

## Digital Communication Channel Equaliser using Single Generalised Neuron

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### ABSTRACT

Equalisation is necessary in a digital communication system to mitigate the effect of inter-symbol interference and other nonlinear distortions. A new reduced complexity approach to digital communication channel equalization is proposed based on a single generalised neuron (GN). Since it uses only a single GN, there is no problem of selection of initial architecture of the neural network giving optimum performance. It has less computational requirements giving rise to reduced training and computation time. The simulation results show that proposed equaliser bit error rate (BER) performance approaches to optimal Bayesian solution.

**Keywords:** Digital communication, inter-symbol interference, channel equaliser, single generalised neuron, artificial neural network.

### 1. INTRODUCTION

Band-limited high speed digital transmission suffers from inter-symbol interference (ISI) and various other noise sources and equalisation is necessary at the receiver to overcome these channel impairments [1]. Figure 1 shows the simplified model of a discrete time transmission model of a communication system. Nonlinear channel model is shown in Fig. 2.

Band-limited communication channels are generally modelled as digital FIR filters represented as

$$H(z) = \sum_{i=0}^N h_i z^{-i} \quad (1)$$

where,  $N$  is the channel order,  $q(k)$  is the additive white Gaussian noise (AWGN). The task of equaliser is to recover an estimate of  $s(k-d)$ , denoted as  $s'(k-d)$ , using the channel output,  $y(k)$ , the present and past values,  $d$  is the equaliser delay and  $k$  is any time instant.

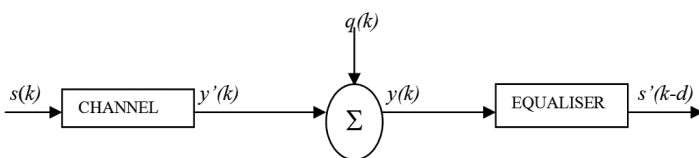


Figure 1. Simplified block diagram of discrete time transmission model.

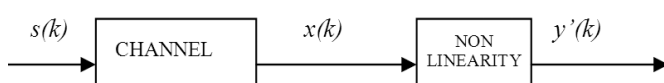


Figure 2. Nonlinear channel model.

The channel output vector can be represented as

$$y(k) = [y(k), y(k-1), \dots, y(k-m+1)]^T \quad (2)$$

where,  $m$  is the order of equaliser.

Traditionally, equalisation has been considered as equivalent to inverse filtering, where a linear transverse equaliser (LTE) is used to invert the channel response and its parameters are adjusted using minimum mean square error (MMSE) criterion[1]. Least mean square error (LMS) algorithm is most commonly used for this purpose. Linear filter-based equalisers under-perform under severe nonlinear and distortion conditions. The maximum likelihood sequence estimation (MLSE)[2] gives nearly optimum results but it requires batch processing of the entire received sequence. Its high computational complexity restricts its use, hence practically not suitable. Equalisation can be considered as a nonlinear classification problem where the job of an equaliser is to assign the received signal to one of the signal constellation. The optimal solution (least misclassification) to this classification problem is given by Bayes theory[1].

The channel input vector for an  $m^{\text{th}}$ , order equaliser is given by

$$s(k) = [s(k), s(k-1), s(k-2), \dots, s(k-m+1-N)]^T \quad (3)$$

and can take  $N_s = 2^{N+m}$  different values, giving rise to  $N_s$  possible values of noiseless channel output vector given by

$$y'(k) = [y'(k), y'(k-1), y'(k-2), \dots, y'(k-m+1)]^T \quad (4)$$

which, is to be divided into two classes

$$Y^+ = \{y'(k)/s(k-d) = 1\}$$

$$Y = \{y'(k)/s(k-d) = -1\} \tag{5}$$

where  $y(k)$ , is a random process having conditional Gaussian density functions centered at each of the  $Y_i^+$  and  $Y_i^-$ , where  $i \in \{1, 2, \dots, N_s/2\}$ .

If the transmitted sequence  $s(k)$  is an independent identically distributed (i. i. d.) and equi-probable binary sequence with values  $\{+1, -1\}$ . For this sequence, the optimal solution to this classification task is given by Bayes theory as [3].

$$s'(k-d) = \text{sgn}(f_B(y(k))) = \begin{cases} +1 & f_B(y(k)) \geq 0 \\ -1 & f_B(y(k)) < 0 \end{cases} \tag{6}$$

$$f_B(k) = \sum_i \exp\left(-\frac{\|y(k) - Y_i^+\|^2}{2\sigma_n^2}\right) - \sum_j \exp\left(-\frac{\|y(k) - Y_j^-\|^2}{2\sigma_n^2}\right) \tag{7}$$

where  $\sigma_n^2$  is the variance of the noise,  $q(k)$ .

Good equalisers for this channel will than approximate the function given by the Eqn (7) which is a nonlinear function. Optimal theoretical solution obtained as above with the knowledge of channel and noise characteristics is known as Bayesian solution.

Nonlinear mapping capability of artificial neural network (ANN) and fuzzy logic make them a suitable choice for the equalisation of nonlinear equalisation. Equalisers based on classification problem attempts to reach optimum performance given by Bayesian equaliser performance in terms of bit error rate (BER). Several equalisers are developed to address this problem using ANN and fuzzy logic. Radial basis function RBF[3] and Multi layer perceptron (MLP)[4-6] equalisers provide good approach to optimal Bayesian solution but at the cost of high computational complexity.

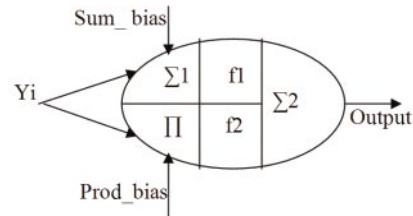
The applications of reduced complexity functional link ANN (FLANN) for channel equalisation have been reported[7-8]. The mathematical requirement of FLANN increases for functional expansion of input data. The Kernel Adaline (KA) equaliser's[9] BER performance approaches to Bayesian solution but the complexity of KA increases with number of training pairs. Recurrent neural network (RNN) with only a few nodes outperforms the LTE. However, the computational complexity of the RNN training algorithm is excessively large, and hence, these networks involve a very long training time when the number of nodes increases. Rule-based Fuzzy equalisers are highly effective in approximating the optimal solution [11]. These fuzzy rules are developed by human experts using input- output data pairs of the channels. In some

cases, the construction of fuzzy rules is difficult when the channel is unknown.

Such structures usually outperform LTE and also compensate for nonlinearities in the channel with varying degree of success Thus, the major problem of such equalisers is the complexity and computational requirement which can further be reduced. The common neuron model has been modified to obtain a generalised neuron (GN) model using fuzzy compensatory operators to reduce the complexity of the structure and overcome the problems such as initial selection of architecture of neural network, giving optimum performance for complex function mapping, which affects the training time requirement and also fault-tolerant capabilities of the ANN [12]. GN has been used successfully for power systems problems [13-14]. A new approach to channel equalisation in digital communication systems using GN is proposed. It has been shown that proposed equaliser BER performance outperforms conventional LTE, approaches to optimal theoretical Bayesian equaliser and comparable to MLP equalisers and requires very low complexity and simple architecture.

**2. GENERALISED NEURON MODEL**

Existing conventional neuron model uses an aggregation function and its transformation through an activation function. It uses generally summation as aggregation with sigmoid, radial basis, tangent hyperbolic or linear limiters, etc, as activation function. Generalised neuron structure[13] shown in Fig. 3 is developed by modifying the conventional neuron structure using fuzzy compensatory aggregation operators along with fuzzy activation functions.



**Figure 3. Structure of generalised neuron.**

Aggregation operation in GN is performed partly by sum ( $\Sigma_1$ ) and partly by product ( $\Pi$ ) functions with weight sharing. Sigmoid function ( $f1$ ) is used as a transformation function for  $\Sigma_1$  part and Gaussian function ( $f2$ ) is used as transformation function for  $\Pi$  part of the structure. The final output is the summation of the  $\Sigma_1$  output and  $\Pi$  output with weights  $W$  and  $(1-W)$ , respectively.  $Y_i$  represents the input vector.

Back-propagation (BP) learning rule used for error minimisation is as follows:

$$W(k) = W(k-1) + \Delta W \tag{8}$$

$$\text{where, } \Delta W = \eta \frac{\partial \text{Error}}{\partial W} + \alpha W(k-1) \tag{9}$$

$\eta$  = learning rate,  $\alpha$  = momentum, weight vector =  $W$  and

Error = error function.

Generally, mean square error (MSE) is used as error function. The details of GN and its learning algorithm [13] has been found and presented here in appendix.

### 3. SIMULATION RESULTS AND DISCUSSION

Following channel model as given in [9-10] is used to simulate the channel.

$$H(z) = 0.3482 + 0.8704z^{-1} + 0.3482z^{-2} \quad (10)$$

This represents linear channel NL=0.

The nonlinear (NL) channel, NL=1 is modelled as

$$y(k) = \tanh(x(k)) + q(k)$$

$$\frac{X(z)}{S(z)} = 0.3482 + 0.8704z^{-1} + 0.3482z^{-2} \quad (11)$$

NL=1, corresponds to the nonlinearity introduced due to saturation of amplifiers used in the transmission systems.

And NL = 2 is modelled as

$$y(k) = x(k) + 0.2x^2(k) - 0.1x^3(k) + q(k)$$

$$\frac{X(z)}{S(z)} = 0.3482 + 0.8704z^{-1} + 0.3482z^{-2} \quad (12)$$

NL = 2 corresponds to the random nonlinear distortions. Equaliser is simulated with  $m = 4$  and  $d = 1$ .

Extensive simulation studies were carried out for the channel equalisation problem using MLP, GN, and a conventional linear equaliser based on classical LMS algorithm. The simulation results have been compared with BER performances of theoretical optimal Bayesian solution. Bayesian performance is theoretically obtained with the prior knowledge of the channel and noise characteristics using Eqn (6). The MLP of 4-4-1 with logsig-tansig-purelin activation functions is used. Selection of detailed structure of ANN used for simulation is mainly by experiment. Selection of type of activation function for each layer, number of hidden layers, and neurons for each layer of MLP, etc are determined by number of experiments to give optimise results with a constraint of hardware complexity. Conventional linear equaliser trained with LMS uses four taps. Back propagation learning algorithm is used to train the network by batch mode.

The parameters used for simulation of GN equalisers are as follows:

- (i) Learning rate - 0.0015
- (ii) Momentum - 0.5
- (iii) Gain scale factor - 1

For a fair comparison, the same learning rate and momentum have been used for the training of equaliser models using MLP and conventional linear equaliser.

Through extensive simulation studies, it has been found that as far as the speed of convergence, it takes only a few hundred training patterns to achieve pretty good results. The training sample size of 1000 is chosen arbitrarily. For training a random sequence of 1000 duobinary signals of

{1, -1}, equiprobable and identically distributed is generated and passed through the channel. Nonlinearities and white Gaussian noise are further introduced. Initial weights are generated randomly. Equalisers are trained using this training sequence to minimise MSE and obtain steady state error. It has been observed during simulation work, that with 1000 number of training samples, 300 epochs are enough

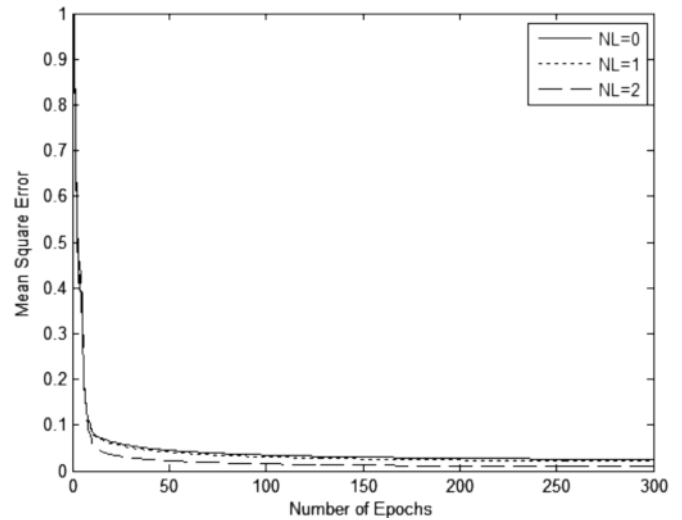


Figure 4. Convergence characteristic of the equalisers for the three channel models at SNR=16 dB.

for the proposed equalisers to reach to steady state error for a variation of signal-to-noise ratio (SNR) from 2-20 dB hence, equalisers are trained for 300 epochs. Figure 4 shows the training performance of the GN-based equalisers for 300 epochs for the three nonlinear channel models at SNR=16dB. Better convergence of error is obtained as the severity of nonlinearity increases. The characteristics show fast and smooth convergence of error for all the three nonlinear channel models. The trained equaliser was then tested using a separate equiprobable, identically distributed sequence with nonlinearities and white Gaussian noise added and generated in the same manner as training sequence. The results were averaged over 10 independent repetitions using testing sample of size 10000 each. SNR was varied between 2-20 dB in steps of 2 dB to ascertain performance under different noise conditions.

Figures 5 to 7 show the plot of BER performance of channel for the channel with NL = 0, NL = 1, and NL = 2, respectively for the various equalisers used in the paper. Superior performance of MLP and GN-based equalisers over the classical LMS-based equaliser is quite evident from these figures.

There is performance degradation of conventional LMS based equalisers as the severity of nonlinearity increases. The performance of MLP and GN-based equaliser is nearly similar to each other for all the three channel models. For most of the cases, GN-based equalisers outperform the MLP-based performance which clearly shows the capability of the GN-based equaliser to reconstruct

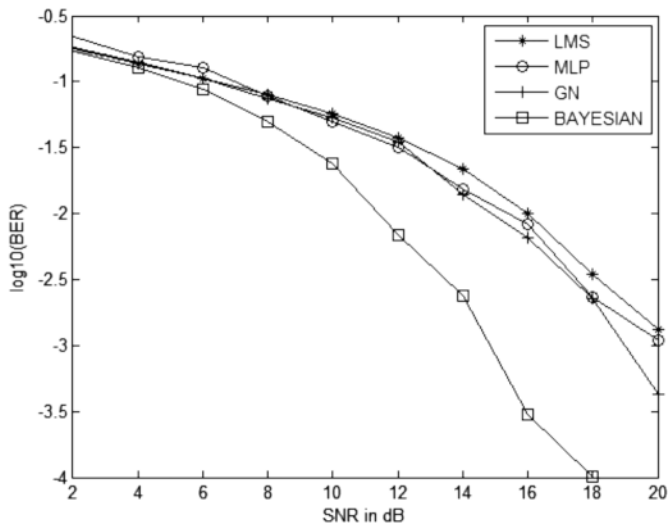


Figure 5. BER performance of the channel NL = 0.

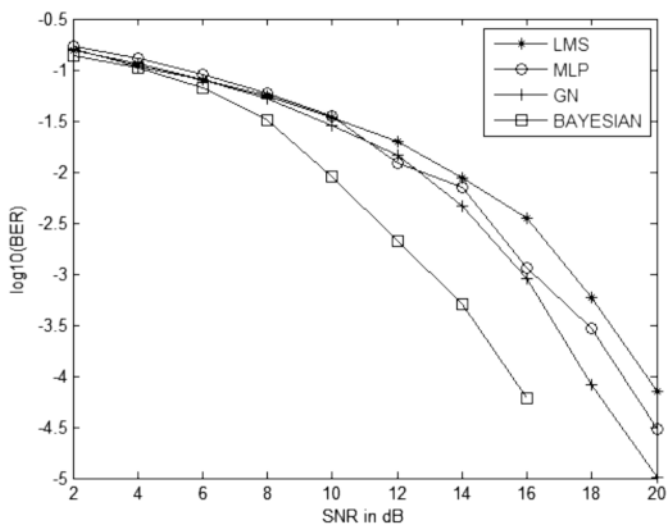


Figure 6. BER performance of the channel NL = 1.

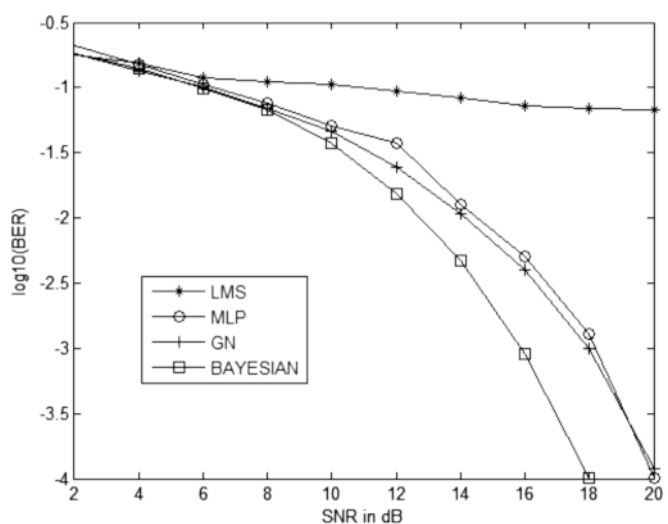


Figure 7. BER performance of the channel with NL = 2.

the received destroyed signals. It requires much simpler complexity for GN equaliser in comparison to MLP equaliser to obtain nearly the same BER. More close approach to optimal theoretical Bayesian solution for GN-based equaliser is achieved, specially for nonlinear channel models as compared with other equalisers throughout the variation of SNR. Computational requirement of LMS-based equaliser is simplest of the three equalisers but its performance is very poor under severe nonlinear conditions. GN-based equalisers have significantly simpler computational requirement and very simple design procedure as compared with MLP-based equalisers while providing good BER performance.

#### 4. CONCLUSIONS

There is no problem of selection of initial architecture of neural network as only a single GN is required. GN-based equalisers have less computational requirements and have simple design procedures. Fast convergence characteristics are obtained because it has a much smaller number of weights to be adopted than common MLP ANN. This neuron provides flexibility and fault-tolerant capability to cope up with the nonlinearities involved, thus giving good BER performance. The simulation result shows that proposed equalisers have advantage of both low computational requirement and good performances and have simple design procedure. Computational requirement is also lower in comparison to other equalisers reported in the literature like equalisers based on fuzzy systems-RBF, FLANN, and RNN. Fast convergence characteristics and low computational requirement make GN-based equalisers attractive alternatives for designing online adaptive equalisers for digital communication systems when the channel characteristics are unknown.

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## Contributors



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**GN Structure and Its Learning Algorithm**

The details of GN and its learning algorithm are adopted[13] and presented as follows:

Let  $Y_i$  represents the input vector to the equaliser which is  $y[k]$  as given by Eqn (2).

The output of  $\Sigma_1$  part of GN

$$O_s = \frac{1}{1+e^{-\lambda_s * sum\_s}} \tag{13}$$

where  $\lambda_s$  is the gain scale factor for  $\Sigma_1$  part of GN,

$sum\_s = \sum W_{si} Y_i + X_{os}$ ,  $X_{os}$  is the bias to  $\Sigma_1$  part and  $W_{si}$  is the weight vector.

The output of  $\mathfrak{D}$  part of GN

$$O_p = e^{-\lambda_p * prod\_p^2} \tag{14}$$

where,  $\lambda_p$  is the gain scale factor for  $\mathfrak{D}$  part of the GN,

$$prod\_p = \prod W_{pi} Y_i * X_{op}$$

$X_{op}$  is the bias to  $\mathfrak{D}$  part and  $W_{pi}$  is the weight vector.

The final output is

$$O_i = W * O_s + (1-W) * O_p \tag{15}$$

where,  $W$  is the weight vector.

The final output  $O_i$  is the estimated output vector  $s'[k-d]$

Steps to train the network till the error reaches to a minimum are as follows.

*Step 1.* Calculate the output for each pair of input using Eqns (13, 14, and 15).

*Step 2.* Calculate the error using the following relation

$$E_i = (O_i - D_i) \tag{16}$$

where,  $D_i$  is the desired output  $s[k-d]$ .

*Step 3.* Calculate the mean square error for all convergence as

$$E = 0.5 * \sum E_i^2 / N \tag{17}$$

where,  $N$  is the total number of training patterns and a multiplication of 0.5 has been taken to simplify the calculations.

*Step 4.* Weights of the networks are updated as follows

(a) Weights associated with  $\Sigma_1$  and  $\Sigma_2$  part of the GN are updated as

$$W(k) = W(k-1) + \Delta W$$

$$\text{where } \Delta W = \eta \delta_k (O_s - O_p) + \alpha W(k-1)$$

and

$$\delta_k = \sum (O_i - D_i) \tag{18}$$

(b) Weights associated with inputs and  $\Sigma_1$  part of the GN are updated as

$$W_{si}(k) = W_{si}(k-1) + \Delta W_{si}$$

$$\text{where } \Delta W_{si} = \eta \delta_s Y_i + \alpha W_{si}(k-1)$$

and

$$\delta_s = \sum \delta_k W (1 - O_s) * O_s \tag{19}$$

(c) Weights associated with inputs and  $\Pi$  part of GN are updated as

$$W_{pi}(k) = W_{pi}(k-1) + \Delta W_{pi}$$

$$\text{where } \Delta W_{pi} = \eta \delta_p Y_i + \alpha W_{pi}(k-1)$$

and

$$\delta_p = \sum \delta_k (1 - W) * (-2 * prod\_p) * O_p \tag{20}$$

where  $\eta$  is the learning rate and  $\alpha$  is the momentum factor, whose value ranges between 0 and 1.