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New Density-based Thermal Conductivity Equation for Snow

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ABSTRACT

More than two hundred thermal conductivity measurements for different snow densities and snow types were carried out *in-situ* at a field research station located in greater Himalayan range of India. These measurements were carried out using a commercially available portable thermal conductivity meter. Thermal conductivity measurements were carried out on the fresh snow, equi-temperature snow, and surface hoar and temperature-gradient snow. Average thermal conductivity of snow varied from 0.08 W/mK (Fresh snow of 120 kg/m^3 density) to 0.32 W/m K (Equi-temperature snow of 420 kg/m^3 density). Based on these measurements, a new density-based thermal conductivity equation is proposed. Using this proposed equation, modeled snowpack temperatures showed closer agreement with the observed data as compared to the predictions based on other well-known empirical and theoretical thermal conductivity equations for snow. This study highlights the advantages and limitations of empirical based thermal conductivity equations over the complex models based on snow microstructure.

Keywords: Empirical, snow microstructure, snowpack, temperature, thermal conductivity

NOMENCLATURE

$\Delta T(r,t)$	Temperature change $\Delta T(r, t)$ at some radial distance 'r' from heater (K)
t	Elapsed time(s)
q	Heater power of a unit length (W/m)
C_E	Euler's constant
λ	Thermal conductivity of the measured medium (W/mK)
α	Thermal diffusivity of the measured medium (m^2/s)
(a,b,c,d)	Coefficients of the empirical thermal conductivity equation [Eqn (6)]
ρ	Density of snow (kg/m^3)

1. INTRODUCTION

The thermal conductivity is one of the most significant parameters governing the thermal gradient within a snowpack. The temperature gradient in turn is responsible for the snow metamorphism, and hence, change in the mechanical strength of the snow. Further, the knowledge of the thermal conductivity of snow can help greatly in developing accurate avalanche-forecasting models. A lot of work has been done in the past on the thermal properties of snow. Schwerdtfeger[1] gave a theoretical expression for the effective thermal conductivity and diffusivity of snow based on the Maxwell's work on the electrical conductivity. In this work, predictions based on this model differ remarkably from the observed values. Lange[2] gave density-based empirical expression for the effective conductivity, which is applicable for a narrow density range of snow. Adams[4], *et al.* presented a theoretical model for the effective thermal conductivity of snow based on series and parallel resistances of ice, air, and water vapour components. This is an idealised model based on uniform packing of spherical snow grains. This is a quite elaborate model but cumbersome to apply in the field conditions due to difficulty in estimating the changes in the snow microstructure parameters like bond size, grain size and pore space. Sturm[5], *et al.* carried out systematically a very large number of thermal conductivity measurements using transient line heat source method and based on that data presented two equations for the effective conductivity of snow. These equations are applicable over the entire range of snow density. However, experiments for this study were conducted at average temperatures below –12 °C. Singh[7] presented a new database on the effective thermal conductivity of snow based on transient probe method but no significant correlation could be obtained for the snow thermal conductivity. Sturm[5] *et al.* presented new 89 point measurements of the conductivity and tried to correlate the effective conductivity with snow metamorphism. In the resent work, a new density-based empirical thermal conductivity equation is proposed based on more than 200 measurements carried out using transient probe instrument in the temperature range from –3 °C to –14 °C.

2. ABOUT THE STUDY AREA

The present study was done at the Patsio Field Station, located at 3800 m above MSL (Latitude: 32.75° N, Longitude:

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77.25° E) in greater Himalayan range of India. This place is marked by very low temperatures (nearby –30° C), dry snow precipitation, and without inhabitants. These features make the climate of this place similar to polar regions.

3. METHODOLOGY FOR THERMAL CONDUCTIVITY EXPERIMENTS

3.1 Theory Behind the Instrument

The instrument used for the present study was a commercially available portable thermal conductivity (Fig. 1) meter 'ISOMET 2104' (Brand name). This has an accuracy of ±5 per cent. The 'ISOMET 2104' is based on transient probe technique. Probe is a 3 mm dia and 120 mm long needle probe in which heat pulse is generated for a time interval and the temperature response influenced by measured infinite medium analysed by means of a temperature sensor connected to the heater. In the case of needle probes, the produced heat flow can be considered to be radial. Temperature change $\Delta T(r, t)$ at some radial distance 'r' from heater is given by solution of Fourier equations, which for $r << \sqrt{(4\alpha t)}$ has form:

$$\Delta T(r,t) = \frac{q}{4\pi\lambda}\left(\ln t + \ln\frac{4\alpha}{C_E\ r^2}\right) \tag{1}$$

where t is elapsed time, q is power of a unit length, C_E is Euler's constant, λ is thermal conductivity, and α is thermal diffusivity of the measured medium.

Thermal conductivity λ can be determined from recorded linear temperature dependence on $\ln(t)$ as follows:

$$\Delta T(r,t) = A\ln(t) + B \tag{2}$$

In the above equation,

$$A = \frac{q}{4\pi\lambda} \tag{3}$$

$$B = \left(\frac{q}{4\pi\lambda}\right)\ln\frac{4\alpha}{C_E\ r^2} \tag{4}$$

From the above two equations:

$$\lambda = \frac{q}{4\pi A} \tag{5}$$

The temperature-time record data was analysed during heating as well as cooling of a sample. For one measurement, heater remains 'ON' for about 5 min and then automatically switches 'OFF' till measurement is complete in about 12 min.

3.2 Testing and Calibration of the Instrument

The instrument measures in two modes, one is optimisation mode in which the instrument automatically selects the heater power for optimum temperature rise (around 10 °C) within the medium and other is fixed-power mode in which the user can select heater power to control the temperature rise of the probe within the medium. For snow and viscous liquids, fixed mode of power was found to be a better choice because by selecting fixed power, the temperature rise within snow, and hence, convection currents within the liquid can be controlled. The instrument was tested by measuring thermal conductivity of ice, soil, wood, glycerol, thermocol, river sand, and cement before testing thermal conductivity of snow. The results are compiled in Table 1. Glycerol being highly viscous fluid, its correct thermal conductivity value at 27 per cent to 30 per cent heater power gave an idea about setting the heater power for snow. For all the measurements, heater power percentage was selected as per the criteria mentioned in Table 2.

However, in cases when snow temperature was –2.5 °C or higher, 15 per cent to 22 per cent heater power was used depending upon the density of the snow. The average temperature rise during all measurements varied from 2.6 °C to 6.4 °C. Temperature rise was confined to reduce snow metamorphism effects during the measurement. In case, temperature rise during any measurement exceeded 0 °C, that measurement was rejected and measurement repeated with lower heater power.

3.3 Data Collection for Snow Conductivity

During January 2004 to February 2004**,** more than 200 thermal conductivity tests were conducted on snow at Patsio. Most of these tests were carried out in the snow pits within the observatory area during clear weather and in no wind or very light wind conditions. In the field, tests

(i) (ii)

(a) (b)

Figure 1. (a) ISOMET 2104 portable thermal conductivity meter, and (b) instrument being used at Patsio in: (i) cold chamber, and (ii) field

Table 1. Test data for the thermal conductivity meter

Material	Density (kg/m^3)	N^a	Thermal conductivity (W/mK)				Avg temp (°C)	Value reported (W/mK)
			Max	Min	Avg	Std Dev		
Ice	917.0	4	2.01	1.91	1.94	0 .048	-15.0	2.2[6]
Glycerol	1245.0	4	0.294	0.281	0.287	0.005	21.0	0.289[6]

Table2. Selection of heater power values for snow

Snow density range (kg/ m^3)	Heater power (%)
100<= & <= 260	25
260< & <=400	30
400< & <=500	35

were carried out by inserting the sensor horizontally at different depths (Fig. 1 (b-ii)) within the snowpack. Few tests were done in a cold chamber at –9 °C , – 6 °C and –3 °C temperatures by cutting rectangular blocks of snow (30 cm × 30 cm × 20 cm) from the pits and then thermally stabilising these within the chamber. Uncohesive snow like TG or surface hoar was directly taken in a rectangular wooden box of size 30 cm × 30 cm × 30 cm and then brought to the cold chamber for testing. Tests on the sieved ET snow were done in the cold chamber by putting sieved snow in a rectangular wooden box of 30 cm × 30 cm × 25 cm size. In all the laboratory and field tests, stable temperature within the snow sample was ensured before starting the conductivity test. Those conductivity measurements were rejected in which the temperature rise and fall was not smooth, i.e., cases when there was sudden decrease of temperature during the heating period and sudden increase of temperature even when heater was switched-off, were rejected. For all the measurements on snow, only one thermal conductivity probe (0.2-1.0 W/m K) was used. All the snow thermal conductivity measurements have been summarised in Table 3.

4. FITTING THE REGRESSION EQUATION TO THE DATABASE

In Fig. 2, variation of thermal conductivity with bulk density for fresh, surface hoar, ET, sugar, and depth hoar types of snow is shown. It was observed that for different types of snow, average value of thermal conductivity of snow was increasing with density. However, in case of depth hoar snow, there was significant scatter in the values of conductivity.

It can be seen that density is playing the major role in controlling the value of thermal conductivity of different types of snow. In case of depth hoar, where variation in conductivity values seems to be governed more by micro-structural variations. Shunting out the conductivity values for depth hoar snow from the database as mentioned in Table 3, a third-degree polynomial equation can be easily fitted through the average conductivity variation with density

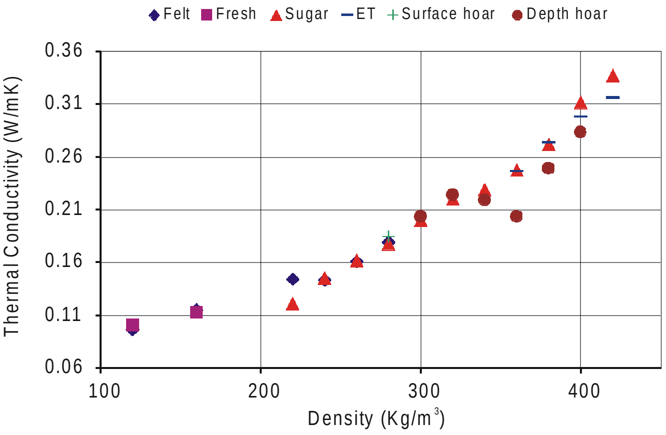

Figure 2. Variation in the thermal conductivity of snow.

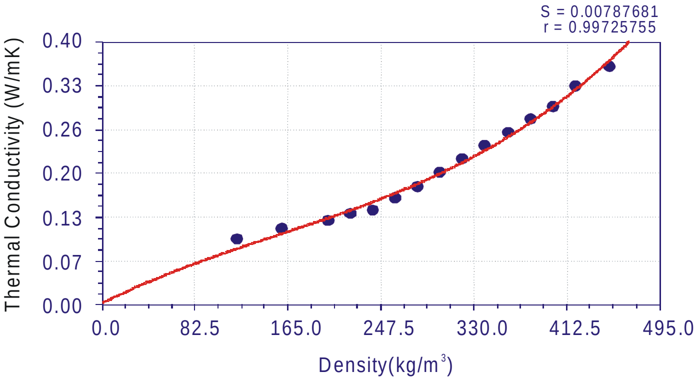

Figure 3. Third-degree density based polynomial equation for thermal conductivity of snow.

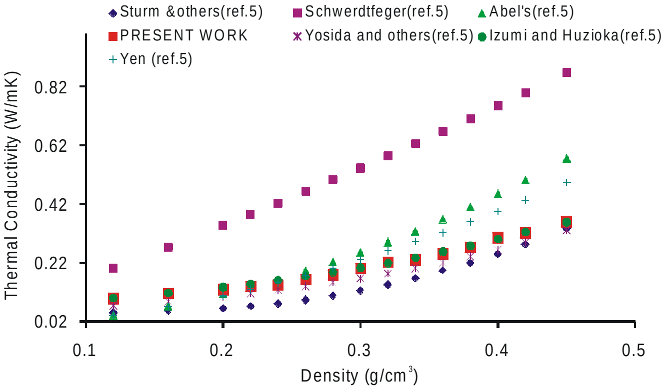

Figure 4. Comparison of the present work with other thermal conductivity models.

Table 3. Thermal conductivity database for snow

Sr.no.	Snow type	Avg. grain size range (mm)	N	ICSSG code[3]	Mean density (kg/m^3) ± 6%	Avg temp (°C) ±1%	Thermal conductivity (W/mK)			
							Min. value	Max. Value	Avg. value	Std. dev.
1.	Fresh	<1	7	1d	120.0	-5.3	0.058	0.091	0.079	0.014
2.	Fresh	<1	2	1d	140.0	-13.8	0.089	0.095	0.092	0.004
3.	Surface hoar	1-2	2	7a	200.0	-8.8	0.126	0.129	0.128	0.002
4.	Surface hoar	1-2	4	7a	280.0	-7.1	0.168	0.209	0.185	0.020
5.	Felt like	0.5-1.5	2	2a	120.0	-8.8	0.082	0.113	0.097	0.022
6.	Felt like	0.5-1.5	5	2a	160.0	-12.7	0.105	0.138	0.115	0.013
7.	Felt like	0.5-1	2	2a	200.0	-13.7	0.127	0.131	0.129	0.003
8.	Felt like	0.5-1	3	2a	220.0	-13.5	0.141	0.148	0.145	0.004
9.	Felt like	<1	13	2a	240.0	-7.0	0.134	0.157	0.143	0.007
10.	Felt like	<1	7	2a	260.0	-6.4	0.154	0.180	0.162	0.009
11.	Felt like	<1	7	2a	280.0	-7.2	0.142	0.205	0.179	0.020
12.	Felt like	<1	1	2a	320.0	-7.8	-	-	0.216	-
13.	Felt like	0.5-1	1	2a	360.0	-7.8	-	-	0.225	-
14.	ET	0.5-1	20	3b	380.0	-7.6	0.247	0.286	0.275	0.011
15.	ET	0.5-1	5	3b	400.0	-7.3	0.293	0.307	0.299	0.006
16.	ET	0.5-1	3	3b	420.0	-7.3	0.310	0.327	0.316	0.009
17.	Sugar	1-2	1	5a	220.0	-12.7	-	-	0.121	-
18.	Sugar	1-2	1	5a	240.0	-5.8	-	-	0.146	-
19.	Sugar	1-2	9	5a	260.0	-6.0	0.144	0.172	0.162	0.009
20.	Sugar	1-2	9	5a	280.0	-9.5	0.154	0.212	0.178	0.020
21.	Sugar	1-2	8	5a	300.0	-8.1	0.184	0.228	0.200	0.016
22.	Sugar	1-2	15	5a	320.0	-6.3	0.195	0.266	0.221	0.015
23.	Sugar	1-2	6	5a	340.0	-4.9	0.201	0.241	0.230	0.015
24.	Sugar	1-2	7	5a	360.0	-3.7	0.239	0.268	0.248	0.011
25.	Sugar	1-2	7	5a	380.0	-3.6	0.244	0.282	0.272	0.017
26.	Sugar	1-2	5	5a	400.0	-3.8	0.302	0.317	0.313	0.006
27.	Sugar	1-2	1	5a	420.0	-4.0	-	-	0.338	-
28.	Depth hoar	1-3	4	5b	300.0	-4.6	0.169	0.226	0.204	0.025
29.	Depth hoar	2-4	3	5b	310.0	-4.9	0.222	0.248	0.231	0.014
30.	Depth hoar	2-4	24	5b	320.0	-6.1	0.134	0.275	0.225	0.033
31.	Depth hoar	2-4	6	5b	340.0	-5.1	0.197	0.251	0.220	0.020
32.	Depth hoar	2-4	1	5b	350.0	-3.6	0.193	0.193	0.193	-
33.	Depth hoar	2-4	4	5b	360.0	-4.8	0.165	0.250	0.205	0.040

for rest of the snow types (Fig. 3). A commonly available curve fitting software was employed for this purpose. The effective thermal conductivity λ (W/mK) is given by the following equation:

$$\lambda = a + b\rho + c\rho^2 + d\rho^3 \qquad (6)$$

ρ = density of snow (kg/m^3)

Correlation coefficient for the above equation is 0.997 and coefficient values are:

$a = 0.00395$, $b = 0.00084$,

$c = -1.7756\text{e}-6$, $d = 3.80635\text{e}-9$

At density of 917 kg/m^3 which is the density of ice, the Eqn (6) predicts thermal conductivity as 2.22 W/mK which is the same as that for ice at 0 °C. This deduction supports the correctness of the equation. As the correlation coefficient for this equation is quite high, it can be used for the prediction of thermal conductivity of snow.

Further, predicted conductivity values based on this equation were compared with other authors work[5], as shown in Fig. 4. From this figure, it is clear that conductivity values predicted by the present regression Eqn (6) are matching well with those predicted by Izumi & Hozioka and Yosida[5], equations. The present values are slightly higher than those predicted by Sturm[5], *et al*. The reason for this is probably high snow temperatures in the present case as compared to (ref.5, p. 26). Values predicted by Schwerdtfeger[1] model seem exceedingly high than all the models considered here, and thus, model needs a check for its correctness. These results are close to Yen and Abel[5] but above a density of 280 kg/m^3, these models predict quite higher values than these data.

5. CONCLUSIONS

A new density-based regression equation has been developed for the prediction of thermal conductivity of snow. This equation is applicable over the entire density range of snow. However, the equation needs to be verified for its accuracy with the experimental data.

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