TESTING FOR DETERMINISTIC AND STOCHASTIC CYCLES IN MACROECONOMIC TIME SERIES

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Abstract

In this paper we use a statistical procedure which is appropriate to test for deterministic and stochastic (stationary and nonstationary) cycles in macroeconomic time series. These tests have standard null and local limit distributions and are easy to apply to raw time series. Monte Carlo evidence shows that they perform relatively well in the case of functional misspecification in the cyclical structure of the series. As an example, we use this approach to test for the presence of cycles in US real GDP.

Keywords: *Deterministic Cycles, Stochastic Cycles, Long Memory*

JEL Classification: *C22*

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1. Introduction

It is a well-known stylised fact that many macroeconomic time series can be specified in terms of a trend, and seasonal and cyclical components. However, while the first two of these components have been widely examined in the empirical literature, little attention has been paid to the cyclical structure of the series. In this paper, we focus on the latter, and use an appropriate version of a testing procedure suggested by Robinson (1994) which enables us to test for cyclical structures of any type in a unified framework. These tests have several distinguishing features compared to other procedures. In particular, they have standard null and local limit distributions, implying that it is not necessary to calculate finite sample critical values based on Monte Carlo simulations. In addition, their limiting distribution is the same regardless of the deterministic components used in the regression model, and therefore they are suitable to test for both deterministic and stochastic (stationary or nonstationary) cycles.

The structure of the paper is as follows: Section 2 outlines alternative approaches to modelling cycles in raw time series, and describes the version of the tests of Robinson (1994) used in the present study; Section 3 reports several Monte Carlo experiments aimed at assessing the performance of these tests under misspecification in the functional form of the cycles; Section 4 presents an empirical application to US real GDP, while Section 5 concludes.

2. Testing for cycles with the tests of Robinson (1994)

Modelling cycles in macroeconomic time series is still rather controversial. Deterministic cycles based on trigonometric functions of time have been proposed for many years. They are based on models of the form:

$$y_t \;=\; \beta_0 \cos \lambda t \;+\; \beta_1 \, sin \lambda t \;+\; u_t, \qquad t \;=\; 1, 2, ... \qquad (1)$$

where β_0 and β_1 are fixed parameters, λ takes a particular value between 0 and π, and u_t is an I(0) process, defined for the purpose of the present paper as a covariance stationary process with a spectral density function that is positive and finite at any frequency.

Stochastic stationary cycles were proposed, amongst others, by Harvey (1985). They are based on autoregressive (AR) processes of the form:

$$y_t \;=\; \phi_1\, y_{t-1} \;+\; \phi_2\, y_{t-2} \;+\; u_t\,, \qquad t \;=\; 1, 2, \ldots, \tag{2}$$

with the roots of the AR polynomial lying outside the unit circle. However, in the last few years, it has been claimed that, similarly to the trend and to the seasonal components, cycles may change or evolve over time, and nonstationary stochastic cycles (or unit root cycles) have also been proposed. Thus, for example, Ahtola and Tiao (1987) developed cyclical unit root tests based on the AR(2) model (2), which, under the null hypothesis

$$H_o\!: \left|\phi_1\right| < 2 \;\text{ and }\; \phi_2 = -1, \tag{3}$$

becomes the cyclical unit root model specified below. More recently, Gray et al. (1989, 1994) extended the cyclical unit root model to the fractional case and considered processes of the form:

$$(1 - 2\,\mu\, L + L^2)^d\; x_t \;=\; u_t\,, \qquad t \;=\; 1, 2, \ldots\,, \tag{4}$$

where the unit root model corresponds to the case of d = 1. They showed that the polynomial in (4) can be expressed in terms of the Gegenbauer polynomial such that for all $d \neq 0$

$$(1 - 2\,\mu\, L + L^2)^{-d} \;=\; \sum_{j=0}^{[T/2]} C_{j,d}(\mu)\, L^j\,, \tag{5}$$

where

$$C_{j,d}(\mu) \;=\; \sum_{k=0}^{\infty} \frac{(-1)^k\,(d)_{j-k}\,(2\,\mu)^{j-2k}}{k!\,(j-2k)!}; \qquad (d)_j \;=\; \frac{\Gamma(d+j)}{\Gamma(d)},$$

where $\Gamma(x)$ stands for the Gamma function, and a truncation will be required below (5) to make (4) operational. Alternatively, we can use the recursive formula:

$$C_{0,d}(\mu) = 1,$$

$$C_{1,d}(\mu) = 2\mu d,$$

$$C_{j,d}(\mu) = 2\mu\left(\frac{d-1}{j}+1\right)C_{j-1,d}(\mu) - \left(2\frac{d-1}{j}+1\right)C_{j-2,d}(\mu), \quad j = 2, 3, ...$$

(see, for instance, Magnus et al., 1966, Rainville, 1960, etc. for further details on Gegenbauer polynomials). Using (5), the process in (4) becomes

$$x_t = \sum_{j=0}^{t-1} C_{j,d}(\mu)\, u_{t-j}, \quad t = 1, 2, ... \qquad (6)$$

and, when d = 1, we have

$$x_t = 2\mu x_{t-1} - x_{t-2} + u_t, \qquad t = 1, 2, ... \qquad (7)$$

which is a cyclical I(1) process with the periodicity determined by μ.[1] Nesting the unit root cyclical model (7) within the fractional structure (4) has some advantages from a statistical viewpoint. Note that testing (3) in (2) produces a radically different behaviour in the limit distribution. Specifically, if ϕ_1 and ϕ_2 in (2) are such that the roots are within the unit circle, the process is stationary, and the limit distribution is, under appropriate transformations, standard normal; if ϕ_1 and ϕ_2 are given by (3), the process contains unit roots and the limit distribution is non-standard; finally, for the remaining values of ϕ_1 and ϕ_2 the limit distribution is explosive. On the other hand, testing the null of d = 1 in (4) does not produce such an abrupt change in the limit behaviour, and the boundary line between stationarity and nonstationarity now corresponds to d = 0.5 (if $|\mu| < 1$).

Robinson (1994) developed a general testing procedure which enables one to test all the above specifications for the cyclical structure in a unified framework. He considers the regression model

[1] Unit root cycles have been examined by Ahtola and Tiao (1987), Chan and Wei (1988), Gregory (1999a, b), and, more recently, by Gil-Alana (2001) and Bierens (2001).

$$y_t \;=\; \beta' z_t \;+\; x_t, \quad t = 1, 2, ... \tag{8}$$

where y_t is the time series we observe; β is a (kx1) vector of unknown parameters; z_t is a (kx1) vector of exogenous regressors that may include, for example, those in (1); and the regression errors x_t are of the form given in (4). Thus, we can consider the model

$$y_t \;=\; \beta_0 \cos\lambda_r t \;+\; \beta_1 \sin\lambda_r t \;+\; x_t, \qquad t \;=\; 1,\, 2,\, ... \tag{9}$$

$$(1 - 2\cos\lambda_r\, L + L^2)^d\, x_t \;=\; u_t\,, \qquad t \;=\; 1, 2, ...\,, \tag{10}$$

where λ_r = 2πr/T and r = T/j, j indicating the number of time periods per cycle. Robinson (1994) proposes a Lagrange multiplier (LM) test of the null hypothesis:

$$H_o:\; d \;=\; d_o, \tag{11}$$

in (9) and (10) for any real value d_o. Specifically, the test statistic is given by:

$$\hat{r} \;=\; \frac{T^{1/2}}{\hat{\sigma}^2}\, \hat{A}^{-1/2}\, \hat{a} \tag{12}$$

where T is the sample size and

$$\hat{a} \;=\; \frac{-2\pi}{T} \sum_j^{*} \psi(\lambda_j)\, g(w_j; \hat{\tau})^{-1} I_{\hat{u}}(w_j); \qquad \psi(w_j) \;=\; \log \left| 2\, \left(\cos w_j \;-\; \cos\, \lambda_r\right) \right|;$$

$$\hat{A} \;=\; \frac{2}{T} \left(\sum_j^{*} \psi(w_j)\psi(w_j)' \;-\; \sum_j^{*} \psi(w_j)\, \hat{\varepsilon}(w_j)'\; x \left(\sum_j^{*} \hat{\varepsilon}(w_j)\, \hat{\varepsilon}(w_j)' \right)^{-1} x \sum_j^{*} \hat{\varepsilon}(w_j)\, \psi(w_j)' \right);$$

$$\hat{\varepsilon}(w_j) \;=\; \frac{\partial}{\partial \tau} \log\; g(w_j; \hat{\tau}); \qquad \hat{\sigma}^2 \;=\; \frac{2\pi}{T} \sum_j^{*} g(w_j; \hat{\tau})^{-1}\, I_{\hat{u}}(w_j); \qquad w_j \;=\; \frac{2\pi\, j}{T}.$$

$g(w_j;\, \tau)$ is the function appearing in the spectral density of u_t: $f(w_j;\, \tau) = (\sigma^2/2\pi)\; g(w_j;\, \tau)$, evaluated at $\hat{\tau}$ = arg min $\sigma^2(\tau)$. Thus, for example, if u_t is a white noise process, g ≡ 1, whilst if u_t is an AR process of the form: $\phi(L)u_t = \varepsilon_t$, then $g = |\phi(e^{i\lambda})|^{-2}$, with $\sigma^2 = V(\varepsilon_t)$, so that the AR coefficients are a function of τ. Finally, $I_{\hat{u}}(w_j)$ is the periodogram of $\hat{u}_t$ defined as:

$$I_{\hat{u}}(\lambda_j) \;=\; \frac{1}{2\pi T} \left| \sum_{j=1}^{T} \hat{u}_t\, e^{i\lambda_j t} \right|^2;$$

$$\hat{u}_t = \rho(L)\, y_t - \hat{\beta}' w_t; \quad w_t = \rho(L)\, z_t; \quad \hat{\beta} = \left(\sum_{t=1}^{T} w_t\, w_t' \right)^{-1} \sum_{t=1}^{T} w_t\, \rho(L)\, y_t,$$

$\rho(L) = (1 - 2\cos\lambda_r L + L^2)^{d_0}$, and the summations on * in the above expressions are over w ∈ M, where M = {w: $-\pi < w < \pi$, $\lambda \notin (\rho_l - w_1, \rho_l + w_1)$, l = 1, 2, …, s}, such that ρ_l, l = 1, 2, …, s < ∞ are the distinct poles of $\psi(w)$ on $(-\pi, \pi]$.

Based on H_o (11), Robinson (1994) established that, under very general conditions,

$$\hat{r} \rightarrow N(0,1) \qquad as \;\; T \rightarrow \infty, \tag{13}$$

and the same limit distribution holds whether or not deterministic regressors are included in (8). Furthermore, he shows that the above test is efficient in the Pitman sense, i.e. that against local alternatives of the form: H_a: $d = d_o + \delta\, T^{-1/2}$, for $\delta \neq 0$, the limit distribution is normal, with variance 1 and mean that cannot (when u_t is Gaussian) be exceeded in absolute value by that of any rival regular statistic. Consequently, we are in a classical large sample testing situation for the reasons outlined by Robinson (1994). A one-sided test of H_o (11) against the alternative:

$$H_a: \;\; d > d_o, \tag{14}$$

will be given by the rule:

"Reject H_o (11) if $\hat{r} > z_\alpha$",

where the probability that a standard normal variate exceeds z_α is α. Conversely, a test of (11) against the alternative:

$$H_a: \;\; d < d_o, \tag{15}$$

will be given by the rule:

"Reject H_o (11) if $\hat{r} < -z_\alpha$".

Using the set-up described by (9) – (11), we can test for different forms of cyclical structure. For example, if we test H_o (11) with $d_o = 0$ and white noise u_t, the null model becomes the deterministic structure described in (1); testing the same null hypothesis with

AR(2) u_t, we have a test for stochastic stationary cycles of the form given by (2). Further, testing H_o with $d_o = 1$ amounts to a test for unit root cycles, regardless of whether or not deterministic structures and/or autocorrelated disturbances are included.

3. Some Monte Carlo evidence

In this section we examine the finite-sample behaviour of the above version of the tests of Robinson (1994) by means of Monte Carlo simulations. In particular, we investigate their size and power properties in the context of deterministic and stochastic (stationary and nonstationary) cycles. In all cases, we generate Gaussian series using the routines GASDEV and RAN3 of Press, Flannery, Teukolsky and Vetterling (1986), with 10,000 replications. The sample sizes are T = 60, 120, 240 and 360 observations, and the nominal size is 5%.

First, we assume that the cyclical structure of the series is purely deterministic and consider a process of the form:

$$y_t \;=\; \cos\lambda_r\, t \;+\; sin\lambda_r\, t \;+\; \varepsilon_t, \qquad t \;=\; 1, 2, \ldots \qquad (16)$$

with r = T/6. We choose this value in view of the fact that cycles in economics seem to occur approximately every six years, and consider alternatives of the form (9) and (10), with d_o = 0, (0.25), 2, and white noise and weakly autocorrelated disturbances.

(Insert Table 1 about here)

The values reported in Table 1 are the rejection probabilities of the one-sided statistic given by $\hat{r}$ in (12). Hence, the values corresponding to $d_o = 1$ and white noise u_t indicate the size of the tests. One can see that there is a bias in the size in favour of alternatives of the form H_o: d < 0, though there is a considerable improvement as the number of observations increases. Specifically, if T = 60, the sizes are 12.7% (against d < 0) and 0.9% (d > 0), whilst they improve to 10.3 and 1.9% respectively when T = 120, and to 8.3% and 3.2% with T = 240. Finally, if T = 360, these values become 6.7% and 4.5%. When the I(0) disturbances are misspecified, there is a higher distortion in the sizes, although again there is an improvement as

T increases. Also, when testing for a unit root (i.e., d = 1) in this context of deterministic cycles, the rejection probabilities are very high, being equal to 1 in practically all cases if T > 60.

Next, we assume that the cyclical structure is stochastic, and model the true process in terms of a stationary AR(2) of the form

$$y_t \;=\; 0.55\, y_{t-1} \;-\; 0.84\, y_{t-2} \;+\; u_t, \qquad t \;=\; 1, 2, \ldots \;. \tag{17}$$

We choose this parameterisation in order to obtain a cyclical structure with cycles occurring approximately every six periods. Note that the spectral density function of a process like (2) is given by:

$$\frac{\sigma^2}{2\pi}\frac{1}{\left|1 - \phi_1 e^{iw} - \phi_2 e^{2iw}\right|^2} \;=\; \frac{\sigma^2}{2\pi}\left(\frac{1}{1 + \phi_1^2 + \phi_2^2 - 2\phi_1(1-\phi_2)\cos w - 2\phi_2\cos 2w}\right)$$

and setting this expression equal to 0 yields:

$$w^* \;=\; \cos^{-1}\left(\frac{-\phi_1(1-\phi_2)}{4\phi_2}\right) \tag{18}$$

implying that j = $2\pi/w^*$. Then, substituting ϕ_1 and ϕ_2 in (18) with, for example, 0.55 and –0.84, leads to j ≈ 6.

(Insert Table 2 about here)

Table 2 reports the results of the same experiment as in Table 1, but assuming that the true process is generated by (17), while the alternatives are of the form given by (10). It can be seen that, similarly to Table 1, there is a size distortion with a bias in favour of alternatives with d < 0. This distortion is higher than in the previous case of white noise u_t, though again it decreases as T increases. When the I(0) disturbances are misspecified, the bias is in the opposite direction, with values close to 0 against H_a: d < 0, and practically equal to 1 if the alternatives are of the form: d > 0. Moving on to the power of the tests with d_o = 1, the rejection probabilities are close to 1 if u_t is correctly assumed to be AR(2), whilst they are relatively low with misspecified disturbances. To sum up, Table 2 seems to suggest that the correct specification of the underlying I(0) autocorrelated disturbances is crucial in the context of stationary stochastic cyclical structures.

(Insert Table 3 about here)

Finally, we assume that the true data generating process contains cyclical unit roots of the form given by (7) with $\mu = \cos w_{T/6}$ and white noise u_t, and again perform the test in (10) with d = 0 and 1. When d = 0, the rejection probabilities are practically 1 if u_t is white noise or AR(1), and slightly lower if the disturbances are AR(2). As for the size (i.e., d = 1), once more we observe a bias in favour of alternatives with d < 1, though, similarly to the previous cases, there is a substantial improvement as the sample size increases.

Overall, the Monte Carlo evidence indicates that the tests of Robinson (1994) are adequate for testing cyclical structures in raw time series, and that, although there is a size distortion when the sample size is small, this tends to disappear as the number of observations increases.

4. An empirical application

The time series analysed in this section is the logarithmic transformation of US real GDP in billion dollars, annually, for the time period 1870 –2000, in 1990 prices. Plots of the original series and its first differences, along with their corresponding correlograms and periodograms, are shown in Figure 1. The original series is rising over time, and its nonstationarity is confirmed by the correlogram (with values decaying very slowly), and the periodogram (with a large peak around the zero frequency). Therefore, we perform several unit root tests at the long run or zero frequency. In particular, we use ADF tests (Dickey and Fuller, 1979), where the null hypothesis is that of a unit root in the process; the KPSS test (Kiatwkoswki et al., 1992), for the null of an I(0) process against the alternative of a unit root; finally, a suitable version of Robinson's (1994) tests (see, e.g., Gil-Alana and Robinson, 1997). In all cases we found evidence of a unit root, and therefore first differences were taken. Their plot suggests that these might be stationary, although a cyclical pattern can also be observed; this is especially clear when looking at the correlogram and the periodogram.

(Insert Figure 1 about here)

Denoting the differenced series by y_t, we use the specification given by (9) and (10) with a constant, i.e., we consider processes of the form

$$y_t \;=\; \alpha \;+\; \beta_1 \cos\lambda_r\, t \;+\; \beta_2\, sin\,\lambda_r\, t \;+\; x_t\,, \qquad t \;=\; 1,\, 2,\, \ldots \qquad (19)$$

$$(1 - 2\cos\lambda_r\, L + L^2)^d\, x_t \;=\; u_t\,, \qquad t \;=\; 1, 2, \ldots \qquad (20)$$

testing H_o (11) with $d_o = 0$ (in Table 4), and $d_o = 1$, (in Table 5), and setting $\lambda_r = 2\pi r/T$, r = T/j, and j = 3, (1), 9, i.e., allowing cycles with a periodicity oscillating between three and nine years. In both Table 4 and 5, we consider separately the cases of (i) $\alpha = \beta_1 = \beta_2 = 0$ a priori (i.e., we assume no regressors in the levels regression (19)); (ii) α unknown and $\beta_1 = \beta_2 = 0$ a priori (i.e.., including an intercept); and (iii) all the coefficients unknown, and present the results based on both white noise and weakly (AR) autocorrelated disturbances.

(Insert Tables 4 and 5 about here)

Table 4 reports the results based on the null hypothesis of d = 0. Starting with the case with all the coefficients unknown, it can be noticed that the non-rejection values occur when j = 5 (in the case of white noise u_t), and when j = 5 or 6 with AR disturbances; very similar results are obtained if $\beta_1 = \beta_2 = 0$ a priori or if all the coefficients are 0. The results based on the null d = 1 (in Table 5) decisively reject the hypothesis of cyclical unit roots for all values of j and all types of disturbances. In fact, the values in Table 5 also indicate that the tests reject the null in favour of alternatives of the form d < 1. Note that the tests are based on the statistic given by $\hat{r}$ in (12), and therefore significant negative values represent evidence of orders of integration smaller than 1.

Going back to the results in Table 4, the similarities between the three cases of no regressors, an intercept, and an intercept and cycles may suggest that these deterministic components are not required. Gil-Alana and Robinson (1997) introduced a joint test for simultaneously testing the need of a linear time trend and the order of integration at the zero frequency. Here, we propose a similar test, but, instead of looking at the zero frequency, we focus on the cyclical roots, and, instead of a linear trend, we consider a deterministic cyclical structure. Thus, we carry out a joint test of:

$$H_o : \; d \; = \; d_o \quad and \quad \beta \; = \; 0, \qquad (21)$$

with $\beta = (\beta_1, \beta_2)'$, against alternatives of form:

$$H_o : \; d \; \neq \; d_o \quad or \quad \beta \; \neq \; 0, \qquad (22)$$

in (19) and (20). This case is not analysed by Robinson (1994), but the LM test can easily be derived as follows:

$$\tilde{S} \; = \; \hat{r}^2 + \sum_{t=1}^{T} \tilde{u}_t w_{2t}' \times \left\{ \sum_{t=1}^{T} w_{2t} \; w_{2t}' - \sum_{t=1}^{T} w_{2t} w_{1t} \left(\sum_{t=1}^{T} w_{1t}^2 \right)^{-1} \sum_{t=1}^{T} w_{1t} \; w_{2t}' \right\}^{-1} \times \; \sum_{t=1}^{T} \tilde{u}_t \; w_{2t}, \quad (23)$$

with $w_t = (w_{1t}, w_{2t}')'$, $w_{1t} = \rho(L)1_t$ and $w_{2t'} = \rho(L)(\cos w_r t, \sin w_r t)'$, and

$$\tilde{u}_t = (1 - 2\cos w_r L + L^2)^{d_o} \, y_t - \tilde{\alpha} \; w_{1t}; \quad \tilde{\alpha} = \left(\sum_{t=1}^{T} w_{1t}^2 \right)^{-1} \sum_{t=1}^{T} w_{1t} \; (1 - 2\cos w_r L + L^2)^{d_o} \, y_t ,$$

and $\hat{r}$ calculated as described in Section 2 but using the $\tilde{u}_t$ just defined. Then, under H_o (21), $\tilde{S} \rightarrow_d \chi_3^2$ as $T \rightarrow \infty$, and (23) is compared to the values of the upper tail of the χ_3^2 distribution.

(Insert Table 6 about here)

In Table 6 we present the statistic (23) for the same values of j and d_o as before. It can be seen that, similarly to the previous tables, the non-rejection values occur when j = 5 and 6, implying that deterministic cycles may not be important when modelling this series. In view of all this evidence, and also taking into account the insignificance of the estimated coefficients in the models based on an intercept and on AR(1) disturbances, we can conclude that the best model specification for the growth rate series is a stationary AR(2) model of the form given by

(2), with estimated coefficients equal to 0.278 and –0.072. According to (18), this implies that the cycles occur approximately every 6.29 periods, which is consistent with the empirical evidence for many other macroeconomic time series.

5. Conclusions

In this paper we have presented a version of the tests of Robinson (1994) that is appropriate to test for cyclical components in raw time series. These tests are very general and suitable for both deterministic and stochastic (stationary and nonstationary) cycles without any change in their standard (normal) limit distribution. We report several Monte Carlo experiments showing that their size is slightly biased for small sample sizes, but approximates the nominal one for higher values of T. Also, we show that these tests have power to detect functional misspecification in the cyclical case. Finally, we applied them to the first differences of the log transformation of US real GDP. We find strong evidence against unit root cycles, and deterministic components also seem to be inappropriate. A simple AR(2) process with stationary complex roots appears to be the best specification for describing the cyclical structure of this series.

The present study can be extended in several ways. For instance, finite-sample critical values for the different forms of cyclical structures could be computed, and the case of non-normal disturbances could also be considered. Further, it might be of interest to obtain point estimates of the fractional differencing parameters for the cyclical components (examples in a semiparametric context are Arteche and Robinson, 2000, and Arteche, 2001). However, this would be much more computationally intensive. Moreover, the emphasis should be put on confidence intervals, rather than point estimates, when preliminary integer differencing appears to be required in order to achieve I(0) stationarity.[2]

[2] Note that the approach used in this paper generates simply computed diagnostics for departures from a particular type of cyclical behaviour; specifically, the results presented in this paper should be interpreted as giving support to models assuming that real output is I(1) with the cyclical structure determined by a stationary AR(2) model.

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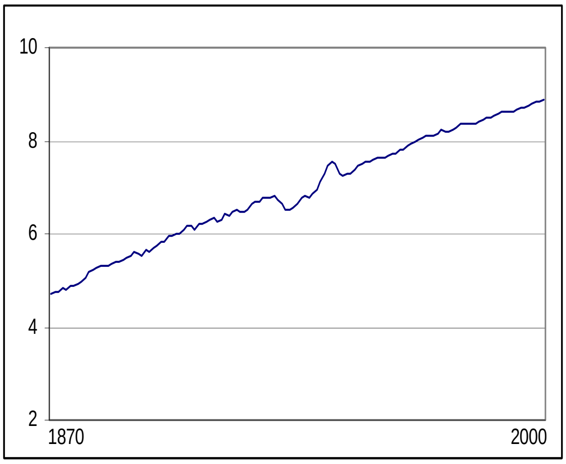
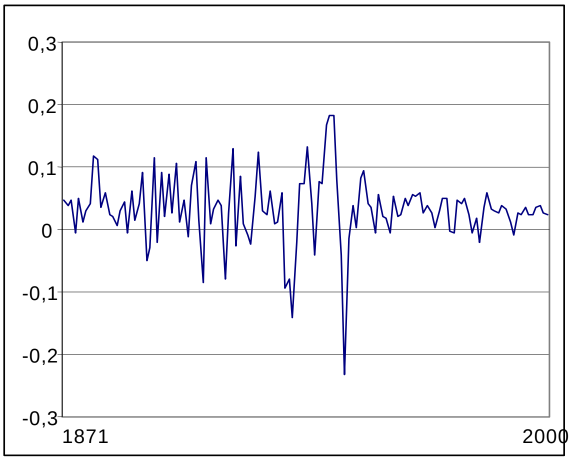
FIGURE 1

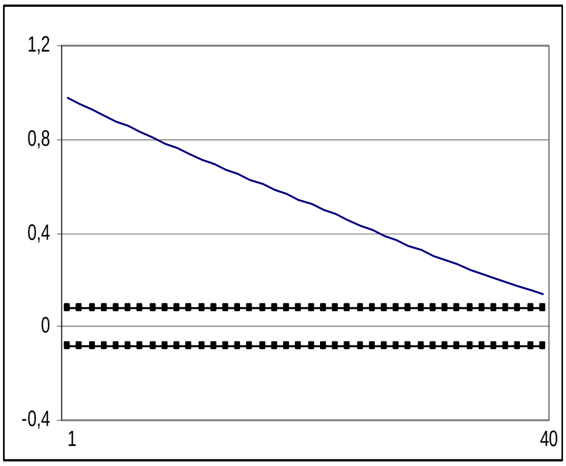
Plots of the original series with their corresponding correlograms and periodograms

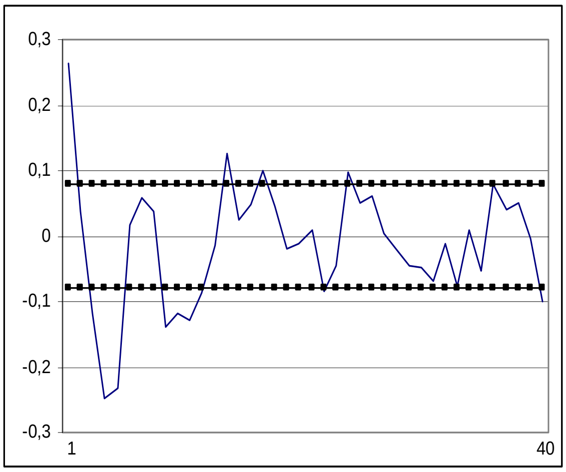
Logarithm of the real U.S. GDP	First differenced series



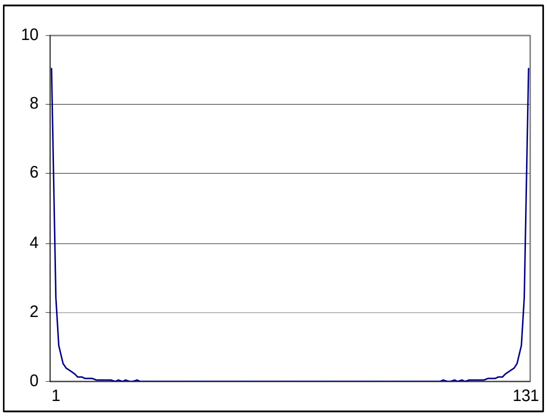


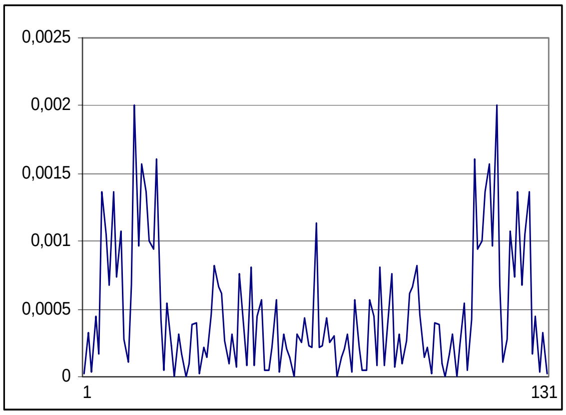
Correlogram GDP	Correlogram first differences





Periodogram GDP	Periodogram first differences





The large sample standard error under the null hypothesis of no autocorrelation is $1/\sqrt{T}$ or roughly 0.08 for the series used in this application.

TABLE 1				
Rejection frequencies of the tests of Robinson (1994) in the context of deterministic cyclical structures				
True model: $y_t \;=\; \cos\lambda_{T/6}\, t \;+\; sin\,\lambda_{T/6}\, t \;+\; \varepsilon_t,$				
Alternatives: $y_t \;=\; \beta_0 \cos\lambda_{T/6}\, t \;+\; \beta_1\, sin\,\lambda_{T/6}\, t \;+\; x_t,$ $(1 - 2\cos\lambda_{T/6}\, L + L^2)^d\, x_t \;=\; u_t,$				
Sample size	Disturbances	H_o: d = 0		H_o: d = 1
		H_o: d < 0	H_o: d > 0	H_o: d < 1
T = 60	White noise	**0.127**	**0.009**	1.000
	AR(1)	0.152	0.004	0.993
	AR(2)	0.164	0.003	0.987
T = 120	White noise	**0.103**	**0.019**	1.000
	AR(1)	0.136	0.016	1.000
	AR(2)	0.149	0.012	0.999
T = 240	White noise	**0.083**	**0.032**	1.000
	AR(1)	0.099	0.026	1.000
	AR(2)	0.105	0.024	1.000
T = 360	White noise	**0.067**	**0.045**	1.000
	AR(1)	0.071	0.044	1.000
	AR(2)	0.073	0.043	1.000

In bold: The sizes of the tests. The nominal size is 95% and 10,000 replications were carried out in each case.

TABLE 2				
Rejection frequencies of the tests of Robinson (1994) in the context of stationary stochastic cyclical structures				
True model: $y_t = 0.55\, y_{t-1} - 0.84\, y_{t-2} + \varepsilon_t$,				
Alternatives: $(1 - 2\cos\lambda_r\, L + L^2)^d\; y_t = u_t$,				
Sample size	Disturbances	H_o: d = 0		H_o: d = 1
		H_o: d < 0	H_o: d > 0	H_o: d < 1
T = 60	White noise	0.000	0.996	0.365
	AR(1)	0.000	0.988	0.317
	AR(2)	**0.261**	**0.000**	0.895
T = 120	White noise	0.000	1.000	0.512
	AR(1)	0.000	1.000	0.506
	AR(2)	**0.198**	**0.004**	0.9976
T = 240	White noise	0.000	1.000	0.677
	AR(1)	0.000	1.000	0.605
	AR(2)	**0.097**	**0.015**	1.000
T = 360	White noise	0.000	1.000	0.775
	AR(1)	0.000	1.000	0.707
	AR(2)	**0.083**	**0.022**	1.000

In bold: the sizes of the tests. The nominal size is 95% and 10,000 replications were carried out in each case.

TABLE 3				
Rejection frequencies of the tests of Robinson (1994) in the context of nonstationary stochastic cyclical structures				
True model: $y_t \;=\; 2\cos\lambda_{T/6}\, y_{t-1} \;-\; y_{t-2} \;+\; \varepsilon_t\,,$				
Alternatives: $(1 - 2\cos\lambda_{T/6}\, L + L^2)^d\; y_t \;= u_t\,,$				
Sample size	Disturbances	H_o: d = 0	H_o: d = 1	
		H_o: d ≠ 0	H_o: d < 1	H_o: d > 1
T = 60	White noise	0.763	**0.136**	**0.015**
	AR(1)	0.768	0.141	0.002
	AR(2)	0.711	0.468	0.004
T = 120	White noise	0.991	**0.102**	**0.022**
	AR(1)	0.995	0.144	0.006
	AR(2)	0.833	0.441	0.033
T = 240	White noise	1.000	**0.080**	**0.039**
	AR(1)	1.000	0.174	0.026
	AR(2)	0.859	0.515	0.113
T = 360	White noise	1.000	**0.061**	**0.048**
	AR(1)	1.000	0.173	0029
	AR(2)	0.984	0.546	0.110

In bold: The sizes of the tests. The nominal size is 95% and 10,000 replications were carried out in each case.

TABLE 4							
Testing of stationary cycles with the tests of Robinson (1994)							
Model: $y_t = \alpha + \beta_0 \cos\lambda_j t + \beta_1 sin\lambda_j t + x_t$, $(1 - 2\cos\lambda_j L + L^2)^d x_t = u_t$, H_o: d = 0							
U_t / j	3	4	5	6	7	8	9
White noise	-3.06	-1.70	**0.12'**	1.90	3.47	4.10	3.38
AR(1)	-2.05	-1.77	**-1.02'**	**-0.52'**	2.31	3.65	3.40
AR(2)	-3.34	-1.98	**-1.65'**	**-1.39'**	2.04	3.11	3.54
Imposing $\beta_0 = \beta_1 = 0$							
U_t / j	3	4	5	6	7	8	9
White noise	-2.92	-1.56'	**0.12'**	2.20	3.88	4.40	3.99
AR(1)	-2.17	-1.71	**-1.02'**	**-0.23'**	3.08	3.65	3.11
AR(2)	-3.45	-1.68	**-1.39'**	**-0.73'**	3.34	3.21	2.95
Imposing $\alpha = \beta_0 = \beta_1 = 0$							
U_t / j	3	4	5	6	7	8	9
AR(1)	-2.99	-1.71	**-1.03'**	**-0.23'**	2.31	3.21	3.90
AR(2)	-3.21	-1.69	**-1.40'**	**-0.74'**	1.98	2.34	2.98

' and in bold: Non-rejection values of the null hypothesis at the 95% significance level.

TABLE 5							
Testing of nontationary (integrated) cycles with the tests of Robinson (1994)							
Model: $y_t = \alpha + \beta_0 \cos\lambda_j t + \beta_1 sin\lambda_j t + x_t,$ $(1 - 2\cos\lambda_j L + L^2)^d x_t = u_t,$ H_o: d = 1							
U_t / j	3	4	5	6	7	8	9
White noise	-7.87	-4.13	-4.67	-6.64	-7.90	-8.58	-8.96
AR(1)	-8.54	-4.32	-11.98	-7.08	-8.08	-8.34	-7.09
AR(2)	-6.59	-4.40	-16.44	-15.06	-10.04	-9.07	-7.85
Imposing $\beta_0 = \beta_1 = 0$							
U_t / j	3	4	5	6	7	8	9
White noise	-7.89	-4.16	-4.69	-6.66	-7.91	-8.59	-8.96
AR(1)	-7.99	-5.26	-11.13	-7.13	-8.00	-8.34	-7.11
AR(2)	-8.04	-4.39	-16.54	-15.67	-8.91	-9.08	-7.88
Imposing $\alpha = \beta_0 = \beta_1 = 0$							
U_t / j	3	4	5	6	7	8	9
White noise	-7.89	-4.16	-4.69	-6.66	-7.91	-8.58	-8.97
AR(1)	-8.43	-5.26	-11.12	-7.23	-7.88	-8.34	-7.09
AR(2)	-8.04	-4.40	-16.56	-15.98	-8.90	-9.08	-7.88

TABLE 6		
Joint test of (21) against (22) in model given by (19) and (20) and white noise u_t		
J	H_o : d = 0 and β = 0	H_o : d = 1 and β = 0
3	10.58	62.30
4	8.46	17.31
5	**0.01'**	22.07
6	**4.86'**	44.39
7	15.21	62.69
8	19.43	73.82
9	15.98	80.45

' and in bold: Non-rejection values for the null hypothesis at 95% significance level.