Defence Science Journal, Vol. 60, No. 1, January 2010, pp. 106-111 © 2010, DESIDOC

# Flexural Vibration Characteristics of Initially Stressed Composite Plates

Rupesh Daripa and M.K. Singha\*

Indian Institute of Technology Delhi, New Delhi–110 016 \*E-mail: maloy@am.iitd.ac.in

## ABSTRACT

The influence of localised in-plane load on the flexural vibration characteristics of isotropic and composite plates have been studied using a four-noded shear flexible high precision plate bending finite element. First, the critical buckling loads of such plates subjected to partial or concentrated compressive loads were calculated, then the linear and nonlinear flexural vibration frequencies were obtained. Limited parametric study was carried out to study the influences of location and distribution of tensile or compressive in-plane load on the vibration frequencies of such plates.

Keywords: Composite plate, partial in-plane load, flexural vibration, nonlinear frequency, loads, stress

# 1. INTRODUCTION

Flexural vibration characteristics of composite plates have attracted the attention of many investigators in the last few decades<sup>1-2</sup>. It has been observed from the existing literature that studies on the vibration characteristics of such plates under concentrated or partial in-plane load are limited, even though such plates find wide application in the thin-walled structural components of different industries.

Leissa and Ayoub<sup>3</sup> used Ritz method, Kaldas and Dickinson<sup>4</sup> applied Raleigh-Ritz method, Kukla and Skalmierski<sup>5</sup> employed power series method, and Gutierrez and Laura<sup>6</sup> used differential quadrature method to understand the vibration characteristics of isotropic rectangular plates subjected to non-uniform loading. Srivastava<sup>7</sup>, *et al.* and Chakrabarty and sheikh<sup>8</sup> employed finite element method to study the linear vibration frequencies of stiffened isotropic, composite, and sandwich plates subjected to in-plane partial edge loads. Recently, Cheung<sup>9</sup>, *et al.* used finite strip method. Chen<sup>10</sup>, *et al.* and Chen and Fung<sup>11</sup> used Runge-Kutta method. and Chen and Doong<sup>12</sup> used Galerkin's method, to study nonlinear vibration characteristics of isotropic and laminated composite plates subjected to initial in-plane stress.

In the present work, a four-noded shear flexible quadrilateral high precision plate bending element developed for the stability analysis<sup>13</sup> has been extended to study the vibration characteristics of isotropic and composite plates under concentrated or partial in-plane load. A complete cubic polynomial shape function was used to interpolate in-plane displacements for better accuracy in capturing the non-uniform stresses near localised load. The nonlinear matrix-amplitude equation<sup>14</sup> has been solved by direct iteration technique to obtain linear and nonlinear vibration frequencies of simply supported square composite plates subjected to tensile or compressive partial in-plane load.

# 2. FINITE ELEMENT FORMULATIONS

The displacement components at a generic point (x, y, z) of a shear deformable quadrilateral plate<sup>13</sup> can be expressed as:

$$u(x, y, z) = u_0(x, y) + z\{-w_{,x} + \gamma_{xz}(x, y)\}$$
  

$$v(x, y, z) = v_0(x, y) + z\{-w_{,y} + \gamma_{yz}(x, y)\}$$
(1)  

$$w(x, y, z) = w_0(x, y)$$

Here,  $u_{0'} v_{0'} w_{0}$  are the mid-surface displacements;  $\gamma_{xz}$  and  $\gamma_{yz}$  are the rotations due to shear; (),  $_x$  and (),  $_y$  represent the partial differentiation wrt x and y;  $\phi_x = -w_{,x} + \gamma_{xz}(x, y)$  and  $\phi_y = -w_{,y} + \gamma_{yz}(x, y)$  are the nodal rotations.

Following the von Karman strain-displacement relation, the in-plane and shear strains can be written as:

$$\begin{cases} \varepsilon_{x} \\ \varepsilon_{y} \\ \varepsilon_{xy} \end{cases} = \begin{cases} u_{o,x} \\ v_{o,y} \\ v_{o,x} + u_{o,y} \end{cases} + \frac{1}{2} \begin{cases} w^{2}, \\ w^{2}, \\ 2w, \\ xw_{y} \end{cases} + \begin{cases} -W, \\ w, \\ wy + y_{yz,y} \\ -2.W, \\ xy + y_{yz,x} + y_{xz,y} \end{cases}$$
$$= \{\varepsilon_{m}\} + z \{\varepsilon_{b}\} \end{cases}$$

$$(2(a))$$

and 
$$\begin{cases} \gamma_{xz} \\ \gamma_{yz} \end{cases} = \begin{cases} \gamma_x \\ \gamma_y \end{cases}$$

The membrane stress resultants  $\{N\}$ , bending stress resultants  $\{M\}$  and shear stress resultants  $\{Q\}$  are expressed as:

(2(b))

$$\{N\} = \{N_{xx,}N_{yy,}N_{xy,}\}^{\mathrm{T}} = [A_{ij}]\{\varepsilon_{m}\} + [B_{ij}]\{\varepsilon_{b}\}$$

$$\{M\} = \{M_{xx,}M_{yy,}M_{xy,}\}^{\mathrm{T}} = [B_{ij}]\{\varepsilon_{m}\} + [D_{ij}]\{\varepsilon_{b}\}$$

$$\{O\} = [S]\{\gamma\}$$

$$(3)$$

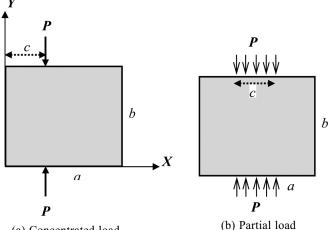
where, [A], [B], [D], and [S] are extensional, extension-

Revised 19 February 2009, Revised 14 July 2009

bending, bending, and shear stiffness coefficients, respectively. For a composite laminate of thickness h, comprising of Nlayers with stacking angles  $\theta_i$  (*i* = 1, 2, ..., *N*) and layer thicknesses  $h_i$  (i = 1, 2, ..., N), the necessary expressions to compute the stiffness coefficients are given by Jones<sup>15</sup>.

A four-noded rectangular high precision plate bending element<sup>13</sup> with the following complete cubic polynomial shape functions for the in-plane and lateral displacements  $(u_{\rho}, v_{\rho}, w_{\rho})$  and linear polynomial shape functions for the shear strains  $(\gamma_{xz}, \gamma_{yz})$  is employed here:

$$\begin{split} & u_0 = [1, x, y, x^2, xy, y^2, x^3, x^2y, xy^2, y^3, x^3y, x^2y^2, xy^3, x^3y^2, x^2y^3, \\ & x^3y^3] \ \{c_i\}, \ i = 1, \ 16 \\ & v_0 = [1, x, y, x^2, xy, y^2, x^3, x^2y, xy^2, y^3, x^3y, x^2y^2, xy^3, x^3y^2, x^2y^3, \\ & x^3y^3] \ \{c_i\}, \ i = 17, \ 32 \\ & w_0 = [1, x, y, x^2, xy, y^2, x^3, x^2y, xy^2, y^3, x^3y, x^2y^2, xy^3, x^3y^2, x^2y^3, \\ & x^3y^3] \ \{c_i\}, \ i = 33, \ 48 \\ & \gamma_{xz} = [1, x, y, xy] \ \{c_i\}, \ i = 49, \ 52 \\ & \chi = [1, x, y, xy] \ \{c_i\}, \ i = 53, \ 56 \end{split}$$



(a) Concentrated load

Figure 1. A rectangular plate of size  $a \times b$  is under a pair of in-plane load P: (a) acting at a distance c from the left edge and (b) uniformly distributed over a partial edge-length 'c' with a pressure P/c per unit length.

where, c are constants and are expressed in terms of nodal displacements  $(u_{0,i}, u_{0,x'}, u_{0,y_i}, v_{0'}, v_{0,x_i}, v_{0,y_i}, v_{0,xy_i}, w, w_{x_i}, w_{y_i}, w_{xy_i}, \gamma_{xz}$  and  $\gamma_{y_2}$ ) in the finite element discretisation. Following standard procedure (minimisation of potential energy), the equation of equilibrium of the plate subjected to in-plane load can be written as:

$$\begin{bmatrix} K_L + \frac{1}{2}N_1(\delta) + \frac{1}{3}N_2(\delta,\delta) \end{bmatrix} \{\delta\} + [M] \{\ddot{\delta}\} = \{F\} \quad (5(a))$$

or 
$$\left[K_L + \frac{1}{2}N_1(\delta) + \frac{1}{3}N_2(\delta,\delta) + \lambda K_G\right] \{\delta\} + [M] \{\ddot{\delta}\} = \{0\}$$
 (5(b))

where,  $K_L$  and M are linear stiffness and mass matrices,  $N_i$  and  $N_i$  are nonlinear stiffness matrices,  $K_G$  is the geometric stiffness matrix due to unit in-plane load,  $\{\delta\}$  is the vector of nodal displacements,  $\{F\}$  is the load vector, and  $\lambda$  is the load parameter.

#### 3. SOLUTION PROCEDURE

The vibration characteristics of composite plates under partial or concentrated inplane load have been studied. The procedures for calculating the critical buckling load and linear/nonlinear frequencies are described.

Step 1-Buckling Analysis: Initially pre-buckling displacements are calculated by linear analysis  $[K_r] \{ \delta \} = \{F\}$  under unit in-plane load P (Fig. 1). Thereafter, the pre-buckling stress resultants  $(N_{xy}, N_{yy}, N_{xy})$  and the corresponding geometric stiffness matrix  $K_{g}$  are calculated. The critical buckling load  $(P_{a})$ , at which Euler type of bifurcation occurs, is obtained from the following eigenvalue equation problem:

$$[K_L + \lambda K_G]\{\delta\} = \{0\} \tag{6}$$

Step 2-Vibration analysis: Assuming a harmonic solution of the form  $\{\delta\} = \{\delta_{\max}\} \sin \omega t$  to the differential eqn 5(b) and following the solution procedure as outlined by Singha and Daripa<sup>14</sup>, the following matrix amplitude equation is obtained:

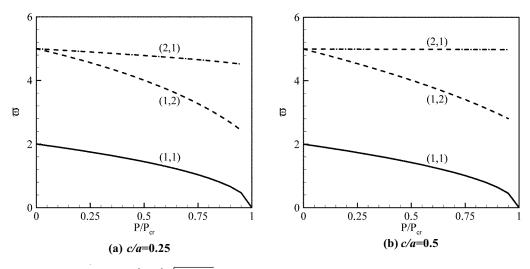


Figure 2. Frequency parameter ( $\omega = \omega a^2 / \pi^2 \sqrt{\rho h/D}$ ) versus in-plane concentrated edge load P, for thin square isotropic plates (a/ *h*=100). Different modes are shown.

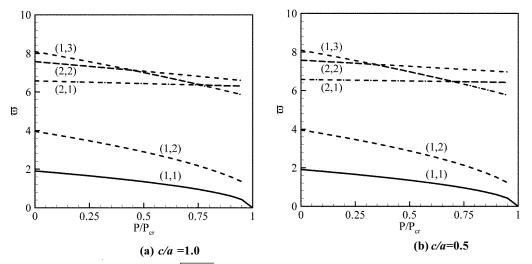


Figure 3. Frequency parameter  $(\varpi = \omega a^2 / \pi^2 h \sqrt{\rho / E_T})$  versus compressive in-plane load *P* distributed over a partial edge-length *c* at the middle for thin square cross-ply  $[0^{\circ}/90^{\circ}/0^{\circ}]$  plates (a/h=100). Different modes are shown.

$$\begin{bmatrix} K_L + \frac{4}{3\pi} N_1(\delta_{\max}) + \frac{1}{4} N_2(\delta_{\max}, \delta_{\max}) + \lambda K_G \end{bmatrix}$$

$$\{\delta_{\max}\} - \omega^2 [M] \{\delta_{\max}\} = \{0\}$$
(7)

Equation (7) was solved iteratively for different values of in-plane load parameter ( $\lambda = P/P_{cr}$ ) to obtain the nonlinear frequency ( $\omega_{NL}$ ) and corresponding mode shape { $\delta_{max}$ }. Thereafter, the governing Eqn 5 (a) was solved by Newmark's time integration technique starting from the initial conditions ({ $\delta$ } = { $\delta_{max}$ } = at t = T/4) to investigate the validity of numerical results obtained from matrix-amplitude Eqn (7).

### 4. RESULTS AND DISCUSSION

Vibration characteristics of simply supported square plates under a pair of compressive in-plane load P are studied. Total in-plane compressive load P is either assumed to be concentrated at a distance c from the left edge or uniformly distributed at the middle of the plate over a partial edge-length c with intensity of pressure P/c per unit length (Fig. 1). The material properties, unless specified otherwise, used in the present analysis were:

 $E_L/E_T = 40.0$ ,  $G_{LT}/E_T = 0.6$ ,  $G_{TT}/E_T = 0.5$ ,  $v_{LT} = 0.25$ ,  $E_T = 1.0$ where, *E*, *G*, and *v* are Young's modulus, shear modulus and Poisson's ratio, respectively. Subscripts *L* and *T* represent the longitudinal and transverse directions, respectively

Table 1. Convergence study of non-dimensional linear frequencies ( $\varpi = \omega a^2 / \pi^2 h \sqrt{\rho / E_T}$ ) of 5-layered [0°/90°/0°/90°/0°] square composite plates (*a/b*=1, *a/h* = 100).

		Mesh			Modes		
		size	1	2	3	4 5	56
		4 × 4	1.91450	3.98667	6.68114	7.68393	8.28908
Simply	Present	6 × 6	1.91413	3.97688	6.66102	7.66131	8.18025
support	study	8 × 8	1.91407	3.97516	6.65750	7.65736	8.16008
	Wang <sup>16</sup>		1.91410	3.97450	6.65670	7.65640	8.15110

Table 2. Convergence study of non-dimensional linear frequencies ( $\omega = \omega a^2 \sqrt{\rho h/D}$ ) of a simply supported square isotropic plate subjected to in-plane concentrated loads (a/b=1, a/h = 100)

	Mess division				
c/a		-1	-0.5	0	0.5
	4×4	26.45861	23.53274	19.73466	14.34803
0.25	8×8	26.40004	23.50172	19.73219	14.37196
	Srivastava <sup>7</sup> , et al.	26.41000	23.54000	19.73000	14.37000
	4×4	27.74434	24.08852	19.73466	14.02267
0.50	8×8	27.72982	24.07947	19.73219	14.75507
	Srivastava <sup>7</sup> , et al.	27.69000	24.06000	19.73000	14.04000

w/h		0.2	0.4	0.6	0.8	1.0
	6×6	1.0330	1.1269	1.2701	1.4502	1.6568
	$8 \times 8$	1.0329	1.1268	1.2701	1.4503	1.6572
Simply supported	10×10	1.0329	1.1268	1.2701	1.4503	1.6572
	Bhimaraddi <sup>17</sup>	1.0290	1.1250	1.2780	1.4650	1.6710

Table 3. Comparison of nonlinear frequency ratio ( $\omega_{NL}/\omega_{L}$ ) of simply supported (SS-1) symmetric cross-ply  $[0^{\circ}/90^{\circ}/0^{\circ}/90^{\circ}/0^{\circ}]$  laminated square plate (a=b, a/h = 10)

wrt the fibres. All the layers are of equal thickness. Fibre orientation is measured from X axis. The simply supported boundary conditions considered here are

Immovable edge (SS-1):  $u_0 = v_0 = w = 0$  at x = 0, a and y = 0, b

Movable edge (SS-2): w = 0 at x = 0, a and y = 0, b $u_0 = 0$  at x = a/2:  $v_0 = 0$  at y = b/2

Before proceeding for the detailed study, the efficacy of the present finite element formulation was tested by carrying out the convergence study for non-dimensional linear free vibration frequencies ( $\varpi = \omega a^2 / \pi^2 h \sqrt{\rho/E_T}$ ) of simply supported (SS-1) square cross-ply composite plates (a/h=1000) in Table 1 and results are compared with the results of Wang<sup>16</sup>. The efficiency of the present finite element in the buckling analysis of rectangular plates under partial in-plane load is established by Daripa and Singha<sup>13</sup> and the same is not repeated here for the sake of brevity. Next, the non-dimensional linear vibration frequencies  $(\varpi = \omega a^2 \sqrt{\rho h/D}, D = Eh^3/12(1-\upsilon^2)$ , Poisson's ratio  $(\upsilon = 0.3)$  of simply supported isotropic square plate under concentrated edge load *P* applied at a distance c (= 0.5a,0.25a) from the left edge are presented in Table 2 along with the available solutions of Srivastava<sup>17</sup>, *et al.* and these match very well. Further, an 8 × 8 mesh is found to be adequate to idealise the full plate.

Table 4. Nonlinear vibration frequencies of square plates subjected to in-plane partial edge loads with moving edges (*a/b*=1, a/h = 100). ( $\varpi = \omega a^2 \sqrt{\rho h/D}$ ) for isotropic plate and ( $\varpi = \omega a^2 / \pi^2 h \sqrt{\rho/E_T}$ ) for composite plate

w/h									
c/a	$P/P_{cr}$	0	0.2	0.4	0.6	0.8	1	1.2	
Isotrop	ic								
	0	1.99928	2.00659	2.02830	2.06383	2.11231	2.17265	2.24366	
	0.25	1.73284	1.74123	1.76608	1.80659	1.86154	1.92947	2.00880	
	-0.25	2.23375	2.24032	2.25984	2.29192	2.33586	2.39061	2.45569	
0.5	0.5	1.41626	1.42644	1.45652	1.50516	1.57039	1.65001	1.74176	
	-0.5	2.44553	2.45155	2.46948	2.49897	2.53944	2.59024	2.65022	
	0.25	1.73144	1.73985	1.76484	1.80554	1.86072	1.92884	2.00879	
	-0.25	2.23527	2.24180	2.26124	2.29319	2.29319	2.39167	2.45641	
1.0	0.5	1.41371	1.42402	1.45441	1.50353	1.56932	1.64952	1.74179	
	-0.5	2.44861	2.45459	2.47237	2.50160	2.54181	2.59223	2.65214	
Cross-p	ly [0º/90º/0º/9	90º/0º]							
	0	1.90872	1.91452	1.93185	1.96039	1.99969	2.04913	2.10801	
	0.25	1.65430	1.66098	1.68087	1.71349	1.75819	1.81406	1.88015	
	-0.25	2.13231	2.13753	2.15312	2.17886	2.21440	2.25931	2.31304	
	0.5	1.35177	1.35992	1.38408	1.42345	1.47684	1.54280	1.61983	
0.5	-0.5	2.33394	2.33872	2.35305	2.37674	2.40954	2.45103	2.50086	
	0.25	1.65300	1.65969	1.67965	1.71239	1.75722	1.81327	1.87954	
	-0.25	2.13401	2.13921	2.15473	2.18036	2.21576	2.26049	2.31404	
1.0	0.5	1.34967	1.35786	1.38217	1.42177	1.47545	1.54176	1.61915	
	-0.5	2.33769	2.34243	2.35663	2.38009	2.41257	2.45373	2.50315	
Angle-p	ly [45°/-45°/4	5°/-45°/45°]							
	0	2.42637	2.43034	2.44221	2.46175	2.48863	2.52248	2.56278	
	0.25	2.11873	2.12322	2.13663	2.15869	2.18897	2.22698	2.27213	
	-0.25	2.69552	2.69915	2.70997	2.72781	2.75240	2.78339	2.82040	
0.5	0.5	1.74985	1.75525	1.77130	1.79764	1.83365	1.87860	1.93169	
	-0.5	2.93754	2.94091	2.95099	2.96759	2.99050	3.01941	3.05398	
	0.25	2.10221	2.10685	2.12067	2.14337	2.17446	2.21340	2.25954	
	-0.25	2.71174	2.71526	2.72579	2.74317	2.76714	2.79740	2.83361	
1.0	0.5	1.71731	1.72306	1.74015	1.76806	1.80601	1.85311	1.90841	
	-0.5	2.96956	2.97276	2.98229	2.99807	2.99807	3.04748	3.08059	

Celebrating Sixty Years of Publication

Next, the variation of non-dimensional linear frequencies  $(\varpi = \omega a^2 / \pi^2 \sqrt{\rho h/D})$  of simply supported (SS-2) isotropic square plates under applied concentrated edge load P at a distance c = 0.5a, 0.25a from the left edge are presented in Fig. 2. It can be observed that frequency parameters decrease with increasing compressive edge load and finally become zero at the corresponding critical buckling load. Frequency parameters of higher modes are also decreasing with increasing compressive edge load (Fig. 2). Thereafter, the variation of frequency parameters  $(\varpi = \omega a^2 / \pi^2 h \sqrt{\rho / E_T})$  of a crossply  $[0^{\circ}/90^{\circ}/0^{\circ}/0^{\circ}]$  thin square (a/h = 100) laminates under partial in-plane edge load is presented in Fig. 3. Similar to the isotropic case, frequency parameters decrease with increasing compressive partial in-plane edge load and become zero at corresponding critical buckling load. Shifting of higher modes can be observed from Fig. 3.

Now, before calculating the nonlinear frequencies of laminated composite plates under partial in-plane load, nonlinear frequency ratios ( $\omega_{NL}/\omega_{L}$ ;  $\omega_{L}$  is the linear frequency) of simply supported composite plates (a/h = 10) with immovable edge are calculated with the present element and compared with Bhimaraddi<sup>17</sup> in Table 3. Material properties for this calculation are taken from the same reference. Good agreement is observed. Thereafter, nonlinear vibration frequencies of square isotropic, cross-ply [0°/90°/0°/90°/0°] and angle-ply [45°/-45°/45°/-45°/45°] laminated composite plates (a/h=100) under partial in-plane load (c/a=0.5,1.0)at the centre are investigated in Table 4. Simply supported plate with movable in-plane boundary condition (SS-2) is considered here. Nonlinear vibration frequencies of the same plate without in-plane load are also shown in the table for comparison. It is observed that the linear vibration frequency decreases with the increase of compressive inplane load, whereas it increases with the increase of tensile (negative P) in-plane load. The nonlinear frequencies always increase with the increase of vibration amplitude (w/h).

However, the degree of hardening nonlinearity decreases with the increase of intensity of compressive in-plane edge load as well as its partial edge length. However, tensile load has reverse effect, as evident from (Table 4).

Thereafter, the governing equation [Eqn 5(b)] is solved by Newmark's time integration technique starting from the initial conditions ({ $\delta$ } = { $\delta_{max}$ } =at *t*=*T*/4) and the dynamic response of transverse displacement ( $w_c/h$ ) at the centre for a simply supported isotropic plate is presented in Fig.4 for different in-plane load parameters ( $P = 0, 0.25P_{cr}, 0.5P_{cr}$ and  $0.75P_{cr}$ ). From the response, it is observed that the response is steady-state and further, the nonlinear time period ( $T_{NL}$ ) obtained from the matrix-amplitude [Eqn (7)] and the dynamic response analysis are the same when the in-plane load is less than the critical buckling load  $P_{cr}$ .

## 4. CONCLUSIONS

Large amplitude flexural vibration characteristics of isotropic and composite plates subjected to localised inplane load are investigated using a high precision shear flexible plate bending element. The influence of magnitude and distribution of tensile or compressive in-plane load on the linear and nonlinear free vibration frequencies of isotropic and composite square plates are investigated in detail. It is observed that, the nonlinear frequencies always increase with the increase of vibration amplitude (w/h). However, the degree of hardening nonlinearity decreases with the increase of intensity of compressive in-plane edge load as well as its partial edge length. The results may be helpful for the designers working in the area of composite structures.

### REFERENCES

1. Kapania, R.K. & Raciti, S. Recent advances in analysis of laminated beams and plates, Part II: Vibration and wave propagation. *AIAA Journal*, 1989, **27**, 935-46.

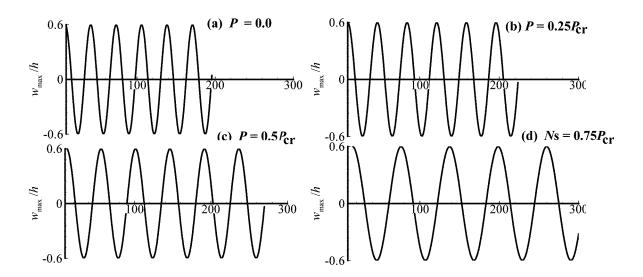


Figure 4. Time history of an isotropic plate under compressive in-plane load.

- 2. Sathyamoorthy, M. Nonlinear vibrations of plates: An update of recent research developments. ASME Appl. Mech. Rev., 1996, **49**, S55-S62.
- Leissa, A.W. & Ayoub, E.F. Vibration and buckling of simply supported rectangular plate subjected to a pair of in-plane concentrated forces. J. Sound Vib., 1988, 127(1) 155–71.
- 4. Kaldas, M.M. & Dickinson, S.M. Vibration and buckling calculation for rectangular plates subjected to complicated in-plane stress distribution by using numerical integration in a Rayleigh–Ritz analysis. *J. Sound Vib.*, 1981, **75**, 151–62.
- 5. Kukla, S. & Skalmierski, B. Free vibration of rectangular plate loaded by a non-uniform in plane force. *J. Sound Vib.*, 1995, **187**(2), 339–43.
- 6. Gutierrez, R.H. & Laura, P.A.A. Use of differential quadrature method when dealing with transverse vibration of a non-uniform stress distribution field. *J. Sound Vib.*, 1999, **220**(4),765–69.
- Srivastava, A.K.L.; Datta, P.K. & Sheikh, A.H. Buckling and vibration of stiffened plates subjected to partial edge loading. *Int. J. Mech. Sci.*, 2003, 45, 73-93.
- 8. Chakrabarti, A. & Sheikh, A.H. Vibration of imperfect composite and sandwich laminates with in-plane partial edge load. *Composite Structures*, 2005, **71**, 199-09.
- 9. Cheung, Y.K.; Zhu, D.S. & Iu V.P. Nonlinear vibration of thin plates with initial stress by spline finite strip method. *Thin-Walled Struct.*, 1998, **32**, 275–87.
- 10. Chen, C.S.; Hwang, J.R. & Doong, J.L. Large amplitude vibration of plates according to a modify higher order deformation. *Int J. Solids Struct.*, 2001, **38**, 8563–583.
- 11. Chen, C.S. & Fung, C.P. Nonlinear vibration of an initially stressed hybrid composite plates. J. Sound Vib., 2004, 74(3-5), 1013-029.
- 12. Chen, L.W. & Doong, J.L. Large amplitude vibration of an initially stressed moderately thick plate. *J. Sound*

*Vib.*, 1983, **89**, 499–08.

- Daripa, R. & Singha, M.K. Stability analysis of Composite Plates under localized in-plane load. *Thin-walled Struct.*, 2009, 47(5), 601-06.
- Singha, M.K. & Daripa, R. Nonlinear Vibration of symmetrically laminated composite skew plates by finite element method. *Int. J. Non-linear Mech.*, 2007, 42, 1144-152.
- 15. Jones, R.M. Mechanics of composite materials. McGraw-Hill, New York, 1975.
- 16. Wang, S. Vibration of thin skew fiber reinforced composite laminates. *J. Sound Vib.*, 1997, **201**, 335-52.
- 17. Bhimaraddi, A. Large amplitude vibrations of imperfect antisymmetric angle-ply laminated plates. J. Sound Vib., 1993, 162(3), 457–70.

# Contributors



**Mr Rupesh Daripa** obtained his BE (Mechanical Engineering) and ME (Engineering Mechanics) from Bengal Engineering College Shibpur in 2002 and 2004 respectively. Thereafter, he worked as Senior Research Fellow in NIOH, Kolkata for one year. He is currently pursuing PhD in solid mechanics in the

Department of Applied Mechanics, IIT Delhi.



**Dr M.K. Singha** obtained his PhD from IIT, Kharagpur in 2002. He is currently working as Assistant Professor in the Department of Applied Mechanics, IIT, Delhi. His areas of interest are stability and dynamics of composite structures, FGM panels, and aero-thermo-elasticity.