205

# A GENERALIZED THREE-PARAMETER BIAXIAL STRENGTH CRITERION FOR CONCRETE

BY

DAVID L. KITTERMAN

B.S., Kansas State University, 1980

---

A MASTER'S THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Mechanical Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1985

Approved by:

C. L. D. Huang
Major Professor

ACKNOWLEDGEMENTS

It is the wish of this investigator to extend his deepest gratitude to the instructors and friends, who through their undaunting efforts have guided me to this stage in my education. A special gratitude is extended to Dr. C.L.D. Huang for his guidance, dedication, patience, and relentless efforts throughout the course of this study, for without his wisdom this investigation could not have been accomplished. Finally, appreciation is expressed to the Department of Mechanical Engineering and Dr. Paul Miller especially for financial and moral support.

TABLE OF CONTENTS

Chapter Page

Introduction
Purpose of Research. . . . . . . . . . . . . . . . . . . . 1

I. Material Characteristics and Experimental Data
Literature Review. . . . . . . . . . . . . . . . . . 6
The Composition of Concrete. . . . . . . . . . . . . . . . 6
The Mechanical Behavior of Concrete. . . . . . . . . . . . 9
Influences of Test Procedure . . . . . . . . . . . . . . . 11
Fracture and Failure of Concrete . . . . . . . . . . . . . 14
Biaxial Test Data for Concrete . . . . . . . . . . . . . . 16
Figures 1.1 through 1.20 . . . . . . . . . . . . . . . . . 24

II. Material Strength Criteria-Literature Review . . . . . . . 34

Introduction . . . . . . . . . . . . . . . . . . . . . . . 34
Classification of Concrete . . . . . . . . . . . . . . . . 37
General Macroscopic Strength Criterion . . . . . . . . . . 38
Concrete Specific Strength Criterion . . . . . . . . . . . 45
Conclusion . . . . . . . . . . . . . . . . . . . . . . . . 52
Figures 2.1 through 2.9. . . . . . . . . . . . . . . . . . 54

III. A Proposed Strength Criterion for Concrete . . . . . . . . 59

Introduction . . . . . . . . . . . . . . . . . . . . . . . 59
Development of the Proposed Strength Criterion . . . . . . 61
Characterization of the Proposed Strength Criterion. . . . 68
Comparison of Proposed to Cubic Strength Criterion . . . . 72
Tables 3.1 through 3.3 . . . . . . . . . . . . . . . . . . 74
Figures 3.1 through 3.21 . . . . . . . . . . . . . . . . . 77

IV. Summary and Conclusions. . . . . . . . . . . . . . . . . . 91

Bibliography . . . . . . . . . . . . . . . . . . . . . . . . . 94

Appendix . . . . . . . . . . . . . . . . . . . . . . . . . . .102

Abstract

Vita

# INTRODUCTION

## PURPOSE OF RESEARCH

Concrete has proven to be one of the most versatile building materials available to date. The high compressive strength of concrete and its ability to cast into almost any shape or size has inspired new areas of design avenues. Concrete is primarily used as a bearing material within construction, although modern designers have used it in new applications such as; pressure vessels for nuclear reactors, undersea vessels for mineral oil storage, off-shore platforms for oil drilling and production, and even ship's hulls. This wider range of applications can be traced to improved methods of dealing with tensile stresses within concrete structures. Steel reinforcement and prestressing methods have allowed designers to incorporate tensile loads, where as prior to these methods concrete was normally used in designs involving only compressive loads.

A paradox arises when considering concrete, despite its time honored and widespread use there is still much that is unknown as to the nature and behavior of concrete when consideration is given to mathematical characterization. Significant research into the study of concrete has only been achieved in the past twenty-five years or so. Research by cement technologists and chemists have established a generally accepted picture of the composition and internal structure of hardened cement paste. The knowledge of the internal structure of any material is essential if its complex mechanical behavior at the engineering level is to be properly understood. These improvements in understanding of concrete structure have provided fertile grounds for development of theories of elastic and inelastic behavior.

Past design methods, when using concrete, have generally involved very simplified idealizations of the material behavior and include large factors of safety. These methods depended on experience which was incorporated into design codes, rules, and recommendations. An exact detailed mathematical description of the materials behavior was in most cases of no practical relevance, as a vehicle for its use was unavailable. This is no longer true with the advent of computer aided design methods, such as finite element analysis. In addition the recent introductions of high strength concretes of 10,000 PSI or more compressive strength capabilities, has changed the design problems. Structural engineers are now considering complex designs involving extreme dimensions and higher safety standards using unusual or severe load conditions. The use of numerical analysis through the finite element method is the most desirable mode of structural analysis under these increasingly demanding restraints.

Computer aided design methods have lagged behind in concrete structural analysis when compared to other well characterized materials. The lack of or inadequacies in descriptive material models, especially for brittle materials such as concrete, have limited the application of the finite element method to concrete structural design. There are additional reasons for lack of widespread use such as the degree of homogeneity to adopt, the type of element to be used, the cohesion factor between concrete and steel, and especially the numerical solution procedure of the resulting non-linear set of equations. Mathematical solutions to these problems have been a continuing area of much needed research.

The inadequacies of present material models for concrete can be traced to its complex, quasi-linear and under certain conditions quasi-plastic states of mechanical response. Thus formation of a mathematically tractable function has been hindered by the very nature of concrete. In an effort to resolve the material characterization problems associated with concrete this investigation focuses on the ultimate strength prediction. The development of such a strength criterion based on sound mathematical procedures and proven material characteristics would be most valuable. Development of such a criterion will greatly enhance computer aided design procedures as well as classical design methods. An improved strength criterion is a powerful tool which will advance the accuracy and safety of modern structures designed using concrete.

The development of any multiaxial strength criterion is best proceeded by experimental testing of the material under combination loads. The experimental results are used to guide development and validate the proposed strength criterion. The multiaxial stress space formed under biaxial load conditions is the primary concern of this investigation. An accurate strength criterion for concrete under biaxial states of stress is required in many design instances such as, the shell of a nuclear power plant or material storage tanks. There have been many experimental results published for the failure of concrete under biaxial states of stress. These results will be reviewed and the most accurate selected for validation of the proposed strength criterion.

The increase in popularity of composite materials has stimulated growth in the area of material characterization. The developments of descriptive strength criteria for anisotropic brittle materials has been accelerated by the increasing popularity of designing systems using fiber-reinforced composite materials. In addition advances in the theories of non-linear continuum mechanics have proven highly applicable in this instance. The tensor function theory approach to the development of descriptive constitutive equations and subsequent strength criteria has proven its capabilities in these areas. These general theories will be investigated and their application to concrete expanded and evaluated.

The application of the tensor function theory approach to the development of a strength function for concrete is followed in this investigation. Concrete is idealized as a composite material adhering to modern material theory. The average response of the individual components are taken to indicate the total response of the material system. Thus the macroscopic approach is adopted and the material is characterized as a homogeneous, isotropic continuous media. The composite nature justifies using the powerful general strength theories often recommended for usage with more exotic materials.

The objective of this investigation is to develop a simple, rational, multiaxial strength criterion for plain concrete at isothermal, static, monotonically increasing load conditions. The proposed strength criterion developed is a general, three-parameter, unified equation capable of predicting failure of concrete under biaxial loading conditions. The proposed criterion satisfies the invariant requirements of coordinate transformation by adhering to the laws of continuum mechanics. The strength function is easily characterized to the material quality through three simple engineering strength tests.

The proposed criterion will be graphically compared and verified against experimental results under biaxial load conditions. In addition the newly proposed criterion is compared to past theories and proven to be superior. The results are highly useful as concrete will remain the major structural material of choice owing to its outstanding technological and economic qualities.

CHAPTER ONE

# MATERIAL CHARACTERISTICS AND EXPERIMENTAL DATA

## Literature Review

### The Composition of Concrete

The heterogeneous nature of concrete creates a complex material which is difficult to characterize. Concrete is a multi-phase material normally classified as a composite. A few of the components in concrete being; course aggregate, sand, unhydrated cement particles, cement gel, capillary and gel pores, pore water, admixtures, and accidentally or deliberately entrapped air voids. Plain concrete is a combination of these components formed into a uniform, evenly distributed mixture. This mixture is cured while being contained in a form, until a minimum strength is obtained. The ultimate strength is very dependent on the total process from initial mixing to the final cure.

The proportions of the components which form the initial concrete mix are commonly expressed in terms of the specific volume or volume fraction:

$$V_c + V_w + V_s + V_{ca} + V_v = 1 \tag{1.1}$$

where, V, represents the relative volume fractions of the subscripts which are: c is unhydrated cement, w is initial water content, s is sand, ca is coarse aggregate, and v is entrapped air. Equation (1.1) is not all inclusive as other components are added from time to time.

also:

$$V_c = \left(\frac{M_c}{G_c}\right) / \left(\frac{M_c}{G_c} + \frac{M_a}{G_a} + \frac{M_s}{G_s} + \frac{M_{ca}}{G_{ca}} + \frac{M_v}{G_v}\right) \tag{1.2}$$

where G represents specific gravity of the subscripted components.

Concrete undergoes a process known as hydration upon initial mixing. Avram et.al. [5] states that the chemical reaction between the cement mineralogical components and water forms the hydration process. This chemical process proceeds forming a hardened cement paste which binds together the fine and coarse aggregate particles.

The ultimate strength of plain concrete is affected by numerous aspects, i.e. the materials of composition, environmental curing conditions, and mixing proportions. To attain the desired ultimate strength of a particular mix two primary influences must be considered; the initial ratios of the respective parts and the environmental cure conditions. The environmental conditions with the greatest effects on concrete strength variations during hardening are temperature and moisture conditions states Avram et.al. [5].

Newman [67] in order to simplify the overall strength characterization of concrete considers hydrated cement pastes, mortars, and concretes as two-phase materials. Each material is composed of coarse particles uniformly distributed and embedded within a reasonably homogeneous matrix. Thus for concrete the primary strength influencing components are mortar and coarse aggregates. The two-phase model suggested is large compared with the maximum size of the particles imbedded in the matrix. By following this reasoning the behavior of the matrix under load can be expressed in terms of the average stresses and strains of a homogeneous and isotropic material.

The overall properties of the two-phase model are highly dependent on the individual characteristics of the mortar and aggregate. The specific mortar properties are dependent on the mix proportions and cement type used. The ultimate strength of the cement paste is best described by Abrams [1] classic law of:

$$S = \frac{A}{B^x} = \text{strength, PSI} \tag{1.3}$$

where, x, represents the water to cement ratio (w/c), 'A' is given as 14,000 PSI, and 'B' is seven (7). Abram's law states that the strength of cement paste is an inverse function of the water to cement ratio.

The strength of concrete is highly dependent on the classifications of the cement type used. The mineralogical constituents of the cement determine its classification. The main cement used for construction is of the Portland variety. Portland cement is available in five different classifications. The classificaitons define the primary usage for the cement and differ mainly in their compound composition and fineness. A good description may be found in Davis et.al. [19] or Avram et.al. [5].

The aggregates are generally inert filler material which consume approximately 80% of the concrete volume. Aggregates are classified by their source, mode of preparation, and mineralogical composition. Natural aggregates consist of hard, non-weathering, frost resistant rocks. The most frequently used artificial aggregates are blast-furnace slag, agglomerated ashes, ceramic particles, expanded clays, fibers and others. The aggregate used depends on the desired concrete characteristics.

Newman [69] presents the properties of concrete materials in tabular form to demonstrate the differences between hardened cement paste and aggregates. The primary emphasis of his comparison is to demonstrate that the macroscopic characteristics are the average of the individual component characteristics which essential are widely different. The factors affecting the intrinsic properties of concrete systems being dependent on the degree of homogeneity, properties of the cement paste, properties of the aggregates, and properties of the aggregate-paste interface.

The final ultimate strength property of a concrete mix is also influenced by other factors. The environmental conditions during the cure period are highly influencial. The longer the period of moist storage and the higher the temperature (within the $40^{o}$ to $100^{o}$ F range [69]) the greater the strength at any age. The strength of moist concrete increases with age. The hydration process between cement and water may not be completed for 25 years or more, states Newman et.al. [7]. The rate of increase in the strength of concrete gradually decreases, tending towards an asymptotic value when the rate of increase reaches zero.

## The Mechanical Behavior of Concrete

The behavior of concrete under loading represents the complex inner response of the material to an external action. The response is due to a large number of influencial factors and is covered well in Avram et.al. [5]. To summarize, the major mechanical aspects of concrete behavior will be addressed here. The stress-strain relations and ultimate strengths of concrete are dependent on various testing conditions including; specimen size, moisture condition and temperature, the state of stress and strain actually induced in the specimens, and the methods by which they are loaded. In addition recent testing has considered the rate of load application in material response.

The influence of secondary characteristics to the response of concrete has been the subject of research in recent years. The need for adequate constitutive equations for representation of material response under various loading and environmental conditions has prompted inclusion of secondary influences as much as possible. Desai and Siriwardance [20] have reviewed recent progress in constitutive equation development for concrete. In past years much of the experimental testing of concrete was performed under short-term, monotonic, static load conditions. Therefore much of the published data has ignored the secondary influences such as, rate of loading, fatigue factors, variations of temperature, and others.

An example of concrete response during short-term, monotonic loading is given in the stress-strain curve of Figure (1.1). The initial portion of the curves are reasonably straight up to about 40 to 60 percent of the ultimate strength. Plain concretes are non-linear and have no readily identifiable elastic limit. Newman et.al [71] has defined two different 'elastic' constants (see Fig. 1.1) as: (1) the tangent modulus of elasticity given by the slope of 'OB' and, (2) the secant modulus of elasticity given at the stress 'A' by the slope of 'OA'. It has also been noted that the modulus of elasticity is a function of the concrete class and is affected by the same factors as the compressive strength. The American Concrete Institute building code suggests the use of the following equation to determine the modulus of elasticity for use in standard calculations [19]:

$$E_c = W^{1.5}\ 33\sqrt{\sigma_c} \tag{1.4}$$

where, $E_c$, represents the modulus of elasticity in P.S.I., W is the weight of the concrete in $lb/ft^3$, and $\sigma_c$ is the uniaxial compressive strength in P.S.I.

The Poisson's ratio for concrete is variable and depends on the load application rate. During relatively low load rates (< 10 to $10^3$ $lb/in^2$ S

[69]) a static Poisson's ratio is considered. The static Poisson's ratio varies during loading from .14 to almost .5. The dynamic Poisson's ratio (load rates $> 10^4$ lb./in$^2$ S [69]) is generally about 20% higher than the static value.

The recorded response of concrete is highly dependent on the loading method and conditions at test time. Thus the stress-strain curve, ultimate strength, modulus of elasticity, and Poisson's ratio determination are highly dependent on the testing techniques.

Influence of Test Procedure

For the measured values of the mechanical properties of concrete to be considered fundamental and unique they must be obtained from tests which have produced known stress conditions in the specimens, independent of any machine or testing effects. Wastiels [99], Nelissen [66], Avram et.al. [5], and Newman-Newman [71] have extensively reviewed the many problems in testing techniques for concrete. These studies concluded that there are two primary influences which affect the accuracy of the mechanical properties determined from experimental data; the specimen geometry and the load application device. The two influences are not mutually exclusive but are interactive.

The induced stress within the test specimen must be easily determined and uniform in the critical zone, so that the exact stress condition at failure is known. The edge effects caused by the loading mechanism can produce uneven loading with or without shear stresses in the boundary zone. The frictional restraint between the specimen and loading platens caused by the differences in the Poisson's ratio prevent the ends of the specimen from expanding laterally, thus inducing shear stresses in the boundary zone. Nelissen [66] reviewed the various methods used to eliminate the edge effects caused by the loading mechanism. He reached the conclusion that the system proposed by Kupfer, Hilsdorf, and Rusch [57] was the most effective. These investigators developed a brush bearing platen for use as a load application device. A representative sketch of this platen is given in Figure (1.2). These platens are best designed for the particular strength quality of concrete to be tested, as discussed in Nelissen [66]. These concrete load application platens consist of a series of closely spaced small steel rods which are flexible enough to allow lateral deformation without inducing shear stresses near the edge of the specimen. For tensile tests the ends of the bars of the platens can be glued to the concrete.

Another popular testing procedure is the hydrostatic pressure systems which are used to induce stresses into a specimen. The surface of the specimen to be loaded is exposed to a fluid under pressure. Pure, normal stresses result on the specimen surface with no induced shear stresses. The investigators using hydrostatic test procedures must insure that the pressurized fluid does not enter into the specimen pores or high tensile stresses could develop causing premature failure. Typically the surface exposed to the pressurized fluid is covered or coated with an impermeable material.

Atkinson [4] describes a hydrostatic test cell for multi-axial loading of cubic concrete specimens by independently controllable pressure chambers, thus any combination of triaxial compressive stresses is obtainable. This pressure cell was used by Gertsle [31] for triaxial and biaxial concrete testing. The hydrostatic system proposed by Atkinson [4] eliminates the undesirable testing condition of non-parallel loading platens that can induce a non-uniform stress distribution within the specimen.

An inherent restraint of hydrostatic loading systems is their inabilities to induce normal tensile stresses. To overcome this handicap combination mechanical and hydrostatic loading mechanisms have been proposed. Newman [67] describes and uses a combination loading system for concrete specimens.

To further the understanding of edge loading effects on concrete specimens Nelissen [66] studied the stress distribution produced within a square cube specimen by use of the finite element method. The results were inconclusive due to Nelissen's admitted lack of adequate material characterization for concrete. Although he did conclude that the brush bearing platens were superior to solid steel platens for load application.

Herrmann [36] addresses the importance of edge effects on concrete specimens from a statistical point of view. He concluded that due to practical limitations samples of concrete are usually far too small to achieve statistical homogeneity. Therefore considerable statistical scatter is inevitable. Herrmann argued that specimens with zero shear stresses and uniform normal stresses on the edges are lower bounds for the determination of mechanical properties. He further concludes that material property results obtained from uniform normal displacement tests are closer to the actual composite properties than those measured from uniform normal stress tests. Thus the mechanical brush bearing platen test is favored over the hydrostatic testing method.

The specimen geometry being used influences the determination of the stresses induced. The stresses are generally calculated from the measured loading force distribution within the critical zone of the specimen. To obtain accurate stresses the specimen shape should be of a type which allows direct calculation based on linear relationships. Nelissen [66] provides a summary of specimen shapes and analyzes the problems associated with calculation of the stress state in each. He concludes that square cube specimens are the most accurate. He uses these in the development of his experimental data investigation.

The Fracture and Failure of Concrete

Concrete specimens subjected to any state of stress can support loads up to 40-60 percent of ultimate without any apparent signs of distress. As the load continues to be increased, soft but distinct noises of internal disruption can be heard until, at about 70-90 percent of ultimate, small fissures or cracks appear on the surface. These cracks spread and interconnect until, at ultimate load, the specimens fracture into separate pieces [52].

Newman and Kotsovos have undertaken a series of papers which further defines the deformational and strength characteristics of concrete [51, 52, 53, 67, 68, 69]. They and other investigators such as Wastiels [99] suggest four distinct stages in the process of crack initiation and propagation. Kotsovos [52] further categorizes the failure mechanism of concrete as being promoted by two separate stress components; crack growth occurring in the direction of the applied maximum principal compressive stress is caused by the deviatoric component, and crack growth of random orientation is caused by the hydrostatic component. Kotsovos concludes that crack growth caused by the deviatoric component of stress eventually becomes unstable, leading to the ultimate collapse. He states that the hydrostatic components of stress leads to delayed crack growth and never becomes unstable.

An inherent characteristic of concrete is that it contains a proliferation of flaws and microcracks which exist even prior to loading.

These preexisting flaws are randomly oriented and distributed throughout in a range of shapes and sizes. Upon loading, these flaws act as stress concentration points which initiate the final cracks which propagate causing complete material failure. The four stages of concrete crack growth defined and generally agreed to in the literature are as follows:

1) A quasi-elastic behavior occurs up to 30-40 percent of ultimate, during which additional microcracks are forming. The hydrostatic and deviatoric components are operating and causing crack growth.

2) Concrete begins to exhibit a plastic behavior between 45-90 percent of ultimate, during which the microcracks begin to branch. This period is known as Localized Stable Crack Growth. The hydrostatic and deviatoric stress components are both operating to cause crack growth.

3) An increasing load continues the stable crack growth as the cracks begin connecting through the cement matrix, occurring between 70-90 percent of ultimate. This stage is known as the Onset of Stable Fracture Propagation (O.S.F.P.). The hydrostatic component of stress tends to delay crack growth caused by the deviatoric component during this stage.

4) The final stage occurs during what is known as the Onset of Unstable Fracture Propagation (O.U.F.P.). At this point unstable crack growth occurs and total ultimate failure follows as the specimen breaks into pieces. This occurs between 70-90 percent of the ultimate. The hydrostatic component of stress tends to stop crack growth during this stage while the deviatoric component promotes unstable crack growth.

Kotsovos [52] proposes an upper and lower bound for definition of concrete failure. The upper bound being O.U.F.P. and the lower bound being O.S.F.P. Analogous properties of these bounds would be the yield point and the ultimate strength when discussing ductile materials.

Biaxial Test Data for Concrete Failure

To verify the proposed failure criterion for concrete under biaxial states of stress a comparison to failure data obtained experimentally must be accomplished. A review of the published literature exposed numerous examples of concrete failure testing. The published literature consisted of uniaxial, biaxial, and triaxially loaded specimens. Since this investigation is primarily concerned with the development of a failure criterion for a biaxial stress state, further evaluation shall be restricted to experimental failure data under biaxial loading only.

Comprehensive studies of the published failure data for concrete in biaxial loading have been accomplished by Wastiels [99] and Nelissen [66]. They conclude that a critical assessment of published experimental data is vital. This is especially important in obtaining failure data for concrete under biaxial states of stress as triaxial conditions are easily induced. As was discussed previously there are numerous problems involved with obtaining the desired state of stress in a specimen. Much of the published data has been shown to be in error. To complement and update the previous reviews of failure data for concrete a summary review of the most favorable data available shall be accomplished here.

In reviewing chapter one up to this point it can be said that concrete is a highly complex, variable, and difficult to characterize material. The properties of plain concrete have been shown to be influenced by its composition, while the mechanical behavior has been shown to be influenced by the testing methods. Indeed the ultimate strength point is not even well defined in the literature. In general the test data available does not adequately define the composition of the concrete mix being tested or the test conditions under which the investigation was performed.

To select the best available concrete failure data this investigation followed the recommendations of Wastiels [99] and Nelissen [66] while adding a few more. The criteria used to select acceptable failure data was culled from the literature and by comparison studies of accepted failure data for biaxial loading. The main criteria used for selection of experimental data was:

1) The stresses within the test specimen must be easily calculable by accurate means. Approximate or non-linear stress distributions should be avoided.

2) Load applicaton to the specimen must be free of edge effects or the specimen critical failure zone must be truely loaded in a known biaxial state.

3) The material properties must be adequately defined so as to permit duplication of the test. This includes the exact type of portland cement, aggregate make-up, environmental conditions during cure and specimen age at test.

4) An adequate description of the loading sequence should be given. The failure testing should have been performed with monotonically increasing, static results as the conclusion.

5) The material failure condition will have been considered at the Onset of Unstable Fracture Propagation (O.U.F.P.). The specimen should have been loaded to complete failure.

6) An adequate number of failure tests should have been performed to allow for an average value for a given failure data point. The failure data obtained must be the statistical average.

7) The failure data must be in general agreement with accepted test results from the literature.

These seven criteria for data acceptance are not totally definitive but are used as a general guideline. A strong reliance on the reviews previously mentioned dealing with experimental data was imperative.

The wide variation of concrete properties would appear to make it impossible to compare different experimental test results. The variation in material mixes and ultimate strengths make direct comparison of the experimental results between investigators impossible. Fortunately, normalization of the experimental data yields comparable results. Thus the experimental failure data for concrete is divided by the specimens uniaxial compressive strength ($\sigma_c$) to normalize the relationships. The failure envelope for the biaxial states of loading appear very similar when normalized, as may be seen in Figures (1.3) through (1.20).

The experimental test results for the failure of concrete have been presented in differing forms throughout the literature. Data has been presented as shear-normal stresses, octahedral shear-normal stresses, and as principal normal stresses. The failure stress points are either presented in the respective plane by plotting or in tabular format. The data is not always normalized and sometimes normalized in an incompatible manner.

In this investigation all experimental data was translated into normal principal stresses. The data was then normalized with respect to the uniaxial compressive strength of the specimen geometry used in the test program. The data was then plotted in the principal stress plane of biaxial loading for comparison. Brittle materials are generally much stronger in compression than in tension, thus in brittle material research compressive stresses are often regarded as positive. Compression will be considered as positive throughout this investigation.

Evaluations of pertinent experimental works for biaxial loaded concrete are now presented:

1) Mchenery and Karni (1958), [63], Figure (1.3)

This investigation tested hollow concrete cylinders of three different strengths by applying axial compression or tension combined with inside pressure. A uniform stress distribution was assumed through the cylinder wall and plasticity theory used to calculate the stress values. In reality a triaxial state of stress is present on the inside of the cylinder and a truely plastic state is non-existent. Experimental results for the tension-compression region were obtained. The accuracy is questionable due to the method of stress calculation and comparison to others.

2) Rosenthal and Glucklich (1970), [83], Figures (1.4) and (1.5)

This investigation obtained experimental data for the complete failure region of two different strengths of concrete. Hollow cylinders were tested in axial compression and internal pressure. A uniform stress distribution was assumed through the cylinder wall and plastic theory used for stress calculations. The experimental data is questionable due to the method of stress calculations. Also the data is inconsistent with the reported shape of the biaxial failure envelope.

3) Bresler and Pister (1957, 1958), [9, 10], Figures (1.6) and (1.7)

This investigation tested four different strengths of concrete in the tension-compression region only. Hollow cylinders were loaded by applying torque to the outer surface and axial compression. A uniform stress distribution was assumed through the wall and a linear-elastic solution was used for stress calculation. The experimental results appear consistent with other works, even though the assumptions are approximate.

4) Goode and Helmy (1967), [34], Figure (1.8)

Two different strength concretes were tested in the tension-compression region only. Hollow cylinders were subjected to torsion and axial loading. The stress distribution was assumed uniform through the wall, and linear-elastic theory used for stress calculation. Results appear very scattered and do not follow the prescribed path of the failure envelope.

5) Isenberg (Johnson & Lowe) (1969), [45], Figures (1.9) and (1.10)

A single strength of concrete was tested in the tension-compression region only. Hollow cylinders were loaded by torsion and axial compression. The stresses were calculated from linear-elastic theory. The results appear consistent with other works. This is one of the most complete, informative investigations using hollow cylinders.

6) Kupfer, Hilsdorf, and Rusch (1969), [57], Figure (1.11)

Three strengths of concrete were tested. Square plates were loaded through brush bearing platens, which these investigators introduced. The complete failure region of all four biaxial quadrants were investigated. The stress distribution in these specimens are well known, as the relation is linear between the force and area of application. The stress equation being, S = P/A, where 'P' is the loading and 'A' is the cross sectional area of the specimen. These results are considered some of the most accurate to date.

7) Pandit and Tanwani (1975), [77], Figure (1.12)

This investigation tested one strength of concrete in the biaxial compression region only. Square plates were loaded by brush bearing platens. This specimen again allows for a linear calculation of stress as in example (6). The experimental results differ from the data of example

(6) at the compression stress ratio of .5 (i.e. $\sigma_1/\sigma_2 = .5$). The other data points are consistent with expected results.

8) Tasuji, Nilson, and Slate (1979), [90], Figure (1.13)

This investigation tested one strength of concrete in the complete biaxial principal stress region. Square plates were loaded through brush bearing platens. This specimen geometry yields stress calculations that are linear as in examples (6) and (7). The experimental data and failure envelope shape are consistent with other results.

9) Weigler and Becker (1963), [102], Figures (1.14) and (1.15)

These investigators tested six different strengths of concrete in the biaxial compression region only. Square plates were loaded through solid platens with soft ductile shims between the specimen and the platens. The results show a larger increase at the stress ratio of one, which is not consistent with other results. The introduction of edge effects are most likely the cause of the poor results.

10) Vile (1965), [97], FIgure (1.16)

This investigator tested one strength of concrete in the complete biaxial principal stress region. Square plates were loaded through solid platens with no friction reducing method. Vile felt the plates were slender enough to insure true biaxial loading in the critical region. Apparently he was correct as his results compare favorably to others.

11) Nelissen (1972), [66], Figures (1.17), (1.18), and (1.19)

This investigator tested two different strengths of concrete for the complete biaxial principal stress region. Cubes (18 x 18 x 13 cm) were loaded through brush bearing platens. This investigation is the most informative, thorough, and detailed of all biaxial tests found. The results appear consistent with other works and the discussion invaluable.

12) Mills and Zimmerman (1970), [64], Figure (1.20)

These investigators tested three strengths of concrete in the biaxial compression region. Only one strength of concrete contained adequate data for evaluation. Solid steel platens with lubricated teflon pads were used for loading. These results appear to give higher values than other works, therefore the data is questionable.

In reviewing the failure data which has been published for biaxial loading of concrete it becomes apparent that there is slight differences in the shape of the normalized failure envelope in Figures (1.3) through (1.20). Following the suggestions of eminent investigators and review of additional data the poor quality results may be eliminated. Therefore the following data sets for concrete in biaxial loading have been chosen to verify the proposed strength function:

1) Kupfer, Hilsdorf, and Rusch [57], Figure (1.11).

2) Tasuji, Nilson, and Slate [90], Figure (1.13).

3) Vile [97], Figure (1.16).

4) Nelissen [66], Figures (1.17), (1.18), and (1.19).

5) Johnson and Lowe [45], Figures (1.9) and (1.10).

Conclusions

The complex characteristics of the composite material concrete can be represented as a two-phase model. At the phenomenological or engineering level the concept of concrete as a two-phase material may be replaced by the assumption that it is homogeneous, isotropic, continuous medium composed of structural elements of identical properties. Strictly speaking, the assumption of homogeneity can be justified only on a statistical basis considering the average properties of the elements or groups in the body.

The experimental strength data published to-date covers a wide range of concrete strength qualities, mix proportions and test methods. A critical assessment of these results is deemed necessary to select failure data which most accurately represents the true failure conditions. The mechanical properties determined from experimental results are highly dependent on the test methodology.

In reviewing the experimental data, which may be considered most accurate, several conclusions can be drawn. The strength of concrete in uniaxial tension is shown to be somewhere between 5 and 11 percent of the absolute value of the strength in uniaxial compression. Also, higher strength concrete (above 4,000 PSI) tends to be on the lower percentage side of the range for its uniaxial tensile strength, generally between 9 and 5 percent of the absolute value of the uniaxial compressive strength. The lower strength concretes tend to be near 10 percent.

The strength of concrete in biaxial compression is between 108 and 120 percent of the uniaxial compressive strength at a load ratio of one, i.e. $\sigma_1/\sigma_2 = 1.0$. This value of biaxial compression tends to be approximately 116 percent on the average.

The maximum biaxial compressive strength for concrete occurs at a stress ratio of approximately .5, (i.e. $\sigma_1/\sigma_2 = .5$). At this ratio the biaxial strength is between 120 to 135 percent of the uniaxial compressive strength.

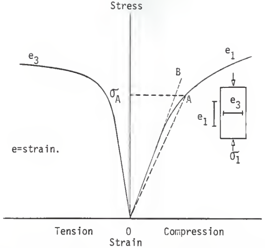



Fig. 1.1 Stress-strain curves for a typical concrete loaded in uniaxial compression.

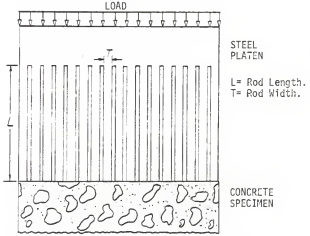
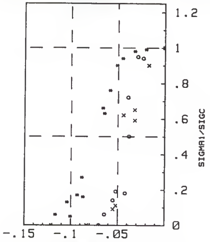



Fig. 1.2 Brush bearing platen for specimen loading.

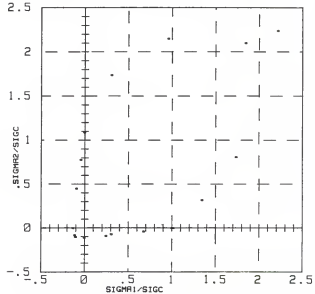
EXPERIMENTAL BIAXIAL FAILURE DATA BY:
McHENERY & KARNI(1958) HOLLOW CYLINDERS
*-SIGC=2380 PSI X-SIGC=4300PSI O-SIGC=5430PSI




TENSION-COMPRESSION FAILURE AREA

Fig. 1.3

EXPERIMENTAL BIAXIAL FAILURE DATA BY:
ROSENTHAL & GLUCKLICH(1970)- o -SIGC=3520PSI




BIAXIAL FAILURE REGION. SIGC= 3520 PSI

Fig. 1.4

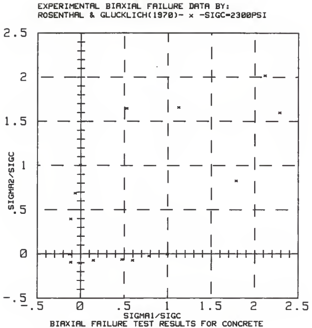

EXPERIMENTAL BIAXIAL FAILURE DATA BY:
ROSENTHAL & GLUCKLICH(1970)- x -SIGC=2300PSI
2.5
2
1.5
1
.5
0
-.5
SIGMA2/SIGC
-.5
0
.5
1
1.5
2
2.5
SIGMA1/SIGC
BIAXIAL FAILURE TEST RESULTS FOR CONCRETE

Fig. 1.5

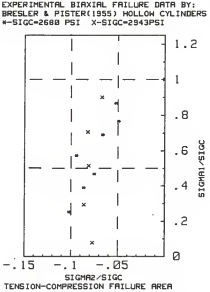
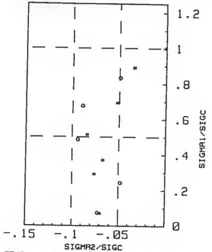

EXPERIMENTAL BIAXIAL FAILURE DATA BY:
BRESLER & PISTER(1955) HOLLOW CYLINDERS
*-SIGC=2680 PSI X-SIGC=2943PSI
1.2
1
.8
.6
.4
.2
0
SIGMA1/SIGC
-.15
-.1
-.05
SIGMA2/SIGC
TENSION-COMPRESSION FAILURE AREA

Fig. 1.6

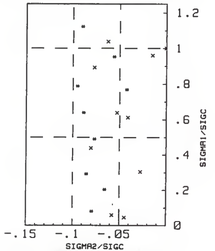
EXPERIMENTAL BIAXIAL FAILURE DATA BY:
BRESLER & PISTER(1955) HOLLOW CYLINDERS
*-SIGC=4273PSI X-SIGC=5497PSI




TENSION-COMPRESSION FAILURE AREA

Fig. 1.7

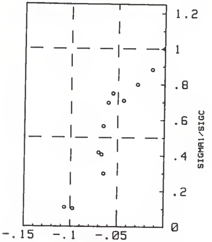
EXPERIMENTAL BIAXIAL FAILURE DATA BY:
GOODE & HELMY(1967) HOLLOW CYLINDERS
*-SIGC=2110 PSI X-SIGC=4469 PSI




TENSION-COMPRESSION FAILURE AREA

Fig. 1.8

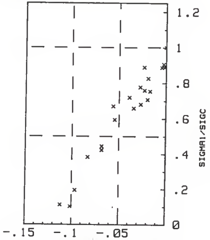
EXPERIMENTAL BIAXIAL FAILURE DATA BY:
ISENBERG(1969) HOLLOW CYLINDERS
0-SIGC=6420 PSI SERIES C




TENSION-COMPRESSION FAILURE AREA

Fig. 1.9

EXPERIMENTAL BIAXIAL FAILURE DATA BY:
ISENBERG(1969) HOLLOW CYLINDERS
0-SIGC=6420 PSI SERIES D




TENSION-COMPRESSION FAILURE AREA

Fig. 1.10

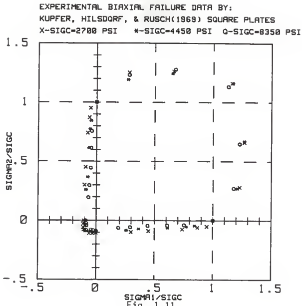



Fig. 1.11

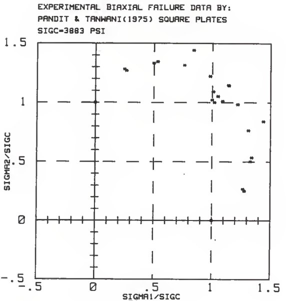



Fig. 1.12

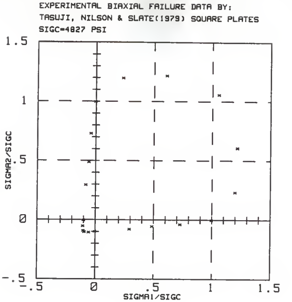



Fig. 1.13

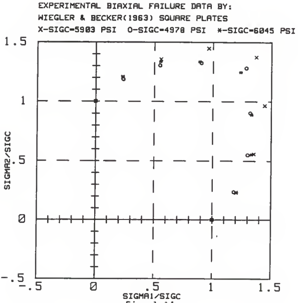



Fig. 1.14

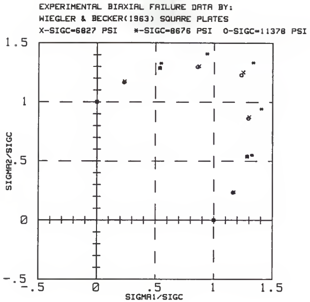



Fig. 1.15

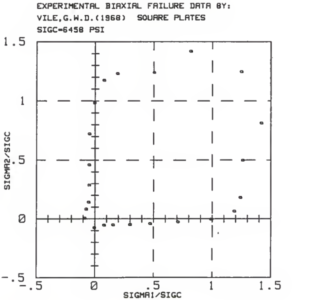



Fig. 1.16

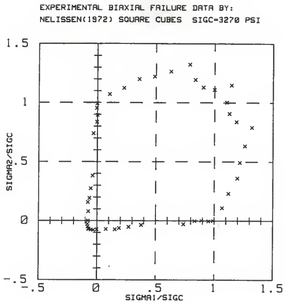



Fig. 1.17

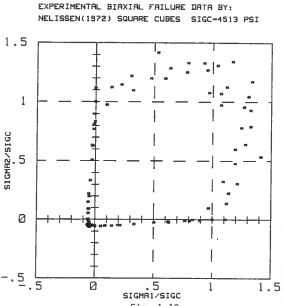



Fig. 1.18

EXPERIMENTAL BIAXIAL FAILURE DATA BY:
NELISSEN(1972) SQUARE CUBES SIGC=4513 PSI

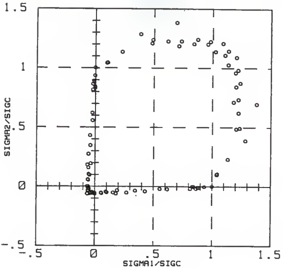
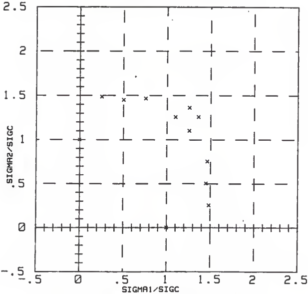



Fig. 1.19

EXPERIMENTAL BIAXIAL FAILURE DATA BY:
MILLS & ZIMMERMAN(1970)-X- SIGC=3340PSI




BIAXIAL FAILURE TEST RESULTS FOR CONCRETE

Fig. 1.20

# CHAPTER II

## MATERIAL STRENGTH CRITERIA

### Literature Review

Introduction

The characterization of a material failure under time-independent, isothermal conditions of multiaxial stress depend upon the material classification at the time of failure. Materials are classified as either brittle or ductile depending upon their state at the time of classification. The state of a material depends on such factors as temperature, pressure, rate of loading, and others. Thus failure classification for any given material is variable and depends on the material state.

Ductile state failure is generally defined as the onset of plastic flow. Plastic flow occurs when a material is loaded beyond its elastic limit or yield point. The material enters the plastic deformation range once the elastic limit is exceeded. Ductile fracture occurs following excedence of the elastic limit and ensuing physical material separation. Generally the failure criterion for a ductile state is defined as a yield criterion [74].

The failure of a material in the brittle state is defined as total fracture occurance before any appreciable plastic flow. A yield point for the brittle state of a material is indefinable, as brittle materials fail catastrophically by complete fracture. Failure in the brittle state is defined by a fracture criterion [74].

The physical process of brittle fracture is uncomparable to that of ductile fracture as they can be shown to be caused by differing mechanisms. Paul [74] concludes that ultimate fracture in the ductile state depends on the strain history and load path, while an understanding of the brittle fracture process is relatively imperfect. Thus the failure mechanics for the material states are incompatible. A yield criterion for ductile materials will not be the same as a fracture criterion for brittle materials.

Paul [74] demonstrates that failure of a material in the ductile state is controlled by a different stress system than brittle state failure. Material in the ductile state yield or undergo plastic flow as a result of shear stresses. Material in the brittle state fail due to a combination of shear and normal stresses. Stress at a point can be defined by a second-order tensor which is separable into normal and shear components. The stress tensor can be the sum of the following; a hydrostatic stress tensor ($\sigma_m$) and a deviatoric stress tensor ($S_{ij}$). The stress condition and its parts can be represented in tensorial notation as:

$$[\sigma_{ij}] = [S_{ij}] + \sigma_m[I] \quad i,j = 1,2,3 \qquad (2.1)$$

$$\sigma_m = 1/3\ (\sigma_{11} + \sigma_{22} + \sigma_{33})$$

The hydrostatic component represents a stress state of equal normal stresses, while the deviatoric component represents a stress state of pure shear.

The failure criterion for a ductile material, is independent of the hydrostatic component of the stress tensor depending only on the stress deviation ($S_{ij}$) [74]. Thus ductile failure criterion are considered pressure-independent. A function for a yield criterion may be expressed as:

$$f(S_{ij}) = 1 \quad i,j = 1,2,3 \tag{2.2}$$

The brittle state of a material has been found to be pressure-dependent [74]. Thus a fracture criterion depends on both the hydrostatic ($\sigma_m$) and the deviatoric ($S_{ij}$) stress components. A fracture function may be expressed as:

$$f(\sigma_{ij}) = 1 \quad i,j = 1,2,3 \tag{2.3}$$

Paul [74] has shown that the failure criterion for isotropic materials, when expressed in the principal stress coordinates ($\sigma_1$, $\sigma_2$, $\sigma_3$) will geometrically represent a failure surface in three-dimensional principal stress space. Equation (2.2), represented geometrically, is shown in Figure (2.1). Figure (2.1) is a cylindrical yield surface with its axis aligned on the hydrostatic axis ($\sigma_1 = \sigma_2 = \sigma_3$). The area within the cylinder is considered to be the elastic region, while the cylinder wall represents the yield surface. The yield surface is independent of the hydrostatic pressure or equivalently the location along the hydrostatic axis, as the cylinder extends indefinitely along both axis directions. Thus Figure (2.1) represents a pressure-independent failure surface.

The failure of a material in a brittle state has been shown to be pressure-dependent, thus it depends on the hydrostatic axis location. A generalized geometric representation of a fracture surface is given in Figure (2.2). This fracture surface represents an open ended cone with its vertex located on the hydrostatic axis in the triaxial tension region. Compression is taken as positive throughout this investigation. The cone surface (fracture surface) demonstrates the dependence on the hydrostatic axis location. The radius from the hydrostatic axis to the cone surface increases as the positive hydrostatic pressure increases, noting that the region inside the cone represents non-failure.

Brittle materials normally exhibit higher absolute strengths in compression than in tension. This phenomenon is referred to as the Bauschinger effect. Thus the merdians of the cone-surface shown in Figure (2.2) do not intersect the compressive principal stress axis at the same absolute value as the tensile principal stress axis.

Equation (2.2) is a specialized condition of equation (2.3) with the hydrostatic component eliminated. Therefore the following definitions apply to both failure criterion:

$$f(\sigma_{ij}) < 1 \quad \text{no failure occurs.} \qquad (2.4)$$

$$f(\sigma_{ij}) = 1 \quad \text{failure will occur.}$$

$$f(\sigma_{ij}) > 1 \quad \text{undefined.}$$

Equations (2.4) states that a stress point may be inside or on the failure surface and that failure occurs only at the surface. These failure surfaces are idealizations (Figures (1.1) and (1.2)), as individual failure surfaces for a given material must be determined through experimentation.

Classification of Concrete

The development of a strength criterion is linked to the materials classification, either brittle or ductile. In chapter one concrete was shown to have variable properties and complex characteristics. The materials properties must be idealized at the present level of investigation to aid classification. The true properties of concrete are neither brittle nor ductile during any given state.

The state of concrete changes depending on its hydrostatic pressure [5]. At low hydrostatic pressures concrete behavior is brittle, but at high hydrostatic pressures it approximates ductile behavior. Additionally there are signs of the yield point phenomenon exhibited by ductile materials, although this can be traced to irreversible crack growth [69]. Concrete also exhibits a pronounced Bauschinger effect upon repeated loadings usually attributed to ductile materials.

The lack of a pronounced yield point, the tendency of the material to fail by complete physical separation, the dependency of the material on the hydrostatic component of stress (pressure-dependent), and the brittle characteristics under low hydrostatic pressures all indicate that concrete is best classified as a brittle material for normal design circumstances. Thus a fracture criterion in the form of equation (2.3) will best represent concrete failure.

## General Macroscopic Brittle Failure Theories

Throughout history there have been numerous failure theories proposed for materials under multiaxial states of stress. In general these theories have been postulated based on physical observations. Paul [74] reviews the historical development of failure theories up to 1968. In his review he discusses the development of the four classic failure criteria and subsequent generalizations of these. As a more comprehensive understanding of the failure process has evolved, so has failure theory development. The latest proposed theories of failure can be shown to be generalizations of the classical theories. The four classical theories of material failure are:

1) The maximum normal stress theory (Rankine's Theory); and its counterpart the maximum normal strain theory (St. Venant's Theory).
2) The maximum shearing stress theory (Tresca's Theory).
3) The maximum strain energy theory (Beltrami's Theory).

4) The maximum distortion energy theory (Huber-Von Mises-Hencky Theory).

These classical failure theories are represented well in many excellent monographs on the subject of material failure, such as Paul [74] or Nadai [65]. Thus they will not be reviewed in detail in this investigation. Paul [74] discusses the limitations of the classical theories and proves their applicability to brittle failure is inadequate. The aforementioned theories are all limited in their usefulness as they are subjected to a number of strong constraints.

Paul [74] states that one of the most useful criterion which can be applied to brittle failure is the Coulomb-Mohr theory. This combined theory assumes that material failure is attributed to shear stresses that are dependent on normal stresses. The Coulomb-Mohr criterion represented in principal stress coordinates as stated by Paul is:

$$\sigma_1/f_t - \sigma_3/f_c = 1 \qquad (2.5)$$

where the material constants $f_t$ and $f_c$ represent the uniaxial material strength in tension and compression, respectively.

The Coulomb-Mohr criterion correlates reasonable well for some materials like soil, but not for others. Equation (2.5) represents a cylindrical cone in three-dimensional stress space as in Figure (2.2). The failure criterion includes the Bauschinger effect and is easily characterized by only two parameters. The Coulomb-Mohr criterion is not an affective failure criterion for two main reasons:

1) The failure criterion is independent of the intermediate principal stress ($\sigma_2$). This ignores stress interactions.
2) The failure criterion does not accurately predict tensile failures.

Paul [75] originally proposed additions of tension cut-offs to the Coulomb-Mohr criterion. His proposals improved failure prediction within the triaxial tensile quadrants, yet did not include needed stress interactions, thus it proves inadequate in all other regions. Paul [74, 75] recognized that the Coulomb-Mohr criterion with tension cut-offs suffered inadequacies and proposed a generalized pyramidal fracture and yield criterion. His improvements over his previous work were to include the effects of the second principal stress, adequate prediction of tensile strength, and allowance for a non-linear meridian surface of failure. Geometrically, his criterion approximates a non-linear, curved failure surface in three-dimensional principal stress space. The three-dimensional shape is a multi-segmented surface made up of adjoining sets of hexagonal pyramids. The vertex being in the tension-tension-tension section on the hydrostatic axis. Paul's criterion is useful only for approximations as it is inaccurate and ignores many stress interactions. The multi-segmented failure surface leads to ambiguities as a definition of the segment section in use is required.

The classic Coulomb-Mohr and Paul's [74, 75] failure theories are useful only for isotropic materials. This is not a problem for concrete as it is considered isotropic, yet it demonstrates the lack of generality for application to anisotropic materials of these theories. These theories will not adequately predict the failure envelope for concrete.

In general most failure criteria with any degree of applicability to homogeneous or quasi-homogeneous anisotropic material are of the maximum distortion energy theory type. Kaminski and Lantz [46] in their critical review of failure criteria for composite materials concluded that nearly all of the maximum distortion energy type of theories refer to principal strengths of the material and ignore the influences of stress interactions. Thus the theories cannot deal with complex materials.

Tang [89] critically reviewed all past failure criterion for their application to a transversely isotropic brittle material. He concluded that the failure criteria which adequately fulfill the necessary requirements are those formed from strength tensor relations based on tensor function representations. The tensor function approach has been suggested for use with anisotropic materials such as fiber-reinforced composites, as the material failure criterion is capable of including any number of stress interaction terms. Stress interaction components allow the failure function to account for differing values of ultimate strengths in tension and compression about each of the material symmetric axis. The tensor function approach can also account for the dependence of the ultimate shear strengths on the sign (direction) of the shear stresses. Recent developments in nonlinear continuum mechanics have enhanced the application of tensor function theory to failure criterion development.

Gol'denblat and Kopnov [33] were among the first to propose the use of strength tensors for an anisotropic strength criterion. They proposed the following form of an equation:

$$f(\sigma_k) = (F_i\sigma_i)^{\alpha} + (F_{ij}\,\sigma_i\,\sigma_j)^{\beta} + (F_{ijk}\,\sigma_i\sigma_j\sigma_k)^{\gamma} + \dots = 1 \qquad (2.6)$$

$$i,j,k = 1,2,..,6$$

where the strength tensors ($F_i$, $F_{ij}$, $F_{ijk}$) are of second, fourth, and sixth rank respectively, and are material parameters. The empirical powers ($\alpha$, $\beta$, $\gamma$) are also material parameters. $\sigma_i$ represents stresses.

Equation (2.6) is one of the most generalized forms for a failure function consisting of polynomial terms and empirical powers based on the strength tensor approach. The linear terms ($F_i\,\sigma_i$) account for normal stresses which describe the difference between positive and negative stress induced failures, i.e. it allows for inclusion of the Bauschinger effect. The quadratic terms ($F_{ij}\sigma_i\sigma_j$) define an ellipsoid in the stress space as well as account for second-order stress interaction terms. The higher order terms account for additional stress interactions as well as allowing the failure surface to describe irregularities. In using equation (2.6) it is simplified as necessary to adequately characterize the material failure surface determined by experimental results.

Tsai and Wu [95] proposed a simplified quadratic tensorial strength criterion based on Gol'denblat and Kopnov [33]. Their stress tensor function for anisotropic material failure is:

$$F(\sigma_i) = F_i\sigma_i + F_{ij}\sigma_i\sigma_j = 1 \qquad (2.7)$$

where the strength tensors ($F_i$, $F_{ij}$) and stresses are identical to those of equation (2.6). Tsai and Wu dropped the higher order terms and set the empirical powers of Gol'denblat and Kopnov to one. Tsai and Wu required their criterion form to be operationally simplier by including fewer material parameters. Being quadratic, equation (2.7), can be solved explicitly while higher order equations may not. The empirical powers of equation (2.6) complicate equation characterization, while they are set to one for equation (2.7). Equation (2.7) includes all necessary stress interaction terms of equation (2.6) allowing for the Bauschinger effect, stress direction effects, and anisotropy of various materials.

Tsai and Wu's [95] equation reduces to the following for an isotropic material when written in principal stress coordinates:

$$F_1(\sigma_1 + \sigma_2 + \sigma_3) + F_{11}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - F_{11}(\sigma_1\sigma_2 + \sigma_1\sigma_3 + \sigma_2\sigma_3) = 1 \qquad (2.8)$$

where the components remain as described in equation (2.7). Equation (2.8) represents a strength criterion based upon tensor theory in quadratic form.

The criterion requires only two parameters ($F_1$, $F_{11}$) to characterize the function to the material. The parameters are determined through simple engineering strength tests by the following relationships:

$$F_1 = \frac{f_c^2 - f_t^2}{f_t f_c + f_t^2 f_c} \quad (2.9)$$

$$F_{11} = \frac{1}{f_t^2} + \frac{f_c^2 - f_t^2}{f_c(f_t f_c^2 + f_t^2 f_c)} \quad (2.10)$$

where the strength parameters $f_t$ and $f_c$ represent the absolute values for uniaxial tensile and compressive strengths, respectively.

Tsai and Wu's [95] formulation is a simplification of Gol'denblat and Kopnov's [33]. Each of these criteria can be reduced to past failure criteria of non-tensorial base by imposing restrictions. Tsai and Wu show this by reducing their function to that of Hills [37], which is used as a yield criterion.

Priddy [79] noted that the biaxial failure envelope presented by Tsai and Wu's [95] equation was always an ellipse. He proved that an elliptical form of a quadratic equation cannot yield accurate correlations with data in both the tension-tension and compression-compression quadrants for some materials. Thus, he proposed including a limited number of mathematically independent cubic terms to Tsai and Wu's equation to improve the failure envelope surface control. Priddy presented his equation in invariant form as:

$$W = 1 + I_1 + I_1 I_2 + fI_3 \quad (2.11)$$

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3$$

$$I_2 = -(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3)$$

$$I_3 = \sigma_1\sigma_2\sigma_3$$

where stress tensor invariants ($I_1$, $I_2$, $I_3$) are represented in principal stress coordinates ($\sigma_1$, $\sigma_2$, $\sigma_3$) and W is the strain energy density for the material, a constant. f is a material parameter.

Priddy [79] states that the quadratic theory of Tsai and Wu [95] contains only one independent data point for each biaxial plane. This limits the equations control in the tension-tension and compression-compression quadrants, thus the interactions cannot be treated independently. Priddy selectively added the invariant $I_3$ and the invariant combination $I_1I_2$ while dropping the $J_2$ term from Tsai and Wu's equation. His arbitrary selection of cubic invariant terms will accommodate only a limited number of materials as the higher order terms required depend on the interaction effects. Furthermore, Priddy failed to properly expand the invariants when writing the full equation in principal stress terms and thus his final equation form was non-invariant.

Tennyson et.al. [94] also addressed the question of expanding a strength tensor based function by selectively adding higher order terms. They reached the conclusion that cubic and higher order terms are often required in composite material failure function development.

Huang [42] determined the second, fourth, and sixth rank strength tensors in the three-dimensional case for each of the material crystal classes from consideration of invariant transformations of the strength function. He based his work on the criterion of strength proposed by Gol'denblat and Kopnov [33]. Based on Huang's [42] earlier determination of invariant properties for orthotropic material symmetry Huang and Kirmser [43] derived a formulation for a strength criterion for a glass-reinforced composite material. The criterion is a quadratic form of tensor function theory using invariants of the material symmetry group and is presented as:

$$f(\sigma_i) = F_1\sigma_1 + F_2\sigma_2 + F_3\sigma_3 + F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{33}\sigma_3^2$$
$$+ 2F_{12}\sigma_1\sigma_2 + 2F_{23}\sigma_2\sigma_3 + 2F_{13}\sigma_1\sigma_3$$
$$+ F_{44}\sigma_4 + F_{55}\sigma_5 + F_{66}\sigma_6 = 1. \qquad (2.12)$$
$$i,j = 1, 2, \ldots 6.$$

where $F_i$ and $F_{ij}$ are strength tensors. The stress invariants are $\sigma_1$, $\sigma_2$, $\sigma_3$, $\sigma_4^2$, $\sigma_5^2$, $\sigma_6^2$.

Equation (2.12) represents a Taylor expansion of the material symmetry group invariants up to second-order. The equation may be further expanded to a cubic by including additional invariant terms of third-order combinations as presented by Huang and Kirmser [43].

Concrete Specific Strength Criteria

A literature search revealed numerous developments in strength criteria for concrete. In general past design criterion consisted of empirically derived nomographs or codes. The introduction of advanced methods for structural analysis have promoted requirements for more accurate material characterization. To meet the needs there have been many curve fitting attempts at development of a strength function [9, 11, 12, 44, 60, 61, 63, 77, 80]. These methods formulate an equation from experimental data. The data is plotted in either the principal stress plane or the shear-normal stress plane and a curve fit performed. Thus the accuracy of the failure equation is highly influenced by the accuracy of the failure data. To use a curve fit function the material coordinate system under consideration must be aligned with the coordinate axis with which the function was developed, thus it is non-invariant. Furthermore due to the complex shape of the failure surface for concrete a single equation, generally does not operate in all quadrants of stress space. Thus a total definition of the failure surface requires several functions to adequately describe the complete region of all stress combinations.

In recent years the trend has been to develop failure functions incorporating invariants of the stress tensor. This form is more suited for usage within computer codes. The invariant form of a failure function tends to include a higher number of stress interaction terms yielding a higher order function capable of descriptive failure surfaces. Surveys of the most applicable strength criterion have been accomplished by Wastiels [98, 99, 100], Ottosen [72], and Newman and Newman [71].

Wastiels [98, 100] has analyzed seventeen different failure criteria for their validity within the compression-compression region of biaxial load conditions. Wastiels considered failure criteria specific to biaxial loading and de-generated triaxial failure criteria into their biaxial form for accuracy comparisons. The purpose of his survey was to defend his presentation of a triaxial failure function for concrete.

Wastiels [98,100] found that many of the failure criteria were inaccurate. He plotted the failure envelopes for the compression-compression region and compared them to experimental data. Wastiels concluded that the best criterion for failure of concrete under biaxial compressive loading conditions was that of Drucker and Prager [23]. Furthermore he concludes that the best failure criteria for triaxial load conditions are those of Ottosen [72], William and Warnke [103] and his criterion [98, 100].

The Drucker-Prager [23] failure criterion was originally proposed for usage with soils. The criterion is a generalized form of the Coulomb-Mohr failure theory. The Drucker-Prager failure function is of the following form:

$$f(\sigma_{ij}) = aI_1 + \sqrt{J_2} = k \quad i,j, = 1,2,3 \tag{2.13}$$

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3$$

$$J_2 = 1/6[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]$$

where the first invariant of the stress tensor $I_1$ and the second invariant of the stress deviator tensor $J_2$ are written in terms of principal stresses ($\sigma_1$, $\sigma_2$, $\sigma_3$). The material parameters are given as 'a' and 'k'.

The Drucker and Prager function is quadratic, invariant and uses two material parameters for characterization. Wastiels only concern was the compression-compression region of the principal stress plane. His limited evaluation fails to recognize the inadequacies of the Drucker and Prager function in the tension-compression and tension-tension quadrants. Equation (2.13) being quadratic, can not adequately describe a complete biaxial failure envelope due to a lack of function parameters in all regions.

The failure criterion for concrete proposed by Chen and Chen [13, 14] is a form of the Drucker and Prager failure function. Wastiels [98, 100] reviewed this criterion but discarded its value as a triaxial failure function. Wastiels stated that the lack of a cubic term, the third invariant of deviator tensor ($J_3$) invalidates Chen and Chen's function for triaxial loading. He proved that the criterion of Chen and Chen is superior to Drucker and Prager yet dismissed it due to the aforementioned reason. Indeed Chen and Chen's proposed criterion is inadequate for triaxial conditions, yet it is quite suitable for biaxial states of stress as the invariant term ($J_3$) vanishes for plane stress conditions.

A typographical error has been published in the presentation of Chen and Chen's [13, 14] criterion within both papers. The error pertains to material parameter characterization. The corrected results are presented in the appendix and will be used throughout this investigation.

Chen and Chen [13, 14] proposed two quadratic equations as their failure criterion. Their proposed criterion is a Taylor expansion of the invariants following tensor theory in the following form:

for compression-compression region:

$$f(\sigma_i) = J_2 + \frac{A_u}{3} I_1 = t_u^2 \qquad i = 1,2,3 \tag{2.14}$$

$$A_u = \frac{f_{bc}^2 - f_c^2}{2f_{bc} - f_c} \qquad t_u^2 = \frac{f_{bc} f_c (2f_c - f_{bc})}{3(2f_{bc} - f_c)}$$

for tension-tension & tension-compression region:

$$f(\sigma_i) = J_2 + \frac{I_1^2}{6} + \frac{A_u}{3} I_1 = t_u^2 \qquad i = 1,2,3$$

$$A_u = \frac{1}{2}(f_c - f_t f_c) \qquad t_u^2 = \frac{1}{6}(f_c f_t)$$

where the first invariant of the stress tensor ($I_1$) and the second invariant of the deviator tensor ($J_2$) are as defined in equation (2.13), also ($f_t$) is the uniaxial tensile strength, ($f_c$) is the uniaxial compressive strength and ($f_{bc}$) is the biaxial compressive strength when $\sigma_1 = \sigma_2$, (absolute values).

In reviewing the failure criterion of Chen and Chen the failure envelopes produced by equation (2.14) have been generated. Figures (2.3), (2.3a), (2.4) and (2.4a) demonstrate the biaxial failure envelope in all regions for the equations within (2.14). The functions are plotted along with the experimental results of Kupfer, Hilsdorf, and Rusch [57] and Nelissen [66]. The failure function describes the biaxial envelope reasonably well in the compression-compression region of Figures (2.3) and (2.4). The tension-compression and tension-tension function presented in equation (2.14) does not represent the experimental envelopes accurately. Closer examination of the tension-compression quadrants within Figures (2.3a) and (2.4a) demonstrate the poor correlation.

The proposed criterion of Chen and Chen is not accurate and requires two independent equations for a complete biaxial failure envelope. The function represents the compression-compression region well, but is inadequate in all others.

Ottosen [72] proposed a four-parameter triaxial failure criterion for concrete based on geometric relationships between the suspected shape of the three-dimensional failure surface for concrete and the stress tensor invariants. He developed a function by using the invariants $I_1$, $J_2$, and cos 3θ. Thus his proposed form was given as:

$$f(I_1, J_2, \cos 3\theta) = 0 \tag{2.15}$$

Ottosen's function is invariant and follows the prescribed shape of the suspected failure surface very well. The function allows for a smooth convex failure surface with curved meridians, which open in the compressive direction of the hydrostatic axis, and the trace in the deviatoric plane changes from nearly triangular to a more circular shape with increasing compressive hydrostatic pressure.

Wastiels [98, 100] and Robutti et. al. [81] critically reviewed the criterion proposed by Ottosen [72]. Wastiels concludes that the criterion over estimates the strength as the material parameters were obtained from poorly coordinated data. The criterion could be improved through better approximations of the parameter values. Ottosen's criterion proves to be overly complex for usage as a biaxial failure criterion as it requires too many experimentally determined data points for characterization, unlike Chen and Chen's [14] criterion.

William and Warnke [103] developed an elliptical equation to describe the three-dimensional failure surface of concrete. Their function development is based on the geometric interpretation of the stress tensor invariants $I_1$ and $J_2$ and the material parameter (cos θ). The invariants were incorporated into the elliptical representation of the three-dimensional failure surface for concrete.

Wastiels [98, 100] and Kotsovos [51] working independently, sought to modify William and Warnke's [103] original equation through improvements in the characterization of the material constants. Wastiels version of William's and Warnke's function was reviewed further and in general all comments also are applicable to Kotsovos's failure criterion as the base equations are identical.

Wastiels [98, 100] presents the failure criterion of William and Warnke [103] in the following form:

$$\frac{t_o}{\theta_c} = \frac{2C(C^2 - T^2)\cos\theta + C(2T - C)[4(C^2 - T^2)\cos^2\theta + 5T^2 - 4TC]^{.5}}{4(C^2 - T^2)\cos^2\theta + (C - 2T)^2} \qquad (2.16)$$

where:

$$C = .12051 - .55128\ \sigma_o/\sigma_c$$

$$T = .25834 - .63917\ \sigma_o/\sigma_c$$

$$\cos\theta = (2\sigma_1 - \sigma_2 - \sigma_3)/3\sqrt{2}\ t_o$$

where ($t_o$) and ($\sigma_o$) are the octahedral shear and normal stresses, respectively and ($\sigma_c$) is the uniaxial compressive strength of concrete.

To review equation (2.16) with the selected biaxial failure data for concrete it was de-generated into its biaxial form and plotted in Figures (2.5) and (2.6). The biaxial failure envelopes generated from equation (2.16) fit the experimental data well in these figures. The failure envelope is identical in both cases as the failure function is not capable of accounting for variations in concrete quality. The parameters T anc C are fixed by the author, Wastiels. These parameters control the function by determining the failure envelope positioning. To consider concrete of different quality the parameters (T, C) must be re-evaluated by producing enough experimental data to perform a reasonably accurate least squares fit, yielding the linear equations 'T' and 'C'.

The failure criteria proposed by Wastiels [98, 100] and Kotsovos [51] both reflect the same problems. The functions are non-invariant and highly complex to characterize for variations of concrete quality. The characterization method of these functions requires voluminous experimental data to obtain the parameters through least squares methods.

One of the most recent proposals for a triaxial failure criterion of concrete is that of Lade [59]. Lade proposed a three-parameter failure criterion based on stress tensor invariants. He has based the development of his criterion for concrete on his previously proposed failure criterion for soils [58], only a modification to allow for tensile stresses has been incorporated. The failure function for concrete proposed by Lade [59] is as follows:

$$F(\sigma_i) = \left(\frac{I_1^3}{I_3} - 27\right)\left(\frac{I_1}{p}\right)^m - N = 0 \tag{2.17}$$

$$I_1 = \bar{\sigma}_1 + \bar{\sigma}_2 + \bar{\sigma}_3$$

$$I_3 = \bar{\sigma}_1\bar{\sigma}_2\bar{\sigma}_3$$

$$\bar{\sigma}_i = \sigma_i + ap \quad ; \quad i = 1,2,3$$

where the first ($I_1$) and the third ($I_3$) invariants of the stress tensor are modified by addition of the (ap) term to the principal stresses ($\sigma_1$, $\sigma_2$, $\sigma_3$), the terms 'm', 'N', and 'a' are matèrial parameters while 'p' represents atmospheric pressure.

The material parameters for equation (2.17) are determined by regression analysis of experimental failure data for concrete. To use equation (2.17) a wide variation of values is given for the material parameters by Lade. He presents a table listing the parameter values calculated for twenty-two sets of experimental data.

In reviewing Lade's [59] failure criterion it was de-generated into its biaxial form and plotted for two sets of experimental data in Figures (2.7) and (2.8). The inaccuracies of equation (2.17) become apparent as the biaxial failure envelope does not fit the experimental data very well. In Figure (2.7) equation (2.17) over estimates the biaxial strength in the compression-compression region and under estimates the strength in the tension-compression region. The failure envelope in Figure (2.8) does not pass through the uniaxial compression stress and underestimates the strength by 20 percent.

The failure criterion proposed by Lade [59] is a three-parameter invariant function, although it is essentially a curve fitting routine. To use the function a complete data set must first be experimentally obtained and a linear regression performed to evaluate the material parameters. This procedure is overly complex for actual usage. In addition Lade's criterion fails to include the second invariant of the stress tensor ($J_2$). Concrete failure has been shown to depend on the shear component of stress, which the second invariant represents. This criterion produces poor results and is overly complex to characterize, thus its value is limited.

Conclusion

In reviewing the proposed failure criteria for concrete under both biaxial and triaxial load conditions a satisfactory criterion is not available. The published criteria suffer from many inadequacies such as, poor agreement with experimental data, non-invariance or overly complex characterization schemes. In general the concrete specific failure criteria are based on invariants of the stress tensor and most are artificially characterized through curve fit routines. These curve fit based functions are highly dependent on the accuracy of the experimental data under which they were developed and cannot easily be characterized for differing strength qualities of concrete.

The composite material approach involving tensor theory based functions appears most promising for comprehensive failure criterion development. The concrete strength criterion proposed by Chen and Chen [14] follows the tensorial approach. Their criterion comply with tensorial theory as they use a Taylor expansion of invariants with strength tensor parameters. This proposed criterion consists of two equations for a complete description of the failure envelope, as both are merely quadratic. As was previously suggested, higher order terms are necessary to allow development of a single equation criterion capable of describing the known biaxial failure envelope.

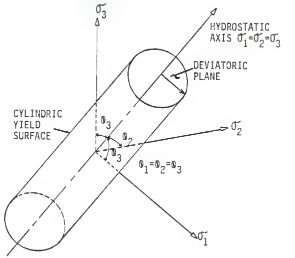



Fig. 2.1 Geometric representation of pressure-independent yield surface.

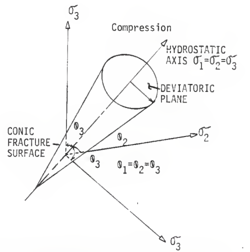
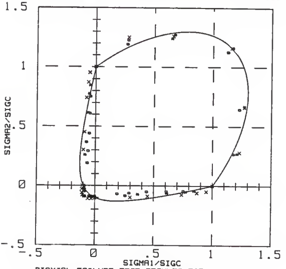



Fig. 2.2 Geometric representation of pressure-dependent fracture surface.

EXPERIMENTAL BIAXIAL FAILURE DATA BY:
KUPFER, HILSDORF & RUSCH(1969) SQUARE PLATES
X-SIGC=2700 PSI *-SIGC=4450 PSI o-SIGC=8350 PSI




BIAXIAL FAILURE TEST RESULTS FOR CONCRETE
CHEN & CHEN FAILURE FUNCTION(1975)
WITH THE CORRECTED COEFFICIENT

Fig. 2.3

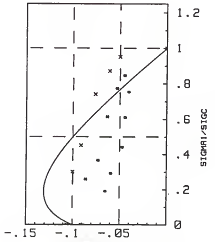
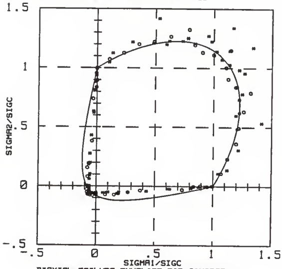



Fig. 2.3a

TENSION-COMPRESSION FAILURE AREA
PROPOSED FAILURE CRITERION OF
CHEN & CHEN (1975)




BIAXIAL FAILURE ENVELOPE FOR CONCRETE
CHEN & CHEN'S PROPOSED FAILURE CRITERION(1975)

Fig. 2.4

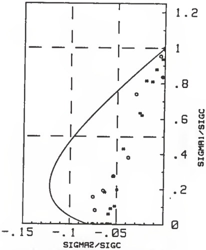
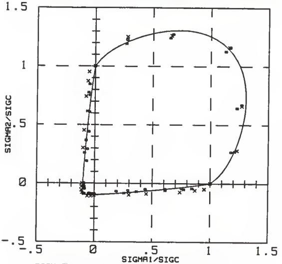



Fig. 2.4a

TENSION-COMPRESSION FAILURE AREA
PROPOSED FAILURE CRITERION OF
CHEN & CHEN (1975)

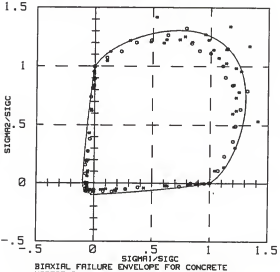
EXPERIMENTAL BIAXIAL FAILURE DATA BY:
KUPFER, HILSDORF & RUSCH(1969) SQUARE PLATES
X-SIGC=2700 PSI *-SIGC=4450 PSI o-SIGC=8350 PSI




BIAXIAL FAILURE ENVELOPE FOR CONCRETE
WASTIELS PROPOSED FAILURE CRITERION(1979)

Fig. 2.5

EXPERIMENTAL BIAXIAL FAILURE DATA BY:
NELISSEN(1972) SQUARE CUBES
O-SIGC=3270 PSI *-SIGC=4513 PSI




BIAXIAL FAILURE ENVELOPE FOR CONCRETE
WASTIELS PROPOSED FAILURE CRITERION(1979)

Fig. 2.6

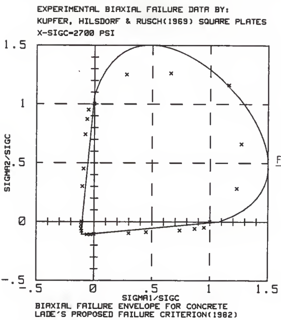

EXPERIMENTAL BIAXIAL FAILURE DATA BY:
KUPFER, HILSDORF & RUSCH(1969) SQUARE PLATES
X-SIGC=2700 PSI
SIGMA2/SIGC
SIGMA1/SIGC
1.5
1
.5
0
-.5
BIAXIAL FAILURE ENVELOPE FOR CONCRETE
LADE'S PROPOSED FAILURE CRITERION(1982)

Fig. 2.7

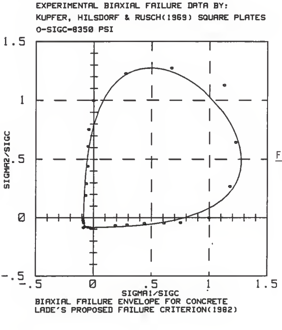

EXPERIMENTAL BIAXIAL FAILURE DATA BY:
KUPFER, HILSDORF & RUSCH(1969) SQUARE PLATES
O-SIGC=8350 PSI
SIGMA2/SIGC
SIGMA1/SIGC
1.5
1
.5
0
-.5
BIAXIAL FAILURE ENVELOPE FOR CONCRETE
LADE'S PROPOSED FAILURE CRITERION(1982)

Fig. 2.8

CHAPTER 3

# A PROPOSED STRENGTH CRITERION FOR CONCRETE

## Introduction

The characteristic properties of concrete have been shown to be those of a complex, multi-phase material which is best studied as a composite. The physical properties in the final state depend on the original mixed proportions and the environmental conditions during cure. Real materials are in general nonhomogeneous, anisotropic, and noncontinuous, as they are composed of groups of elements formed into a large number of discrete particles. However there is a dimensional level of aggregation (the phenomenological or engineering level) at which the concept of the element structure can be replaced by a homogeneous, isotropic, continuous medium composed of structural elements of identical properties. The mechanical characteristics of concrete are best idealized at the macroscopic level for engineering design applications. The assumption of homogeneity can be justified only on a statistical basis when considering the average properties of the elements in the body.

The mechanics of the failure mechanism for concrete were shown to be initiated by numerous microscopic flaws or cracks inherent within the concrete matrix. The average influence of these microscopic flaws, as viewed from macroscopic theory, reveal distinct levels of change in the mechanical behavior of concrete. As the stress level increases the mechanical behavior changes from quasi-elastic to plastic, with two distinct points of departure. The initial discontinuity point is at the onset of stable fracture propagation while the ultimate strength is reached at the onset of unstable fracture propagation. The hydrostatic and deviatoric components of the localized stress have been shown to delay and propagate the internal crack growth, respectively.

The development of a strength criterion for a material depends on its state at or during failure conditions, either it is brittle or ductile. The mechanical response and failure mode of concrete is best classified as a brittle material at normal hydrostatic loads. Strength characterization of most brittle materials is dependent on the hydrostatic as well as the deviatoric component of stress, while ductile material characterization is independent of the hydrostatic component. Thus a fracture criterion for concrete must depend on the complete stress tensor and is represented as previously given by equation (2.3).

$$f(\sigma_{ij}) = 1 \qquad i,j, = 1,2,3 \tag{2.3}$$

In reviewing the strength criteria of the previous chapter the most applicable forms follow equation (2.3), yet none prove totally satisfactory. The strength criteria reviewed were shown to lack compliance with experimental results, require strict adherence to a given material property coordinate system (non-invariance), and/or require complex methodology for material parameter characterization. These criteria have for the most part been formulated within the framework of the classical theories of plasticity, which are subjected to a number of strong constraints. These approaches lack generality and pertinance, and they tend to be complex mathematically.

In recent years with the introduction of materially complex, anisotropic, fiber-reinforced composites, more appropriate methods for material characterization have been sought. In the field of non-linear continuum mechanics there has been continuous developments following more powerful approaches to these problems. In reviewing the recently proposed general strength criteria, the continuum mechanics approach has been most notable. The application of general and explicit tensorially based scalar-valued or tensor-valued functions have proven to be highly useful towards developing strength criteria and constitutive equations. Many investigations have shown the value of using tensor function theory in these applications.

The continuum mechanics approach to material characterization based on tensor function theory appears highly useful for concrete. The composite nature and complex failure mechanism of concrete dictate a need for a more powerful approach to strength criterion development. The purpose of this chapter is to demonstrate the utility of tensor function theory as applied to concrete failure prediction. The general results are applicable to any quasi-elastic brittle material, but for the purpose of concrete characterization a specific criterion is developed.

## Development of the Proposed Strength Criterion

The development of a strength criterion for the prediction of the ultimate strength of concrete under multiaxial loading should be formulated from the systematic theories of modern continuum mechanics. The criterion should be validated by accurate and well organized experimental data for the determination of the failure surface for concrete. A strength criterion to predict the failure of concrete is by necessity governed by the failure mechanisms. These failure mechanisms must be related mathematically, forming a failure function.

The tensor function technique of non-linear continuum mechanics associated with a unified approach to constitutive equation development is logically applicable to strength criterion formulation. These functions satisfy the invariance requirement under a group of orthogonal transformations specific to the material symmetry. In addition tensor function theory allows inclusion of any number of stress interaction terms, which gives the theory broad applicability to anisotropic material characterization. Thus the tensor function approach to strength criterion development through non-classical means results in a novel approach and a much improved criterion.

A general strength function has been shown to be expressible as:

$$f(\sigma_{ij}) = 1 \qquad i,j, = 1,2,3 \tag{3.1}$$

where ( $\sigma_{ij}$) are stress components referred to an arbitrary coordinate system. This form of the failure function in equation (3.1) was followed by past investigators presented in chapter two. In general these fracture criteria are functions of the applied stress, but were non-invariant, i.e. William-Warnke [103], Wastiels [98, 100], Kotsovos [51], and others.

A strength function for a given material symmetry (isotropic for concrete) must be invariant under a group of transformations of coordinates, $\{t_{ij}\}$. This insures the scalar polynomial function of the strength criterion is single-valued as indicated by equation (3.1). Additionally it is known that failure is a physical phenomenon which is totally independent of coordinates. Thus the requirement of invariance states:

$$f(\bar{\sigma}_{ij}) = f(\sigma_{ij}) \qquad i,j, = 1,2,3 \tag{3.2}$$

where ($\bar{\sigma}_{ij}$) represents the transformed stress components, also:

$$\bar{\sigma}_{ij} = t_{ir}\, t_{js}\, \sigma_{rs} \qquad i,j,r,s = 1,2,3 \tag{3.3}$$

The strength function is required to be invariant with respect to the material symmetry group . The material symmetry group or isotropy group of a material is defined as the group of transformations of the material coordinates which leave the constitutive equations invariant (Malvern [62]). Invariant quantities for each system of anisotropic materials have been obtained by Smith and Rivlin [88], and Huang [42]. Huang determined the second, fourth, and sixth rank strength tensors in the three-dimensional case for each of the crystal classes from consideration of invariant transformations of the strength function. The invariants for the isotropic material case have been determined from these investigators and are presented in principal stress coordinates. The invariant terms for the isotropic material symmetry case are as follows:

$$
\begin{aligned}
I_1 &= \sigma_1 + \sigma_2 + \sigma_3 \\
I_2 &= -(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) \\
I_3 &= \sigma_1\sigma_2\sigma_3
\end{aligned}
\tag{3.4}
$$

Any strength function for an isotropic material in the form of equation (3.1) is expressible as:

$$f(I_1, I_2, I_3) = 1 \tag{3.5}$$

By coincidence the invariants of the material symmetric class (isotropic) are also the invariants of the stress tensor. The deviatoric tensor is obtained by subtracting the mean normal stress from each of the diagonal elements of the stress tensor. Thus the invariants of the deviatoric tensor are related to the invariants of the stress tensor and material symmetry invariants. The deviatoric invariants are also considered invariants of the isotropic material symmetry. They are expressible in principal stress coordinates as:

$$
\begin{aligned}
J_2 &= 1/6[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] \\
J_3 &= (\sigma_1 - \sigma)(\sigma_2 - \sigma)(\sigma_3 - \sigma)
\end{aligned}
\tag{3.6}
$$

where: $\sigma = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$

The principal directions of the deviatoric tensor are the same as those of the stress tensor, since both represent directions perpendicular to planes having no shear stresses. Therefore any strength function is expressible as a polynomial in the invariant quantities as:

$$f(I_1, J_2, J_3) = 1 \tag{3.7}$$

The proposed strength function of Chen and Chen [14] followed the invariant tensorial function form of equation (3.7). They proposed a two-equation strength criterion using the material invariants discussed. Their proposed strength criterion is as follows:

for the compression-compression region,

$$f(I_1, J_2) = J_2 + \frac{A_u}{3} I_1 = t_u^2 \tag{3.8a}$$

for all other regions,

$$f(I_1, J_2) = J_2 - \frac{1}{6} I_1^2 + \frac{A_u}{3} I_1 = t_u^2 \tag{3.8b}$$

where $A_u$ and $t_u^2$ are material parameters as shown in chapter two equation (2.14).

The strength functions of (3.8) are a combination of invariants. The functions are of quadratic form. The quadratic form has been addressed in chapter two and shown to be inadequate in its definition of the failure envelope for the biaxial principal stress plane ($\sigma_1 - \sigma_2$). The order of the polynomial based on tensor function theory was discussed by Huang [42, 43], Tennyson et.al. [94], Ottosen [72], and Priddy [79]. They suggested higher order terms are necessary to include additional stress interactions. The quadratic form at best can describe a conic curve which cannot yield accurate correlations with experimental data in all four quadrants of the biaxial plane.

Any continuous function is expressible as a polynomial function of the invariants up to a desired order of the polynomial. The requirement of higher order terms within the strength function (3.7) leads to the following modification:

$$f(I_j^{(i)}) = 1 \tag{3.9}$$

where $I_j^{(i)}$ denotes the usage of invariant quantities, I, in the i-th degree and the j-th element. The combination of the invariants proposed for an isotropic material are as follow:

$$\begin{aligned} &\text{first degree: } I_1 \\ &\text{second degree: } I_1^2,\ J_2 \\ &\text{third degree: } I_1^3,\ I_1J_2, J_3,\ I_1^2\sqrt{J_2} \end{aligned} \tag{3.10}$$

System (3.10) represents terms which may be required to form a cubic strength function for isotropic materials.

A combination of the invariant system has been proposed by Cui [18] for usage with concrete. Based on tensor function theory Cui combined the invariant terms into a polynomial combination of the cubic invariants as follows:

$$\begin{aligned} f(I_j^{(i)}) = A_1I_1 + A_{11}I^2 + A_{22}J_2 + A_{111}I_1^3 + A_{122}I_1J_2 \\ + A_{333}J_3 \mp A_{112}I_1^2\sqrt{J_2} = 1 \end{aligned} \tag{3.11}$$

where all 'A's' represent material parameters of the strength tensor. These parameters are determined from seven independent engineering strength tests of the concrete being characterized. The proposed cubic function of equation (3.11) represents a complete set of tensor generators for the isotropic material case up to the third degree.

This investigation is concerned with a biaxial stress condition which causes failure. The proposed cubic equation (3.11) is reduced to six material parameters under a biaxial loading condition. The third invariant of the deviator tensor ($J_3$) is not independent but is a polynomial combination of three other invariant combinations:

$$J_3 = I_3 + \frac{1}{3}I_1J_2 - \frac{1}{27}I_1^3 \tag{3.12}$$

where $I_3 = \emptyset$ in the biaxial state of stress. Thus equation (3.11) is rewritten in the following form for a biaxial state of stress:

$$A_1I_1^2 + A_{11}I_1^2 + A_{22}J_2 + A_{111}I_1^3 + A_{122}I_1J_2 \mp A_{112}I_1^2\sqrt{J_2} = 1 \quad (3.13)$$

The cubic term '$A_{112}$' requires a sign change to completely characterize the four quadrants comprising the biaxial region of stress. The sign of '$A_{112}$' remains positive (+) in all quadrants but the tension-tension quadrant where the sign changes to negative (-). The change was found necessary to allow for a continuous failure envelope in the tension-tension region.

The proposed strength criterion of equation (3.13) requires six independent material constants ($A_1$, $A_{11}$, $A_{111}$, $A_{22}$, $A_{122}$, $A_{112}$), thus to characterize the cubic equation (3.13) to a given strength quality of concrete six independent engineering tests are required. A viable, simple strength criterion should be characterized by the least number of engineering tests which produce a criterion with acceptable accuracy. Therefore this investigation has sought to simplify the proposed equation by Cui through elimination of terms with slight influence.

The strength function presented by Chen and Chen [14] can be shown as a special case of the proposed cubic function of equation (3.13). Their criterion for the compression-compression region is re-written in the following form:

$$\frac{1}{t_u^2} J_2 + \frac{A_u}{3t_u^2} I_1 = 1 \quad (3.14)$$

Comparison of equation (3.14) to the cubic equation of (3.13) reveals the similarities. Four higher order terms in equation (3.13) have been eliminated and the following material parameters are equivalent:

$$A_{22} = \frac{1}{t_u^{\ 2}} \tag{3.15}$$

$$A_1 = \frac{A_u}{3t_u^{\ 2}}$$

A similar analogy can be drawn for the equation of the tension-tension and tension-compression region (3.8b) presented by Chen and Chen. Therefore the proposed strength function of Chen and Chen is merely a reduced form of the cubic function proposed by Cui, equation (3.13).

The strength criterion proposed by Cui requires six independent material strength tests, while the criterion of Chen and Chen requires only three. Through judicious selection of terms from comparison of the two strength criteria, a unified, three-parameter, cubic strength function is proposed. The proposed function is similar to that of Chen and Chen but it includes additional higher-order invariant combinations. The proposed function, recommended for use as a fracture criterion for concrete subjected to biaxial states of stresses, is given as follows:

$$f(I_j^{(i)}) = \frac{a_1}{3} I_1 + A I_1^{\ 2} + \frac{a_3}{27} I_1^{\ 3} + J_2 = B^2 \tag{3.16}$$

where $a_1$, $a_3$, and $B^2$ are material parameters which are determined through simple engineering material tests, and 'A' is a constant value.

The strength criterion of equation (3.16) represents a simple fracture function which satisfies the invariant requirement of isotropic material symmetry. The strength criterion is based on the continuum mechanics approach of tensor function theory. The form presented is analytically simplier than previously proposed criteria which were based on classical plasticity approaches to fracture definition. In addition the proposed criterion represents a complete and unified function capable of fully describing the biaxial failure envelope.

Characterization of the Proposed Strength Criterion

The proposed failure function presented in equation (3.16) is characterized to a given strength quality of concrete by the three material parameters $a_1$, $a_2$, and $B^2$. These parameters are determined from equations based on simple engineering material tests. The engineering material tests required are; uniaxial compressive strength ($f_c$), uniaxial tensile strength ($f_t$), and the biaxial compressive strength ($f_{bc}$).

To solve for the material parameters the proposed fracture function is re-written in terms of the stress conditions imposed within the test specimen during the engineering tests. The parameters are determined by the simultaneous solution of the three resulting linearly independent equations. The material parameters are determined from the following set of equations:

Uniaxial Compression:

$$\sigma_1 = -f_c \quad ; \quad \sigma_2 = \sigma_3 = 0 \tag{3.17}$$

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3 = -f_c \tag{3.18}$$

$$J_2 = \frac{1}{6}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right] = \frac{f_c^2}{3} \tag{3.19}$$

substitute $I_1$ and $J_2$ into equation(3.16) yielding:

$$\frac{-a_1}{3} f_c + Af_c^2 - \frac{a_3}{27} f_c^3 + \frac{f_c^2}{3} = B^2 \tag{3.20}$$

Uniaxial Tension:

$$\sigma_1 = f_t \quad ; \quad \sigma_2 = \sigma_3 = 0 \tag{3.21}$$

$$I_1 = f_t \tag{3.22}$$

$$J_2 = \frac{f_t^2}{3} \tag{3.23}$$

substitute $I_1$ and $J_2$ into equation (3.16) yielding:

$$\frac{a_1}{3} f_t + A f_t^2 + \frac{a_3}{27} f_t^3 + \frac{f_t^2}{3} = B^2 \quad (3.24)$$

Biaxial Compression:

$$\sigma_1 = \sigma_2 = -f_{bc} \quad ; \sigma_3 = 0 \quad (3.25)$$

$$I_1 = -2f_{bc} \quad (3.26)$$

$$J_2 = \frac{f_{bc}^2}{3} \quad (3.27)$$

substitute $I_1$ and $J_2$ into equation (3.16) yielding:

$$\frac{-2a_1}{3} f_{bc} + 4A f_{bc}^2 - \frac{8a_3}{27} f_{bc}^3 + \frac{f_{bc}^2}{3} = B^2 \quad (3.28)$$

These results yield three equations with three unknown material parameters. The three equations (3.20), (3.24), and (3.28) are solved by the method of equivalent simultaneous linear equations and the material parameters are given in terms of the simple material strengths as:

$$a_1 = \frac{[(3A + 1)(f_t^2 - f_c^2)(8f_{bc}^3 - f_c^3) - 3A(f_c^2 - 4f_{bc}^2)(f_t^3 + f_c^3) - (f_c^2 - f_{bc}^2)(f_t^3 + f_c^3)]}{(2f_{bc} - f_c)(f_t^3 + f_c^3) - (f_t + f_c)(8f_{bc}^3 - f_c^3)} \quad (3.29)$$

$$a_3 = \frac{-a_1(f_t + f_c) - 3(A + 1/3)(f_t^2 - f_c^2)}{(f_t^3 + f_c^3)} \quad (3.30)$$

$$B^2 = Af_c^2 - \frac{a_3 f_c^3}{27} + \frac{f_c^2}{3} - \frac{a_1 f_c}{3} \quad (3.31)$$

where $f_c$, $f_t$, and $f_{bc}$ are absolute values. Therefore by measuring or estimating the three material strengths $f_c$, $f_t$, and $f_{bc}$, the proposed fracture criterion is completely characterized.

The function constant 'A' of equation (3.16) is the coefficient of the squared first invariant term ($I_1^2$). The value of the constant 'A' has been determined to be independent of concrete strength quality, but does have an important influence upon the failure envelope shape. Determination of the value for 'A' has been accomplished by comparison of the failure envelope generated from equation (3.16) to the failure envelope implied through experimental data.

In general the value of 'A' influences the overall shape of the biaxial failure envelope. The regions influenced to the greatest extent by this value have been found to be the tension-tension and tension-compression quadrants of the principal stress plane. The value of the constant which yields the highest accuracy within these regions has been determined to be:

$$A = -.34 \qquad (3.32)$$

Comparison to experimental data by using true strength properties ($f_c$, $f_t$ and $f_{bc}$) demonstrates the usefulness of the proposed failure criterion. In Figures (3.1), (3.2), and (3.3) the failure envelope generated from equation (3.16) is presented with the experimental data of Kupfer, Hilsdorf, and Rusch [57] and Nelissen [66]. The value of 'A' is as given in (3.32). The material parameters ($a_1$, $a_3$ and $B^2$) are calculated from equations (3.29), (3.30), and (3.31), respectively, using the true experimental values for $f_c$, $f_t$, and $f_{bc}$. Figure (3.2) is an enlargement of the tension-tension quadrant of Figure (3.1). The biaxial failure envelopes generated from equation (3.16) agree with the experimental envelopes reasonably well. Although both predicted failure envelopes (Figs. 3.1 and 3.3) tend to overestimate the strength of concrete in biaxial compression, except at the equal biaxial compression point ($\sigma_1 = \sigma_2$, $\sigma_3 = 0$).

The strength predicted within the biaxial compression region can be improved if the value of 'A' is changed within this quadrant. The value of the constant which yields the highest accuracy within the biaxial compression quadrant has been determined to be:

$$A = .20 \tag{3.33}$$

The increased accuracy of the proposed failure criterion, by changing the constant value 'A' in the compression-compression region, is demonstrated by comparison to experimental data. The experimental data selected from chapter one are compared with the final form of the proposed strength function in Figures (3.4) through (3.8). The accuracy and ability of the function to conform to variations in the failure envelope shape due to strength quality variations, is demonstrated in these figures.

The advantage of the proposed strength function in comparison to previously presented criteria becomes apparent in Figures (3.9) through (3.12). These figures demonstrate the accuracy of the proposed criterion with respect to the criteria of Wastiels [98, 100] and Chen and Chen [14]. Figure (3.9) presents the three fracture criteria in comparison with the experimental data indicated. The proposed criterion proves to be as accurate as the others in the biaxial compression quadrant. The advantage of the proposed criterion is clearly demonstrated in Figures (3.10) and (3.11), where it proves to be of higher accuracy in the tension-tension and tension-compression regions than the criterion of Chen and Chen. The strength criterion proposed by Wastiels is incapable of failure prediction in the tension-tension region, thus it is not indicated in Figure (3.11). Wastiels recommends using the maximum stress criterion in this region.

The strength criterion proposed by Wastiels appears to predict the failure envelope extremely well in all but the tension-tension region of Figure (3.11). In reality his strength function is artificial and cannot account for slight variations in the shape of the failure envelope due to differing qualities of concrete. This lack of failure envelope control is evident when compared to the experimental data of Nelissen [66] in Figure (3.12). In this case Wastiels criterion overestimates the strength in the entire biaxial compression region. In addition Wastiels criterion is not an invariant form.

The proposed strength function of equation (3.16) is a single, unified equation, fracture criterion for concrete in biaxial states of stress. The fracture criterion proves to be mathematically simple and of higher accuracy than the criterion of Chen and Chen, while requiring the same number of material parameters.

Comparison of the Three-Parameter with the Six-Parameter Criterion

The proposed three-parameter cubic strength criterion of equation (3.16) is a reduced form of the six-parameter cubic strength function given in equation (3.13), i.e.

$$A_1 I_1 + A_{11} I_1^2 + A_{22} J_2 + A_{111} I_1^3 + A_{122} I_1 J_2 \mp A_{112} I_1^2 \sqrt{J_2} = 1 \quad (3.13)$$

The higher order invariant terms $I_1 J_2$ and $I_1^2 \sqrt{J_2}$ are dropped and the material parameter '$A_{11}$' becomes a constant value 'A'. These changes were accomplished by judicious analysis of the six parameters within equation (3.13). To evaluate the effects of the reduced form of the original cubic function (3.13) a comparison of failure envelopes is accomplished.

The strength function of equation (3.13) is a polynomial combination of the invariant terms up to the cubic. This function is satisfied at any combination of stress states which cause failure. The six material parameters ($A_1$, $A_{11}$, $A_{22}$, $A_{111}$, $A_{122}$, $A_{112}$) are material constants determined through strength tests. These parameters are linearly independent, thus six independent experimental strength tests are required. The six parameters are determined by simultaneous solution of the six linearly independent equations formed by solving equation (3.13) for each strength test stress condition.

The experimental failure data of Kupfer, Hilsdorf, and Rusch [57] is again used for evaluation of the six-parameters of equation (3.13), based on the concrete tested. These investigators tested three different compressive strengths of concrete. Each strength requires exclusive material parameters within equation (3.13) for accurate failure envelope representation. The six-parameters obtained are presented in table (3.1) for each strength quality of concrete tested. For comparison the equivalent material parameters for the proposed three-parameter equation (3.16) are also given in table (3.2) for $A = -.34$, and table (3.3) for $A = .20$.

| Concrete Strength (PSI) | $A_1$ | $A_{11}$ | $A_{22}$ | $A_{111}$ | $A_{122}$ | $A_{112}$ |
|---|---|---|---|---|---|---|
| 2700 | -8.762 | -33.923 | 89.000 | -6.165 | -110.481 | 98.745 |
| 4450 | -8.729 | -39.995 | 138.408 | -17.500 | -250.285 | 181.025 |
| 8350 | -8.351 | -43.690 | 164.366 | -22.948 | -328.088 | 226.142 |

TABLE 3.1 SIX-PARAMETER CRITERION COEFFICIENTS

| Concrete Strength (PSI) | $A_1$ | $A_{11}$ | $A_{22}$ | $A_{111}$ |
|---|---|---|---|---|
| 2700 | 11.274 | -33.028 | 97.142 | -12.922 |
| 4450 | 11.274 | -33.028 | 97.142 | -12.922 |
| 8350 | 11.272 | -31.735 | 93.338 | -12.894 |

TABLE 3.2 THREE-PARAMETER CRITERION COEFFICIENTS (A = -.34)

| Concrete Strength (PSI) | $A_1$ | $A_{11}$ | $A_{22}$ | $A_{111}$ |
|---|---|---|---|---|
| 2700 | 10.043 | 4.425 | 22.123 | .756 |
| 4450 | 10.043 | 4.425 | 22.123 | .756 |
| 8350 | 10.047 | 4.409 | 22.045 | .710 |

TABLE 3.3 THREE-PARAMETER CRITERION COEFFICIENTS (A = .20)

<u>NOTE</u>: for Tables (3.2) and (3.3)

$A_1 = a_1/3B^2$ ; $A_{11} = A/B^2$ ; $A_{22} = a_3/27B^2$ ; $A_{111} = 1/B^2$

The cubic function (eq. 3.13) is represented using the related six material parameters from table (3.1) in Figures (3.13) through (3.15). The proposed three-parameter strength criterion with $A = -.34$ for all regions, is present also in these figures. The complete cubic function (eq. 3.13) proves to be highly accurate, as it complies with the implied experimental biaxial failure envelope extremely well.

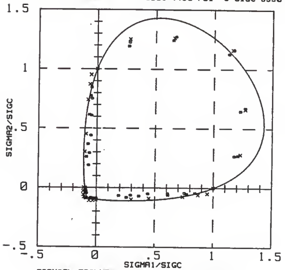
For additional comparison the complete cubic strength function of equation (3.13) is compared to the reduced form of the three parameter strength criteria proposed in this investigation using both values of 'A' and Chen and Chen in Figures (3.16) through (3.18). Additionally Wastiels criterion is compared to the cubic strength function (3.13) and the reduced three-parameter form proposed in this investigation in Figures (3.19) through (3.21).

The cubic function proposed by Cui proves to be of the highest accuracy when compared with all others. The biaxial failure envelope generated from Cui's cubic function complies with the experimentally determined envelope exceptionally well, as can be seen within Figures (3.16) through (3.21). In further demonstration of this function's capabilities the regions of tension-tension and compression-tension are enlarged in Figures (3.16) through (3.21). The function fit to the data points is superior to any previously presented functions for failure prediction.

The proposed three-parameter function of this investigation given in equation (3.16) proves very accurate. The proposed strength criterion is generated within Figures (3.16) through (3.21) along with the criterion of Cui. The values for the constant term 'A' are changed to conform to the active quadrant, as previously recommended. The proposed three-parameter function is shown to be only slightly less accurate than the complete six-parameter function. Indeed within the region of greatest design concentration (compression-compression) the three-parameter function is equally as accurate.

The complete cubic function of Cui [18] proves to be of the highest accuracy, although the requirement of six engineering tests to characterize the function for each strength quality of concrete may be prohibitive in many design instances. The proposed strength criterion requires only three material parameters. These parameters are obtained from three simple engineering material tests. Furthermore, two of the engineering tests may be estimated based on experimentally proven biaxial stress conditions of failure. The biaxial compressive and uniaxial tensile strengths of concrete have been shown to be known percentages of the uniaxial compressive strength as discussed in the conclusion of chapter one. Higher failure envelope accuracy is achieved if the exact values are obtained, but only a slight loss of accuracy occurs if an intelligent estimate is made. Thus the proposed strength function can completely characterize the biaxial failure region of a concrete with only a single uniaxial compression strength test.

EXPERIMENTAL BIAXIAL FAILURE DATA BY:
KUPFER, HILSDORF & RUSCH(1969) SQUARE PLATES
X-SIGC=2700 PSI *-SIGC=4450 PSI o-SIGC=8350 PSI

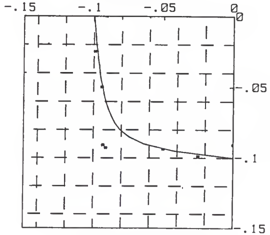



BIAXIAL FAILURE ENVELOPE FOR CONCRETE
PROPOSED FAILURE CRITERION-A*I1^2 A=-.34

Fig. 3.1

EXPERIMENTAL BIAXIAL FAILURE DATA BY:
KUPFER, HILSDORF & RUSCH(1969)-*-SIGC=4450PSI

SIGMA2/SIGC




TENSION-TENSION FAILURE REGION.
PROPOSED CRITERION WITH A*I1^2. A=-.34

Fig. 3.2

EXPERIMENTAL BIAXIAL FAILURE DATA BY:
NELISSEN(1972) SQUARE CUBES
O-SIGC=3270 PSI *-SIGC=4513 PSI

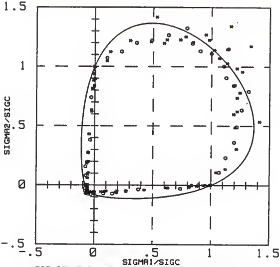



Fig. 3.3

BIAXIAL FAILURE ENVELOPE FOR CONCRETE
PROPOSED FAILURE CRITERION-A*I1^2 A=-.34

EXPERIMENTAL BIAXIAL FAILURE DATA BY:
KUPFER, HILSDORF & RUSCH(1969) SQUARE PLATES
X-SIGC=2700 PSI *-SIGC=4450 PSI o-SIGC=8350 PSI

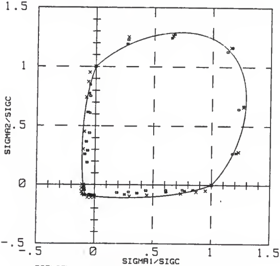



Fig. 3.4

BIAXIAL FAILURE ENVELOPE FOR CONCRETE
PROPOSED FAILURE CRITERION-A*I1^2
T-C,T-T A=-.34 C-C A=.2

EXPERIMENTAL BIAXIAL FAILURE DATA BY:
NELISSEN(1972) SQUARE CUBES
O-SIGC=3270 PSI *-SIGC=4513 PSI

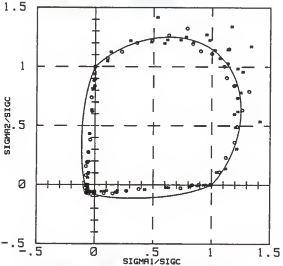



Fig. 3.5

BIAXIAL FAILURE ENVELOPE FOR CONCRETE
PROPOSED FAILURE CRITERION-A*I1^2
T-C,T-T A=-.34 C-C A=.2

EXPERIMENTAL BIAXIAL FAILURE DATA BY:
TASUJI, SLATE & NILSON(1978) SQUARE PLATES
X-SIGC=4827

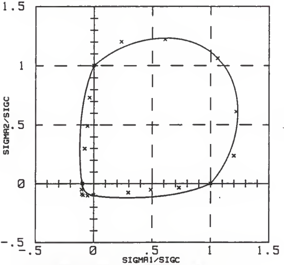
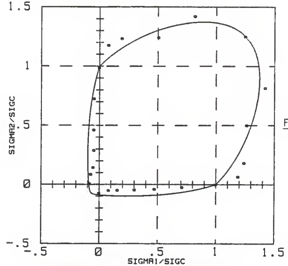



Fig. 3.6

BIAXIAL FAILURE ENVELOPE FOR CONCRETE
PROPOSED FAILURE CRITERION-A*I1^2
T-C,T-T A=-.34 C-C A=.2

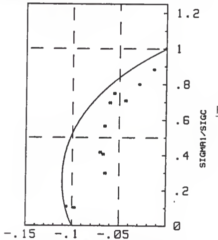
EXPERIMENTAL BIAXIAL FAILURE DATA BY:
VILE, G.W.(1968) SQUARE PLATES
O-SIGC=6458 PSI




BIAXIAL FAILURE ENVELOPE FOR CONCRETE
PROPOSED FAILURE CRITERION-A*I1^2
T-C,T-T A=-.34 C-C A=.2

Fig. 3.7

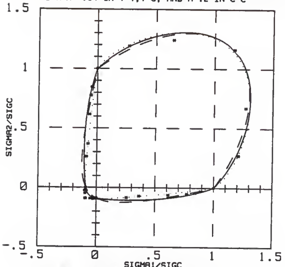
EXPERIMENTAL BIAXIAL FAILURE DATA BY:
JOHNSON & LOWE(1969) HOLLOW CYLINDERS
*-SIGC=6420 PSI




TENSION-COMPRESSION FAILURE AREA
PROPOSED FAILURE CRITERION OF
A*I1^2. T-C,T-T A=-.34

Fig. 3.8

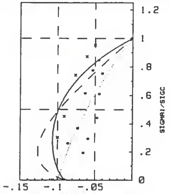
EXPERIMENTAL FAILURE DATA BY: KUPFER, HILSDORF
& RUSCH(1969) SQUARE PLATES *-SIGC=4450 PSI
SOLID LINE-PROPOSED 3-PARAMETER FAILURE FUNCTION
WITH A=-.34 IN T-T,T-C; AND A=.2 IN C-C




FAILURE ENVELOPES FOR BIAXIAL STRESS
DASHED LINE - CHEN & CHEN(1975)
DOTTED LINE - J. WASTIELS(1979)

Fig. 3.9

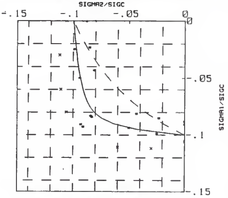
EXPERIMENTAL FAILURE DATA BY:
KUPFER, HILSDORF & RUSCH(1969) SQUARE PLATES
X-SIGC=2700 PSI *-SIGC=4450 PSI O-SIGC=8350 PSI




TENSION-COMPRESSION FAILURE REGION
SOLID LINE - PROPOSED 3-PARAMETER
A=-.34, T-T,T-C
DASHED LINE - CHEN & CHEN(1975)
DOTTED LINE - J. WASTIELS(1979)

Fig. 3.10

EXPERIMENTAL FAILURE DATA BY:
KUPFER, HILSDORF & RUSCH(1969) SQUARE PLATES
X-SIGC=2700 PSI *-SIGC=4450 PSI O-SIGC=8350 PSI




TENSION-TENSION FAILURE REGION.
SOLID LINE-PROPOSED CRITERION WITH A=1I^2. A=-.34
DASHED LINE-CHEN & CHEN'S CRITERION

Fig. 3.11

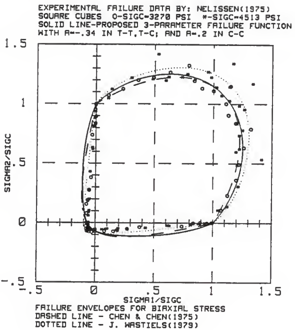

EXPERIMENTAL FAILURE DATA BY: NELISSEN(1975)
SQUARE CUBES O-SIGC=3270 PSI *-SIGC=4513 PSI
SOLID LINE-PROPOSED 3-PARAMETER FAILURE FUNCTION
WITH A=-.34 IN T-T,T-C; AND A=.2 IN C-C
1.5
1
.5
0
-.5
SIGMA2/SIGC
-.5
0
.5
1
1.5
SIGMA1/SIGC
FAILURE ENVELOPES FOR BIAXIAL STRESS
DASHED LINE - CHEN & CHEN(1975)
DOTTED LINE - J. WASTIELS(1979)

Fig. 3.12

EXPERIMENTAL FAILURE DATA BY: KUPFER, HILSDORF & RUSCH(1969) SQUARE PLATES X-SIGC=2700 PSI
SOLID LINE - PROPOSED THREE-PARAMETER FUNCTION WITH A=-.34

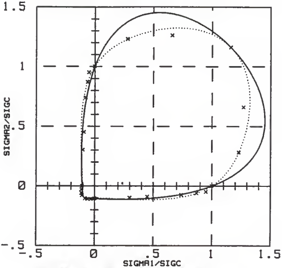



Fig. 3.13

FAILURE ENVELOPES FOR BIAXIAL STRESS
DOTTED LINE - PROPOSED CUBIC FUNCTION (Cui)

EXPERIMENTAL FAILURE DATA BY: KUPFER, HILSDORF & RUSCH(1969) SQUARE PLATES *-SIGC=4450 PSI
SOLID LINE - PROPOSED THREE-PARAMETER FUNCTION WITH A=-.34

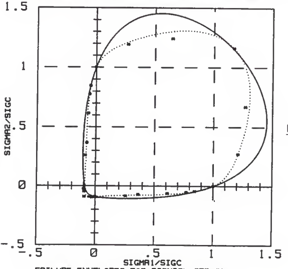
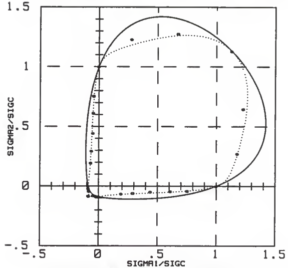



Fig. 3.14

FAILURE ENVELOPES FOR BIAXIAL STRESS
DOTTED LINE - PROPOSED CUBIC FUNCTION (Cui)

EXPERIMENTAL FAILURE DATA BY: KUPFER, HILSDORF
& RUSCH(1969) SQUARE PLATES O-SIGC=8350 PSI
SOLID LINE - PROPOSED THREE-PARAMETER FUNCTION
WITH A=-.34




FAILURE ENVELOPES FOR BIAXIAL STRESS
DOTTED LINE - PROPOSED CUBIC FUNCTION (Cui)

Fig. 3.15

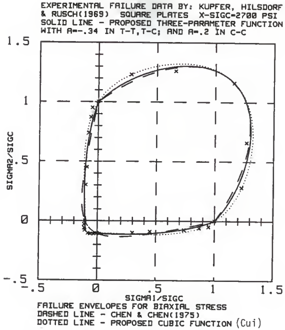



Fig. 3.16a

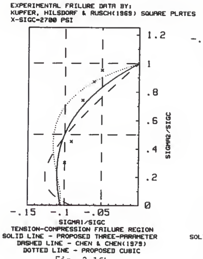



Fig. 3.16b

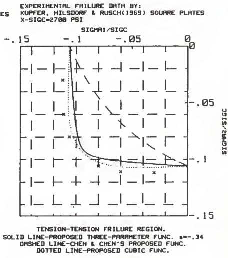



Fig. 3.16c

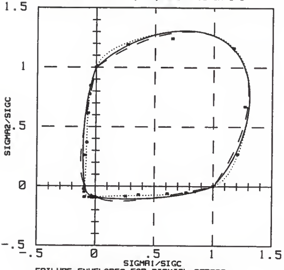



Fig. 3.17a

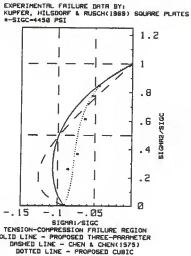



Fig. 3.17b

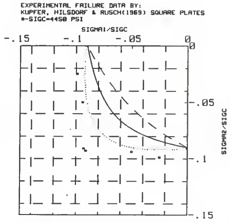



Fig. 3.17c

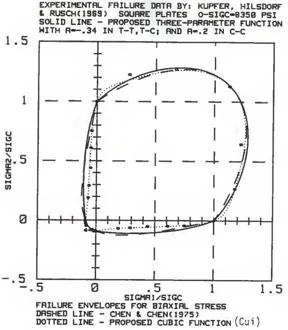



Fig. 3.18a

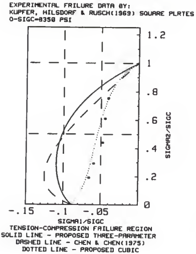



Fig. 3.18b

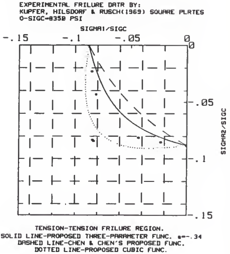



Fig. 3.18c

EXPERIMENTAL FAILURE DATA BY: KUPFER, HILSDORF & RUSCH(1969) SQUARE PLATES X-SIGC=2700 PSI
SOLID LINE - PROPOSED THREE-PARAMETER FUNCTION
WITH A=-.34 IN T-T,T-C; AND A=.2 IN C-C

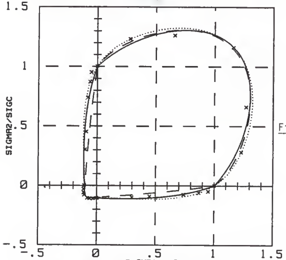



Fig. 3.19a

FAILURE ENVELOPES FOR BIAXIAL STRESS
DASHED LINE - WASTIELS(1979)
DOTTED LINE - PROPOSED CUBIC FUNCTION (Cui)

EXPERIMENTAL FAILURE DATA BY:
KUPFER, HILSDORF & RUSCH(1969) SQUARE PLATES
X-SIGC=2700 PSI

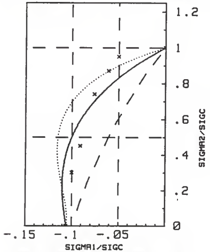



Fig. 3.19b

TENSION-COMPRESSION FAILURE REGION
SOLID LINE - PROPOSED THREE-PARAMETER
DASHED LINE - WASTIELS(1979)
DOTTED LINE - PROPOSED CUBIC

EXPERIMENTAL FAILURE DATA BY: KUPFER, HILSDORF & RUSCH(1969) SQUARE PLATES *-SIGC=4450 PSI
SOLID LINE - PROPOSED THREE-PARAMETER FUNCTION
WITH A=-.34 IN T-T,T-C; AND A=.2 IN C-C

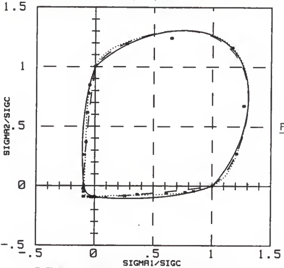



Fig. 3.20a

FAILURE ENVELOPES FOR BIAXIAL STRESS
DASHED LINE - WASTIELS(1979)
DOTTED LINE - PROPOSED CUBIC FUNCTION(Cui)

EXPERIMENTAL FAILURE DATA BY:
KUPFER, HILSDORF & RUSCH(1969) SQUARE PLATES
*-SIGC=4450 PSI

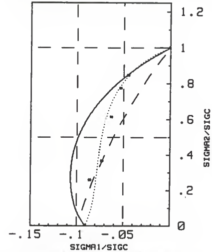



Fig. 3.20b

TENSION-COMPRESSION FAILURE REGION
SOLID LINE - PROPOSED THREE-PARAMETER
DASHED LINE - WASTIELS(1979)
DOTTED LINE - PROPOSED CUBIC

EXPERIMENTAL FAILURE DATA BY: KUPFER, HILSDORF & RUSCH(1969) SQUARE PLATES O-SIGC=8350 PSI
SOLID LINE - PROPOSED THREE-PARAMETER FUNCTION WITH A=-.34 IN T-T,T-C; AND A=.2 IN C-C

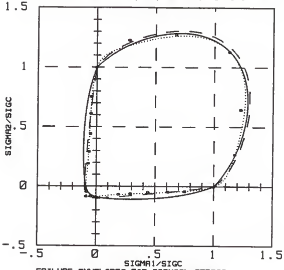



Fig. 3.21a

FAILURE ENVELOPES FOR BIAXIAL STRESS
DASHED LINE - WASTIELS(1979)
DOTTED LINE - PROPOSED CUBIC FUNCTION

EXPERIMENTAL FAILURE DATA BY:
KUPFER, HILSDORF & RUSCH(1969) SQUARE PLATES
O-SIGC=8350 PSI

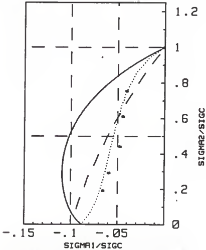



Fig. 3.21b

TENSION-COMPRESSION FAILURE REGION
SOLID LINE - PROPOSED THREE-PARAMETER
DASHED LINE - WASTIELS(1979)
DOTTED LINE - PROPOSED CUBIC

# CHAPTER FOUR

# SUMMARY AND CONCLUSION

## Summary

This investigation has proposed an improved strength criterion for prediction of concrete failure under biaxial loading. The variable characteristics and complex composition of concrete dictate a need for a comprehensive mathematical strength function. The simplest classical theories applied to multiaxial states of stress are not acceptable.

A review of the literature uncovered numerous accounts of experimental data for characterization of the biaxial failure envelope. Further investigation revealed a critical assessment of the experimental data was required, as much of it is in error. Five sets of experimental observations were selected in order to validate the newly proposed strength criterion.

In reviewing strength criteria published to date several pertinent investigations were discussed. The strength criteria proposed for concrete were found to lack suitability for general usage. The expanding field of nonlinear continuum mechanics and the growth in popularity of fiber-reinforced composite materials has promoted research in material characterization through the use of tensorial function theory. The theory has found wide acceptance in development of constitutive equations and strength criteria for anisotropic brittle materials. The proposed strength criterion reflects the recent developments in tensor theory as applied to strength functions.

The proposed strength criterion for concrete under a biaxial state of stress is:

$$\frac{a_1}{3} I_1 + AI_1^{\ 2} + \frac{a_3}{27} I_1^{\ 3} + J_2 = B^2 \tag{3.16}$$

where $a_1$, $a_3$, and $B^2$ are determined from equations (3.29), (3.30), and (3.31), respectively. The values for the constant term 'A' are:

for the compression-compression region:

$A = .20$

for all other regions:

$A = -.34$

The proposed cubic strength function, equation (3.16), is a unified, invariant, three-parameter equation for the prediction of concrete failure. The criterion complies with mathematical and physical requirements imposed upon a strength function for concrete. The features which confirm this compliance are:

1) The function is a unified, single equation, strength criterion. The function is highly descriptive and capable of accurately predicting failure in all biaxial states of stresses. The biaxial failure envelope conforms to the experimentally determined shape exceptionally well.

2) The function is scalar and invariant. The criterion will predict failure in any given coordinate system with equal accuracy, for the isotropic material symmetry class.

3) The cubic strength function is mathematically simple. The function may be used within computer based analysis or by classical methods.

4) The strength function is easily characterized for a given quality of concrete. Three material strength properties are required for complete characterization. These properties can be obtained from simple engineering

material tests or estimated through experience from previous material tests.

5) The function is dependent on both the hydrostatic and deviatoric components of the stress tensor. Mathematically the first invariant ($I_1$) is equivalent to hydrostatic pressure, and the second invariant of the deviator tensor ($J_2$) is equivalent to the shear state of stress.

Conclusion

The failure envelope within the biaxial stress plane for the proposed strength criterion has demonstrated accurate prediction of failure for all regions. In comparison to previously proposed strength criteria the accuracy is comparable or higher. The newly proposed criterion demonstrates mathematical superiority as it is easily characterized to the given strength quality of a particular concrete, were as past strength criterion require complex statistical methods of characterization.

BIBLIOGRAPHY


1. Abrams, D.A., "Design of Concrete Mixtures", Chicago, Lewis Institute, Structural Materials Research Laboratory, Dec. 1918, pp. 20, Bulletin No. 1.

2. Admas, M. and Sines, G., "Methods for Determining the Strength of Brittle Materials in Compressive Stress States", J. of Test and Eval., Vol. 4, No. 6, Nov. 1976 pp. 383-396.

3. Andenaes, E., Gerstle, K., and Ko, H., "Response of Mortar and Concrete to Biaxial Compression", J. of the Engineering Mechanics Division, A.S.C.E., Vol. 103, No. EM4, Proc. Paper 13115, Aug. 1977, pp. 515-526.

4. Atkinson, R.H. and Ko, H., "A Fluid Cushion, Multiaxial Cell for Testing Cubical Rock Specimens", Int. J. of Rock Mech. Min., Sci. and Geo. Mech. Abst., Vol. 10, pp. 351-361. Pergamon Press, 1973.

5. Avram, G. et.al., Concrete Strength and Strains, Elsevier Scientific Publishing Co., 1981.

6. Bernhardt, C.J., "Discussion of Strength of Concrete Under Combined Stresses", J. of A.C.I. Proc., Vol. 55, No. 9, Mar. 1959, pp. 1035-1041.

7. Beskos, D.E., "Fracture of Plain Concrete Under Biaxial Stresses", Cem. Concr. Res., Vol. 4, No. 6, Nov. 1974, pp. 979-985.

8. Bremer, F., and Steinsdorfer, F., "Bruchfestigkeiten und Bruchverformung Von Beton unter Mehraxialer Belastung Bei Raumtemperatur", Deutscher Ausschuss Für Stahlbeton, Heft 263, Berlin 1976.

9. Bresler, B. and Pister, K.S., "Strength of Concrete Under Combined Stresses", J. of A.C.I., Proc., Vol. 55, No. 3, Sept. 1958, pp. 321-345.

10. Bresler, B. and Pister, K.S., "Failure of Plain Concrete Under Combined Stresses", Trans. Am. Soc. Civil Eng., Vol. 122, 1957, pp. 1049-1068.

11. Buyukozturk, O., Nilson, A., and Slate, F., "Stress Strain Response and Fracture of a Concrete Model in Biaxial Loading", J. of A.C.I., Proc., Vol. 68, Aug. 1971, pp. 590-599.

12. Carino, N. and Slate, F., "Limiting Tensile Strain Criterion for Failure of Concrete", J. of A.C.I., Proc., Vol. 73, 1976, pp. 160-165.

13. Chen, A.C.T., "Constitutive Relations of Concrete and Punch Indentation Problems", Thesis presented to Lehigh University, Bethlehem, PA, Ph.D., 1973.

14. Chen, A.C.T. and Chen, W.F., "Constitutive Relations for Concrete", J. of Eng. Mech. Div., Aug. 1975, pp. 465-480.

15. Chen, W.F., "Extensibility of Concrete and Theorems of Limit Analysis", J. of Eng. Mech. Div., Proc. of A.S.C.E., Vol. 96, No. EM3, June 1970, pp. 341-352.

16. Chen, W.F., and Yuan, R.L., "Tensile Strength of Concrete: Double-Punch Test", A.S.C.E., J. of Struct. Div., Vol. 106, No. 8, Aug. 1980, pp. 1673-1693.

17. Cornet, I. and Grassi, R.C., "Study of Theories of Fracture Under Combined Stresses", J. of Basic Eng., Vol. 83, No. 1, 1961, pp. 39-44.

18. Cui, L., "Cubic Strength Function for Concrete", unpublished report, 1985, Kansas State University.

19. Davis, E., Troxell, G.E. and Wiskocil, C.T., The Testing and Inspection of Engineering Materials, Appendix E., McGraw-Hill Book Co., 3rd, 1964.

20. Desai, C.S. and Siriwardane, H.J., Constitutive Laws for Engineering Materials with Emphasis on Geological Materials, Prentice Hall, 1984.

21. Dougill, J.W., "An Approximate Assessment of the Effects of Heterogeneity on the Strength and Mode of Failure of Concrete Systems", Structure, Solid Mech. and Eng. Design, Proc. of the Southampton 1969 Civil Eng. Materials Conf., Edited by M. Te'eni, Wiley-Interscience, 1971.

22. Drucker, D.C., "A More Fundamental Approach to Stress-Strain Relations", Proc., First United States National Congress for Applied Mechanics, A.S.M.E., 1951, pp. 487-491.

23. Drucker, D.C. and Prager, W., "Soil Mechanics and Plasticity Analysis of Limit Design", Quarterly of Applied Mathematics, Vol. X, No. 2, 1952, pp. 157-165.

24. Drucker, D.S., Palmer, A.C. and Maier, G., "Normality Relations and Convexity of Yield Surfaces for Unstable Materials or Structural Elements", J. of Applied Mech., Vol. 34, June 1967, p. 464.

25. Ehrenburg, D.O., "Fracture Criteria for Brittle Materials", J. of Testing and Evaluation, Vol. 4, No. 3, May 1976, pp. 200-208.

26. Ehrenburg, D.O., "Biaxial Analogs of Triaxial Stresses", J. of Testing and Evaluation, Vol. 7, No. 1, Jan. 1979, pp. 18-23.

27. Foppl, A., Reports from the Laboratory for Engineering Mechanics, No. 27 and 28, Technischen Hochschule, Munchen, 1899 and 1900.

28. Forman, G.W., "A Distortion Energy Failure Theory for Orthotropic Materials", J. of Eng. for Industries, Trans. of A.S.M.E., Vol. 94, Nov. 1972, pp. 1073-1078.

29. Fumagalli, E. et.al., "Strength of Concrete Under Biaxial Compression", J. of A.C.I., Proc., Vol. 62, No. 9, Sept. 1965, pp. 1187-1198.

30. Gerstle, K.H., "Material Behavior Under Various Types of Loading", Proc. of the Workshop on High Strength Concr., Univ. of Ill., at Chicago Circle, Dec. 2-4, 1979, pp. 43-78.

31. Gerstle, K.H., et.al., "Behavior of Concrete Under Multiaxial Stress States", J. of Eng. Mech. Div. A.S.C.E., Vol. 106, No. EM6, Dec. 1980, p. 1383.

32. Gerstle, K.H., "Simple Formulation of Biaxial Concrete Behavior", J. of A.C.I., Proc., Vol. 78, No. 1, Jan.-Feb. 1981, pp. 62-68.

33. Gol'denblat, I.I. and Kopnov, V.A., "Strength of Glass-Reinforced Plastics in the Complex Stress State", Mekhanika Polimerov, Vol. 1, No. 2, Mar.-Apr. 1965, pp. 70-78.

34. Goode, C.D. and Helmy, M.A., "The Strength of Concrete Under Combined Shear and Direct Stress", Mag. of Concrete Research, Vol. 19, No. 59, June 1967, pp. 105-112.

35. Hannant, D.J. and Frederick, C.O., "Failure Criteria for Concrete in Compression", Mag. of Concrete Research, Vol. 20, No. 64, Sept. 1968, pp. 137-144.

36. Herrmann, L.R., "Effect of End Conditions on Test Results for Concrete", Cem. Concr. Res., Vol. 8, No. 1, Jan. 1978, pp. 25-36.

37. Hill, R., "A Theory of the Yielding and Plastic Flow of Anisotropic Metals", Proceedings of the Royal Society, Series A, Vol. 193, 1948, pp. 281.

38. Hoffman, O., "The Brittle Strength of Orthotropic Materials", J. of Composite Materials, Vol. 1, No. 2, April 1967, pp. 200-205.

39. Hruban, I. and Vitek, B., "Failure Theory of Concrete in a Biaxial State of Stress", Structure, Solid Mech. and Eng. Design, Proc. of the Southampton 1969 Civil Eng. Materials Conf., Edited by M. Te'eni, Wiley-Interscience, 1971.

40. Hu, K.K. and Swartz, S.E., "A Proposed Generalized Material Failure Theory", Proc. of the 15th Midwestern Mech. Conf., Univ. of Ill. at Chicago Circle, Chicago, IL, March 23-25, 1977, pp. 144-147.

41. Hu, K.K., Swartz, S.E. and Huang, C.L., "A Proposed Generalized Constitutive Equation for Nonlinear Para-Isotropic Materials", Internal Report, Dept. of Civil Eng., Kansas State University, Manhattan, KS, 1978.

42. Huang, C.L., "On Strength Functions for Anisotropic Materials", Proc. Symmetry, Similarity and Group-Theoretic Methods in Mechanics, The Univ. of Calgary, Canada, 1974.

43. Huang, C. and Kirmser, P.G., "A Criterion of Strength for Orthotropic Materials", Fibre Science Technology, Vol. 8, 1975, pp. 103-112.

44. Iyengar, K.T., et.al., "Strength of Concrete Under Biaxial Compression", J. of the American Concrete Institute, Vol. 62, No. 2, Feb. 1965, pp. 239-249.

45. Johnson, R.P. and Lowe, P.G., "Behavior of Concrete Under Biaxial and Triaxial Stress", International Conference on Structure, Solid Mechanics and Engineering Design. Civil Eng. Materials, Southampton Univ., 21-25 Apr., 1969, pp. 1039-1051.

46. Kaminski, B.E. and Lantz, R.B., "Strength Theories for Anisotropic Materials", Composite Materials: Testing and Design, ASTM STP 400 (1969), pp. 160-9.

47. Kaplan, M.F., "Strains and Stresses of Concrete at Initiation of Cracking and Near Failure", J. of A.C.I., Proc., Vol. 60, No. 7, July 1963, pp. 853-879.

48. Ko, H. and Sture, S., "Three-Dimensional Mechanics Characterization of Anisotropic Composites", J. of Composite Materials, Vol. 8, Apr. 1974, pp. 178-190.

49. Kobayashi, S. and Koyanagi, W., "Failure Criterion of Concrete Subjected to Multi-Axial Compression", J. of Society for Materials Science, Japan, Vol. 16, 1967, pp. 897-902.

50. Kotsovos, M.D., "Effect of Stress Path on the Behavior of Concrete Under Triaxial Stress States", J. of A.C.I., Vol. 76, No. 2, Feb. 1979, pp. 213-223.

51. Kotsovos, M.D., "Mathematical Description of the Strength Properties of Concrete Under Generalized Stress", Mag. of Concrete Res., Vol. 31, No. 108, Sept. 1979, pp. 151-158.

52. Kotsovos, M.D., "Fracture Processes of Concrete Under Generalized Stress States", Materials and Structures: Research and Testing, Vol. 12, No. 72, Nov.-Dec. 1979, pp. 431-437.

53. Kotsovos, M.D. and Newman, J.B., "Behavior of Concrete Under Multiaxial Stress", J. of A.C.I., Proc., Vol. 74, No. 9, Sept. 1977, pp. 443-446.

54. Kotsovos, M.D. and Newman, J.B., "Mathematical Description of the Deformational Behavior of Concrete Under Complex Loading", Mag. of Concr. Res., Vol. 31, No. 107, June 1979, pp. 77-90.

55. Krishnaswamy, K., "Strength of Microcracking of Plain Concrete Under Triaxial Compression", J. of A.C.I., Proc., Vol. 65, Oct. 1968, pp. 856-862.

56. Kupfer, H. and Gerstle, K., "Behavior of Concrete Under Biaxial Stresses", Proc. of A.S.C.E., Vol. 99, No. EM4, Aug. 1973, pp. 853-866.

57. Kupfer, H., Hilsdorf, H. and Rusch, H., "Behavior of Concrete Under Biaxial Stresses", J. of A.C.I., Proc., Vol. 66, No. 8, Aug. 1969, pp. 656-666.

58. Lade, P.V., "Elasto-Plastic Stress-Strain Theory for Cohesionless Soil with Curved Yield Surfaces", International Journal of Solids and Structures, Vol. 13, Pergamon Press, Inc., N.Y., N.Y., Nov. 1977, pp. 1019-1035.

59. Lade, P.V., "Three-Parameter Failure Criterion for Concrete", J. of Eng. Mech. Div., Proc. of the A.S.C.E., Vol. 108, No. EM5, Oct. 1982, pp. 850-863.

60. Liu, T., Nilson, A. and Slate, F., "Stress-Strain Response and Fracture of Concrete in Uniaxial and Biaxial Compression", J. of A.C.I., Proc., Vol. 69, May 1972, pp. 291-295.

61. Mahmood, N. and Hannant, D., "The Strength of Concrete Subjected to Compression-Compression-Tension Stress Systems", J. of Testing and Evaluation, Vol. 3, No. 2, Mar. 1975, pp. 107-112.

62. Malvern, L.E., Introduction to the Mechanics of a Continuous Medium, Prentice-Hall, Inc., 1969.

63. McHenery, D. and Karni, J., "Strength of Concrete Under Combined Tensile and Compressive Stress", J. of A.C.I., Vol. 54, No. 10, Apr. 1958, pp. 829-840.

64. Mills, L. and Zimmerman, R., "Compressive Strength of Plain Concrete Under Multiaxial Loading Conditions", J. of A.C.I., Proc., Vol. 67, Oct. 1970, pp. 802-807.

65. Nadai, A., Theory of Flow and Fracture of Solids, McGraw-Hill Book Co., Second Edition, Vol. 1, 1950.

66. Nelissen, L., "Biaxial Testing of Normal Concrete", Heron, Vol. 18, No. 1, 1972.

67. Newman, J.B., "Apparatus for Testing Concrete Under Multiaxial States of Stress", Magazine of Concrete Research, Vol. 26, No. 88, Sept. 1984, pp. 229-238.

68. Newman, J.B., and Kotsovos, M.D., "Failure Criteria for Concrete Under Combinations of Stress", Proc. of the 2nd International Conference on Mechanical Behavior of Materials, 16-20 Aug. 1976, pp. 1431-1435.

69. Newman, K., "The Structure and Properties of Concrete - An Introductory Review", Proc. of an International Conference, London, Sept. 1965. The Structure of Concrete and its Behavior Under Load, Cement and Concrete Association, London, 1968.

70. Newman, K., "Criteria for the Behavior of Plain Concrete Under Complex States of Stress", Proc. of an International Conference, London, Sept. 1965. The Structure of Concrete and its Behavior Under Load, Cement and Concrete Association, London, 1968.

71. Newman, K. and Newman, J.B., "Failure Theories and Design Criteria for Plain Concrete", Structure, Solid Mech. and Eng. Design, Proc. of the Southampton 1969 Civil Eng. Materials Conf., Edited by M. Te'eni, Wiley-Interscience, 1971, pp. 963-995.

72. Ottosen, N., "A Failure Criterion for Concrete", J. of the Engineering Mechanics Division, Proc. of the A.S.C.E., Vol. 103, No. EM4, Aug. 1977, pp. 527-535.

73. Palaniswamy, R. and Shah, S.P., "Fracture and Stress-Strain Relationship of Concrete Under Triaxial Compression", J. of Structural Div., Proc. of the A.S.C.E., Vol. 100, 1974, pp. 901-916.

74. Paul, B., "Macroscopic Criteria for Plastic Flow and Brittle Fracture", Fracture, An Advance Treatise, Edited by H. Liebowitz, Vol. II, Ch. 4, Academic Press, 1968.

75. Paul, B., "A Generalized Pyramidal Fracture and Yield Criteria", International Journal of Solids and Structures, Vol. 4, 1968, pp. 175-196.

76. Paul B., and Mirandy, L., "An Improved Fracture Criterion for Three-Dimensional Stress States", J. of Engineering Materials and Technology, April 1976, pp. 159-163.

77. Pandit, G. and Tanwani, N., "Behavior of Concrete in Biaxial Compression", Indian Concrete Journal, Vol. 49, No. 2, Feb. 1975, pp. 39-45.

78. Pletka, B.J. and Wiederhorn, S.M., "Comparison of Failure Predictions by Strength and Fracture Mechanics Techniques", J. of Material Science, Vol. 17, No. 5, May 1982, pp. 1247-1268.

79. Priddy, T.G., "A Fracture Theory for Brittle Anisotropic Materials", J. of Engineering and Technology, Vol. 96, Apr. 1974, pp. 91-96.

80. Richart, F.E., Brandtzaeg, A. and Brown, R.L., "The Behavior of Plain and Spirally Reinforced Concrete in Compression", Univ. of Illinois Engineering Experiment Station, Bulletin No. 190, Apr. 1929, p. 72.

81. Robutti, G., Ronzoni, E. and Ottosen, N.S., "Failure Strength and Elastic Limit for Concrete: A Comparitive Study", Transactions of the International Conference of Structural Mechanics and Reactor Technology, 5th, Vol. H. Structural Engineering of Prestressed Reactor Pressure Vessels, Berlin, Ger. Aug. 13-17, 1979, Pub. 1B. North-Holland Publ. Co., Amsterdam, Neth., 1979.

82. Romstad, K.M., Taylor, M.A. and Herrmann, L.R., "Numerical Biaxial Characterization for Concrete", J. of Engineering Mechanics Div., Proc. of A.S.C.E., Vol. 100, No. 5, Oct. 1974, pp. 935-948.

83. Rosenthal, I. and Glucklich, J., "Strength of Plain Concrete Under Biaxial Stress", J. of A.C.I., Proc., Vol. 67, No. 1970, pp. 903-914.

84. Rüsch, H., "Physical Problems in the Testing of Concrete", Library Translations, No. 86, Cement and Concrete Association, 1959, pp. 1-21.

85. Saucier, K.L., "Equipment and Test Procedure for Determining Multiaxial Tensile and Combined Tensile-Compressive Strength of Concrete", Tech. Report C-74-1, U.S. Army Engineer Waterways Experiment Station, Vicksburg, Miss., Mar. 1974.

86. Schickert, G. and Winkler, H., "Results of Tests Concerning Strength and Strain of Concrete Subjected to Multiaxial Compressive Stresses", Deutscher Ausschuss für Stahlbeton, Heft 277, Berlin 1977.

87. Smith, G.M., "Failure of Concrete Under Combined Tensile and Compressive Stresses", J. of A.C.I., Proc., Vol. 50, No. 8, Oct. 1953, pp. 137-140.

88. Smith, G.F. and Rivlin, R.S., "The Strain-Energy Function for Anisotropic Elastic Materials", Trans. Am. Math. Soc., 88, pp. 175-193, 1958.

89. Tang, P.Y., "A Recommendation of a Triaxial Failure Theory for Graphite", General Atomic Report GA-A15333, May 1979.

90. Tasuji, M.E., Slate, F.O. and Nilson, A.H., "Stress-Strain and Fracture of Concrete in Biaxial Loading", J. of A.C.I., Proc., Vol. 75, July 1978, pp. 306-312.

91. Tasuji, M.E., Nilson, A.H. and Slate, F.O., "Biaxial Stress-Strain Relationships for Concrete", Mag. of Concrete Research, Vol. 31, No. 109, Dec. 1979, pp. 217-224.

92. Taylor, M.A., "General Behavior Theory for Cement Pastes, Motars, and Concrete", J. of A.C.I., Proc., Vol. 68, No. 10, Oct. 1971, pp. 756-762.

93. Taylor, M.A. and Patel, B.K., "Influence of Path Dependency and Moisture Conditions on the Biaxial Compression Envelope for Normal Weight Concrete", J. of A.C.I., Proc., Vol. 71, No. 12, Dec. 1974, pp. 627-633.

94. Tennyson, R.C., et.al., "Application of the Cubic Polynomial Strength Criterion to the Failure Analysis of Composite Materials", J. of Composite Materials Supplement, Vol. 14, 1980, pp. 28-41.

95. Tsai, S.W. and Wu, E.M., "A General Theory of Strength for Anisotropic Materials", J. of Composite Materials, Vol. 5, 1971, pp. 58-80.

96. Valente, G., "Ultimate Strength Criteria of Concrete Under Biaxial and Triaxial Loading", Transactions of the International Conference of Structural Mechanics and Reactor Technology, 5th, Vol. H, Structural Engineering of Prestressed Reactor Pressure Vessels, Berlin, Ger., Aug. 13-17, 1979, Pub. 1b, North-Holland Publ. Co., Amsterdam, Neth. 1979., pap. H2.4, 10 p.

97. Vile, G.W.D., "The Strength of Concrete Under Short-Term Static Biaxial Stress", Proc. of an International Conference, London, Sept. 1965, The Structure of Concrete and its Behavior Under Load, Cement and Concrete Association, London, 1968.

98. Wastiels, J., "Failure Criteria for Concrete Under Multiaxial Stress States", International Association of Bridge and Structural Engineers, Report of the Working Commissions, Vol. 29, Technical University of Denmark, 1978-1979.

99. Wastiels, J., "Behavior of Concrete Under Multiaxial Stresses: A Review", Cement and Concrete Research, Vol. 9, No. 1, Jan. 1979, pp. 35-46.

100. Wastiels, J., "Failure Criteria for Concrete Subjected to Multiaxial Stresses", International Report, Vrije Universiteit Brussel, Pleinlaan 2, B-1050 Brussel, Belgium, 1981.

101. Wästlund, G., "New Evidence Regarding the Basic Strength Properties of Concrete", Benton, Vol. 22, No. 3, Stockholm, Sweden, 1937, p. 189.

102. Weigler, H. and Becker, G., "Investigation into the Strength and Deformation Properties of Concrete Subjected to Biaxial Stresses", Deutscher Ausschuss für Stahlbeton, Berlin, West Germany, 1963, p. 66.

103. William, K. and Warnke, E., "Constitutive Model for the Triaxial Behavior of Concrete", International Association of Bridge and Structural Engineers, Proc., Seminar or Concrete Structures Subjected to Triaxial Stresses; Paper III-7, Bergamo, Italy, May 17-19, 1974, Vol. 19, 1975.

104. Wittman, F.H., Fracture Mechanics of Concrete, Developments in Civil Engineering, Vol. 7, Elsevier, 1984.

105. Zweben, C.H., "Failure Analysis of Unidirectional Composites Under Combined Axial Tension and Shear", J. of Applied Mechanics and Physics of Solids, Vol. 22, 1974, p. 193.

# APPENDIX

## Correction of Chen and Chen's Material Parameters

In the course of this investigation a typographical error was discovered in the published works of Chen and Chen [13, 14]. The material parameter equations used to characterize the strength criterion proposed by these investigators are in error. Erroneous strength properties are predicted if the published material parameter equations are followed. The strength of concrete in the biaxial compression region is grossly overestimated using the published values.

The incorrect material parameters for the compression zone are given as:

$$\frac{t_u^2}{(f_c')^2} = \frac{3\bar{f}_{bc}' - 2\bar{f}_{bc-}'^{\,2}}{3(\bar{f}_{bc}' - 1)} \tag{1}$$

and;

$$\frac{t_o^2}{(f_c')^2} = \frac{1}{3}\,\frac{2\bar{f}_c^{\,3} - 3\bar{f}_{bc}^{\,2}\,\bar{f}_c + 2\bar{f}_c^{\,2}}{2\bar{f}_{bc} - \bar{f}_c} \tag{2}$$

These material parameters were re-evaluated based on the published strength criterion equation proposed by Chen and Chen for the compression zone. The corrected material parameter equations are:

$$\frac{t_u^2}{(f_c')^2} = \frac{2\bar{f}_{bc}' - \bar{f}_{bc}'^{\,2}}{3(2\bar{f}_{bc}' - 1)} \tag{3}$$

and;

$$\frac{t_o^{\,2}}{(f_c')^2} = \frac{\bar{f}_{bc}\bar{f}_c\,(2\bar{f}_c - \bar{f}_{bc})}{3(2\bar{f}_{bc} - \bar{f}_c)} \tag{4}$$

The corrected material parameters of equations (3) and (4) were tested and found to be correct. The corrected material parameters were used exclusively throughout this investigation when discussing or plotting the strength criterion proposed by Chen and Chen. The additional material parameter equations were also evaluated and found to be correct.

# VITA

DAVID L. KITTERMAN

Candidate for the Degree of

Master of Science

Thesis: A Generalized Three-Parameter Biaxial Strength Criterion For Concrete

Major FIeld: Mechanical Engineering

Biographical:

Personal Data: Born Kansas City, Kansas, December 1, 1954; son of David W. and Elizabeth M. Kitterman.

Education: Received a Bachelor of Science degree from Kansas State University, with a major in Agricultural Engineering, in May 1980.

U.S. Air Force, Electronic Repair Technician, October 1974.

Professional Organizations: American Society of Mechanical Engineers, Vice-President of Veterans on Campus, "Engineer-in-Training" certified, June 20, 1980.

Professional Experience: Test Engineer/Stress Analyst for Dart Truck Company, Kansas City, Missouri, May 1980 through August 1981; Mechanical Project Engineer, Owens-Corning Fiberglas, Kansas City, Kansas, August 1981 through May 1982; Chief Design Engineer, TUMAC Industries, Inc., Colorado Springs, Colorado, August 1982 through August 1983 and May 1984 through August 1984.

# A GENERALIZED THREE-PARAMETER BIAXIAL STRENGTH CRITERION FOR CONCRETE

BY

DAVID L. KITTERMAN

B.S., Kansas State University, 1980

---

AN ABSTRACT OF A MASTER'S THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Mechanical Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1985

ABSTRACT

A general, three-parameter, unified strength criterion for the prediction of concrete failure under biaxial states of stress is developed. From the laws of continuum mechanics the criterion is formulated in terms of the invariant quantities of the stress tensor associated with isotropic material symmetry. The proposed function satisfies the invariant requirements of coordinate transformation, stress interaction requirements, and is easily characterized. The function requires only three simple engineering strength tests, estimated or measured, to completely characterize it to any strength quality of concrete.

The strength criterion is validated using selected experimental failure results for concrete under biaxial stress states. In addition the proposed criterion is graphically compared to past strength criteria for concrete. The proposed strength criterion is proven to be highly pertinent and useful, as it represents the experimentally determined biaxial failure envelope more accurately than previously proposed criteria.