MEASURING HALF-LIVES*

USING A NON-PARAMETRIC BOOTSTRAP APPROACH

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Abstract

In this paper we extend the Murray and Papell (2002) study by using a non-parametric bootstrap approach which allows for non-normality, and focusing on quarterly real exchange rate in twenty OECD countries in the post-1973 floating period. We run Augmented Dickey-Fuller (ADF) regressions, and estimate the half-lives (and confidence intervals) from the corresponding impulse response functions. Further, we use an approximately median-unbiased estimator of the autoregressive parameters, and report the implied point estimates and confidence intervals. We find that accounting for non-normality results in even higher estimates of the degree of persistence of PPP deviations, but, as in Murray and Papell (2002), the confidence intervals are so wide that no strong conclusions are warranted on the existence of a PPP puzzle.

Keywords: *Purchasing Power Parity (PPP), Half-Lives, Approximately Median-Unbiased Estimator, Non-Parametric Bootstrap*

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1. Introduction

In a recent paper, Murray and Papell (2002) argue that previous studies finding slow speed of adjustment of real exchange rates to their purchasing power parity (PPP) level, which cannot be entirely justified in terms of nominal rigidities (the "PPP puzzle" – see Rogoff, 1996), have used inappropriate techniques to measure the degree of persistence[1]. Specifically, they calculate half-lives of PPP deviations on the basis of the estimated autoregressive (AR) parameter, thereby not accounting for serial correlation; they use least squares (LS) estimates, which are biased downwards in small samples; they report point estimates, but not confidence intervals.

To address these issues, Murray and Papell (2002) consider AR(p) processes and calculate half-lives directly from the impulse response function; use median-unbiased estimation (see Andrews, 1993, and Andrews and Chen, 1994) to correct for small sample bias; and supplement the point estimates of half-lives with bootstrap confidence intervals. In their analysis, they rely on a parametric approach, based on generating artificial time series from an i.i.d. normal distribution. However, as they point out, further research is needed on the sensitivity of their results with respect to departures from normality.

In this paper we focus on this issue, and extend the Murray and Papell (2002) study by taking a (residual-based) non-parametric bootstrap approach which allows for non-normality, and focusing on quarterly real exchange rates in twenty OECD countries in the post-1973 floating period. We run Augmented Dickey-Fuller (ADF) regressions, and estimate the half-lives (and confidence intervals) from the corresponding impulse response functions, as, unlike AR(1) processes, higher order AR processes are not characterised by a constant rate of decay, and therefore estimates obtained from the AR coefficient are not valid (see Murray and Papell, 2002). Further, we use an approximately median-unbiased estimator of the AR parameters (as opposed to an exact one, which would be appropriate in AR(1) models) in order to correct for small sample bias (see Andrews and Chen, 1994), and report the implied point estimates and confidence

[1] Rogoff (1996) describes a "remarkable consensus" of 3-5 year half-lives of PPP deviations, and only slightly shorter ones in the post-1973 floating period (see, e.g., Papell, 1997).

intervals. The simple non-parametric technique we use enables us to account for non-normality in all cases.

The layout of the paper is the following. Section 2 briefly describes the estimation technique and the bootstrap procedure. Section 3 reports the empirical results. Section 4 offers some concluding remarks.

2. Empirical Methodology

As stressed in Murray and Papell (2002), in the presence of serial correlation, the half-lives calculated from the slope coefficient in a Dickey-Fuller (DF) regression are not a valid measure of persistence, and ADF equations should be estimated instead. Specifically, consider the AR(p) model:

$$q_t = a + bq_{t-1} + \sum_{i=1}^{p} \theta_i \Delta q_{t-1} + \mu_t \qquad (1)$$

Following Inoue and Kilian (2002), one can obtain point estimates of half-lives directly from the impulse response function. Consider the following AR(p) DGP:

$$\phi(\Psi) q_t = \alpha + \mu_t \qquad (2)$$

with Ψ being a lag operator. The process q_t can be represented as:

$$q_t = \alpha + bq_{t-1} + \theta_1 \Delta q_{t-1} + \theta_2 \Delta q_{t-2} + \ldots + \theta_{p-1} \Delta q_{t-1+1} + \mu_t \qquad (3)$$

$$b = \phi_1 + \phi_2 + \ldots + \phi_p = 1, \ \theta(1) = 1 - \theta_1 - \theta_2 - \ldots - \theta_{p-1}$$

Inoue and Kilian (2002) point out that equation (3) can be written as a linear combination of b and θ_i. Specifically, $\phi_1 = b + \theta_1$, $\phi_j = \theta_j - \theta_{j-1}$ and $\phi_p = -\theta_{p-1}$. They show that, although the bootstrap method is not valid for the unit root parameter b in (3) when $b=1$ and $\alpha=0$, nevertheless, it is asymptotically valid for the slope parameters ϕ_i, linear combinations of which are the parameters of interest when measuring half-lives. This is why, although the bootstrap estimator has a random limit distribution, the rate at which it converges is so fast (i.e. $T^{3/2}$), that any linear combination of bootstrap estimators of the coefficients of the lagged first-differenced variables will be consistent. Hence the bootstrap point estimates and confidence intervals of half-lives based on the impulse response function are asymptotically valid.

In the empirical analysis we use the following specification of the AR(p) model:

$$q_t = \alpha + \sum_{i=1}^{p+1} \phi_i q_{t-i} + \mu_t \tag{4}$$

$\phi_1 = b + \theta_1$, $\phi_j = \theta_j - \theta_{j-1} \ldots \phi_{p+1} = -\theta_p$ for $j = 2,\ldots,p$,

and calculate half-lives and confidence intervals for the real exchange rate based on the impulse response function (i.e. $\phi_{1,}\phi_2,\ldots,\phi_{p+1}$).

To correct the LS estimates for small sample bias, we use a median-unbiased estimator (see Andrews, 1993, and Andrews and Chen, 1994). By definition, an estimator is median-unbiased if the distance between itself and the true parameter being estimated is on average the same as that from any other value in the parameter space. Simulation techniques need to be used for obtaining the estimates, as analytic forms are not available. Exactly median-unbiased estimators can be computed for AR(1), but not for AR(p) models. In the latter case, they depend on the unknown true values of the parameters θ_i in equation (1). However, approximately median-unbiased estimates can be obtained by means of iterative procedures (see Andrews and Chen, 1994).

Finally, we construct confidence intervals to measure the uncertainty surrounding our point estimates of half-lives.

3. Empirical Results

We use the same quarterly data as in Murray and Papell (2002), namely CPI-based, real exchange rates from 1973:1 to 1998:2 for 20 OECD countries, with the US dollar as the numeraire currency. The data are from the IMF's International Financial Statistics.

Insert Table 1

Table 1 reports half-lives calculated from the impulse response function (HL_{IRF}) in the ADF regressions with lag length *k*, and the associated 95% confidence intervals (CI). One can see that individual point estimates are, generally, higher than those reported by Murray and Papell (2002). However, the estimated median half-life is 2.15 years, which is the same their value of 2.15. The median lower and upper bounds are 1.3 and 3.24 years respectively. This compares to their values of 1.14 and 4.04 years.

Table 2 reports point estimates of the first autoregressive parameter (ϕ_{MU}), and two sets of estimates of half-lives, based on this parameter ($HL_{\phi,MU}$) and on the impulse response function ($HL_{IRF,MU}$), as well as the corresponding 95% confidence intervals (CI),

Insert Table 2

where in all cases an approximately median unbiased estimator is used, and *k* is the lag length.

It can be noticed that in most cases the point estimates based on the ϕ_{MU} parameter are higher, and the confidence intervals wider, than in Murray and Papell (2002). The median estimate of half-lives calculated from ϕ_{MU} is 3.33 years, instead of 2.39 as in their study. As for the lower bound of the confidence intervals, our median estimate is 0.87 years compared to 0.74, whilst the upper bound is infinite in most cases, as also found by Murray and Papell (2002).[2]

A similar picture emerges when the half-lives are calculated from the impulse response function. The individual point estimates are still higher, in most cases, than those reported in Murray and Papell (2002). The median estimate is 3.79 years, as

[2] Infinite upper bounds are a common finding in the literature, even when Bayesian methods are used (see, e.g., Kilian and Zha, 2002).

opposed to 3.07 years in their study, and the lower bound of the confidence interval is 1.15 years, rather than 1.24. The upper bound is again found to be infinite.

4. Conclusions

In this paper we have examined the sensitivity to non-normality of the quarterly estimates of half-lives of PPP deviations reported in the study of Murray and Papell (2002). Specifically, we have adopted the same methodology to account for serial correlation, sampling uncertainty, and small sample bias, but have also allowed for non-normality by using a residual-based non-parametric bootstrap method. Focusing, as in Murray and Papell (2002), on the results from our preferred specification (i.e. approximately median unbiased estimates from the impulse response function), we find that accounting for non-normality affects the estimated degree of persistence of PPP deviations. In fact, our country-by-country estimates are higher than those reported by Murray and Papell (2002), and the median estimates might also be seen as providing evidence of a "PPP puzzle" (Rogoff, 1996). However, as in Murray and Papell (2002), the estimated confidence intervals are so wide that strong conclusions on whether or not half-lives are inconsistent with PPP are not really warranted. Further research is required to establish whether the slow convergence of real exchange rates to PPP, and the implied "PPP puzzle", can be considered robust empirical findings.

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Table 1

OLS Half-Lives in ADF Regression		
Country	**HL_{IRF}**	**95% CI**
Australia	3.65	[2.9 3.9]
Austria	1.4	[0.4 2.15]
Belgium	2.2	[1.3 4.3]
Canada	0.25	[0.25 0.25]
Denm.	2.4	[2.15 2.65]
Finland	3.65	[3.4 3.9]
France	1.4	[0.65 2.65]
Germ.	2.5	[1.8 4.4]
Greece	2.01	[1.05 3.98]
Ireland	1.15	[0.4 1.4]
Italy	1.89	[1.01 2.98]
Japan	2.65	[2.15 2.9]
Netherl.	1.9	[0.5 5.9]
N. Zeal.	1.15	[0.4 1.4]
Norway	2.15	[1.4 2.65]
Portugal	3.9	[3.65 4.15]
Spain	2.9	[2.65 3.4]
Sweden	3.15	[2.9 3.65]
Switzerl.	1.9	[0.25 5.4]
UK	2.15	[0.7 2.8]

Table 2

Approximately Median Unbiased Half-Lives						
Country	K	ϕ_{MU}	95% CI	$HL_{\phi,MU}$	95% CI	$HL_{IRF,MU}$	95% CI
Australia	3	0.97	[0.83 1]	5.7	[0.93 ∞]	6.4	[1.15 ∞]
Austria	4	0.96	[0.82 1]	4.24	[0.87 ∞]	4.15	[1.15 ∞]
Belgium	4	0.95	[0.83 1]	3.38	[0.93 ∞]	3.9	[1.4 ∞]
Canada	6	0.91	[0.83 0.97]	1.84	[[0.93 5.7]	2.9	[1.9 6.15]
Denm.	3	0.95	[0.83 1]	3.38	[0.93 ∞]	3.9	[1.4 ∞]
Finland	7	0.91	[0.77 0.99]	1.84	[0.66 17.2]	2.4	[1.4 17.4]
France	4	0.96	[0.84 1]	4.24	[0.99 ∞]	4.4	[1.4 ∞]
Germ.	4	0.96	[0.83 1]	4.24	[0.93 ∞]	4.65	[1.15 ∞]
Greece	4	0.96	[0.82 1]	4.24	[0.87 ∞]	4.4	[0.9 ∞]
Ireland	7	0.92	[0.75 1]	2.08	[0.6 ∞]	1.65	[1.15 ∞]
Italy	4	0.93	[0.77 1]	2.39	[0.66 ∞]	2.65	[0.9 ∞]
Japan	3	0.98	[0.85 1]	8.58	[1.07 ∞]	9.4	[1.65 ∞]
Netherl.	4	0.93	[0.8 1]	2.39	[0.78 ∞]	2.9	[1.15 ∞]
N. Zeal.	3	0.9	[0.79 0.97]	1.64	[0.74 5.7]	2.4	[1.15 6.15]
Norway	7	0.9	[0.75 1]	1.64	[0.61 ∞]	1.9	[0.9 ∞]
Portugal	8	0.97	[0.84 1]	5.69	[0.99 ∞]	6.4	[1.4 ∞]
Spain	8	0.95	[0.82 1]	3.38	[0.87 ∞]	4.15	[1.4 ∞]
Sweden	8	0.93	[0.81 1]	2.39	[0.82 ∞]	3.15	[1.15 ∞]
Switzerl.	4	0.9	[0.71 1]	1.64	[0.51 ∞]	1.65	[0.25 ∞]
UK	7	0.9	[0.75 0.91]	1.64	[0.61 1.84]	2.4	[1.15 18.15]