thus, in this study, the aging concrete has been modelled as a thermo-rheologically simple material. The characteristic time of the material τ is independent of the ambient temperature history. However, the coefficient B measuring the instantaneous f increases with temperature. So, at elevated temperatures, the rate of creep is higher. It implies that the total creep exhibited by material at higher ambient temperature is higher.

3. DISCUSSIONS

Microprestress theory, a special case of the microprestress solidification theory valid for fully-hydrated concrete, provides the mechanism for its aging basic creep. Disjoining pressure acting on the micropore walls caused by hindered adsorbed water introduces internal tensile stress in the chemical bonds bridging the micropore walls. Such internal tensile stress in the randomly oriented bonds has been called microprestress[14]. Viscous creep flow in concrete under the action of applied macrostress states is assumed to be caused by the viscous shear slips between the opposite micropore walls. The physical mechanism responsible for the dependence of viscosity on microprestress has been identified as the continuous breakage and reformation of the micropore bonds carrying the microprestress. The material viscosity is assumed to decrease with increase in microprestress. Material viscosity increases due to microprestress relaxation with age[11,12].

Experimental data on concrete shows that sudden increase or decrease of temperature or moisture content results in an increase of its creep rate. These effects are called transitional thermal creep and Pickett effect, respectively. As per the microprestress-solidification theory, in both the cases, the microprestress increases resulting in observed higher rate of creep[15,16].

The proposed physical mechanisms along with empirical data imply the following relations used for predicting the observed aging creep:

(a) The rate of creep is inversely proportional to the instantaneous viscosity, which in turn is inversely proportional to the instantaneous magnitude of microprestress.

(b) The microprestress is inversely proportional to time elapsed since casting of concrete.

(c) Thus, the viscosity of concrete is established to be a linear function of its age.

Despite the detailed description of the physical mechanisms informing the microprestress theory, the physical mechanisms explaining the dependence of rate of creep upon the applied macrostress and the frequency of bond breakage and restoration have not been identified. Also, the physical mechanism responsible for microprestress relaxation is not clear. As if in response to these lacunae in their proposed physical model, the authors introduced, in the same paper, another one stated in terms of shear stress relaxation for the same purpose[11]. It can be observed that this alternative physical model does not even invoke the concept of microprestress.

Transitional thermal creep and Pickett effect can also be explained by the proposed aging Maxwell model as follows:

(a) Sudden change in ambient temperature or relative humidity introduces thermal and hygral gradients in the specimen.

(b) As per Eyring effect, the increase in strain energy associated with the resulting thermal and hygral stresses lowers the activation energy.

(c) Consequently, the rate of bond breakage/restoration, and so the rate of creep increases.

In this paper, a better constitutive model for aging creep of mature concrete based upon explicitly stated assumptions has been presented. The proposed physical mechanism responsible for time-dependent response of elastic concrete is motivated by Drozdov's adaptive link mechanism. The instantaneous rate of creep has been derived to be proportional to the sustained stress as well as to the instantaneous fractional rate of bond breakage and restoration. For general state of sustained stress, the rate of creep is obtained as linear function of stress. Equivalently, the rate of creep for any strain component is proportional to the corresponding elastic strain component. Explicit expression for the mobility tensor, an inverse of the viscosity tensor of the material, has been derived in terms of its compliance tensor and the instantaneous fractional rate of bond breakage/restoration. For time-dependent stress of strain, the constitutive equation resembles that for an aging Maxwell material capable of exhibiting creep, recovery, and relaxation.

The physical mechanism for aging is motivated by the understanding that the instantaneous fractional rate of bond breakage and restoration is proportional to the number of still unbroken bonds. Thus, at constant temperature, the effect of temperature is incorporated using Arrhenius principle. Like Drozdov's thermorheologically simple material, the resulting model exhibits the thermal shift of origin of time scale with change in ambient temperature.

The model predicts same creep coefficient for all strain tensor components and the same fractional rate of relaxation for all the stress tensor components. Empirical calibration of the model using available data[12] yields the values of parameters B, λ, τ and F_0 as 0.003/day, 0.0015/day, 667 days and 2, respectively.

4. CONCLUSIONS

In this study, the ambiguities in the microprestress theory for aging viscoelastic behaviour of concrete have been exposed. Physical models for creep and aging of concrete have been constructed. Using these models, a better constitutive model for time-dependent behaviour of aging isotropic linear elastic concrete subjected to arbitrary stress, strain, and temperature histories has been presented.

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