Defence Science Journal, Vol. 62, No. 4, July 2012, pp. 228-235, DOI: 10.14429/dsj.62.792

Application of Finite Difference Time Domain to Calculate the Transmission Coefficient of an Electromagnetic Wave Impinging Perpendicularly on a Dielectric Interface with Modified MUR-I ABC

Biswajeet Mukherjee* and Dinesh K. Vishwakarma

PDPM-Indian Institute of Information Technology Design & Manufacturing, Jabalpur, India
**E-mail: biswajeet.26@gmail.com*

ABSTRACT

MATLAB codes were implemented in this study for a one dimension wave formulation using the computational technique of finite difference time domain (FDTD) method. The codes have then been verified under two cases, one a simple one dimensional wave impinging perpendicularly on a dielectric layer from air interface and second is a one dimensional wave impinging momentarily on a small dielectric slab. The transmission coefficients under both the cases have also been verified. For the former case, there is a constant transmission coefficient irrespective of the frequency of the electromagnetic wave impinging on it and for the latter; there is a sinusoidal type variation due to multiple reflections along the wall of the dielectric slab. In the course of this implementation of the codes a novel technique to implement the absorbing boundary condition (ABC) on the dielectric interface has also been devised based on the Mur-I ABC which has been verified for a dielectric of dielectric constant $4\epsilon_o$. The implementation of the codes presents a recapitulation of the evolution of FDTD from Yee's Algorithm to the latest modifications in the ABC.

Keywords: Finite difference time domain, one dimensional wave, transmission coefficient, absorbing boundary condition

1. INTRODUCTION

Computer-aided design of passive RF and microwave components has advanced slowly but steadily over the past four decades. However, in the last decade the rising demand for highly integrated transceivers for wireless, RF and optical applications, low cost radio frequency integrated circuits (RFICs), and multi-Gigahertz processors has generated tremendous requirement for efficient and accurate modelling of on-chip and off-chip passive components and interconnects. In system integration, the impact of packaging and signal integrity issues are other aspects that need attention to achieve proven designs/goals. Electromagnetic solvers are considered as the best tools for such analysis in RF and high speed designs and are equally important in parasitic extraction space.

Parasitic include interconnect parasitics as well as coupling among passive devices, interconnects themselves and ground planes. Furthermore, the layout density and GHz range operating frequency introduces numerous high frequency effects including time retardation, the skin effect, substrate effects and frequency resonances. It is essential to accurately predict parasitics and the numerous high frequency effects while designing RF circuits.

Various electromagnetic solvers are available in the current market to analyse, two dimension (2-D), two and a half dimension (2.5-D), which covers the wire bonds of an RFIC and three dimension (3-D) structures. There is always a trade-off between the accuracy and efficiency of these solvers. Designers as well as tool developers require the knowledge of different solving algorithms, methods, and applications of 2-D to 3-D solvers in the design to make an exact trade-off for efficient and accurate modelling. Amongst the many computational techniques present to implement the solver method, one way is finite difference time domain (FDTD). Finite difference time domain is one such solver technique which gives dynamic simulation using the finite differencing method efficiently implementing the Maxwell's equations. The superiority of FDTD over other computational techniques is the fact that it covers a very wide spectral range unlike FEM, MoM, etc. Amongst the other advantages of FDTD, a major key point is the simplicity of its implementation. Still at 2-D analysis level, it is comparatively faster and gives more accurate results.

One of the key areas of research in FDTD has been identified as efficient implementation of perfectly matched layer (PML) and absorbing boundary condition (ABC) under dispersive material[9]. Dispersive metal behaviour for the description of frequency dependent behaviour of the susceptibility function of materials has also been studied using FDTD. Modified PML technique for such materials is an active area of research. FDTD also becomes computationally intensive when it comes to 3-D simulation due to excessive meshing which gives rise to truncation or staircase error. Even at THz frequency the PML implementation[3] with dispersive metal introduces error in the calculations. Truncation error remains due to the residual reflections generated by ABC. Some techniques[12] have been devised using geometry rearrangement technique (GRT),

Received 14 February 2011, revised 14 June 2012, online published 06 May 2012

where by estimating the boundary reflection; the characteristic impedance calculations can be corrected.

This study addresses the development of a one dimension-based code and further identifies better ABC to increase the efficiency of the algorithm to be implemented. Authors successfully tested the code on a one dimension wave impinging perpendicularly on a dielectric interface and also applied on an EM wave propagating through a dielectric slab and successfully calculated the transmission coefficients for both the cases. The results have also been matched with the theoretical solutions generated through it.

2. YEE LATTICE AND FINITE DIFFERENCE TIME DOMAIN

The FDTD method has become a popular tool for the world of electromagnetic solvers due to its simplicity of implementation and easier comprehensibility. The FDTD method, first introduced by Yee[1] in 1966 and later developed by Allen Taflove[2], is a direct solution to the Maxwell's time dependent curl equations. The finite difference time domain method has widespread applications in the fields of:

(a) Aperture penetration
(b) Antenna/radiation problems
(c) Microwave circuits
(d) Eigen value problems
(e) EM absorption in human tissues (bioelectromagnetics).

In 1966, Yee originated a set of finite difference equations for the time-dependent Maxwell's curl equation systems for the lossless materials $\sigma^*=0$ and $\sigma=0$.

The Yee algorithm may be summarised as:

- It solves for both electric and magnetic fields in time and space using the coupled Maxwell's curl equations, instead of simplifying electric and magnetic field alone with a wave equation.
- It centers in *E* and *H* components in three dimensional spaces so that every *E* component is surrounded by four other *H* components circulating it and similarly, every *H* component is circulated by four other *E* components.
- It also centers its *E* and *H* components in time, in what is called as leapfrog arrangement. According to the leapfrog method, all *E* fields are computed at a given point of time stepping using previously stored values of *H* field and similarly at the next time step, all values of *H* fields are calculated using previously stored values of *E* fields. This process continues until all time steps are over.

3. ONE DIMENSION FORMULATION AND IMPLEMENTATION

The time dependent Maxwell's curl equations is given as under:

$$\nabla \times E = -\frac{\partial B}{\partial t}$$
$$\nabla \times H = J_f + \frac{\partial D}{\partial t} \tag{1}$$

For static case and for one dimension we have only one component of electric field say E_z and correspondingly we have *Hy*. Implementing the central difference method we get,

$$\frac{E_z^{n+1/2}(k) - E_z^{n-1/2}(k)}{\Delta t} = -\frac{1}{\varepsilon_0}\frac{H_y^n(k+1/2) - H_y^n(k-1/2)}{\Delta x}$$
$$\frac{H_y^{n+1}(k+1/2) - H_y^n(k+1/2)}{\Delta t} = -\frac{1}{\mu_0}\frac{E_z^{n+1/2}(k+1) - E_z^{n+1/2}(k)}{\Delta x} \tag{2}$$

The above is the simplest representation of a one dimensional wave equation.

3.1 Implementation Issues

In the Eqn (2), time is specified by the superscripts, i.e., *n* which actually means a time $t = \Delta t \bullet n$. We have to keep in mind that we have to discretize everything for formulation into the computer. The term 'n + 1' means one time step later. The terms in parentheses represent distance, i.e., *i* actually means the distance $z = \Delta z \bullet i$, since Δz represents the discretisation in the spatial *z* axis. The concept of interleaving between the *E* & *H* field by Yee for a one dimensional wave is as shown in Fig.1

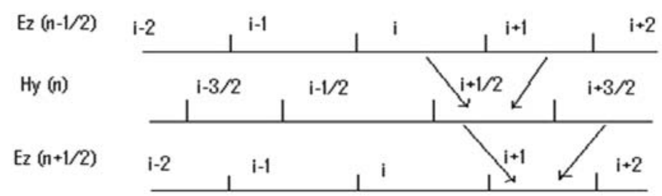

Figure 1. Time stepping in E & H for one dimension field.

Figure 1 tells that every calculation of *Hy* is dependent on the previously stored values of *Ez* and similarly every *Ez* is calculated based on the previously calculated values of *Hy*. Although the one dimension scalar wave equation can be solved directly by centered 2nd differences, it is not robust for solutions of problems that depend on *E* & *H*. Hence the Yee lattice is always a better choice for dynamic approximation[14].

It is worthy to note that while implementing the codes, time is implicit and the space coordinates are to be declared explicitly. Also while implementing the half time step concept through a programming style, we convert the '*n*+1/2' into '*n*' and '*n*–1/2' into '*n*–1'. Hence the sets of equations gets accordingly modified and the corresponding MATLAB codes[5] are written as:

Ez(*i*) = *Ez*(*i*)+(*dt*/(*eo***dx*))*(*Hy*(*i*)-*Hy*(*i*-1)) (3)

Hy(*i*) = *Hy*(*i*)+(*dt*/(*uo***dx*))*(*Ez*(*i*+1)-*Ez*(*i*)) (4)

The above iteration runs for the time steps mentioned in *n* and for all the spatial coordinates specifies by *i*. Here we have chosen *i* to vary for a 500 time steps.

Since we have a regular rectilinear geometry, hence, Cartesian coordinate system has been successfully used. However, for curved surfaces[7] using rectilinear coordinate system would give rise to truncation and approximation error. So, we need to apply FDTD in cylindrical or circular coordinate system.

3.2 Problem Statement

The above FDTD code has been developed for a simple electromagnetic problem, with a one dimension wave impinging perpendicularly on a loss less dielectric slab. When a plane wave from one medium impinges on a different medium, it is partly reflected and partly transmitted. The proportion of the incident wave that is reflected or transmitted depends on the consecutive parameters (ε, μ, σ) of the two media involved. Here we have assumed that the incident wave plane is normal to the boundary between the media.

The reflection coefficient (Γ) and transmission coefficient (τ) are calculated as:

$$\Gamma = \frac{H^r}{H^i} = -\frac{E^r}{E^i} = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2} \qquad (5)$$

$$\tau = \frac{H^t}{H^i} = -\frac{\eta_1 E^t}{\eta_2 E^i} = \frac{2\eta_2}{\eta_2 + \eta_1} \qquad (6)$$

where $\eta_1 = \sqrt{(\mu_1/\varepsilon_1)}$ and $\eta_2 = \sqrt{(\mu_2/\varepsilon_2)}$. Assuming perfect dielectrics[15] $\mu_1 = \mu_2 = \mu_o$,

$$\frac{E^r}{E^i} = \frac{\sqrt{\varepsilon_1} - \sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}} \qquad (7)$$

$$\frac{H^r}{H^i} = \frac{\sqrt{\varepsilon_2} - \sqrt{\varepsilon_1}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}} \qquad (8)$$

$$\frac{E^t}{E^i} = \frac{2\sqrt{\varepsilon_1}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}} \qquad (9)$$

$$\frac{H^t}{H^i} = \frac{2\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}} \qquad (10)$$

3.2.1 Algorithm for the Implementation and Simulation

In order to exactly simulate the above problem we have to place our source at the boundary of the considered frame and then generate the pulse. In MATLAB, we have used a sine modulated Gaussian pulse as the source generated at the spatial position i = 1. The complete spatial region has been divided into 500 spatial points and the pulse is generated for duration of 100 time steps or 5 ns. The wavelength in free space for operation is λ_0 = 0.3m. Now $dx = \lambda_0/20$ and $dt = dx/c$, where c is the speed of light, i.e. 3 x 10^8 ms^{-1}. Thus, we get dx = 0.015 m and dt = 0.05 ns. The MATLAB equivalent is

```
Ez(1) = (sin(2*3.14*f0*n*dt))*
        exp(–(n*dt–tc)*(n*dt–tc))/(2*sigma*sigma));
```

As Maxwell's equations clearly state that every electric field component has an associated magnetic field component also, hence though the excitation is of an electric field, the magnetic field is consequently generated.

The source wave would now travel through the free space. For the implementation of the problem statement, authors assumed that air is present from i = 1 to i = 250 and i = 251 to i = 500 is a dielectric with permittivity $\varepsilon_r = 4\varepsilon_0$. Hence, as the EM wave impinges on this dielectric surface there are going to be some reflections and some transmission of the incident wave in both electric and magnetic fields. The reflected and transmitted wave would now travel towards the computational boundaries to simulate the real time environment. As the first dielectric is air itself, hence in the dielectric 1, the phase velocity is

$$v_p = c \qquad (11)$$

but in the dielectric the phase velocity will decrease by a fraction of square root of the relative dielectric constant of the dielectric medium. Hence in the $4\varepsilon_0$ i.e. dielectric 2, the phase velocity of the electromagnetic wave reduces by:

$$v_p = c/\sqrt{\varepsilon_r} = c/\sqrt{4} = c/2 \qquad (12)$$

The same thing can well be seen in the simulation. The simulation also shows that since the reflected wave is in the air medium hence it covers more distance in a fixed time whereas the transmitted wave covers shorter distance in the same stipulated time period.

To record the observations of the different wave conditions, author placed a probe at spatial position of i = 350. This probe records all data pertaining to transmitted wave that

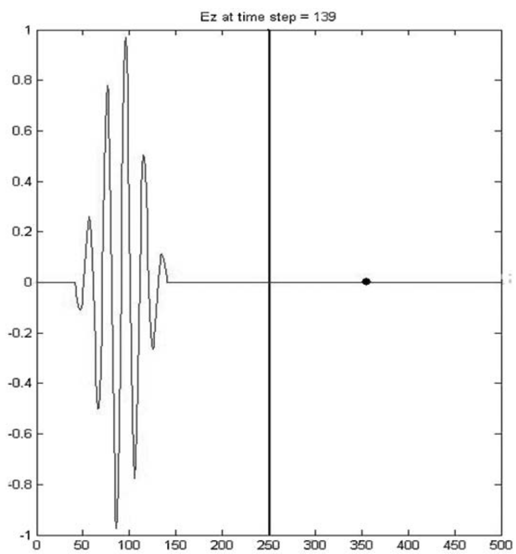

Figure 2. Sine modulated Gaussian pulse as the incident wave at 139 time steps. The dark line at i = 250 shows the variation in the dielectrics. From i = 1 to i = 250 is air and i = 251 to i = 500 is dielectric of permittivity $4\varepsilon_0$. At i = 350 the dot represents the placement of the probe.

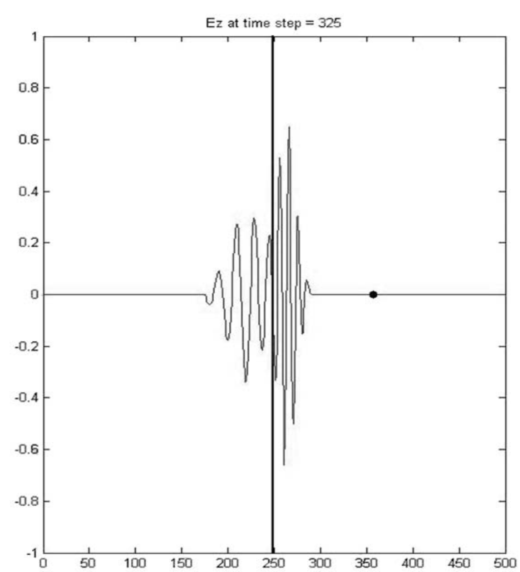

Figure 3. Reflection and transmission from the dielectric interface in electric and magnetic field from i = 251.

passes through the dielectric medium 2. The same is illustrated in Figs. 2 and 3.

3.3 Implementation on Dielectric Slab and Results

This problem statement verifies the correctness of the code. Suppose, we have an electromagnetic wave which falls perpendicularly on a dielectric slab of some finite thickness. Now, once the EM wave strikes the dielectric from air medium, it will have some transmission and some reflection. The transmitted wave would continue its journey inside the dielectric medium. At the edge of the dielectric medium, again there will be partly reflection and partly transmission.

Hence, this process would continue for a very long time, until the value of the wave trapped inside the dielectric considerably reduces. In such a case when we measure the Transmission coefficient, we find that it follows a periodical pattern due to constant transmissions and reflections. The above has been verified using the one dimensional code for FDTD implementation for a dielectric slab of $4\varepsilon_0$ and whose width is 50 spatial steps or $5\lambda_g$. The probe to record all observations is placed at $i = 350$.

However, the absorbing boundary condition applied at both the ends is specifically terminated in air. This is shown in Fig. 4.

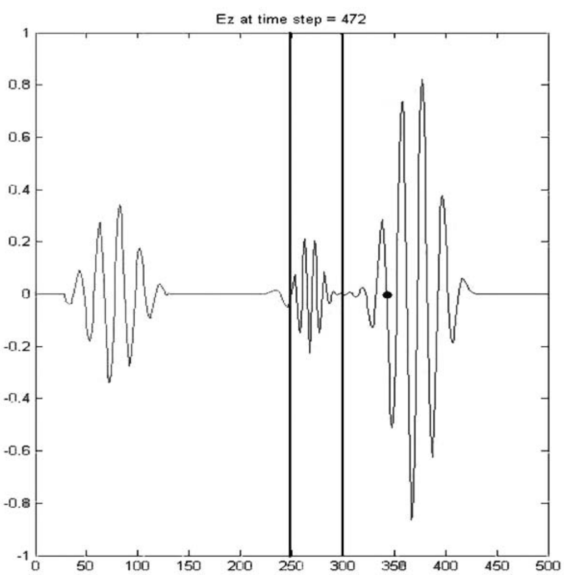

Figure 4. The trapped wave in the dielectric medium between spatial coordinates of i = 250 and i = 300. Further, the reflected and transmitted waves generated from the dielectric medium are also clearly visible. The reflected and the transmitted wave head towards the end of computational domain considered. The dielectric slab has a permittivity of $4\varepsilon_0$. The probe to record data of transmitted wave is placed at i=350.

3.4 Radiation or Absorbing Boundary Condition

A perfectly matched layer (PML) Condition[3], generates minimum reflection and hence generates almost accurate results compared to other methods. However, PML is generally applied in very complex structures where extremely high accuracy is anticipated. While calculating the specific absorption rate (SAR)[4], the object, which is the head needs to be placed between the PML and then put to simulation for accurate results.

In our case, we have implemented the first order Mur radiation boundary[8] which is comparatively an easier method of implementation and generates reasonably accurate results for the defined problem statement. Now suppose that we are simulating a simple plane wave propagating in the forward direction, we need to place the Mur I ABC at the last node of the electric field (M) as follows:

$$E_{zM}(n+1) = E_{zM-1}(n) + \frac{c\Delta t - \Delta z}{c\Delta t + \Delta z} E_{zM-1}(n+1) - E_{zM}(n) \tag{13}$$

To place the first order Mur radiation boundary at node 1 or the first electric field node on the left:

$$E_{z1}(n+1) = E_{z2}(n) + \frac{c\Delta t - \Delta z}{c\Delta t + \Delta z} E_{z2}(n+1) - E_{z1}(n) \tag{14}$$

Assuming the magic time step of $c\Delta t = \Delta z$, the maximum allowed by the stability condition, the truncation conditions reduce to:

$$E_{zM}(n+1) = E_{zM-1}(n) \tag{15}$$

$$E_{z1}(n+1) = E_{z2}(n) \tag{16}$$

All the above equations imply that an electromagnetic wave traveling in free space (with magic time step) with the velocity of light c, will take one time step to cross one step in the space discretization. Hence, Eqn (15) is a perfect implementation in the case when we are simulating the ABC for the reflected wave.

Comparison to time domain integral equation (TDIE) method, FDTD-based PML holds more validity on approximation involving multiple or in-homogeneous medium[10]. Moreover, FDTD is a better implementation for complex body or shape analysis than TDIE. For curved surfaces[6] gives a novel way of applying ABC-based on Liao's method and discusses minimizing reflections with better approximations at the curved surfaces or corners of a computational grid. A recent Mur based ABC[13] uses 1/3rd time stepping method for one dimensional FDTD code and so is generating 3 steps of iteration instead of conventional 2 step method. This in turn exhibits extremely low reflections from the boundary.

However, in the case of transmitted wave, the electromagnetic wave is travelling from the air to dielectric medium and hence the ABC needs suitable modifications. It has been well cited that inside the dielectric of $4\varepsilon_0$, velocity of the electromagnetic wave is restricted to $c/2$, where c is the speed of light in free space. Now bearing in mind the analogy just mentioned, with magic time stepping and $c/2$ velocity of propagation, the EM wave would take two time steps to cover a step in space discretization. The same sounds reasonable since Sadiku[14] also mention that, if we simulate the lattice truncation in a dielectric medium of refractive index m, ABC is:

$$E_{z0}(n) = E_{z1}(n-m) \tag{17}$$

$$E_{zM}(n) = E_{zM-1}(n-m) \tag{18}$$

Following Eqn (13), we can assume that ABC over a dielectric interface may be given as:

$$E_{zM}(n) = E_{zM-1}(n-\sqrt{\varepsilon_r}) \quad (19)$$

The above equation has been simulated for our mentioned problem statement and it works with reasonable accuracy.

Figure 5 shows that Mur I ABC works properly for the air interface region and the ABC applied at the right or the terminating edge of the dielectric where the transmitted wave is truncated. Further it illustrates that there is minimal or no reflection taking place from the edges on the application with the new boundary condition. The simulation is completed after 970 steps or 48.5 ns. Hence it has been verified that the radiation boundary condition devised over the dielectric interfaces work well within limits.

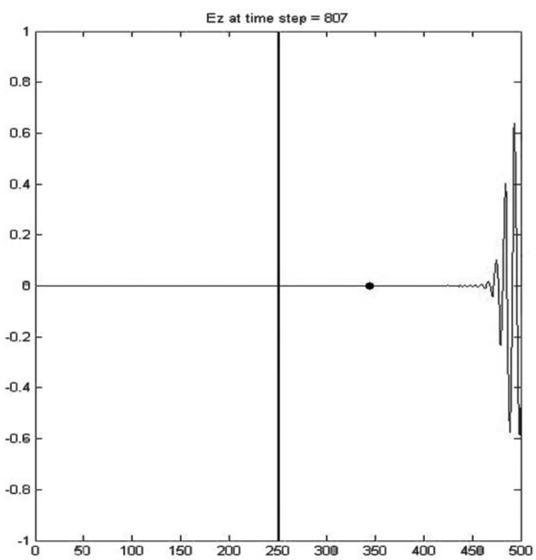

Figure 5. Beginning of the absorption at the dielectric interface end.

4. RESULTS AND DISCUSSIONS

The FFTs calculated of the recorded wave for the mentioned case of 3.2 are as shown in Fig. 6. The transmission coefficient is calculated by simply taking a ratio of the Fourier transform of the Transmitted pulse to that of the Fourier transform of the Incident pulse. The theoretical values are calculated using Eqns (7) to (10) with the data $\varepsilon_1 = \varepsilon_0$ and $\varepsilon_2 = 4\varepsilon_0$, we find out that the ratio for transmission coefficients have the following values:

$$\frac{E^t}{E^i} = \frac{2\sqrt{\varepsilon_1}}{\sqrt{\varepsilon_1}+\sqrt{\varepsilon_2}} = 0.667$$

$$\frac{H^t}{H^i} = \frac{2\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}+\sqrt{\varepsilon_2}} = 1.333$$

The ratio of the Fourier transforms of electric and magnetic fields compared with the theoretical values and its comparison with the Mur I ABC are as shown in Figs 7 and 8.

Figures 7 and 8 show that more accuracy is achieved with the modified Mur I ABC compared to the implementation of Mur I ABC. The above results clearly show that the simulation results are well within range of the theoretical value. We had specified the wavelength of 0.3 m or 1 GHz frequency and we can verify that we get accurate results till 1 GHz.

The implementation of the dielectric slab problem has been verified using the following equation:

$$t = \frac{t_{12}t_{23}c^{i\beta}}{1+r_{12}r_{23}e^{2i\beta}} \quad (20)$$

The above equation is a standard result and verified[16]. Based on the above result, our simulation also matches correctly and result also matches appropriately. Figure 9 shows the Fourier transforms of the incident and reflected waves and Fig. 10 shows the transmission coefficients calculated.

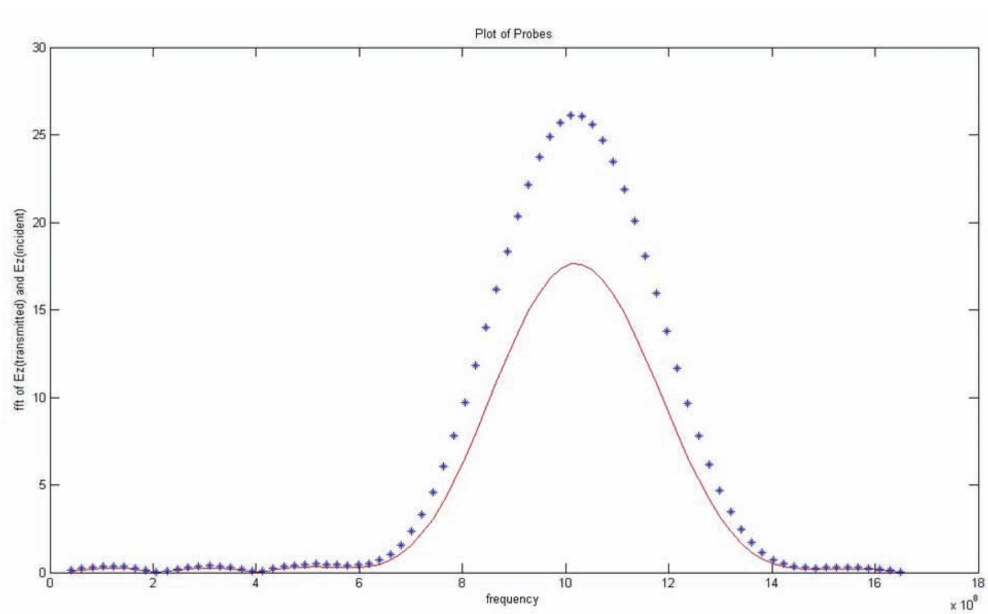

Figure 6. The dotted lines are the FFT of the incident pulse and hard line is FFT of transmitted of electric field.

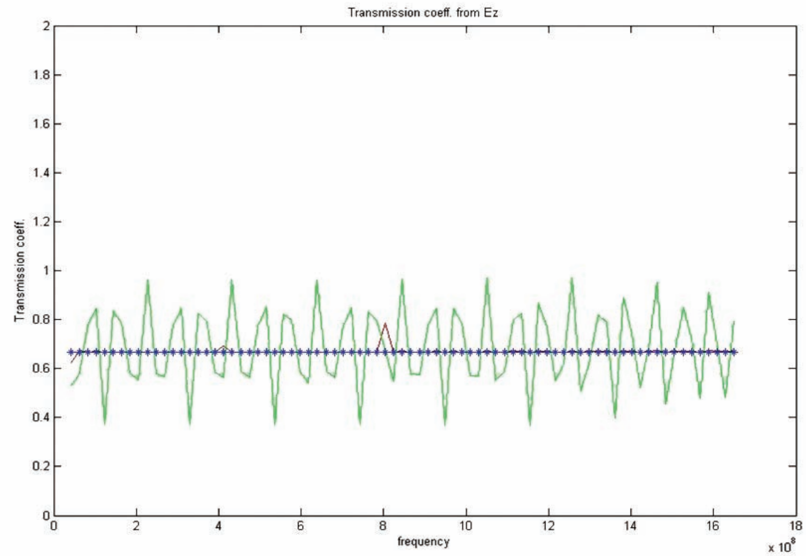

Figure 7. The star marked line is transmission coefficient using the theoretical value and straight line (red) is from simulation for the electric field using proposed ABC and the green line is from simulation for electric field using Mur I ABC termination.

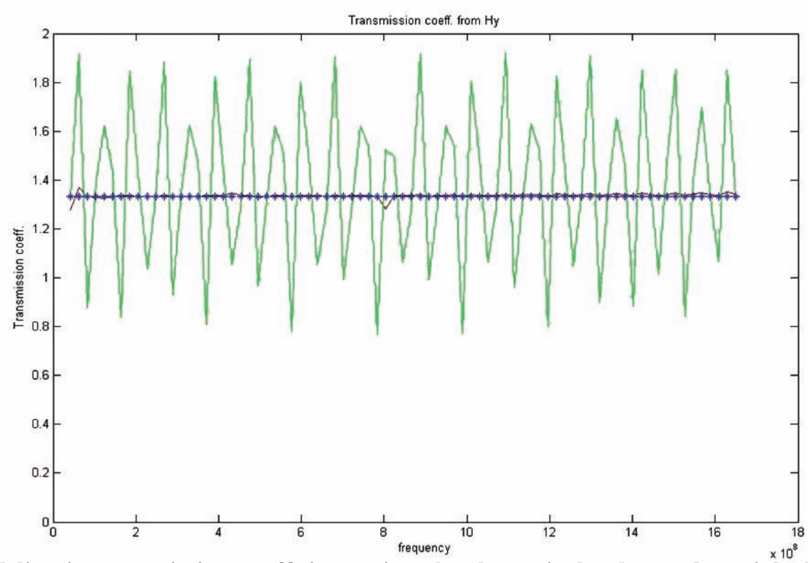

Figure 8. The star marked line is transmission coefficient using the theoretical value and straight line (red) is from simulation for the magnetic field using proposed ABC and the green line is from simulation for magnetic field using Mur I ABC termination.

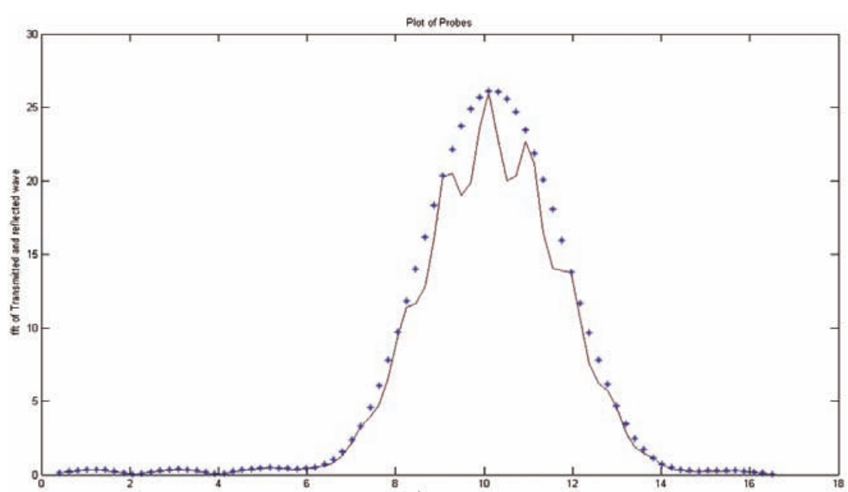

Figure 9. The dotted lines represent the Fourier transform of the incident wave and the hard line represents the Fourier transform of the transmitted wave recorded from $i = 350$ spatial point of the electric fields.

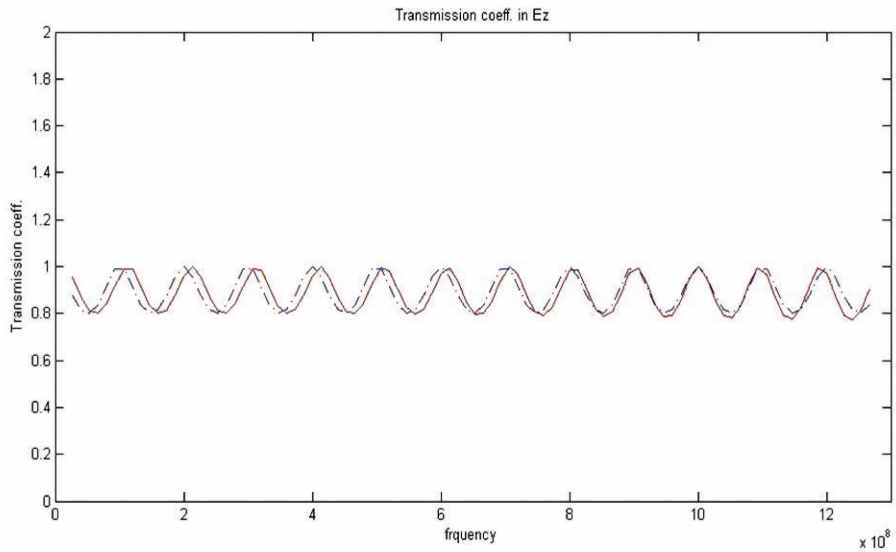

Figure 10. Transmission coefficient as calculated from the simulation for the electric fields shows a periodical behaviour due to multiple transmissions. Red coloured straight lines shows simulation results and blue colored dotted lines are the theoretically calculated results.

5. CONCLUSIONS

This study implements the concepts of finite difference time domain method to a one dimensional electromagnetic wave problem. While implementing the absorbing boundary condition using the first order Mur condition, we have been able to demonstrate and devise a new way of terminating an electromagnetic wave on a dielectric interface only. Verification of the codes developed on MATLAB have been simulated and testified with the theoretical results available to prove that the codes developed give substantially suitable results. The new technique of the ABC implementation on the dielectrics has been tested for a particular case of $4\varepsilon_0$ and verified to get substantially accurate results. Verification is also done by calculating the transmission coefficient of a wave trapped in a dielectric slab piece.

REFERENCES

1. Kane, S Yee. Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media. *IEEE Trans. Antennas Wave Propagation*, 1966, **14** (3), 302-07.
2. Taflove, A. Application of the finite difference time domain method to sinusoidal steady state electromagnetic penetration problems. *IEEE Trans. Electromagnetic Compat.*, 1980, **22**(3), 191-202.
3. Berenger, J.P. A Perfectly matched layer for the absorption of electromagnetic waves. *J. Computational Phy.,* 1994, **144**(2), 185-200.
4. Ali, Md. Fauk & Ray, Sudhabindu. SAR analysis in a spherical inhomogeneous human head model exposed to radiating dipole antenna for 500MHz-3GHz using FDTD method. *Int. J. Microwave Optical Techno.,* 2009, **4**(1), 35-40.
5. Sullivan, Dennis M. More on onedimensional simulation. *In* Electromagnetic Simulation using the FDTD method. Wiley-IEEE Press, 2000. pp. 19-47.
6. Yu, Tong Bin; Qian, Zu Ping & Lu, Chun Lan. A modified Liao's absorbing boundary condition for FDTD simulations in 2-D cylindrical coordinates with rotational symmetry. *In* the IEEE 2007 International Symposium on Microwave, Antenna Propagation and EMC Technologies for Wireless Communications. 2007. pp. 845-847.
7. Jurgens, T.G.; Taflove, A.; Umashankar, K.R. & Moore, T.G. Finite difference time domain modelling of curved surfaces. *IEEE Trans. Antennas Propagation*, 1992, **40**(4), 357-366.
8. Mur, G. Absorbing boundary conditions for the finite difference approximation of time-domain electromagnetic field equations. *IEEE Trans. Eletromagn. Compat.*, 1981, **23**(4), 377-382.
9. Pernics, Wolfram Hans Peter; Payne, Frank P. & Gallagher, Dominie F.G. An efficient multi-domain spectral algorithm for the simulation of dispersive metallic structures in the time domain. *Optical Quantum Electronics*, 2007, **39**(10-11), 877-89.
10. Sachdeva, Niraj; Rao Sadasiva M. & Balakrishnan, N. A comparison of FDTD-PML with TDIE. *IEEE Trans. Antennas Propagation*, 2002, **50**(11), 1609-614.
11. Taflove, Allen & Hagness, Susan C. Computational electrodynamics-The finite difference time domain method. Artech House, 2005.
12. Naishadham, Krishna & Lin, Xing Ping. Minimization of reflection error caused by absorbing boundary condition in the FDTD simulation of planar transmission lines. *IEEE Trans. Microwave Theory Techniques*, 1996, **44**(1).
13. Liang, Feng; Wang, Gaofeng; Lin, Hai & Wang, Bing Zhong. Mur absorbing boundary condition for three step 3-D LOD FDTD method. *IEEE Microwave Wireless Component Letters*, 2010, **20**(11), 41-46.
14. Sadiku, Mathew N.O. Numerical techniques in electromagnetics. Ed. 2. CRC press, 2009, pp. 159-186.
15. Sadiku, Mathew N.O. Elements of electromagnetics. Ed. 3. Oxford University Press, 2001, pp. 440-50.

16. Born, Max & Wolf, Emil. Basic properties of electromagnetic field. *In* Principles of Optics. Ed. 7. Cambridge University Press, 1999, pp. 54-70.

Contributors

Mr Biswajeet Mukherjee received his MTech (Microwave Electronics) from University of Delhi, Delhi in 2009 and is currently pursing his PhD from IIT Bombay, Mumbai. Presently, he is working as a Research Engineer with PDPM Indian Institute of Information Technology Design & Manufacturing, Jabalpur. He has published more than 5 research papers in journals and conference proceedings. His current area of research includes: Computational electromagnetics using FDTD method.

Dr Dinesh Kumar Vishwakarma received his ME (Electronics and Comm. Engg) from Anna University, Chennai in 2000 and PhD (Electronics and Comm. Engg.) from Indian Institute of Science, Bangalore in 2005. Presently working as a Assistant Professor at PDPM Indian Institute of Information Technology, Design & Manufacturing, Jabalpur, India. His research area includes: Applied photonics, surface plasmon and metamaterials, antenna theory, analysis & design, and electromagnetics.