Maximum Likelihood Estimator for Bearings-only Passive Target Tracking in Electronic Surveillance Measure and Electronic Warfare Systems

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ABSTRACT

Maximum likelihood estimator is a suitable algorithm for passive target tracking applications. Nardone, Lindgren and Gong introduced this approach using batch processing. In this paper, the batch processing is converted into sequential processing for real-time applications like passive target tracking using bearings-only measurements. Adaptively, the variance of each measurement is computed and is used along with the measurement in such a way that the effect of false bearings can be reduced. The transmissions made by radar on a target ship are assumed to be intercepted by an electronic warfare (EW) system of own ship. The generated bearings in intercept mode are processed through maximum likelihood estimator (MLE) to find out target motion parameters. Instead of assuming some arbitrary values, pseudo linear estimator outputs are used for the initialisation of MLE. The algorithm is tested in Monte-Carlo simulation and its results are presented for two typical scenarios.

Keywords: Maximum likelihood estimator, pasive target tracking, target tracking, ESM system, EW system.

1. INTODUCTION

Surveillance is the most important facet of maritime warfare and is undertaken by active as well as passive sensors. Active methods of surveillance require electromagnetic (EM)/acoustic transmissions to be made by the surveillance platform and hence, are susceptible to interception by others. So in certain tactical situations, it becomes necessary to resort to passive techniques as electronic surveillance measures (ESM) provide instantaneous measurement of emitter bearing but not its range. In the ocean environment, two-dimensional bearings-only target motion analysis is generally used. Ownship monitors noisy radar bearings from a transmitting target, which is assumed to be traveling at uniform velocity. The bearing measurements generated are corrupted with noise. An EW receiver system on an ownship processes these measurements and finds out target motion parameters - viz., range, course, bearing and speed of the target. Here the measurement is nonlinear, making the whole process nonlinear. Added to this, since bearing measurements are extracted from single EW system, the process remains unobservable until ownship executes a proper manueuvre.

Nardone, Lindgren & Gong[1] used maximum likelihood estimator (MLE) in batch processing to estimate target motion parameters. In this paper, the MLE in batch processing is converted into sequential processing. All the elements of covariance matrix are represented recursively in terms of measurement equation. These terms are known as

RECURSIVE SUMS and are maintained throughout the algorithm. This approach avoids the computational complexity by computing only the incremental values for every new bearing measurement. These incremental values are used to update the RECURSIVE SUMS in covariance matrix. Only few RECURSIVE SUMS are to be updated on the arrival of new bearing measurement. This method does not increase computational burden with more number of samples.

The MLE requires some initial estimate as bearings-only measurements are available. For bearings-only target tracking[2], Ristic, *et al.,* have suggested to use a number of filters in parallel with different initialisations covering all possible ranges, speeds, and courses of the target. Then the number of filters required is in the order of thousands, and hence, the computation time for the filters becomes significant. To reduce the number of filters and computation time, pseudo linear estimator's (PLE) outputs[3] have been used for initialisation of MLE. PLE is developed from least square estimator in such a way that it is recursive and does not require any initial estimate. As PLE generates bias in the estimates, its use is restricted to generate a reasonably accurate estimate for initialisation of MLE. The accuracy of this algorithm is improved by adaptively calculating the weightage of each measurement in terms of its variance and simultaneously using this with the measurement. This work can be considered as an alternate to contributions by Clark[4] *et al.,* Aidala[5], Song & Speyer[6], Grossman[7] and Rao[8] and it generates accurate and fast convergent results

and provides good insight into the bearing-only tracking problems.

Finally Monte-Carlo simulation is carried out and the algorithm is tested against several tactical geometries. For illustration, the results of two tactical scenarios are presented and it has been observed that the results are satisfactory.

2. MATHEMATICAL MODELLING

The target and ownship movements are modelled as follows.

Let $X_s(k)$ be a state vector with target velocity and position components and is given by

$$X_s(k) = [\dot{x}_t(k) \quad \dot{y}_t(k) \quad x_t(k) \quad y_t(k)]^T \qquad (1)$$

The bearing is the angle from ownship's platform to the target, and is given by

$$B(k) = \tan^{-1}(r_x(k)/r_y(k)) \qquad (2)$$

where $r_x(k)$ and $r_y(k)$ are the relative range components at instant k. The measured bearing, $B_m(k)$ is given by

$$B_m(k) = B(k) + \gamma(k) \qquad (3)$$

where $\gamma(k)$ is error in the measurement and this error is assumed to be zero mean Gaussian with variance, σ^2. Let there be k measurements $B_m(1), B_m(2), \dots B_m(k)$ available and these measurements are represented by

$$Z = [B_m(1), B_m(2), \dots, B_m(k)]^T \qquad (4)$$

The likelihood function of the measurements is given by

$$L = \left(\frac{1}{(2\pi\sigma)^2}\right)^{\frac{k}{2}} \prod_{i=1}^{n} e^{\frac{-(B(i)-B_m(i))^2}{2\sigma^2}} \qquad (5)$$

Maximum likelihood estimator (MLE) finds out the components of $X_s(k)$, (which are functions of bearing measurement) for which the measurements are most likely. Maximisation of log L or L gives the same results. As it is easy to maximise log L than that of L, here maximisation of log L is used.

$$\log L = \frac{k}{2} \log\left(\frac{1}{2\pi\sigma^2}\right) + \sum_{i=1}^{k} \frac{-(B(i)-B_m(i))^2}{2\sigma^2} \qquad (6)$$

In eqn (6), $\frac{k}{2}\log\left(\frac{1}{2\pi\sigma^2}\right)$ is constant. Maximisation of log L is maximisation of

$$-\sum_{i=1}^{k} \frac{(B(i)-B_m(i))^2}{2\sigma^2} \text{ and it is minimisation of}$$

$$\sum_{i=1}^{k} (B(i)-B_m(i))^2$$

Let e_{MLE} be

$$e_{MLE} = -\sum_{i=1}^{k} \frac{(B(i)-B_m(i))^2}{2} \qquad (7)$$

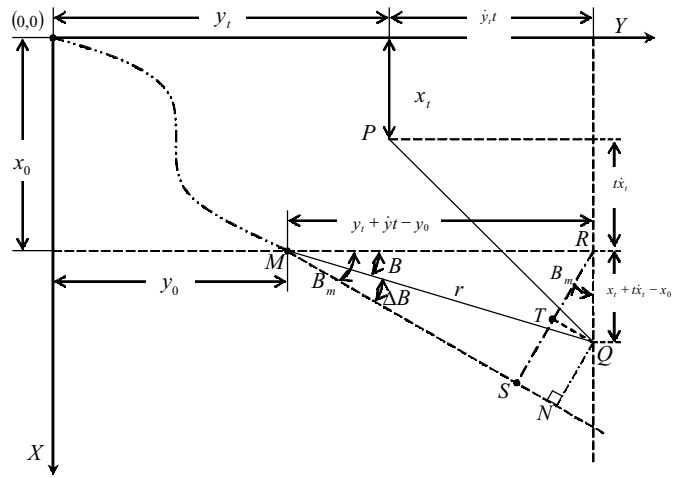Differentiating wrt X_S and equating to zero, we obtain

Figure 1. Target motion analysis problem.

$$\frac{\partial e_{MLE}}{\partial X_S} = -\sum_{i=1}^{k} \left[(B(i)-B_m(i)) \frac{\partial(B(i)-B_m(i))}{\partial X_S(i)} \right] = 0 \qquad (8)$$

Let us consider the measurements B & B_m as shown in Fig.1. Let the target be initially at $P(x_t, y_t)$ and the ownship be at origin. During t seconds, the target travelled to Q and the ownship to $M(x_0, y_0)$. For convenience k is dropped for the time being. Using Fig.1, the following can be written

From $\triangle MRQ$,

$$\tan B = \frac{x_t + t\dot{x}_t - x_0}{y_t + t\dot{y}_t - y_0} \qquad (9)$$

$\triangle MRS$,

$$\sin B_m = \frac{RS}{y_t + t\dot{y}_t - y_0}$$

$$RS = (y_t + t\dot{y}_t - y_0)\sin B_m \qquad (10)$$

$\triangle RTQ$, $\cos B_m = \frac{RT}{y_t + t\dot{y}_t - y_0}$;

$$RT = (x_t + t\dot{x}_t - x_0)\cos B_m \qquad (11)$$

$$QN = (RS - RT) = \begin{bmatrix} (y_t + t\dot{y}_t - y_0) \\ \sin B_m - (x_t + t\dot{x}_t - x_0)\cos B_m \end{bmatrix} \qquad (12)$$

and

$$\sin(B - B_m) = \frac{-QN}{r} \qquad (13)$$

It is assumed that the error in the measurement is of small magnitude, then

$$
\begin{aligned}
B - B_m &= \frac{-QN}{r} \\
&= \frac{(y_t + t\dot{y}_t - y_0)\sin B_m - (x_t + t\dot{x}_t - x_0)\cos B_m}{r} \\
&= \frac{+r_x \cos B_m - r_y \sin B_m}{r}
\end{aligned} \qquad (14)
$$

Let us derive the derivatives of $(B-B_m)$ wrt X_s

$$\frac{\partial}{\partial X_s}(B-B_m) = \frac{\partial}{\partial X_s}\left[\tan^{-1}\frac{r_x}{r_y} - B_m\right]$$

$$= \frac{\partial}{\partial X_s}\left(\tan^{-1}\frac{r_x}{r_y}\right) = \frac{1}{r^2}\left[r_y\frac{\partial r_x}{\partial X_s} - r_x\frac{\partial r_y}{\partial X_s}\right] \quad (15)$$

and

$$(B-B_m)\frac{\partial}{\partial X_s}(B-B_m) = \left(-\frac{QN}{r}\right)\left(\frac{1}{r^2}\left[r_y\frac{\partial r_x}{\partial X_s} - r_x\frac{\partial r_y}{\partial X_s}\right]\right)$$

$$= \frac{1}{r^3}(r_x\cos B_m - r_y\sin B_m)\left[r_y\frac{\partial r_x}{\partial X_s} - r_x\frac{\partial r_y}{\partial X_s}\right] \quad (16)$$

In general notation

$$\sum_{i=1}^{k}\left[(B(i)-B_m(i))\frac{\partial(B(i)-B_m(i))}{\partial X_S(i)}\right] =$$

$$\sum_{i=1}^{k}\left\{\begin{array}{c}\frac{1}{r^3(i)}(r_x\cos B_m(i) - r_y(i)\sin B_m(i)) \\ \left(r_y(i)\frac{\partial r_x(i)}{\partial X_S(i)} - r_x(i)\frac{\partial r_y(i)}{\partial X_S(i)}\right)\end{array}\right\} \quad (17)$$

Let us analyse Eqn. (17) when $i=1$

$$(B(1)-B_m(1))\frac{\partial(B(1)-B_m(1))}{\partial X_S(1)} = \frac{1}{r^3(1)}\left(\begin{array}{c}r_x(1)\cos B_m(1) \\ -r_y(1)\sin B_m(1)\end{array}\right)$$

$$\left(r_y(1)\frac{\partial r_x(1)}{\partial X_S(1)} - r_x(1)\frac{\partial r_y(1)}{\partial X_S(1)}\right) \quad (18)$$

where $r_x(1) = x_t(0) + \dot{x}_t t - x_0(1)$ $\quad$ (19)

$$\frac{\partial r_x(1)}{\partial \dot{x}_t} = t, \ \frac{\partial r_x(1)}{\partial \dot{y}_t} = 0, \frac{\partial r_x(1)}{\partial x_t(1)} = 1, \ \frac{\partial r_x(1)}{\partial y_t(1)} = 0$$

Similarly, $r_y(1) = y_t(0) + \dot{y}_t t - y_0(1)$ $\quad$ (20)

$$\frac{\partial r_y(1)}{\partial \dot{x}_t} = 0, \frac{\partial r_y(1)}{\partial \dot{y}_t} = t, \frac{\partial r_y(1)}{\partial x_t(1)} = 0, \frac{\partial r_y(1)}{\partial y_t(1)} = 1$$

Substituting Eqns. (19) and (20) in eqn. (18)

$$(B(1)-B_m(1))\frac{\partial(B(1)-B_m(1))}{\partial X_S(1)} =$$

$$\frac{1}{r^3(1)}(r_x(1)\cos B_m(1) - r_y(1)\sin B_m(1))\begin{bmatrix} tr_y(1) \\ -tr_x(1) \\ r_x(1) \\ -r_y(1) \end{bmatrix} \quad (21)$$

when

$$(B(2)-B_m(2))\frac{\partial(B(2)-B_m(2))}{\partial X_S(2)} =$$

$$\frac{1}{r^3(2)}(r_x(2)\cos B_m(2) - r_y(2)\sin B_m(2))\times$$

$$\left(r_y(2)\frac{\partial r_x(2)}{\partial X_s(2)} - r_x(2)\frac{\partial r_y(2)}{\partial X_s(2)}\right) \quad (22)$$

where $r_x(2) = x_t(0) + 2\dot{x}_t t - x_0(2)$

$$\frac{\partial r_x(2)}{\partial \dot{x}_t} = 2t, \frac{\partial r_x(2)}{\partial \dot{y}_t} = 0, \frac{\partial r_x(2)}{\partial x_t(2)} = 1,$$

$$\frac{\partial r_x(2)}{\partial y_t(2)} = 0 \quad (23)$$

Similarly, $r_y(2) = y_t(0) + 2\dot{y}_t t - y_0(2)$ $\quad$ (24)

$$\frac{\partial r_y(2)}{\partial \dot{x}_t} = 0, \frac{\partial r_y(2)}{\partial \dot{y}_t} = 2t, \frac{\partial r_y(2)}{\partial x_t(2)} = 0, \frac{\partial r_y(2)}{\partial y_t(2)} = 1$$

Substituting eqn. (23) & (24) in Eqn (22)

$$(B(2)-B_m(2))\frac{\partial(B(2)-B_m(2))}{\partial X_S(2)} =$$

$$\frac{1}{r^3(2)}(r_x(2)\cos B_m(2) - r_y(2)\sin B_m(2))\begin{bmatrix} 2tr_x(2) \\ -2tr_y(2) \\ r_x(2) \\ -r_y(2) \end{bmatrix} \quad (25)$$

Similarly

$$B(k)-B_m(k)\frac{\partial B(k)-B_m(k)}{\partial X_s} =$$

$$\frac{(r_x(k)\cos B_m(k) - r_y(k)\sin B_m(k))}{r^3(k)}\begin{bmatrix} r_y(k)kt \\ -r_x(k)kt \\ r_x(k) \\ -r_y(k) \end{bmatrix} \quad (26)$$

Using Eqns. (21), (25) and (26), Eqn. (17) can be written as

$$\sum_{i=1}^{k} B(i)-B_m(i)\frac{\partial B(i)-B_m(i)}{\partial X_s} =$$

$$\begin{bmatrix} \sum_{i=1}^{k}\frac{r_y(i)it}{r^3(i)}(r_x(i)\cos B_m(i) - r_y(i)\sin B_m(i)) \\ \sum_{i=1}^{k}\frac{-r_x(i)it}{r^3(i)}(r_x(i)\cos B_m(i) - r_y(i)\sin B_m(i)) \\ \sum_{i=1}^{k}\frac{r_x(i)}{r^3(i)}(r_x(i)\cos B_m(i) - r_y(i)\sin B_m(i)) \\ \sum_{i=1}^{k}\frac{-r_y(i)}{r^3(i)}(r_x(i)\cos B_m(i) - r_y(i)\sin B_m(i)) \end{bmatrix} = 0 \quad (27)$$

Let us expand the first row of vector in Eqn. When

$$\sum\frac{r_y(i)it}{r^3(i)}(r_x(i)\cos B_m(i) - r_y(i)\sin B_m(i)) = 0 \quad (28)$$

$(r_x(1)\cos B_m(1) - r_y(1)\sin B_m(1))$ can be expanded as

$$(r_x(1)\cos B_m(1) - r_y(1)\sin B_m(1)) =$$

$$\left(x_t(0) + \dot{x}t - x_0(1)\right)\cos B_m(1) - \left(y_t(0) + \dot{y}t - y_0(1)\right)\sin B_m(1)$$

$$= \begin{bmatrix} t\cos B_m(1) & -t\sin B_m(1) & \sin B_m(1) & -\cos B_m(1) \end{bmatrix} \begin{bmatrix} \dot{x}_t \\ \dot{y}_t \\ x_t(0) \\ y_t(0) \end{bmatrix}$$

(29)

Using Eqn. (29) in Eqn. (28), one can write that

$$\begin{bmatrix} \frac{r_y(1)t^2}{r^3(1)}\cos B_m(1) & \frac{-r_y(1)t^2}{r^3(1)}\sin B_m(1) & \frac{r_y(1)t}{r^3(1)}\cos B_m(1) & \frac{-r_y(1)t}{r^3(1)}\sin B_m(1) \\ \frac{-r_x(1)t^2}{r^3(1)}\cos B_m(1) & \frac{r_x(1)t^2}{r^3(1)}\sin B_m(1) & \frac{r_x(1)t}{r^3(1)}\cos B_m(1) & \frac{-r_x(1)t}{r^3(1)}\sin B_m(1) \\ \frac{r_y(1)t}{r^3(1)}\cos B_m(1) & \frac{-r_y(1)t}{r^3(1)}\sin B_m(1) & \frac{r_y(1)}{r^3(1)}\cos B_m(1) & \frac{-r_y(1)}{r^3(1)}\sin B_m(1) \\ \frac{-r_x(1)t}{r^3(1)}\cos B_m(1) & \frac{r_x(1)t}{r^3(1)}\sin B_m(1) & \frac{r_x(1)}{r^3(1)}\cos B_m(1) & \frac{-r_x(1)}{r^3(1)}\sin B_m(1) \end{bmatrix} * \begin{bmatrix} \dot{x}_t \\ \dot{y}_t \\ x_t(0) \\ x_t(0) \end{bmatrix}$$

$$\begin{bmatrix} \frac{r_y(1)t^2}{r^3(1)}\cos B_m(1) & \frac{-r_y(1)t^2}{r^3(1)}\sin B_m(1) & \frac{r_y(1)t}{r^3(1)}\cos B_m(1) & \frac{-r_y(1)t}{r^3(1)}\sin B_m(1) \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_t \\ \dot{y}_t \\ x_t(0) \\ y_t(0) \end{bmatrix} = \frac{r_y(1)t}{r^3(1)}\left(x_0(1)\cos B_m(1) - y_0(1)\sin B_m(1)\right)$$

(30)

Similarly, the other three equations can be written as follows when $i=1$

$$= \begin{bmatrix} \frac{r_y(1)t}{r^3(1)} \\ \frac{-r_x(1)t}{r^3(1)} \\ r_y(1) \\ -r_x(1) \end{bmatrix} \left\{ x_0(1)\cos B_m(1) - y_0(1)\sin B_m(1) \right\}$$

(31)

When $i=1, 2, 3 \ldots k$

$$\begin{bmatrix} \sum_{i=1}^{k}\frac{r_y(i)(it)^2\cos B_m(i)}{r^3(i)} & \sum_{i=1}^{k}\frac{-r_y(i)(it)^2\sin B_m(i)}{} & \sum_{i=1}^{k}r_y(i)(it)\cos B_m(i) & \sum_{i=1}^{k}-r_y(i)it\sin B_m(i) \\ \sum_{i=1}^{k}-r_x(i)(it)^2\cos B_m(i) & \sum_{i=1}^{k}r_x(i)(it)^2\sin B_m(i) & \sum_{i=1}^{k}-r_x(i)(it)\cos B_m(i) & \sum_{i=1}^{k}r_x(i)(it)\sin B_m(i) \\ \sum_{i=1}^{k}r_y(i)(it)\cos B_m(i) & \sum_{i=1}^{k}-r_y(i)(it)\sin B_m(i) & \sum_{i=1}^{k}r_y(i)\cos B_m(i) & \sum_{i=1}^{k}-r_y(i)\sin B_m(i) \\ \sum_{i=1}^{k}-r_x(i)(it)\cos B_m(i) & \sum_{i=1}^{k}r_x(i)(it)\sin B_m(i) & \sum_{i=1}^{k}-r_x(i)\cos B_m(i) & \sum_{i=1}^{k}r_x(i)\sin B_m(i) \end{bmatrix} *$$

$$\begin{bmatrix} \dot{x}_t \\ \dot{y}_t \\ x_t(0) \\ x_t(0) \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{k}\frac{r_y(i)}{r^3(i)}\left\{x_0(i)\cos B_m(i) - y_0(i)\sin B_m(i)\right\}(it) \\ \sum_{i=1}^{k}\frac{-r_x(i)}{r^3(i)}\left\{x_0(i)\cos B_m(i) - y_0(i)\sin B_m(i)\right\}(it) \\ \sum_{i=1}^{k}\frac{r_y(i)}{r^3(i)}\left\{x_0(i)\cos B_m(i) - y_0(i)\sin B_m(i)\right\} \\ \sum_{i=1}^{k}\frac{-r_x(i)}{r^3(i)}\left\{x_0(i)\cos B_m(i) - y_0(i)\sin B_m(i)\right\} \end{bmatrix}$$

(32)

2.1 Sequential Processing

So far, the variance of noise in the measurements is assumed to be constant for clarity of the concepts. Now this assumption is relaxed. The amount of error in the bearing is usually high and is of the order of 2 degree r.m.s. The measurement is assumed to be available at 1 Hz rate. It is not possible to track the target using ESM measurements with the above-mentioned order of noise. Hence, the measurements are averaged over a fixed period, say 20 s to reduce the variance of the errors in the measurements by 4.5 times. At the same time, the variance of measurements over 20 s is also found out and is used along with the averaged measurement in the filter equations. This leads to the auto editing of the measurements, that means, if the measurement is good (i.e., variance of the error in the measurement is less); more weightage to that measurement is given when compared to that of other measurements.

The treatment is now extended from batch processing to sequential processing using RECURSIVE SUMS (CSUMS, SSUMS etc.). The effect of all the bearing measurements in each element of the matrices in Eqn. (32) is maintained in the form of RECURSIVE SUMS. Whenever a new bearing measurement is available, only calculations pertaining to the newly arrived measurement are carried out and added to the RECURSIVE SUMS. The target state vector at any instant is used to find out target motion parameters at that instant. Detailed procedure for the development of RECURSIVE SUMS is given by Rao[3].

The aim is to find out the solution at the instant of $(k+1)$. For this purpose, all range components in Eqn. (32) are to be translated to time $(k+1)$. Let us use the format of well known transformation of equations

$$X_s(k+1) = \phi(k+1,0)X_s(0) \quad (33)$$

where $\phi(k+1,0)$ is transient matrix between $X_s(k+1)$ and $X_s(0)$. The transformation in eqn. (33) is used for range components as follows.

$$\begin{bmatrix} r_x(k+1) \\ r_y(k+1) \end{bmatrix} = r(k+1) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ (k+1)t & 0 & 1 & 0 \\ 0 & 1 & 0 & (k+1)t \end{bmatrix} \begin{bmatrix} \dot{x}_t \\ \dot{y}_t \\ r_x(0) \\ r_y(0) \end{bmatrix}$$

$$- \begin{bmatrix} 0 \\ 0 \\ x_0(1) \\ y_0(1) \end{bmatrix} = \phi(k+1,0)r(0) - b(1)$$

(34)

where $b(1) = \begin{bmatrix} 0 \\ 0 \\ x_0(1) \\ y_0(1) \end{bmatrix}$ (35)

Similarly

$r(k+1) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ (k)t & 0 & 1 & 0 \\ 0 & 1 & 0 & (k)t \end{bmatrix} \begin{bmatrix} \dot{x}_t \\ \dot{y}_t \\ r_x(1) \\ r_y(1) \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ x_0(2) \\ y_0(2) \end{bmatrix} = \phi(k,1)r(1) - b(2)$ (36)

where $b(2) = \begin{bmatrix} 0 \\ 0 \\ x_0(2) \\ y_0(2) \end{bmatrix}$ (37)

$r(k+1) = \phi(k-1,2)r(2) - b(3)$
$= \phi(k-2,3)r(3) - b(4)$
$\vdots$

$r(k+1) = \phi(1,k)r(k) - b(k)$ (38)

Substituting for r_x and r_y components, and carrying out straight forward manipulations, the R.H.S of eqn. (27) can be written as follows.

PHI * $X_s(k+1)$ = G (39)

where

$$PHI = \begin{bmatrix} \dot{y}_t*CSUMS[3]+ \\ y_t(0)*CSUMS[2]- \\ CSUMS[6] \end{bmatrix} \begin{matrix} -\dot{y}_t*SSUMS[3]- \\ y_t(0)*SSUMS[2]+ \\ SSUMS[9] \end{matrix} \begin{matrix} \dot{y}_t*CSUMS[2]+ \\ y_t(0)*CSUMS[1]- \\ CSUMS[5] \end{matrix} \begin{matrix} -\dot{y}_t*SSUMS[2]- \\ y_t(0)*SSUMS[1]+ \\ SSUMS[8] \end{matrix}$$

(matrix rows)

Row 1: $\dot{y}_t*CSUMS[3]+y_t(0)*CSUMS[2]-CSUMS[6]$, $-\dot{y}_t*SSUMS[3]-y_t(0)*SSUMS[2]+SSUMS[9]$, $\dot{y}_t*CSUMS[2]+y_t(0)*CSUMS[1]-CSUMS[5]$, $-\dot{y}_t*SSUMS[2]-y_t(0)*SSUMS[1]+SSUMS[8]$

Row 2: $-\dot{x}_t*CSUMS[3]-x_t(0)*CSUMS[2]+CSUMS[9]$, $\dot{x}_t*SSUMS[3]+x_t(0)*SSUMS[2]-SSUMS[6]$, $-\dot{x}_t*CSUMS[2]-x_t(0)*CSUMS[1]+CSUMS[8]$, $\dot{x}_t*SSUMS[2]+x_t(0)*SSUMS[1]-SSUMS[5]$

Row 3: $(1,3)$, $(1,4)$, $\dot{y}_t*CSUMS[1]+y_t(0)*CSUMS[0]-CSUMS[4]$, $\dot{y}_t*SSUMS[1]-y_t(0)*SSUMS[0]+SSUMS[7]$

Row 4: $(2,3)$, $(2,4)$, $-\dot{x}_t*CSUMS[1]-x_t(0)*CSUMS[0]+CSUMS[7]$, $\dot{x}_t*SSUMS[1]+x_t(0)*SSUMS[0]-SSUMS[4]$

(40)

$$G = \begin{bmatrix} \dot{y}_t*[CSUMS[9]-SSUMS[9]]+y_t(0)*[CSUMS[8]-SSUMS[8]]-GSUMS[0] \\ -\dot{x}_t*[CSUMS[9]-SSUMS[9]]-x_t(0)*[CSUMS[8]-SSUMS[8]]+GSUMS[2] \\ \dot{y}_t*[CSUMS[8]-SSUMS[8]]+y_t(0)*[CSUMS[7]-SSUMS[7]]-GSUMS[1] \\ -\dot{x}_t*[CSUMS[8]-SSUMS[8]]-x_t(0)*[CSUMS[7]-SSUMS[7]]+GSUMS[3] \end{bmatrix}$$

(41)

The SUMS and other terms in *PHI* and *G* Matrices are given as follows. Initially all SUMS are zero. After obtaining measurement at time k, the SUMS are given by

$P(k) = x_0(k)\cos B_m(k)/\sigma^2(k) - y_0(k)\sin B_m(k)/\sigma^2(k)$ (42)

$CSUMS[0]_k = \cos B_m(k)/\sigma^2(k) + CSUMS[0]_{k-1}$

$CSUMS[4]_k = y_0(k)*\cos B_m(k)/\sigma^2(k) + CSUMS[4]_{k-1}$

$CSUMS[7]_k = x_0(k)*\cos B_m(k)/\sigma^2(k) + CSUMS[7]_{k-1}$

$CSUMS[3]_k = -(kt)^3*CSUMS[0]_k + 3(kt)^2 CSUMS[1]_{k-1}$
$\qquad -3ktCSUMS[2]_{k-1} + CSUMS[3]_{k-1}$

$CSUMS[2]_k = (kt)^2*CSUMS[0]_k - 2ktCSUMS[1]_{k-1} + CSUMS[2]_{k-1}$

$CSUMS[1]_k = -kt*CSUMS[0]_k + CSUMS[1]_{k-1}$

$CSUMS[6]_k = (kt)^2*CSUMS[4]_k - 2ktCSUMS[5]_{k-1} + CSUMS[6]_{k-1}$

$CSUMS[5]_k = -kt*CSUMS[4]_k + CSUMS[5]_{k-1}$

$CSUMS[9]_k = (kt)^2*CSUMS[7]_k - 2ktCSUMS[8]_{k-1} + CSUMS[9]_{k-1}$

$CSUMS[8]_k = -kt*CSUMS[7]_k + CSUMS[8]_{k-1}$ (43)

$SSUMS[0]_k = \sin B_m(k)/\sigma^2(k) + SSUMS[0]_{k-1}$

$SSUMS[4] = x_0(k)*\sin B_m(k)/\sigma^2(k) + SSUMS[4]_{k-1}$

$SSUMS[7]_k = y_0(k)*\sin B_m(k)/\sigma^2(k) + SSUMS[7]_{k-1}$

$SSUMS[3]_k = -(kt)^3*SSUMS[0]_k + 3(kt)^2 SSUMS[1]_{k-1}$
$\qquad -3ktSSUMS[2]_{k-1} + SSUMS[3]_{k-1}$

$SSUMS[2]_k = (kt)^2*SSUMS[0]_k - 2ktSSUMS[1]_{k-1} + SSUMS[2]_{k-1}$

$SSUMS[1]_k = -kt*SSUMS[0]_k + SSUMS[1]_{k-1}$

$SSUMS[6]_k = (kt)^2*SSUMS[4]_k - 2ktSSUMS[5]_{k-1} + SSUMS[6]_{k-1}$

$SSUMS[5]_k = -kt*SSUMS[4]_k + SSUMS[5]_{k-1}$

$SSUMS[9]_k = (kt)^2*SSUMS[7]_k - 2ktSSUMS[8]_{k-1}$
$\qquad + SSUMS[9]_{k-1}$

$SSUMS[8]_k = -kt*SSUMS[7]_k + SSUMS[8]_{k-1}$ (44)

$GSUMS[1]_k = y_0(k)*P(k) + GSUMS[1]_{k-1}$

Table 1. Scenario chosen for evaluation of the algorithm

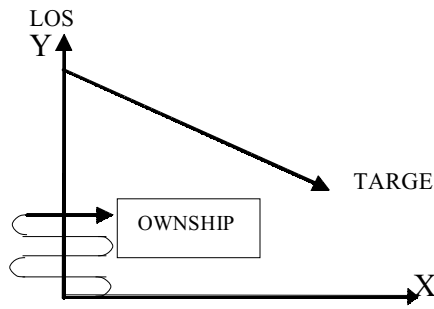|	Parameters						Convergence time (s)		
Scenario	Initial range (m)	Initial bearing (deg)	Target speed (m/s)	Target course (deg)	Ownship speed (m/s)	RMS error in bearing	Range	Course	Speed
1	18520	210	10.3	45	10.3	2	760	960	040
2	20000	145	10.3	300	10.3	2	520	1160	820

Figure 2. Ownship in S-manueuvre on LOS.
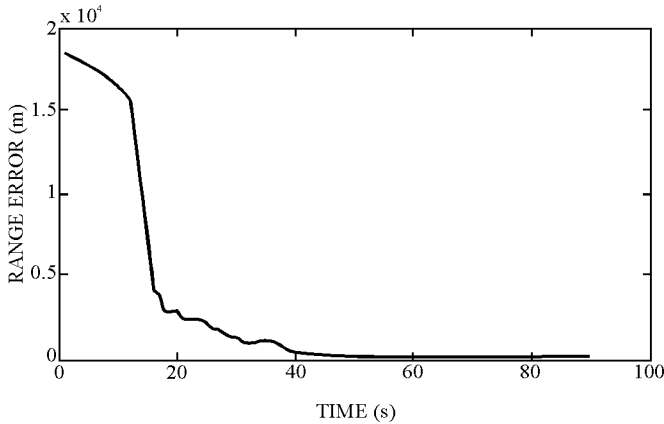

Figure 4(a). Error in range estimate.

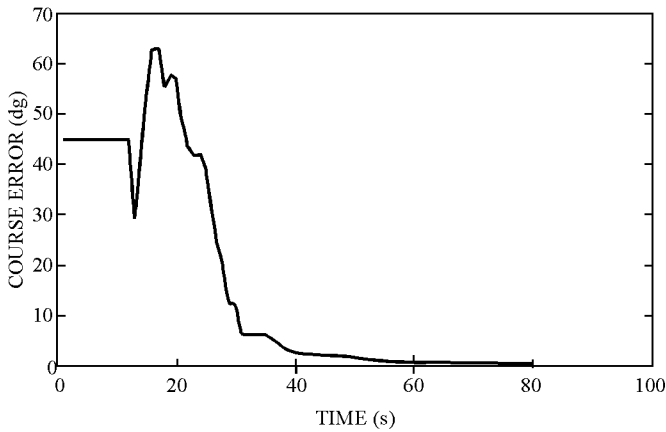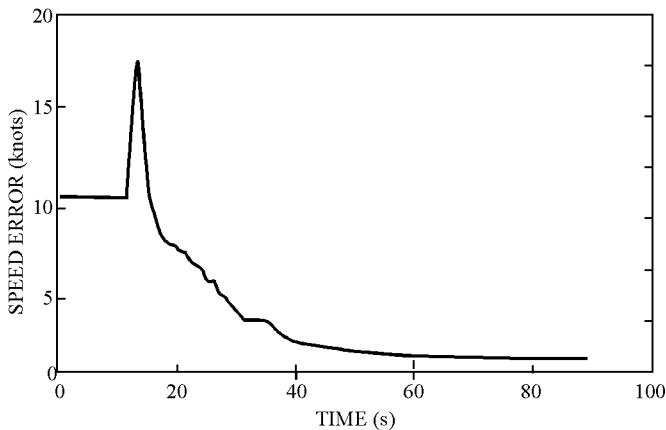Figure 3(a). Error in range estimate.

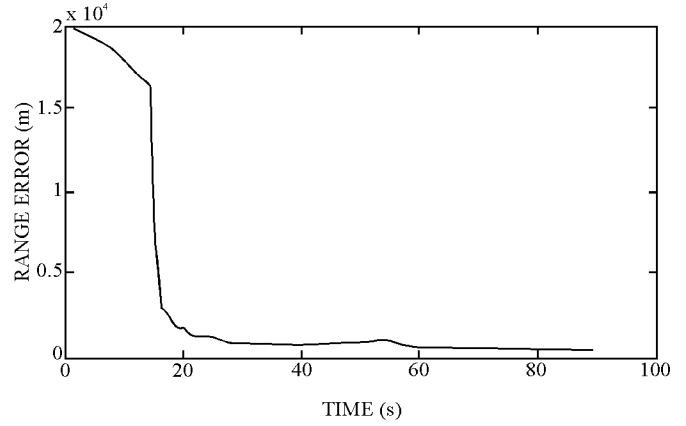

Figure 4(b). Error in course estimate.
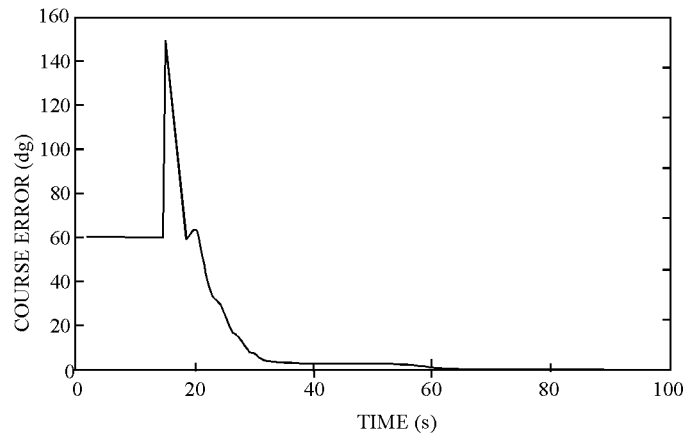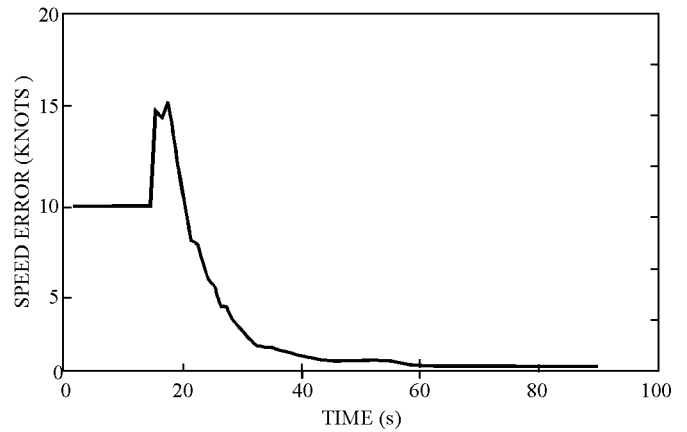

Figure 3(b). Error in course estimate.

Figure 4(c). Error in speed estimate.

$$GSUMS[3]_k = x_0(k) * P(k) + GSUMS[3]_{k-1}$$

$$GSUMS[0]_k = -kt * GSUMS[1]_k + GSUMS[0]_{k-1}$$

$$GSUMS[2]_k = -kt * GSUMS[3]_k + GSUMS[2]_{k-1} \qquad (45)$$

3. SIMULATION AND RESULTS

Simulator is developed to create target, ownship, and measurements. It is assumed that the initial position of the

Figure 3(c). Error in speed estimate.

ownship is at the origin and bearing is considered wrt true north $0°$ to $360°$, clockwise positive. Target and ownship movements are updated every second. All one-second samples are corrupted by additive zero mean Gaussian noise, as shown in Table 1. The EW receiving system is assumed to be available on ownship and two typical scenarios are considered for evaluation of algorithm. It is also assumed that the bearing measurements are available continuously every second. The bearing measurements are pre-processed over 20 s to reduce the variance. The solution is updated every 20 s. During the updating interval time, the solution will be extrapolated at the rate of 1 s. The ownship is assumed to be doing *'S'* manoeuver on the line-of-sight at a turning rate of $3°$/s, as shown in Fig. 2. When the solution from PLE is stabilised, which is usually after first manoeuver of ownship, it is used as initial estimate for MLE. Then onwards, the estimated state vector is used to find out target motion parameters and to update the various SUMS for further processing. For evaluation, it is assumed that the errors allowed in the estimated target motion parameters are 11 per cent in range estimate, six degrees in course estimate, and twenty per cent in speed estimate. The estimates in Monte-Carlo simulation over 50 runs are shown in Figs 3 and 4. It is observed that range, course, and speed estimates with required accuracies are obtained from around 17th minute onwards.

For conceptual clarity, the target is assumed to be moving at constant velocity. For tracking a manoeuvring target, a fixed number of latest measurements in sliding window can be used to track a manoeuvring target. (Through Monte-Carlo simulation, the author noticed that minimum 15 measurements in sliding window are required for this purpose.)

4. LIMITATIONS OF THE ALGORITHM

The algorithm cannot provide good results when the measurement noise is beyond $3°$ rms. When the target is going away wrt the ownship by $> 60°$, the bearing rate decreases substantially with the increase in number of samples. In such situation, it is very difficult to track the target. In general, these two situations are constraints to any type of filtering technique.

5. CONCLUSIONS

Maximum likelihood estimator is proposed to develop a passive tracking filter to track a ship. In this paper the algorithm developed by Nardone[1], is extended with sequential processing. Here RECURSIVE SUMS are introduced and updated whenever the new bearing measurement is available. The estimated target state vector at any instant is used to calculate the target motion parameters at that instant and to update the SUMS for improved solution at the next instant. PLE outputs are used for the initialisation of MLE.

From the simulation, it is observed that the results are satisfactory. Hence, MLE is recommended as an effective approach for bearings-only passive target tracking.

REFERENCES

1. Nardone, S.C.; Lindgren, A.G. & Gong, K.F. Fundamental properties and performance of conventional bearings-only target motion analysis. *IEEE Trans. Autom. Contr.*, 1984, **29**(9), 775-87.
2. Ristic, Branko; Arulapalam, Sanjeev & Gordon, Niel. Beyond the Kalman filter-particle filters for tracking applications. Artech House, USA, 2004.
3. Rao, S.K. Pseudo linear estimator for bearings-only passive target tracking. *IEE Proc. Radar Sonar Navig.*, 2001, **148**(1), 16-22.
4. Clark, J.M.C.; Vinter, R.B. & Yaqoob, M.M. The shifted Rayleigh filter for bearings-only tracking. *In* International Conference on Information Fusion, 2005, Vol. 1. pp. 1-8.
5. Aidala, V.J. & Hammel, S.E. Utilisation of modified polar coordinates for bearings-only tracking. *IEEE Trans. Autom. Contr.*, 1983, **28**(3), 283-94.
6. Song, T.L. & Speyer, J.L. A stochastic analysis of a modified gain extended Kalman filter with applications to estimation with bearing-only measurements. *IEEE Trans. Autom. Cont.*, 1985, **30**(10), 940-49.
7. Grossman, W.; Bearings only tracking: A hybrid coordinate system approach. *Journal of Guidance*, 1994, **17**(3), 451-57.
8. Rao, S.K.; Modified gain extended Kalman filter with application to bearings-only passive maneuvering target tracking. *IEE Proc. Radar Sonar Navig.*, 2005, **152**(4), 239-44.

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