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SHORT COMMUNICATION

Analysis of Dielectric Loss in a Helix Slow-wave Structure

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ABSTRACT

Equivalent circuit analysis of a helix slow-wave structure was carried out and closed form expressions were derived for the shunt capacitance and shunt conductance per unit length of the transmission-line equivalent circuit of the structure. These equivalent circuit parameters were interpreted for the dielectric attenuation constant of the slow-wave structure. The analysis was computationally simple and showsed excellent agreement with published results. The analysis was furthered for predicting the dielectric loss in typical C-Ku band and Ka band helical slow-wave structures, and variation of dielectric loss with temperature.

Keywords: Circuit attenuation, circuit RF loss, dielectric attenuation, dielectric loss, equivalent circuit analysis, helix slow-wave structure, SWS, helix travelling-wave tube, TWT

NOMENCLATURE

a	Mean helix radius
α_d	Dielectric attenuation constant
b	Radius of the electron beam
β	Axial propagation constants of the structure
C	Shunt capacitance per unit length of the structure
ε_0	Permittivity of free-space
ε_r^*	Complex relative permittivity of individual dielectric support rod
$\varepsilon_{r,eff}^*$	Effective complex relative permittivity of the smeared out dielectric tube
ε_r'	Relative permittivity of individual dielectric support rod
$\varepsilon_{r,eff}'$	Effective relative permittivity of the smeared out dielectric tube
ε_r''	Damping part of the complex permittivity of individual dielectric support rod
$\varepsilon_{r,eff}''$	Damping part of the effective complex permittivity of the smeared out dielectric tube
G	Shunt conductance per unit length of the structure
γ	Radial propagation constant of the structure
I_n	Modified Bessel function of first-kind of order n
K_n	Modified Bessel function of second-kind of order
p	Helix pitch
w	Width of the helix tape

1. INTRODUCTION

Helical slow-wave structure (SWS), continues to be important in high-efficiency wideband electronic warfare (EW) travelling-wave tubes (TWTs) [1,2]. However, reduction of RF losses in the helix SWS has always been a challenge to the TWT designer in achieving higher efficiency, especially at millimeter-wave frequency range, where the RF losses in the structure increase appreciably. A helix SWS typically consists of a metallic tape-helix supported inside a metallic barrel by azimuthally symmetrically placed dielectric support rods (Fig. 1). RF losses occurring in such a structure are contributed by both conductivity and dielectric losses. Estimation of conductivity losses in helical SWS has been widely treated in the literature [3,9] by theoretical approaches based on field analysis, and by measurement as well. However, the analysis of dielectric losses in helical SWS has received very little attention in the past, except an early work of Hagmann [10] and a recent one by Duan, *et al.*[11]. Work of Hagmann [10] treated dielectric loss in a coaxially placed dielectric material at the centre of the helix relevant for the design of applicators for microwave hyperthermia. Analysis of Duan, *et al.*[11] proposes a rigorous tape-helix model that gives dielectric attenuation constant as a solution of a transcendental dispersion relation containing complex variables, and one cannot use the dispersion characteristics obtainable from any other model to compute the dielectric loss using their approach. In this paper, a simple method is

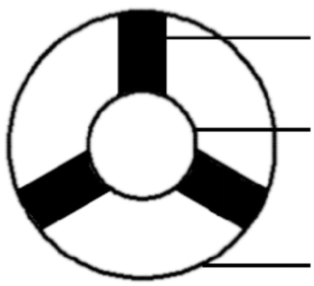

Figure 1. Schematic of a typical helix slow-wave structure.

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proposed based on equivalent circuit analysis to find the dielectric attenuation constant (α_d) of the helical SWS, and a closed form expression for the dielectric attenuation constant has been derived that provides direct relationship with the structure-dimensions and material properties. The present analysis follows the equivalent circuit model of helix SWS by Basu [2,12] and Kumar, *et al.* [13] and improves upon the same by incorporating the actual helix-tape geometry and the surface current density on the helix over the actual width of the tape, that were ignored in the earlier approach. The method consists in finding the shunt capacitance (C) and shunt conductance (G) per unit length of the transmission line equivalent to the SWS, and then interpreting these parameters for the dielectric attenuation constant of the structure. The formula thus arrived at has been benchmarked against published results of Duan, *et al.* [11] and excellent agreement was found. The analysis has been subsequently applied in predicting the dielectric loss in typical C-Ku band and Ka band helical SWSs, and variation of dielectric loss with temperature.

2. ANALYSIS

Considering propagation of azimuthally symmetric fundamental forward space-harmonic mode in the helical SWSs, with variation of RF quantities as $exp(j\omega t - j\beta z)$, one may write the TM mode field components in the different regions of the structure (region-I inside the helix and region-II outside the helix), as [2]:

$$\begin{aligned} E_{z1}\{\gamma r\} &= A_1 I_0\{\gamma r\} \\ H_{\theta 1}\{\gamma r\} &= \left(\frac{j\omega\varepsilon_0}{\gamma}\right) A_1 I_1\{\gamma r\} \\ E_{z2}\{\gamma r\} &= A_2 I_0\{\gamma r\} + B_2 K_0\{\gamma r\} \\ H_{\theta 2}\{\gamma r\} &= \left(\frac{j\omega\varepsilon_0\varepsilon^*_{r,eff}}{\gamma}\right)\left(A_2 I_1\{\gamma r\} - B_2 K_1\{\gamma r\}\right) \end{aligned} \tag{1}$$

Where, A_n and B_n are field constants to be evaluated by enforcing the boundary conditions. Meanings of other parameters follow as those given in the nomenclature of symbols. Here, one considers the discrete dielectric support rods to be smeared out as an equivalent lossy-dielectric tube between the helix and the barrel having an effective complex relative permittivity of $\varepsilon^*_{r,eff} = \varepsilon'_{r,eff} - j\varepsilon''_{r,eff}$ with $\varepsilon'_{r,eff}$ as the effective relative permittivity of the smeared out dielectric tube and $\varepsilon''_{r,eff}$ to include the effects of the conductivity of the lossy dielectric. There are two boundaries for this problem: one at the helix (at $r=a$), and the other at the perfectly conducting barrel (at $r=b$). The boundary conditions are given as:

$$\begin{aligned} E_{z2}\{\gamma b\} &= 0 \\ E_{z1}\{\gamma a\} &= E_{z2}\{\gamma a\} \\ H_{\theta 2}\{\gamma a\} - H_{\theta 1}\{\gamma a\} &= \left(\frac{w}{p}\right)\left(\frac{1}{2\pi a}\right) I_{z,Tape} \end{aligned} \tag{2}$$

Where, $I_{z,Tape}$ is the axial tape helix current introduced following Rowe [3,9] that incorporates the actual helix-tape geometry and the surface current density on the helix over the actual width of the tape. Now, eliminating the constants by enforcing the boundary conditions in Eqn (2) in the set of field Eqn (1), one gets the axial electric field component at the helix as:

$$E_z\{\gamma a\} = jM\left(\frac{\gamma^2}{\omega}\right)\left(1+\left(\varepsilon^*_{r,eff}-1\right)N\right)^{-1} I_{z,\,tape} \tag{3}$$

Where, M and N are intermediate variables. Now, the axial electric field intensity $E_z\{\gamma a\}$ can be expressed in terms of a scalar potential of the structure (V) and a vector potential A of the structure using the time-harmonic gradient relation and the Lorentz condition [2], which on elimination of the axial component of the vector potential A with the help of time harmonic dependence $exp(j\omega t - j\beta z)$ gives

$$E_z\{\gamma a\} = j\left(\frac{\gamma^2}{\beta}\right)V \tag{4}$$

From current telegraphist's equation, one gets the expression of scalar potential V in terms of $I_{z,tape}$ which after substituting in Eqn (4) one gets the expression for axial electric field component as

$$E_z\{\gamma a\} = j\left(\frac{\gamma^2}{\omega}\right)\left(\frac{1}{C_e - jG_e/\omega}\right) I_{z,\,tape} \tag{5}$$

Now, comparing Eqns (5) and (3), and equating the real and imaginary components, one gets the expressions for the shunt capacitance (C) and shunt conductance (G) per unit length of the transmission line equivalent to the SWS, given as

$$\mathrm{C} = \mathrm{M}\left(1+\left(\varepsilon'_{\mathrm{r,eff}}-1\right)\mathrm{N}\right) \text{ and } G = M\omega\varepsilon''_{r,eff}N$$

Following the transmission line definition of dielectric attenuation constant, one may express the dielectric attenuation constant (α_d) of the SWSs in terms of the shunt capacitance (C) and shunt conductance (G) per unit length of the transmission line equivalent to the SWS, given as

$$\alpha_d = \frac{1}{2}\left(\frac{\beta}{\omega}\right)\left(\frac{G}{C}\right) = \frac{1}{2}\left(\frac{\beta\varepsilon''_{r,eff}}{\left(\varepsilon'_{r,eff}-1\right)+\frac{1}{N}}\right) \tag{6}$$

$$\text{with } N = \gamma a\, I_0\{\gamma a\} K_1\{\gamma a\}\left(1+\frac{I_1\{\gamma a\}K_0\{\gamma b\}}{K_1\{\gamma a\}I_0\{\gamma b\}}\right)$$

One can now obtain β and hence γ occurring in Eqn (6) through field analysis based on sheath- or tape-model, or using a commercial 3-D electromagnetic simulation tool such as HFSS, MAFIA or CST Microwave Studio, and proceed with the computation.

3. RESULTS AND DISCUSSION

For benchmarking of the approach, we considered a practical helix SWS, for which the dimensions and dielectric attenuation values are published in the literature [11]. The variation of dielectric attenuation constant of the structure against loss-tangent of the dielectric material of the individual support rod is shown in Fig. 2 vis-à-vis published theoretical results of Duan, *et al*[11]. The present simple method shows excellent agreement against that of a rigorous and computation intensive tape-helix model.

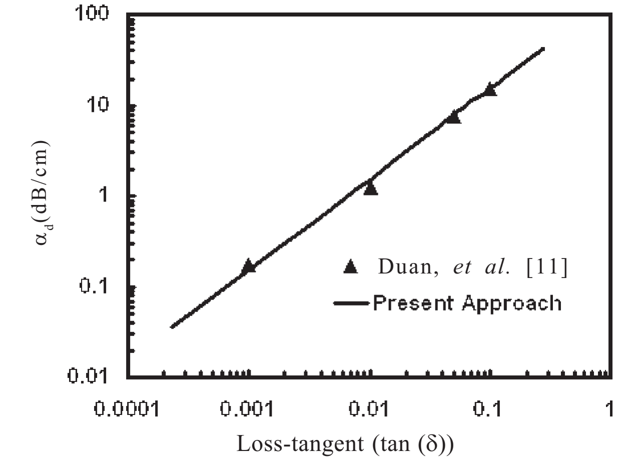

Figure 2. Variation of dielectric attenuation constant against loss-tangent ($tan(\delta)$) of discrete dielectric support rod as computed using the present analysis and compared to those published in the literature [11] at 10 GHz.

The analysis was then applied in comparing the performance of two typical SWSs, one operating in C-Ku frequency band (SWS-1) and the other operating in Ka band (SWS-2) [14-15]. The equivalent structures with BeO and APBN support rods (for both SWS-1 and SWS-2 configurations) were considered to be having suitably tailored support rod widths to have the same dispersion characteristics for BeO and APBN supported structures such that the value of $\varepsilon'_{r,eff}$ remains the same for both the BeO-and APBN rod supported options of each of configurations SWS-1 and SWS-2.

The dielectric properties used for the computations are given in Table 1 and 2. The structure having BeO-support rod shows appreciably high dielectric loss compared to that with APBN support rod (Fig. 3). Moreover, for the beryllia rod supported structures, the attenuation per meter increases by around 3 dB at 40 GHz (SWS-2) compared to its value at 18 GHz (SWS-1).

Table 1. Typical dielectric properties at 10 GHz, 25 ˚C

Material	Dielectric constant (ε'_r)	Loss tangent ($\tan\delta = \varepsilon''_r \varepsilon'_r$)
BeO	6.7	4×10^{-4}
APBN	5.15	1.5×10^{-4}

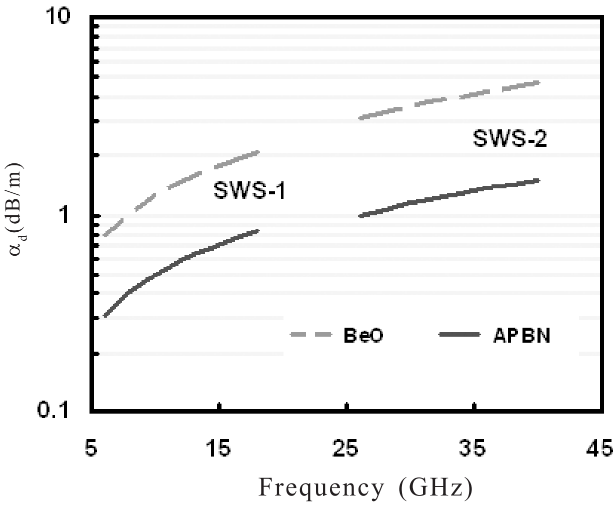

Figure 3. Variation of dielectric attenuation constant versus frequency for a C-Ku band (SWS-1) and a Ka band (SWS-2) slow-wave structures computed for the dielectric properties at 25 ˚C.

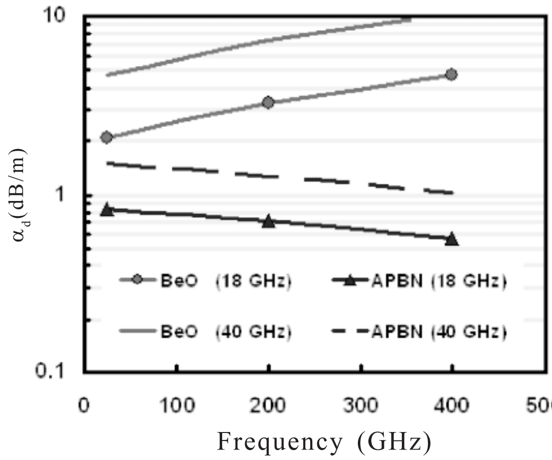

Figure 4. Variation of dielectric attenuation constant versus temperature for a C-Ku band (SWS-1) and a Ka band (SWS-2) slow-wave structures computed for the upper band-edge frequencies.

Table 2. Approximate nomograms of dielectric properties

Material	Rate of variation with temperature and frequency ($\Delta\varepsilon'_r/\varepsilon'_r$, $\Delta\varepsilon''_r/\varepsilon''_r$) (Temperature < 500 °C, Frequency < 50 GHz)			
	Dielectric constant (ε'_r)		Loss tangent ($\tan\delta = \varepsilon''_r/\varepsilon'_r$)	
	with temperature	with frequency	with temperature	with frequency
BeO	6.3×10^{-5} / ^{0}C	-1.5×10^{-4} / GHz	3.3×10^{-3} / ^{0}C	8.50×10^{-3} / GHz
APBN	0	0	-8.5×10^{-4} / ^{0}C	0

With increase in temperature, the dielectric loss in the BeO rod-supported structure increases, while that of the APBN rod-supported structure decreases with the increase in the temperature (Fig. 4). This is due to the fact that the loss tangent of BeO increases sharply with temperature while that of APBN decreases with increase in temperature (Table 2). The dielectric constant of the materials, however, changes very little with practical helix temperature variation and frequency of operation, and has very little effect on the dielectric attenuation constant unless the dielectric loading of the structure is affected.

4. CONCLUSION

A simple analysis has been developed for calculating the dielectric attenuation constant in a helical SWS. Benchmarking of the analysis also has been carried out against published results with excellent accuracy. Typical C-Ku band and Ka band helix SWSs have been analysed and dielectric loss behaviours at different frequencies and operating temperatures have been presented. The present simple analysis is expected to be useful as a handy design tool for the microwave tube community.

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