THEORETICAL REACTOR KINETIC MODELS AND EXPERIMENTAL VERIFICATION

by

MARVIN KEITH DRAKE

B. S., Kansas State University, 1959

A THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Nuclear Engineering

KANSAS STATE UNIVERSITY
OF AGRICULTURE AND APPLIED SCIENCE

1960

TABLE OF CONTENTS

INTRODUCTION

Nature of the Study

The control of a neutron chain reaction is probably the most important aspect of atomic energy. The ability to control a nuclear reactor permits the usefulness of such a device. As is well known, the controllability of a neutron chain reaction is primarily due to the existence of delayed neutrons, i.e., neutrons which are born a considerable length of time (relative to neutron lifetimes) after the fission process. The time behavior of a nuclear reactor must be well understood in order that the very concept of nuclear energy not become a menace to mankind. The study of reactor kinetics has progressed from early in the Manhattan Project and many attempts have been made to understand and solve reactor kinetics problems. In many instances, in order to solve the kinetics problems, each reactor must be treated individually and even then the problems may be extremely difficult or impossible.

The usefulness of any theory lies in its ability to predict mathematically the characteristic behavior with which it is concerned. Therefore, kinetics theory must be able to describe the time behavior of a neutron chain reaction. This paper investigates the characteristics, limitations, and utility of some of the presently used theories. The theory utilized in this paper is very similar to that discussed in treatises such as Glasstone and Edlund (8). The theory was applied to reactor systems for the prediction of the neutron density time dependence where both

step and finite changes of reactivity were assumed. This paper will demonstrate the agreement found between theoretical and experimental determination of the reactivity worth of reactor control rods, measured both with the positive period method and with the rod drop method.

The particular reactor of interest here is a TRIGA reactor (see PLATE I) (designed and built by the General Atomic Division of General Dynamics Corp.). Since Kansas State University will install a TRIGA Mark II reactor in 1961, it is important to know beforehand the nature of the kinetic analysis that will be required to predict the behavior of such a reactor. Because of the extreme variation in complexity of the equations for the various kinetic models, it is important to realize the simplest model which will suitably describe the time behavior of the reactor.

Since almost all solutions of the various kinetic equations must be solved with the aid of a computer, either digital or analog, it is important to realize an approximate solution which will accurately predict the kinetic behavior of a reactor. Therefore, solutions of the kinetic equations using both three and six delayed neutron groups were developed and solved with the aid of the Kansas State University IBM-650 computer.

Since the TRIGA reactor has a control rod drop time of approximately 0.35 seconds from "full out" to "full in", the step change in reactivity assumption is not really justifiable when considering neutron flux predictions for times comparable to the rod drop time. A theoretical development is given for

EXPLANATION OF PLATE I

General view of the TRIGA reactor, looking into the reactor tank at the core. The ionization chambers are shown just outside of the core reflector.

PLATE I

reactivity insertion as a function of time. The resulting analytical prediction for the neutron flux as a function of time computed from the one group model was compared with the experimentally measured neutron flux after a change in reactivity.

Nomenclature

n - Neutron density, neutrons/cm^3.

Φ - Thermal neutron flux, neutrons/cm^2. sec.

D - Diffusion coefficient, cm.

∇^2 - Laplacian operator

$\underline{r}$ - Spatial coordinate

t - Time, seconds

β - Fraction of total neutrons that are delayed

β_i - Fraction of neutrons in the ith delayed group

k_{eff} - Effective multiplication constant

k - Infinite medium multiplication constant

λ_i - Decay constant of precursor of the delayed neutrons of the ith group

C_i - Precursor density of ith group, atomic nuclei/cm^3.

l - Prompt neutron lifetime

B^2 - Buckling

Σ_a - Macroscopic absorption cross section, cm^{-1}.

τ - Fermi age, cm^2.

v - Neutron velocity, cm./sec.

L - Diffusion length, cm.

ρ - Reactivity

ω_i - Root of the characteristic equation

p - Resonance escape probability

Literature Survey

One of the first nuclear scientists to publish an extensive derivation of the "pile-kinetic" equations was Hurwitz (11). He described the methods used to solve the pile-kinetic equations for step changes in reactivity and when reactivity was a slowly varying function of time.

Kimel, et. al., (14) presented a theoretical development for the time behavior of neutron density as a function of step changes in reactivity. The space independent kinetic equations were based on the Fermi continuous slowing down model and used six groups of delayed neutrons. Experimental measurements were made with the Argonaut Reactor at Argonne National Laboratory. Reactivity worths of the control rods of the Argonaut were measured experimentally using the theoretical development. Both positive period measurements and negative rod drops measurements were made and good agreement was obtained between the two methods, indicating that the theoretical development could be used with the Argonaut reactor.

Smets (17) presented a solution for the one energy group bare reactor kinetic equations when the reactivity was a function of time. The time dependence of the reactivity was assumed to be either linear, exponential, or a reciprocal. Smets combined the basic space independent differential equations in the form

$$D\left[\ell + \sum_{i=1}^{m} \frac{\beta_i}{D+\lambda_i}\right] n(t) = \left[1 - \sum_{i=1}^{m} \frac{\beta_i D}{D+\lambda_i}\right] k_{eff}(t) \cdot n(t) \qquad (1)$$

where

ℓ = prompt neutron lifetime,

β = fraction of total neutrons that are delayed,

β_i = fraction of neutrons in the ith delayed group,

n = neutron density, neutron/cm^3,

λ_i = decay constant of precursor of the delayed neutrons of the ith group,

k_{eff} = effective multiplication constant,

m = number of delayed neutron groups,

and D, in this case was the differential operator d/dt. Solutions of Eq. (1) in the form of Laplace type contour integrals were obtained. Smets also found that the kinetic equations could be integrated for any piecewise analytic function, k_{eff} (t), which consisted of straight lines, exponentials, and inverse functions but not the sum of any two or more such functions.

Toppel (20) discussed the errors in determining the magnitude of a step change in reactivity by means of measuring the stable reactor period. He found that for certain reactivities the neutron population would not be represented by a single exponential until several minutes after a positive step was made. Toppel also considered a reactor with a source and showed that a longer wait time was required for the true exponential neutron density distribution than with a source free reactor. A typical

point from Toppel's work showed that in the case of a positive insertion of reactivity, $\rho = 5.0 \times 10^{-4}$, and when the initial effective multiplication constant was 1.000, 100 seconds were required for the reactor to be within one per cent of being on a stable period. In contrast, if the initial reactivity was 0.99995 and then the reactivity was inserted, a wait time of approximately 300 seconds was required for the reactor to be within one per cent of being on a stable period.

Friedman (5) discussed the use of an "effective" fraction of delayed neutrons in place of the true fraction of neutrons. He stated that the fraction of delayed neutrons should be increased because the delayed neutrons have a larger nonleakage probability than prompt fission neutrons, thus a delayed neutron is more effective in producing another fission than is a prompt neutron. Friedman went on to say that if U^{238} was present in the reactor fuel, the fraction of delayed neutrons should be increased due to the increased fraction of delayed neutrons from U^{238}. An expression was given for the effective fraction of delayed neutrons for the ith precursor group (5).

$$\beta_{eff_i} = \frac{p_{delayed}}{p_{prompt}} \left[\frac{\beta_i^{25}}{\epsilon^{28}} + \frac{\nu^{28}}{\epsilon^{25}} \left(\frac{\delta^{28}}{\epsilon^{28}} \right) \beta_i^{28} \right] \qquad (2)$$

Where

$\beta_i^{(25)}_{(28)}$ ≡ fraction of neutrons which are from the i-th precursor due to fission from $U^{(25)}_{(28)}$.

$p^{(\text{delayed})}_{(\text{prompt})}$ ≡ nonleakage probability of $^{(\text{delayed})}_{(\text{prompt})}$ neutrons

$\nu^{(28)}_{(25)}$ ≡ average number of neutrons per fission from $U^{(28)}_{(25)}$

$\delta^{28} \equiv \frac{U^{28} \text{ fissions}}{U^{25} \text{ fissions}}$

ϵ^{28} ≡ fast fission factor of U^{238}

Ewen and Wick (4) extended the work of Friedman (5) pertaining to evaluating the effective fractions of delayed neutrons. They gave results of several cores and showed the difference in core reactivity evaluation by using their calculated effective fraction of delayed neutrons. Keepin, et. al., (13) measured the period, relative abundance, and absolute yield of delayed neutrons from "fast" fission for six nuclides including U^{235} and from thermal fission for three nuclides including U^{235}. The "Godiva" reactor was used as the neutron source. Six exponential periods were found to be necessary and sufficient for an optimum least-square fit of the experimental data. Keepin's work on determination of delayed neutron parameter is the latest presented and is probably the most reliable analytical study that has been made on the delayed neutron phenomenon to date. Many persons have experimentally measured sets of delayed neutron data and most of the sets are slightly different. Keepin

(12) presented a list of experimentally measured sets of delayed neutron constants and presented an excellent description of the delayed neutron phenomenon.

Skinner and Cohen (16) showed that a two delayed neutron group model represented, to a fair degree of accuracy, the kinetic behavior of a reactor when a positive insertion of reactivity was considered. They also demonstrated that at least three groups must be employed for the delayed neutron model used to describe the time behavior of neutron density in a reactor when negative reactivities are inserted into the core. The principal advantages given for using the reduced delayed neutron group models were: 1) the number of differential equations to be solved was reduced, 2) the solutions of the kinetic equations required less computer time, 3) control design was greatly simplified. The following is a list of three group constants as given by Skinner and Cohen (16).

Table 1. Reduced delayed neutron group constants for thermal fission of U^{235}.

Group	λ_i (sec.$^{-1}$)	β_i/β
1	0.01244	0.033
2	0.03694	0.346
3	0.0632	0.621

Most methods of solving the reactor kinetic differential equations include the assumption that the neutron flux can be separated into two variables, space and time. Solutions of the

kinetic equations are difficult to obtain when the separation process is not performed. Garabedian and Leffert (6) solved the kinetic equations where it was assumed that the spatial distribution contained a time function. They found, in the case examined, that the space function reached an asymptotic shape within 0.005 seconds after a flux perturbation was caused by a sudden reactivity change.

THEORETICAL DEVELOPMENT

Since the theory used by this paper is very similar to that found in references (8), (11), and (7), the detailed development and solution of the reactor kinetics equations is found in Appendix - A. The kinetics model and the associated assumptions are given along with the resulting equations.

A reactor system is described by the time dependent thermal diffusion equation and the precursor rate equations. The assumptions used are listed as follows:

1) the reactor is homogeneous,
2) the reactor is bare, i.e., has no reflector,
3) the delayed neutron energy spectrum is the same as that of the prompt fission neutrons,[1]
4) there is no fission from U^{238},
5) the spatial distribution of the neutron flux may be represented by the fundamental mode of the wave equation,

---

[1]This assumption need not be made (see Appendix - A).

6) the delayed neutron precursor spatial distribution is proportional to the spatial distribution of the neutron flux,

7) the Fermi age equation may be used as a source of thermal neutrons for the thermal diffusion equations.

The thermal neutron balance written around a volume element of the reactor is

$$D\nabla^2\Phi(\underline{r},t) - \Sigma_a\Phi(\underline{r},t) + S(\underline{r},t) = \frac{\partial n(\underline{r},t)}{\partial t} = \frac{1}{v}\frac{\partial\Phi(\underline{r},t)}{\partial t} \quad (3)$$

The precursor rate equation is

$$\frac{\partial C_i(\underline{r},t)}{\partial t} = -\lambda_i C_i(\underline{r},t) + \frac{\beta_i \Sigma_a \Phi(\underline{r},t)\, k}{\rho} \quad (4)$$

The space and time variables of Eq.'s (3) and (4) are separated and the resulting space independent kinetic equations are

$$\left[(1-\beta)\, k_{eff} - 1\right]\frac{n(t)}{\ell} + \sum_{i=1}^{6}\lambda_i H_i(t) = \frac{dn(t)}{dt} \quad (5)$$

and

$$\frac{dH_i(t)}{dt} = -\lambda_i H_i(t) + \frac{k_{eff}\, n(t)\, \beta_i}{\ell} \quad (6)$$

By assuming a solution for the set of seven linear differential equations of the form

$$n(t) = A' e^{\omega t} \quad (7)$$

and

$$H_i(t) = B_i' e^{\omega t} \tag{8}$$

a characteristic equation may be obtained as

$$\omega = \frac{\rho - \beta}{\ell(1-\rho)} + \frac{1}{\ell(1-\rho)} \sum_{i=1}^{6} \frac{\beta_i \lambda_i}{\omega + \lambda_i} \tag{9}$$

or

$$\rho = \frac{\ell\omega}{1+\ell\omega} + \frac{1}{1+\ell\omega} \sum_{i=1}^{6} \frac{\lambda_i \beta_i}{\omega + \lambda_i} \,. \tag{10}$$

The neutron density as a function of time is expressed as

$$\frac{n(t)}{n(0)} = \frac{\phi(t)}{\phi(0)} = \sum_{j=1}^{7} A_j e^{\omega_j t} \tag{11}$$

where the coefficients, A_j's, are

$$A_j = (1-\rho) \left[\frac{\ell + \sum_{i=1}^{6} \frac{\beta_i}{\omega_j + \lambda_i}}{\ell(1-\rho) + \sum_{i=1}^{6} \frac{\beta_i \lambda_i}{(\omega_j + \lambda_i)^2}} \right] . \tag{12}$$

Three groups of delayed neutrons are used as an approximation for the usual six groups. This simplifies the solution of

the kinetic differential equations but they are of the same general form as Eq.'s (7) and (8).

When reactivity is a function of time, the differential equations are, in some instances, very difficult to solve; therefore, an approximation may be made where it is assumed that the six groups of delayed neutrons may be represented by one group of delayed neutrons. McPhee (15) developed a solution to the approximated one group kinetic model and a similar model is developed in Appendix - B. McPhee's equations were developed for a linear time dependent reactivity insertion whereas the kinetic development given in Appendix - B, of this paper, is based on the assumption of reactivity insertion rate empiricized to

$$k_{off}(t) = 1 - A(1-e^{-bt^n}). \qquad (13)$$

A is a parameter determined by the final value of the reactivity change and b and n are constants determined empirically.

COMPUTED DATA

General

The solutions of the differential equations for several kinetics models were programmed for the Kansas State University IBM-650 computer. Several neutron parameter models were used. First, the usual equations using six groups of delayed neutrons were programmed and two sets of neutron parameters were used, Hughes' (10) and Keepin's (13). Next, an approximation of the

six groups of delayed neutrons by three groups was made. The three delayed neutron parameters were chosen such that tho approximation would predict the same time dependent neutron flux after a step change in reactivity as did the six group model. Finally, the kinetic equations were approximated with one group of delayed neutrons and were solved for reactivity as a function of time.

The equations of interest (see Appendix - A) for the six group analysis with step inputs of reactivity were the characteristic equation,

$$\rho = \frac{l\omega}{l\omega+1} + \frac{1}{l\omega+1} \sum_{i=1}^{6} \frac{\lambda_i \omega}{\omega+\lambda_i} \tag{9}$$

and the equation of neutron flux as a function of time normalized to tho critical flux just before the step change in reactivity

$$\frac{\phi(t)}{\phi(0)} = \sum_{j=1}^{7} A_j e^{\omega_j t} \tag{11}$$

An IBM-650 program was written to compute roots of Eq. (9) and to compute the A_j's of Eq. (11). The computed data were used to analyze the experimental results from both the TRIGA reactor and the Argonaut reactor; therefore two different prompt neutron lifetimes, 2.0×10^{-4} seconds for the Argonaut and 8.0×10^{-5} seconds for the TRIGA reactor, were used in the

computations. The parameter variation in each set of data was reactivity.

Delayed Neutron Parameters

The two delayed neutron parameter models used in the theoretical computations were those as given by Keepin (13) and by Hughes (10). The two sets of neutron parameters gave slightly different sets of data. The following tables list the delayed neutron constants used in the analytical computations. A list of several other sets of delayed neutron parameters was made by Keepin (12).

Table 2. Keepin's delayed neutron data for thermal fission of U^{235}.

Group	Halflife (Sec.)		λ_i(Sec.$^{-1}$)	β_i/β
1	55.72	1.28	0.01244	0.033 ± 0.003
2	22.72	0.71	0.03051	0.219 ± 0.009
3	6.22	0.23	0.1114	0.196 ± 0.022
4	2.30	0.09	0.3014	0.395 ± 0.011
5	0.61	0.063	1.1363	0.115 ± 0.009
6	0.236	0.025	3.0137	0.042 ± 0.008

$\beta = 0.0064$

Table 3. Hughes' delayed neutron data for thermal fission of U^{235}.

Group	Halflife (Sec.)		λ_i (Sec.$^{-1}$)	β_i/β	
1	55.6	0.2	0.01246	0.034	0.009
2	22.0	0.2	0.03149	0.220	0.023
3	4.51	0.1	0.1537	0.282	0.017
4	1.52	0.05	0.456	0.319	0.017
5	0.43	0.05	1.612	0.112	0.011
6	0.05	0.02	13.86	0.033	0.009

$\beta = 0.00755$

Reactivity Parameter

Computed data were calculated over wide ranges of reactivity, from -$0.001 to -$10.0 for negative reactivities and from $0.001 to $1.20 for positive reactivities (see PLATES II through XXI).

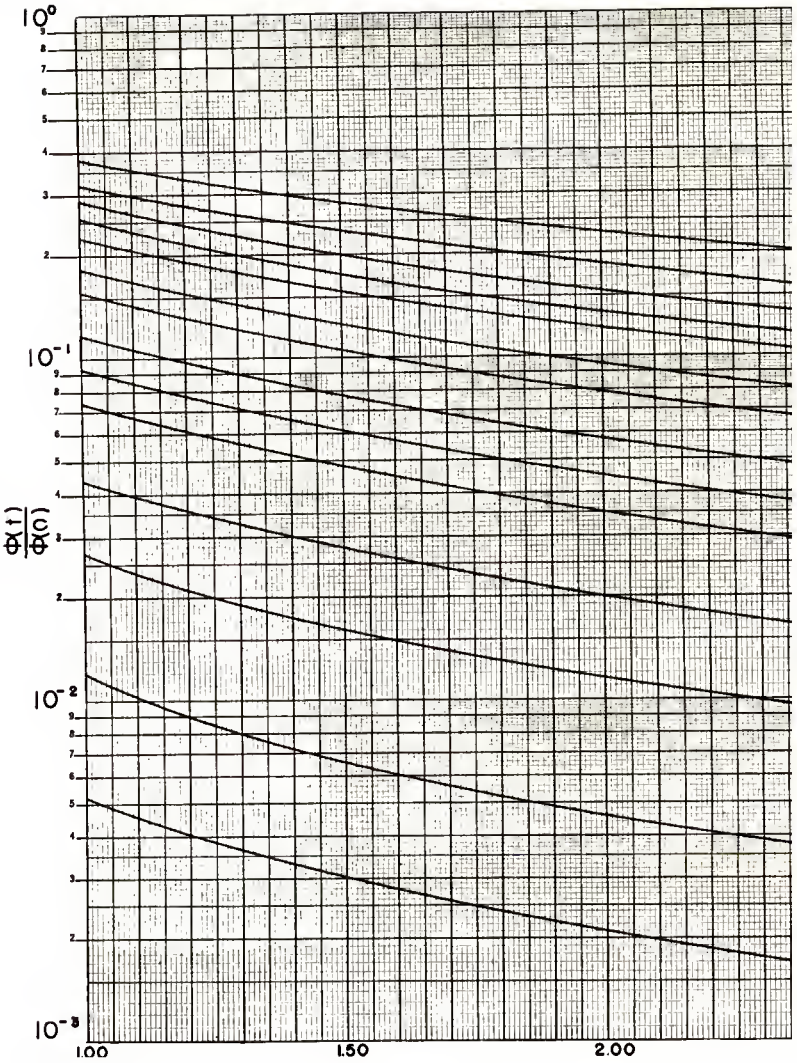
EXPLANATION OF PLATE II

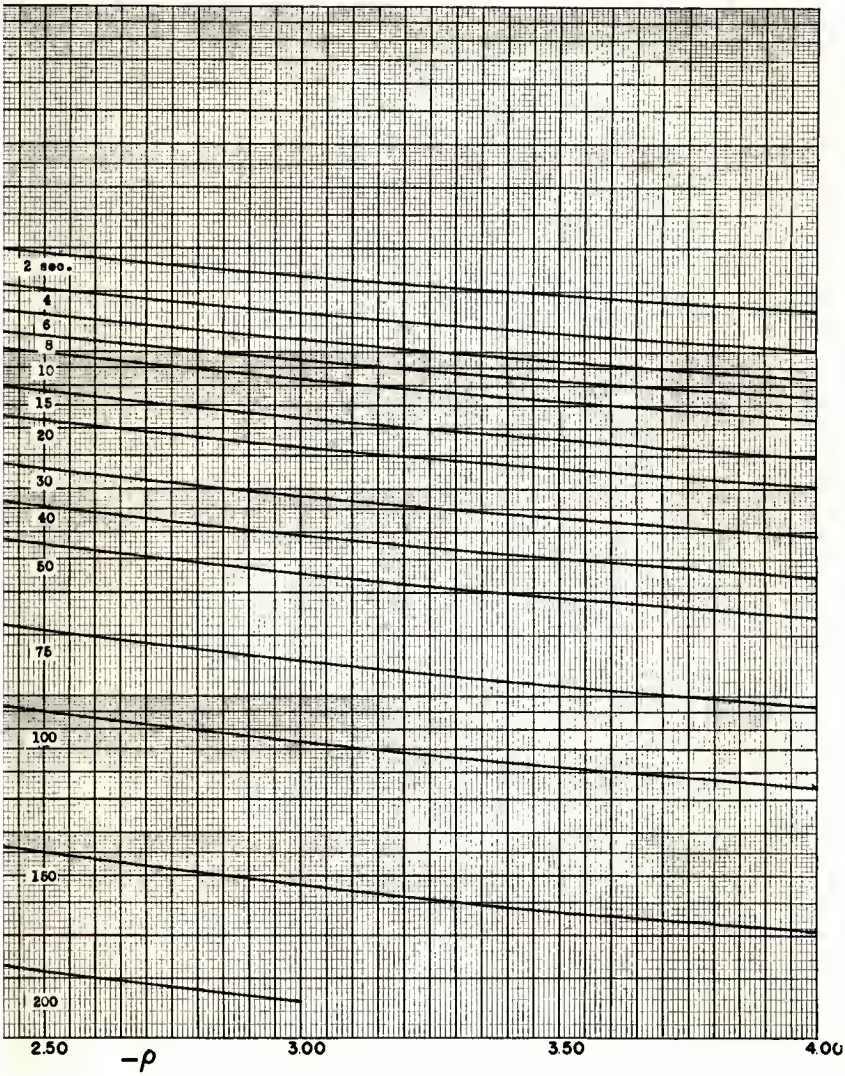
Flux ratio, $\phi(t)/\phi(o)$, vs. step changes of negative reactivity, ρ (dollars), for various times, t (seconds), after the reactivity change.

The prompt neutron lifetime, $\ell = 8.0 \times 10^{-5}$ seconds.

Hughes' delayed neutron parameters.

Data computed from Eq. (11).


10^{0}
10^{-1}
10^{-2}
10^{-3}
$\frac{\Phi(t)}{\Phi(0)}$
1.00
1.50
2.00


2 sec.
4
6
8
10
15
20
30
40
50
75
100
150
200
2.50
3.00
3.50
4.00
−ρ

EXPLANATION OF PLATE III

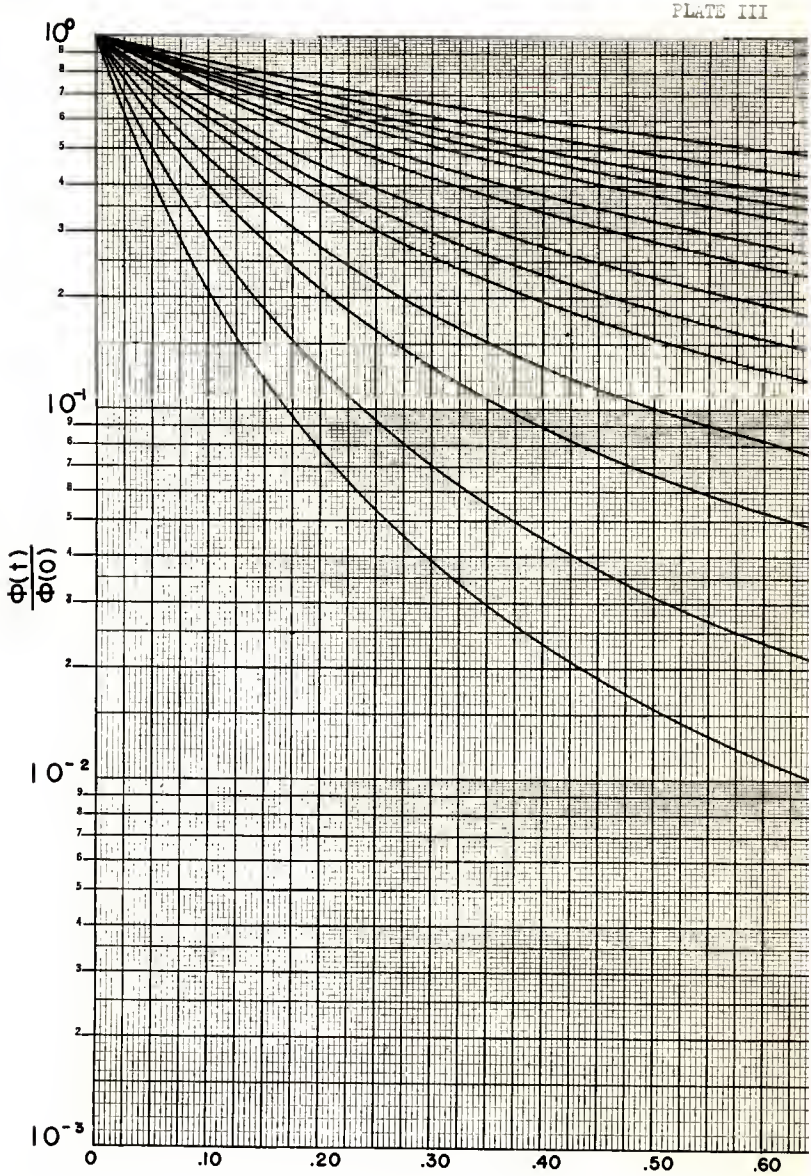
Flux ratio, $\phi(t)/\phi(o)$, vs. step changes of negative reactivity, ρ (dollars), for various times, t (seconds), after the reactivity change.

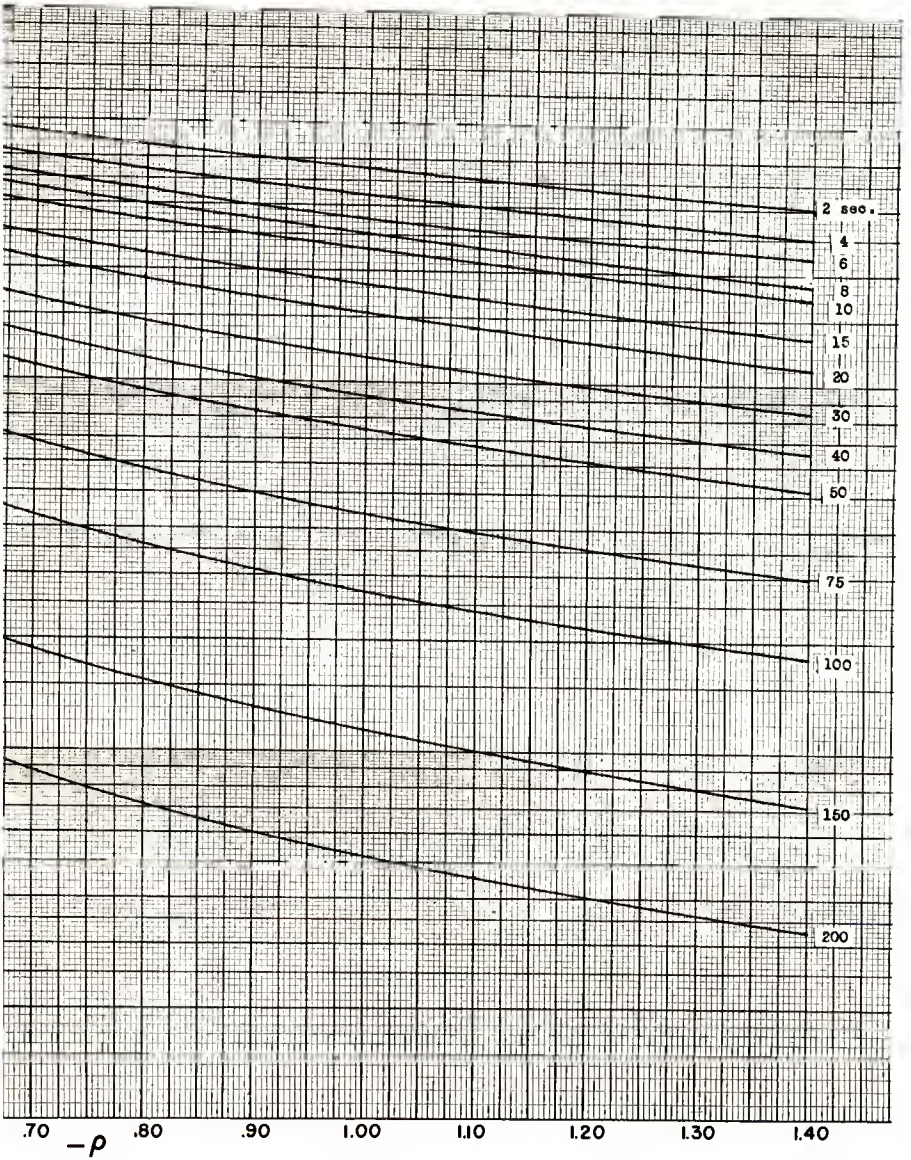
The prompt neutron lifetime, $\ell = 8.0 \times 10^{-5}$ seconds.

Hughes' delayed neutron parameters.

Data computed from Eq. (11).

PLATE III




2 sec.
4
6
8
10
15
20
30
40
50
75
100
150
200
.70
−ρ
.80
.90
1.00
1.10
1.20
1.30
1.40

EXPLANATION OF PLATE IV

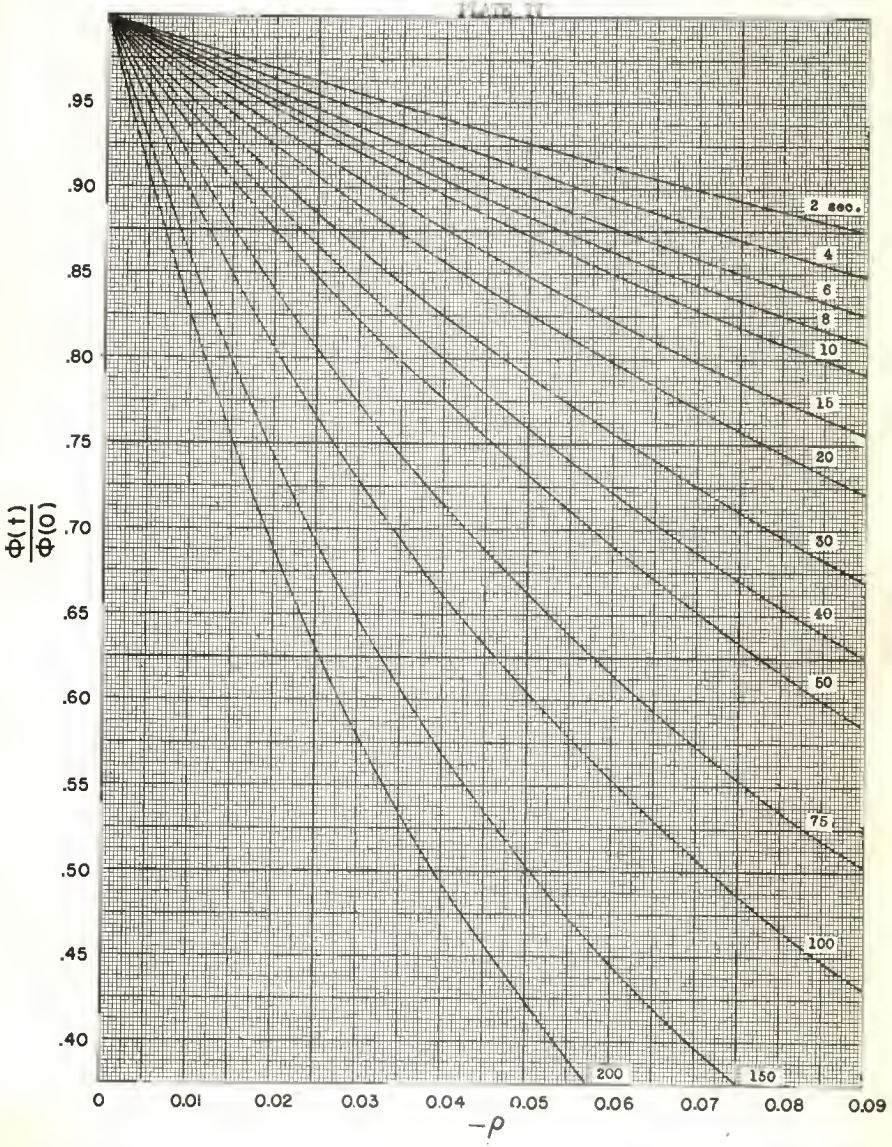
Flux ratio, $\phi(t)/\phi(o)$, vs. step changes of negative reactivity, ρ (dollars), for various times, t (seconds), after the reactivity change.

The prompt neutron lifetime, $\ell = 8.0 \times 10^{-5}$ seconds.

Hughes' delayed neutron parameters.

Data computed from Eq. (11).

PLATE II


.95
.90
.85
.80
.75
.70
.65
.60
.55
.50
.45
.40
$\frac{\Phi(t)}{\Phi(0)}$
2 sec.
4
6
8
10
15
20
30
40
50
75
100
150
200
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
$-\rho$

EXPLANATION OF PLATE V

Negative reactivity, ρ (dollars), vs. roots, ω (seconds^{-1}), of the characteristic equation for step changes of reactivity.

All roots, ω, are negative in value.

The λ's indicate the decay constants of the neutron precursor groups.

Prompt neutron lifetime, $\ell = 8.0 \times 10^{-5}$ second.

Hughes' delayed neutron parameters.

Data computed from Eq. (9).

10^{0}
10^{-1}
10^{-2}
10^{-3}
ρ
ω_1
ω_2
ω_3
10^{-4}
10^{-3}
10^{-2}
10^{-1}
λ_1
λ_2
λ_3

ω_4
ω_5
ω_6
ω_7
λ^4
λ^5
λ^6
10^0
10^1
10^2
ω_j

EXPLANATION OF PLATE VI

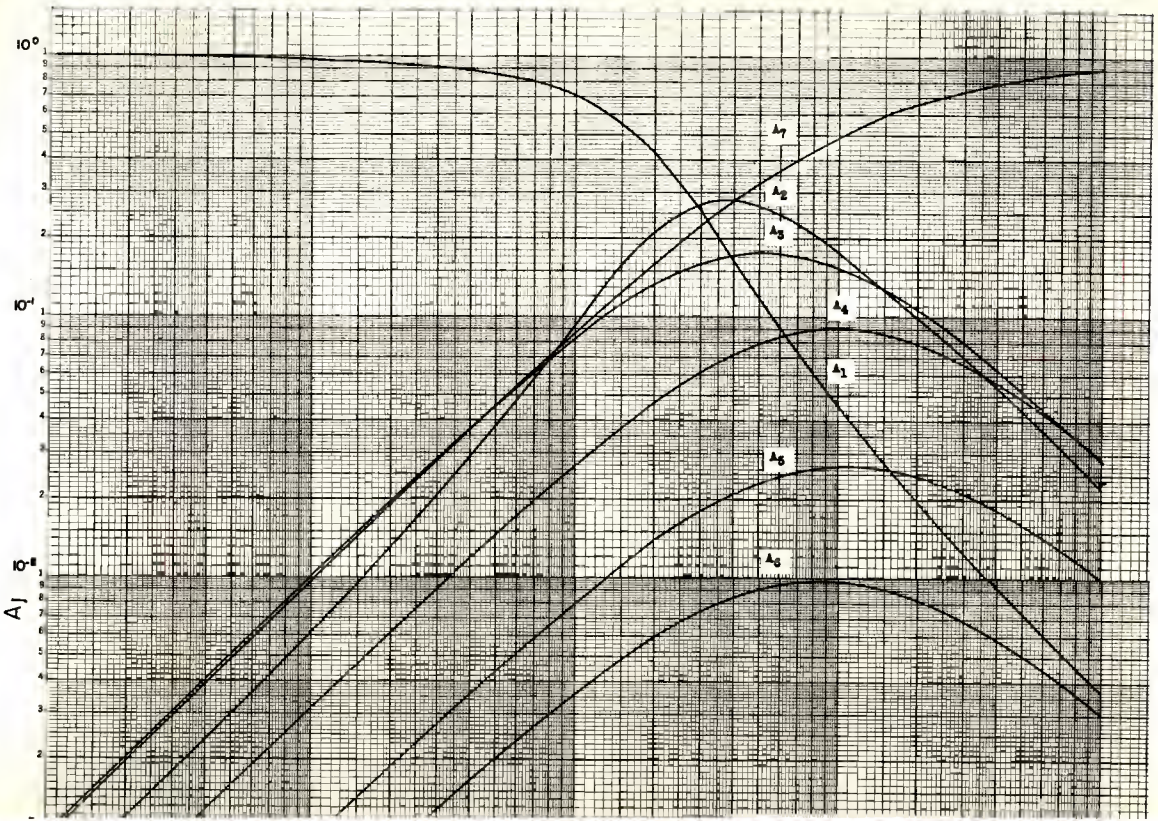
Dimensionless coefficients, A_j's, of the flux equation vs. negative reactivity, ρ (dollars), for step changes in reactivity.

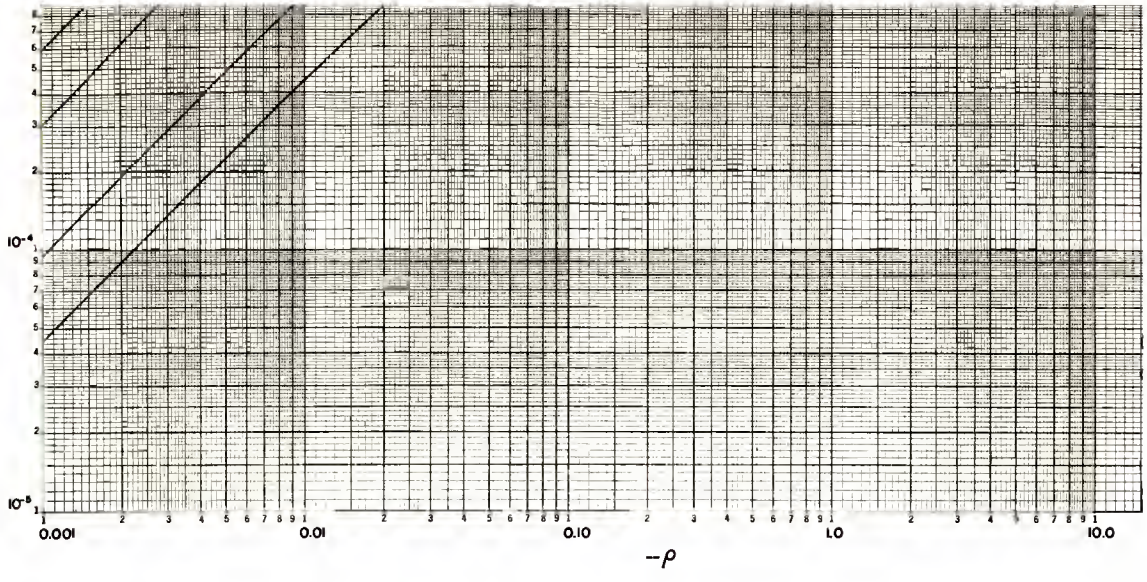
Prompt neutron lifetime, $\ell = 8.0 \times 10^{-5}$ seconds.

Hughes' delayed neutron parameters.

Data calculated from Eq. (12).

PLATE VI




10^{-4}
10^{-5}
0.001
0.01
0.10
1.0
10.0
$-\rho$

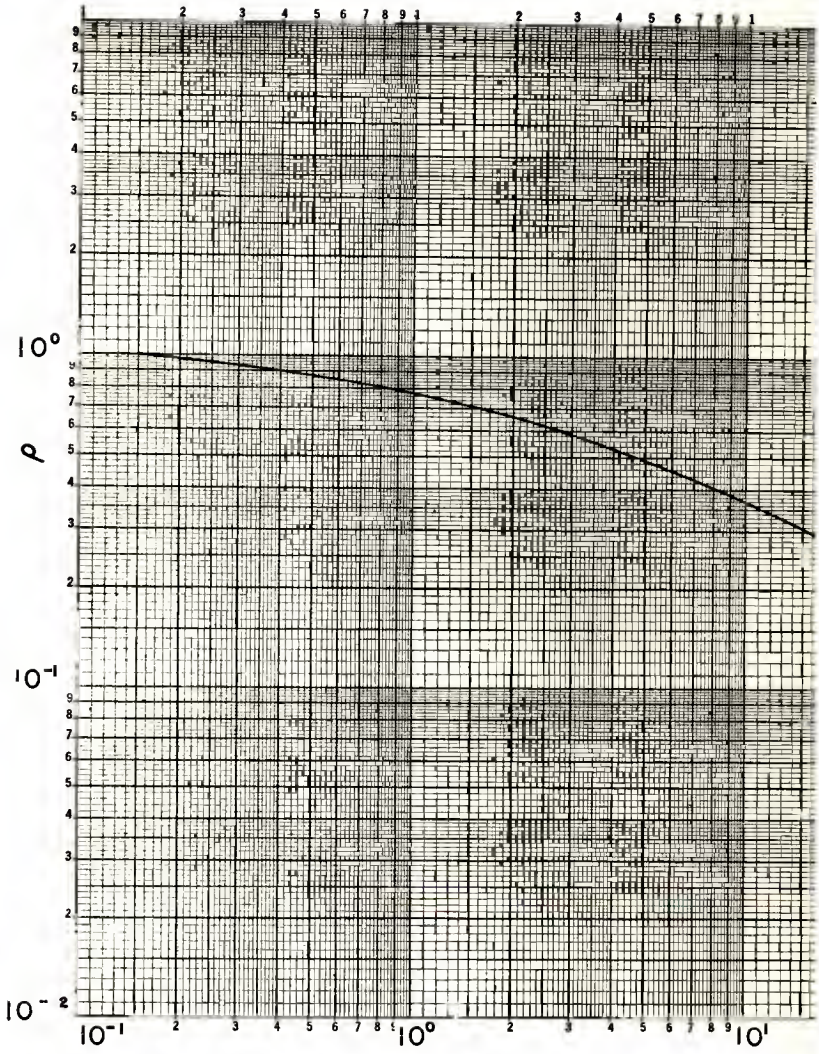
EXPLANATION OF PLATE VII

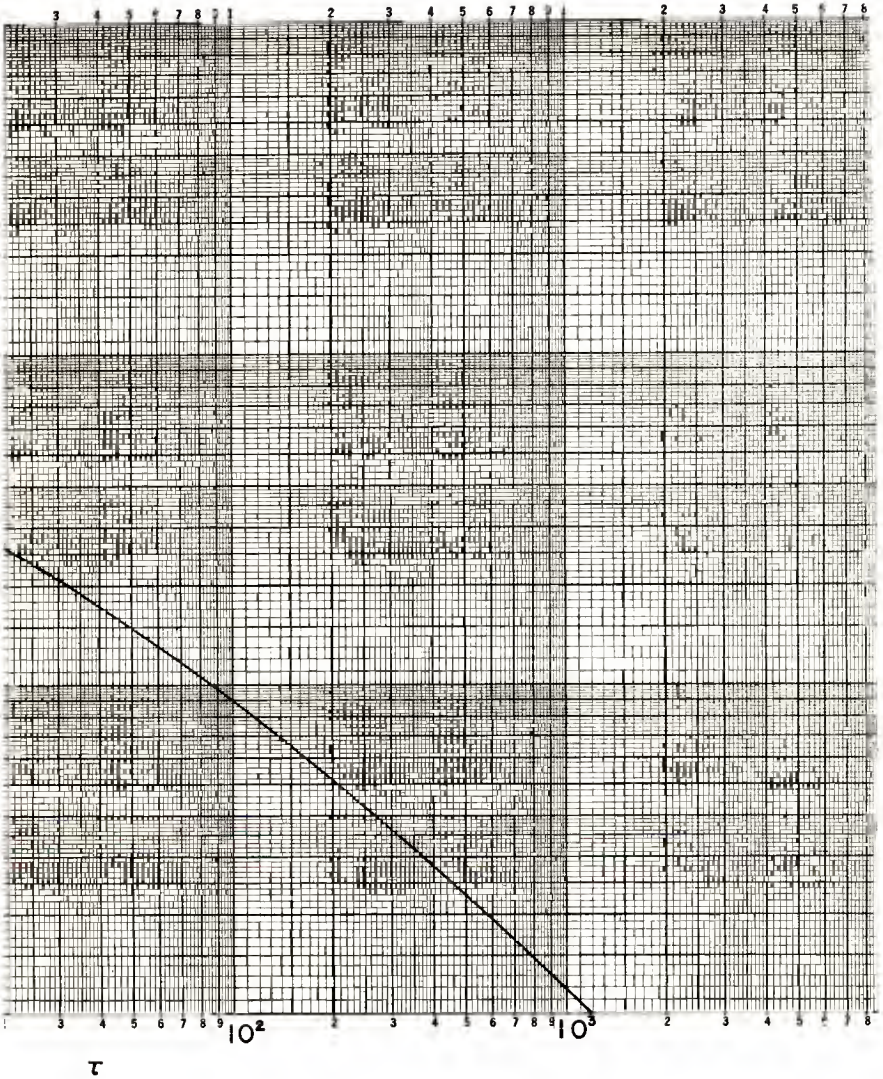
Positive reactivity, ρ (dollars), vs. positive asymptotic period, τ (seconds).

Prompt neutron lifetime, $\ell = 8.0 \times 10^{-5}$ seconds.

Hughes' delayed neutron parameters.

Data computed from Eq. (9).


ρ
10⁰
10⁻¹
10⁻²
10⁻¹
10⁰
10¹


10^2
10^3
τ

EXPLANATION OF PLATE VIII

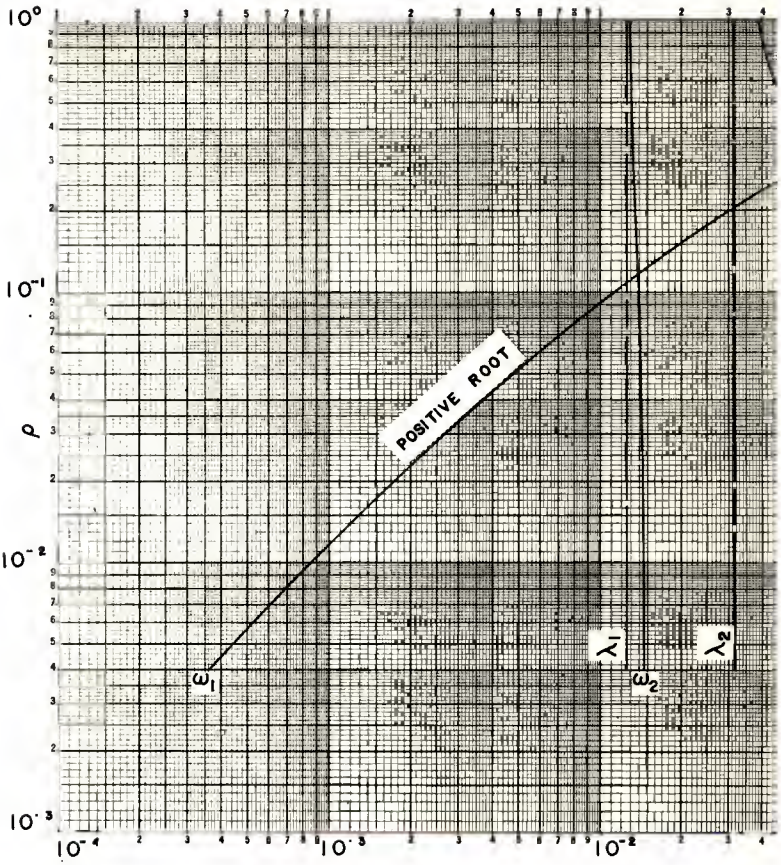
Positive reactivity, ρ (dollars), vs. roots, ω's (seconds^{-1}) of the characteristic equation for step changes in reactivity.

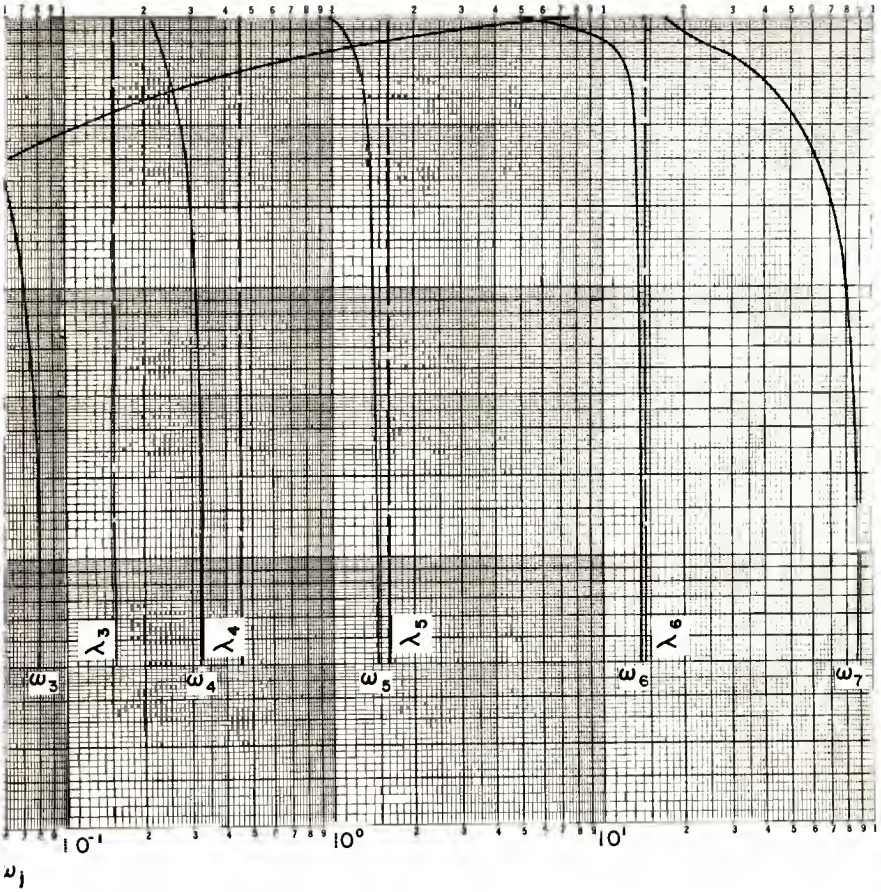
All roots are negative except the indicated positive root.

The prompt neutron lifetime, $\ell = 8.0 \times 10^{-5}$ seconds.

Hughes' delayed neutron parameters.

Data computed from Eq. (9).


POSITIVE ROOT
ρ
ω_1
ω_2
λ_1
λ_2


ω_3
ω_4
ω_5
ω_6
ω_7
λ_3
λ_4
λ_5
λ_6
10^{-1}
10^{0}
10^{1}
ω_i

EXPLANATION OF PLATE IX

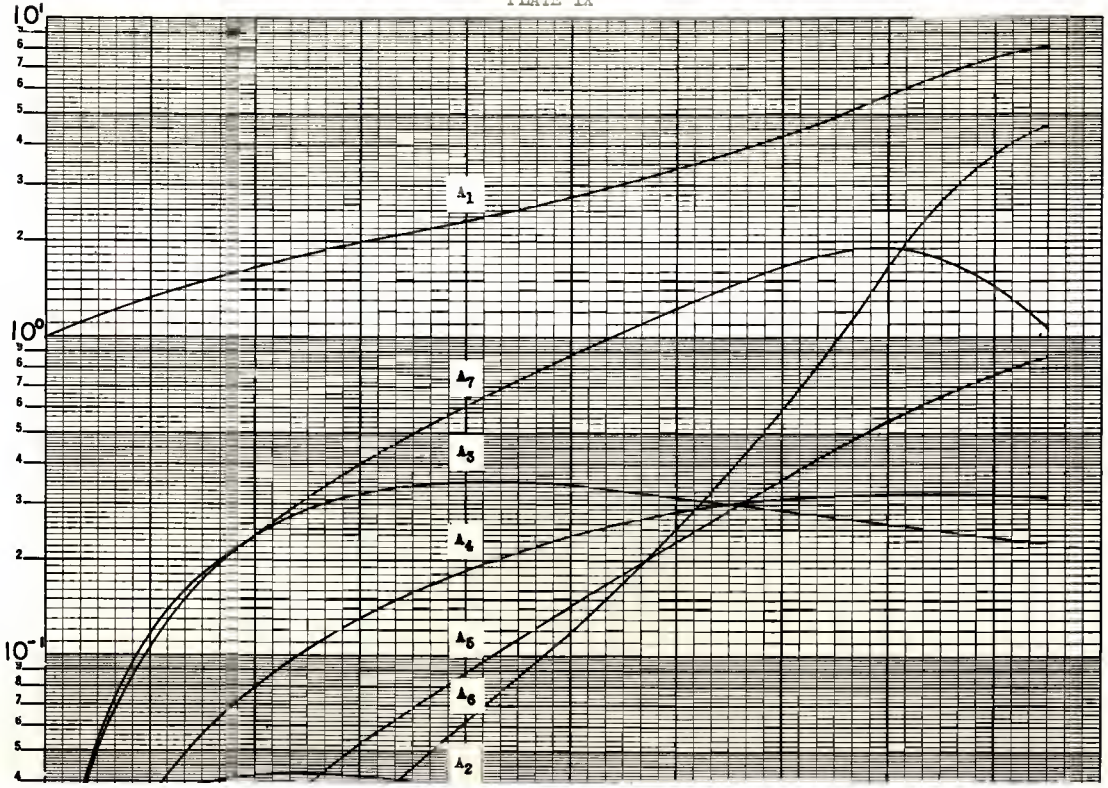
Dimensionless coefficients, A_j's of the flux equation vs. positive reactivity, ρ (dollars), for step changes in reactivity.

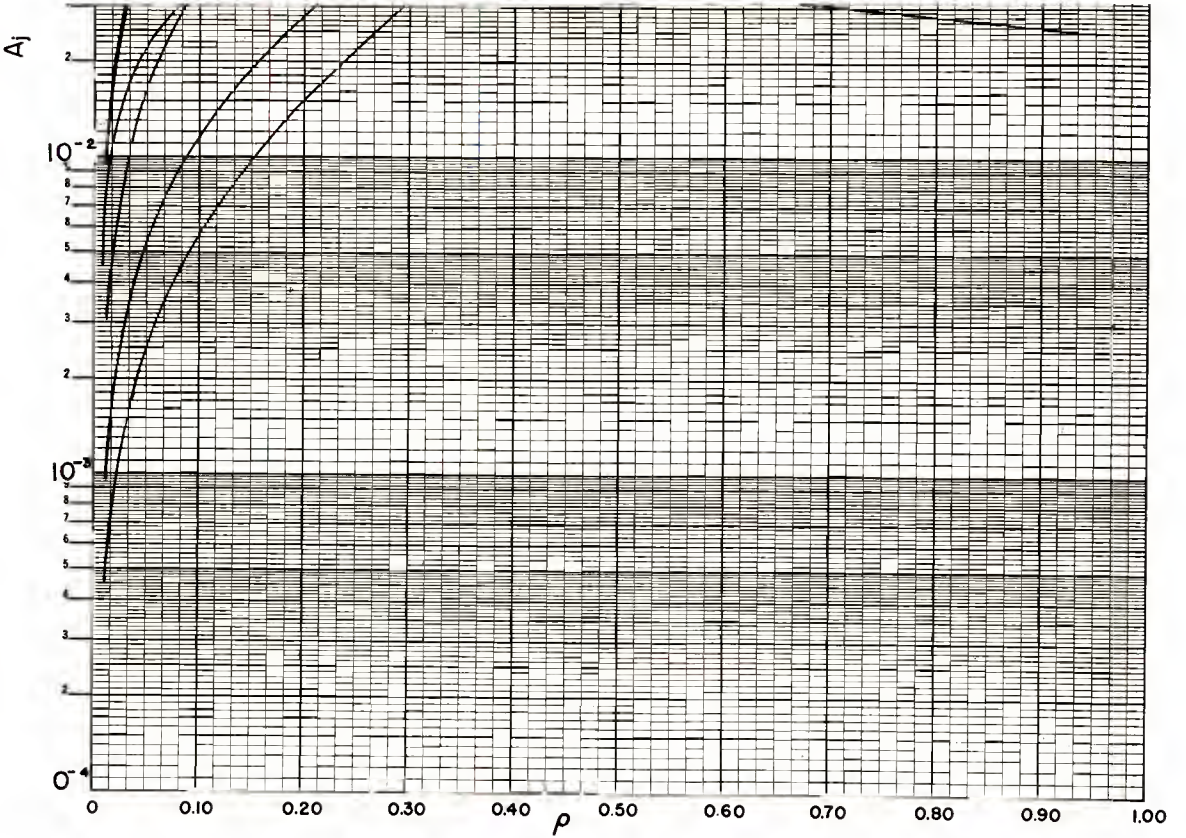
Prompt neutron lifetime, $\ell = 8.0 \times 10^{-5}$ seconds.

Hughes' delayed neutron parameters.

Data computed from Eq. (12).

PLATE IX




A_j
10^{-2}
10^{-3}
10^{-4}
0
0.10
0.20
0.30
0.40
ρ
0.50
0.60
0.70
0.80
0.90
1.00

EXPLANATION OF PLATE X

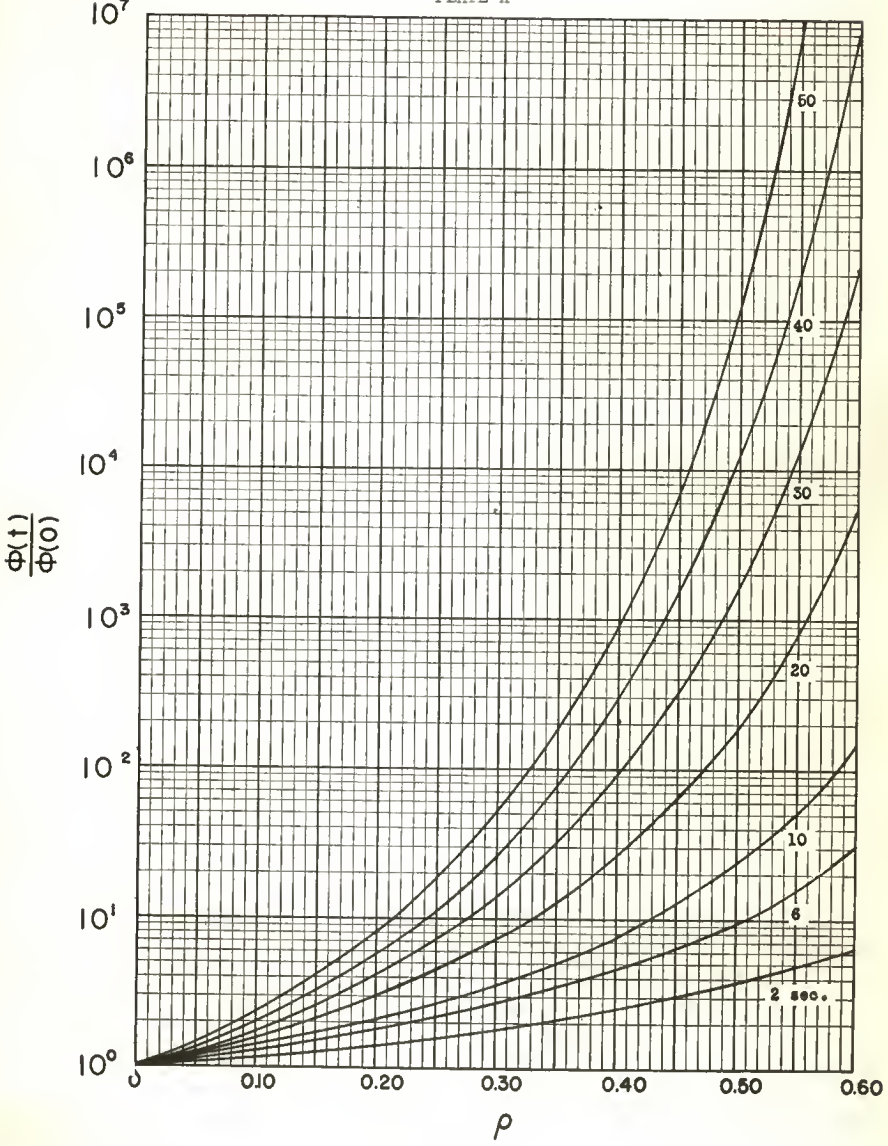
Flux ratio, $\phi(t)/\phi(o)$, vs. step changes of positive reactivity, ρ (dollars), for various times, t (seconds), after the reactivity step.

The prompt neutron lifetime, $\ell = 8.0 \times 10^{-5}$ seconds.

Hughes' delayed neutron parameters.

Data computed from Eq. (11).

PLATE X


10^7
10^6
10^5
10^4
10^3
10^2
10^1
10^0
$\frac{\Phi(t)}{\Phi(0)}$
60
40
30
20
10
6
2 sec.
0
0.10
0.20
0.30
0.40
0.50
0.60
ρ

EXPLANATION OF PLATE XI

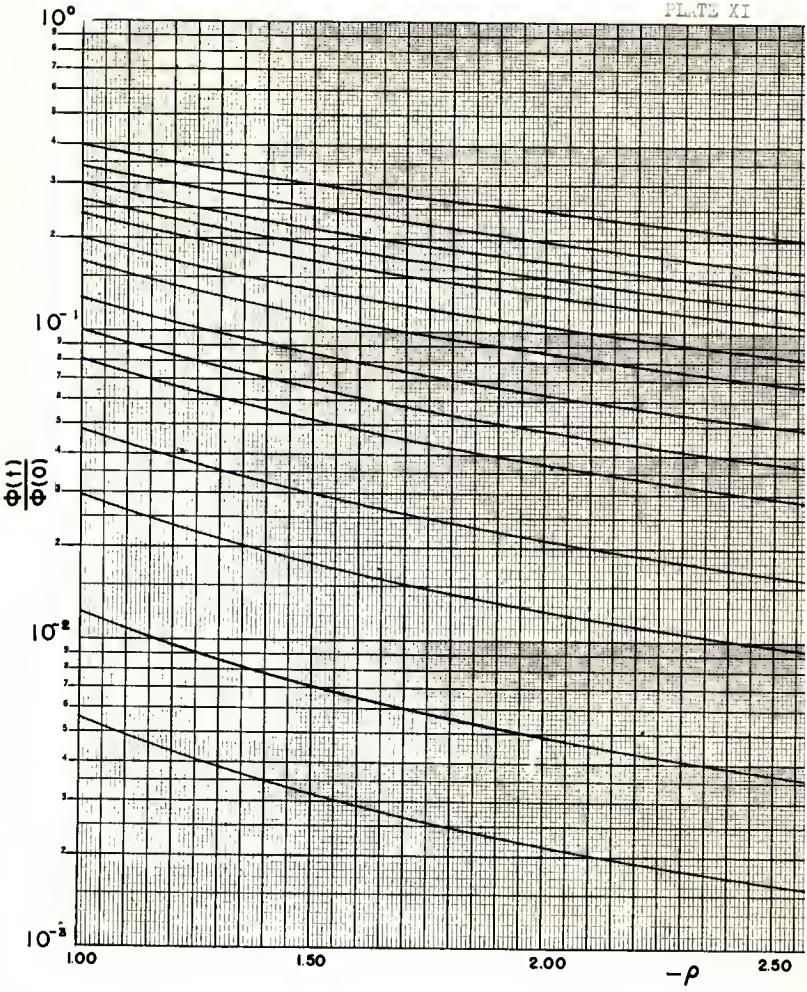
Flux ratio, $\phi(t)/\phi(0)$, vs. step changes of negative reactivity, ρ (dollars), for various times, t (seconds), after the reactivity step.

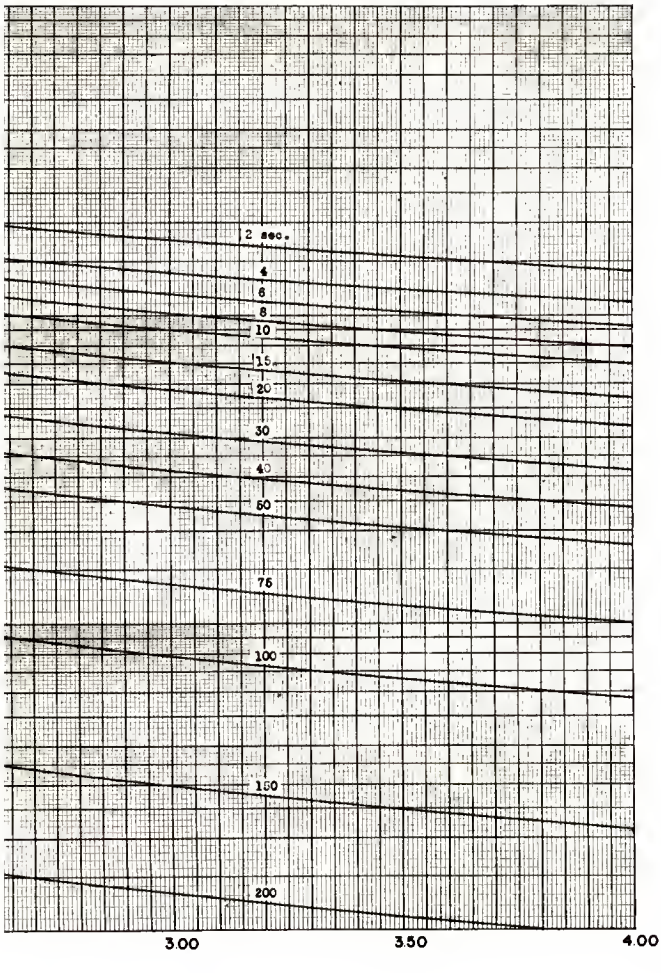
The prompt neutron lifetime, $\ell = 8.0 \times 10^{-5}$ seconds.

Keepin's delayed neutron parameters.

Data computed from Eq. (11).

PLATE XI




2 sec.
4
6
8
10
15
20
30
40
60
75
100
150
200
3.00
3.50
4.00

EXPLANATION OF PLATE XII

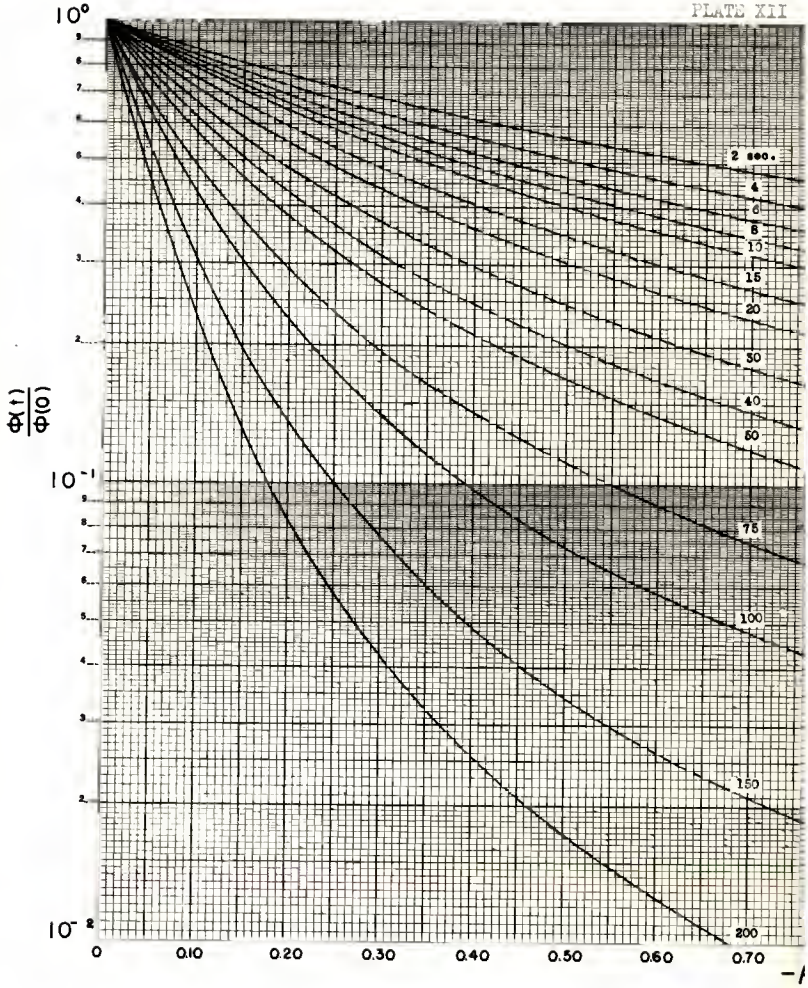
Flux ratio, $\phi(t)/\phi(o)$, vs. step changes of negative reactivity, ρ (dollars), for various times, t (seconds), after the reactivity step.

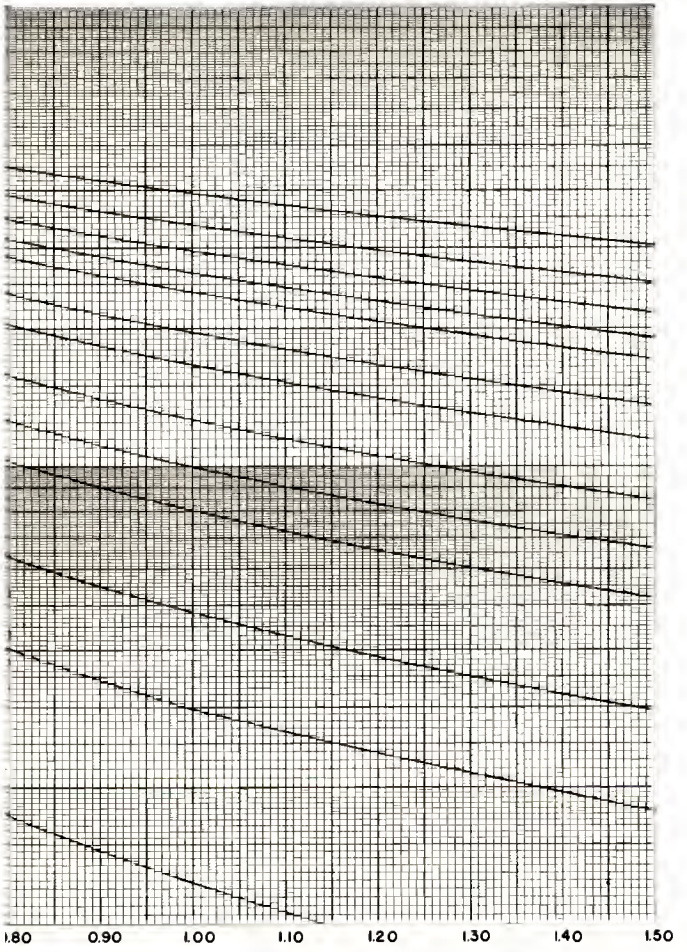
The prompt neutron lifetime, $\ell = 8.0 \times 10^{-5}$ seconds.

Keepin's delayed neutron parameters.

Data computed from Eq. (11).

PLATE XII


$\frac{\Phi(t)}{\Phi(0)}$
10^{0}
10^{-1}
10^{-2}
2 sec.
4
6
8
10
15
20
30
40
60
75
100
150
200
0
0.10
0.20
0.30
0.40
0.50
0.60
0.70


1.80
0.90
1.00
1.10
1.20
1.30
1.40
1.50

EXPLANATION OF PLATE XIII

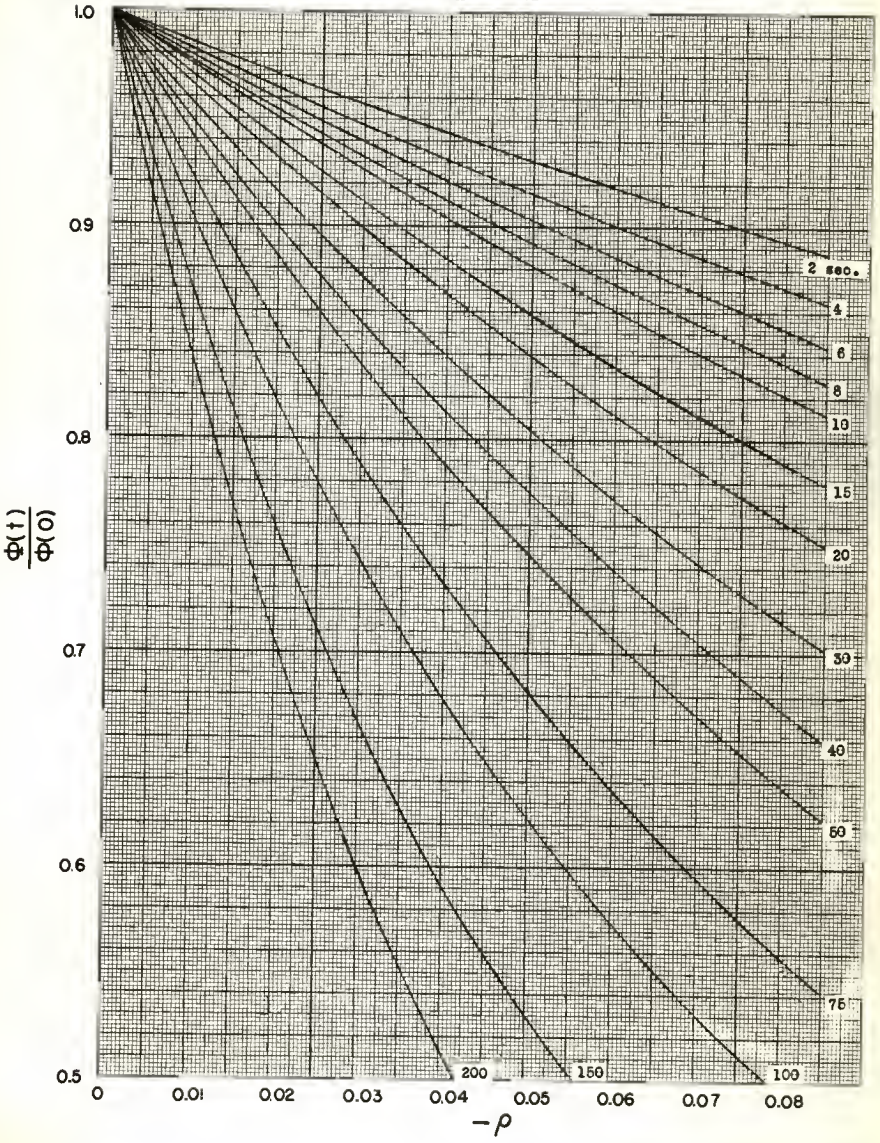
Flux ratio, $\phi(t)/\phi(o)$, vs. step changes of negative reactivity, ρ (dollars), for various times, t (seconds), after the reactivity step.

The prompt neutron lifetime, $\ell = 8.0 \times 10^{-5}$ seconds.

Keepin's delayed neutron parameters.

Data computed from Eq. (11).

PLATE XIII


1.0
0.9
0.8
0.7
0.6
0.5
$\frac{\Phi(t)}{\Phi(0)}$
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
$-\rho$
2 sec.
4
6
8
10
15
20
30
40
50
75
100
150
200

EXPLANATION OF PLATE XIV

Negative reactivity, ρ (dollars), vs. roots, ω's (seconds), of the characteristic equation for step changes of reactivity.

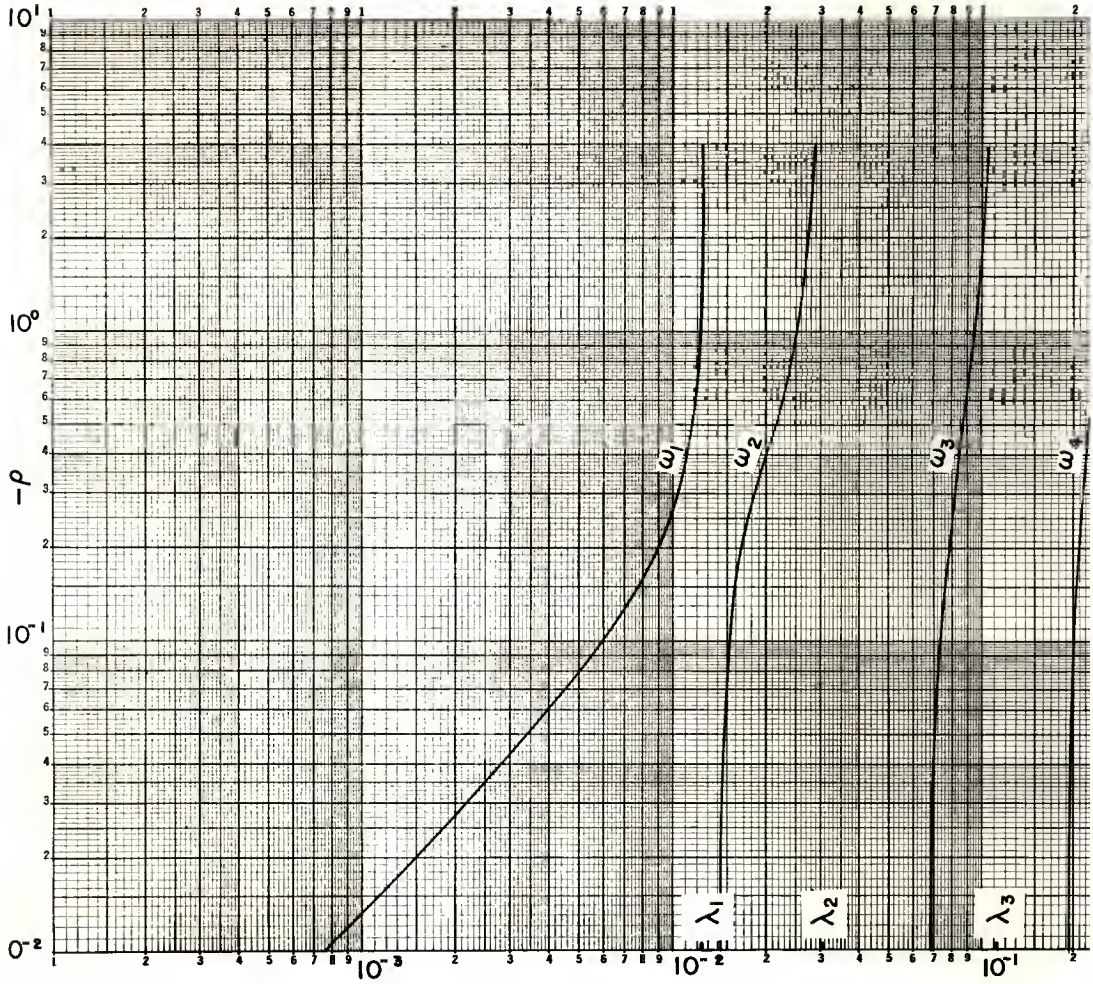
All roots are negative.

The λ's indicate the decay constants of the neutron precursor groups.

Prompt neutron lifetime, $\ell = 8.0 \times 10^{-5}$ seconds.

Keepin's delayed neutron parameters.

Data computed from Eq. (9).


10^1
10^0
10^{-1}
10^{-2}
$-\rho$
ω_1
ω_2
ω_3
ω_4
λ_1
λ_2
λ_3
10^{-3}
10^{-2}
10^{-1}

ω5
ω6
ω7
λ4
λ5
λ6
10°
10¹
10²
ωj

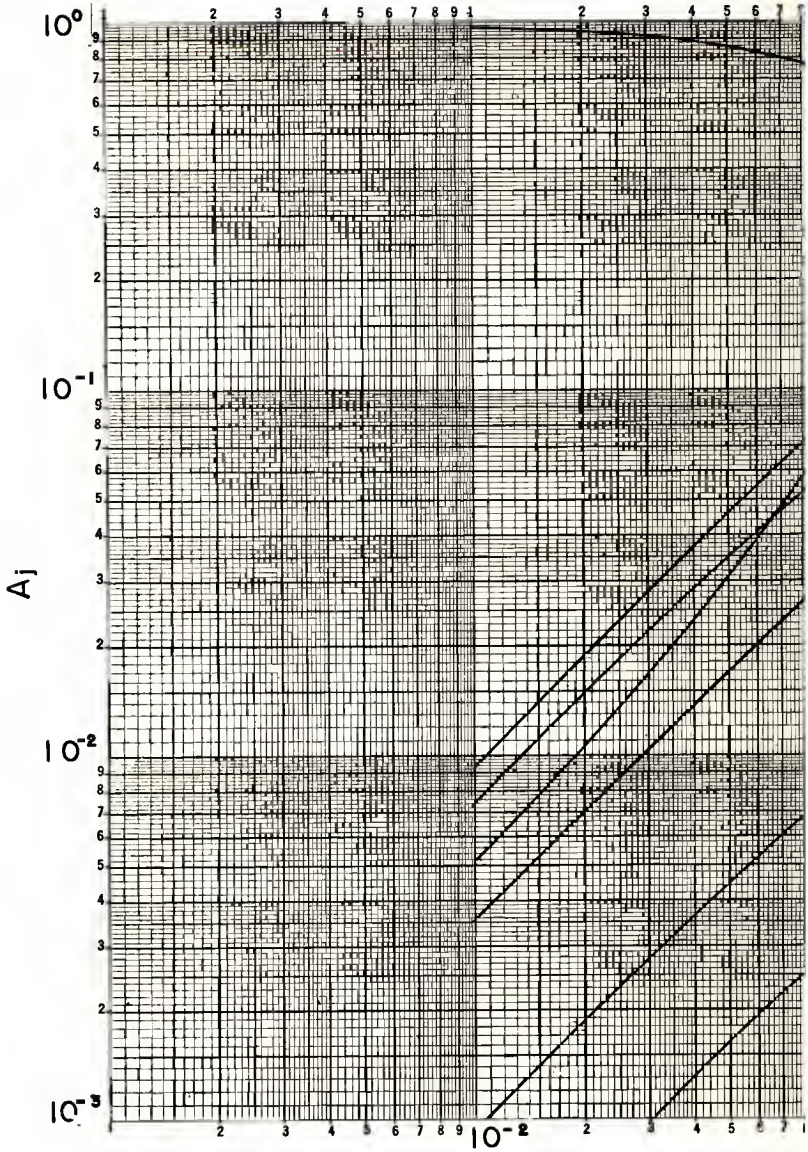
EXPLANATION OF PLATE XV

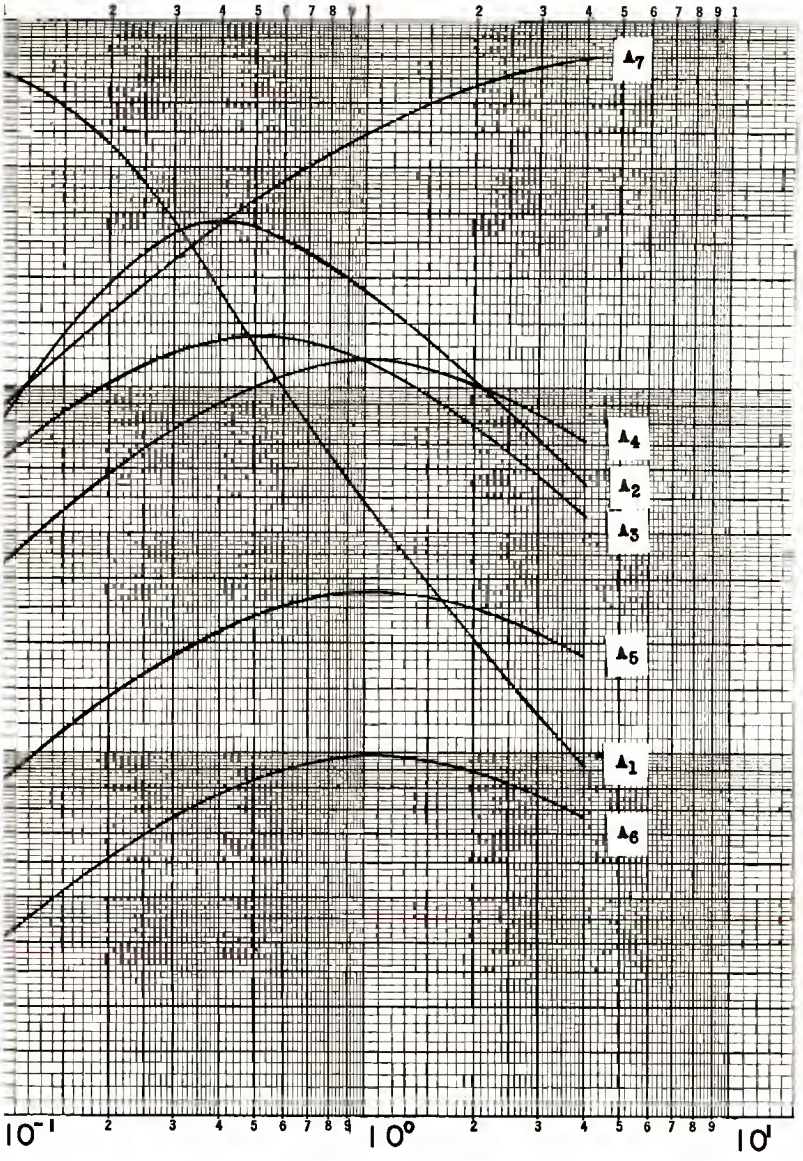
Dimensionless coefficients, A_j's, of the flux equation vs. negative reactivity, ρ (dollars), for step changes in reactivity.

The prompt neutron lifetime, $\ell = 8.0 \times 10^{-5}$ seconds.

Keepin's delayed neutron data.

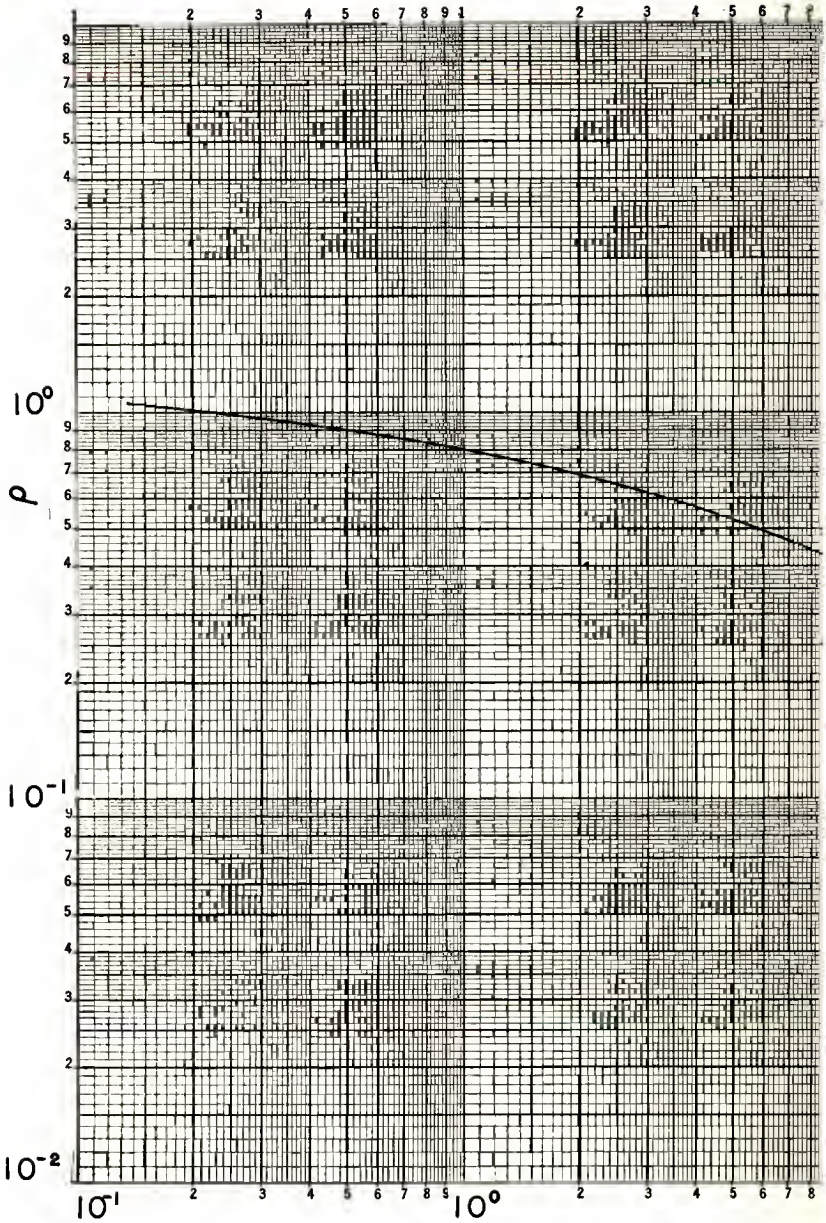
Data computed from Eq. (12).

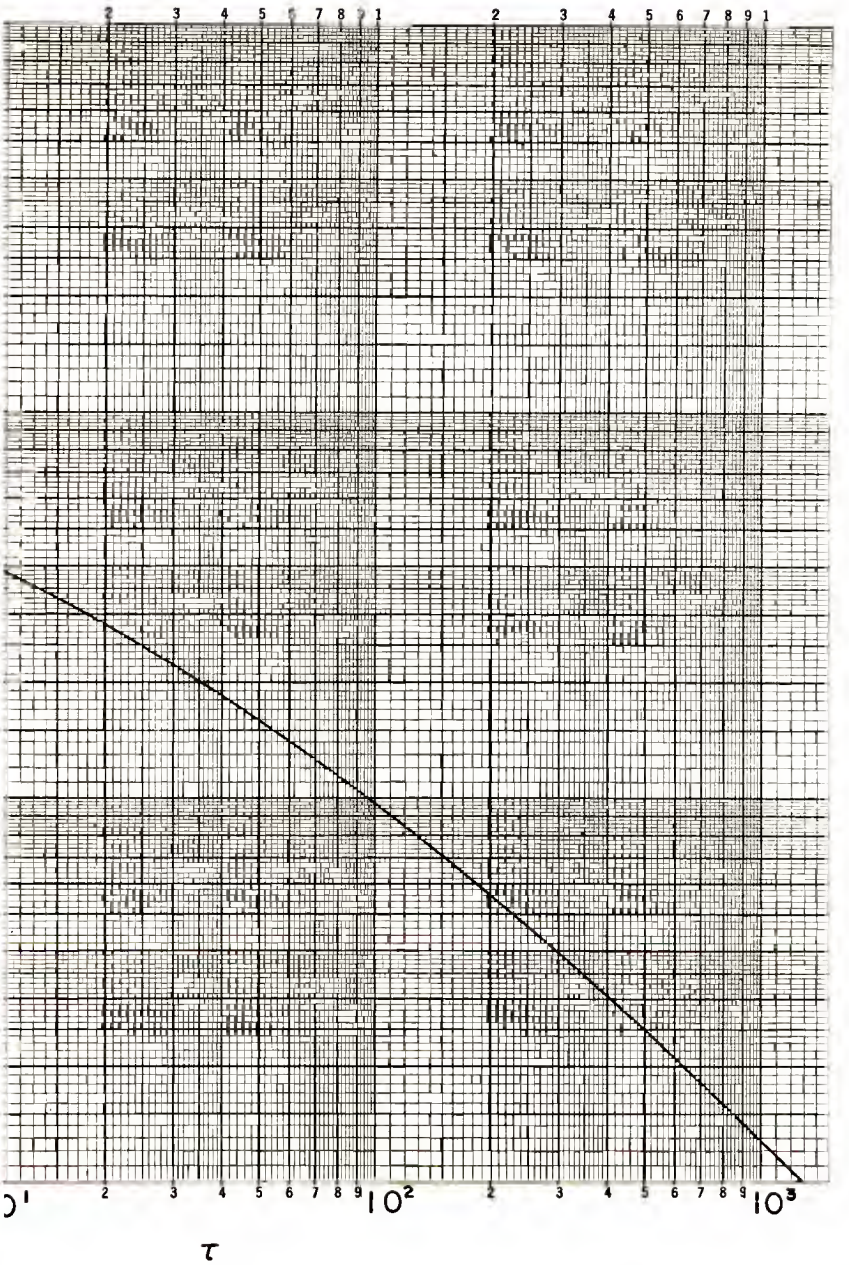

10^{0}
10^{-1}
10^{-2}
10^{-3}
A_j
10^{-2}


A7
A4
A2
A3
A5
A1
A6
10⁻¹
10⁰
10¹
-ρ

EXPLANATION OF PLATE XVI

Positive reactivity, ρ (dollars), vs.
positive asymptotic period, τ (seconds).
Prompt neutron lifetime, $\ell = 8.0 \times 10^{-5}$ seconds.
Keepin's delayed neutron parameters.
Data computed from Eq. (9).


ρ
10⁰
10⁻¹
10⁻²
10⁻¹
10⁰


2 3 4 5 6 7 8 9 1
2 3 4 5 6 7 8 9 1
2 3 4 5 6 7 8 9
10^2
2 3 4 5 6 7 8 9
10^3
τ

EXPLANATION OF PLATE XVII

Positive reactivity, ρ (dollars), vs. roots, ω's (seconds), of the characteristic equation for step changes of reactivity.

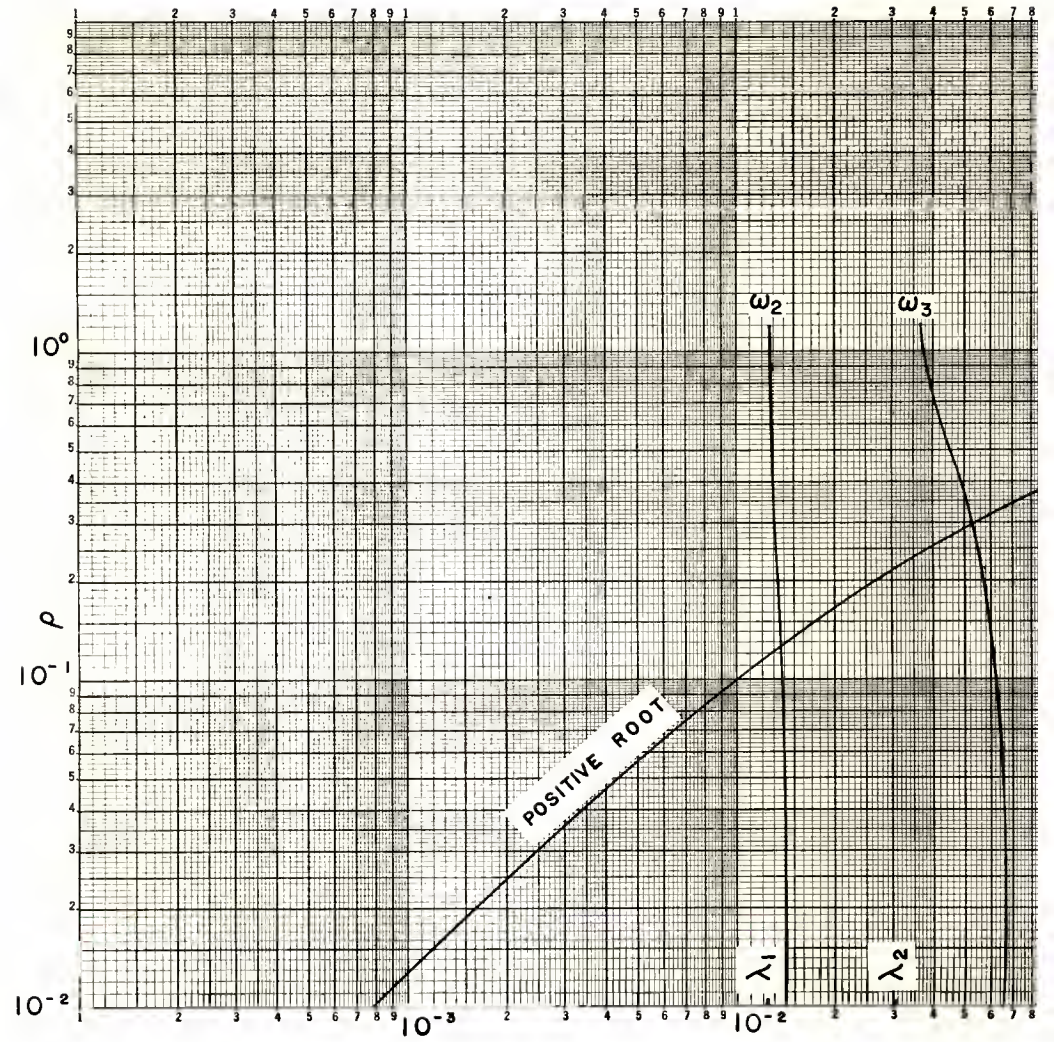
All roots are negative except the indicated positive root.

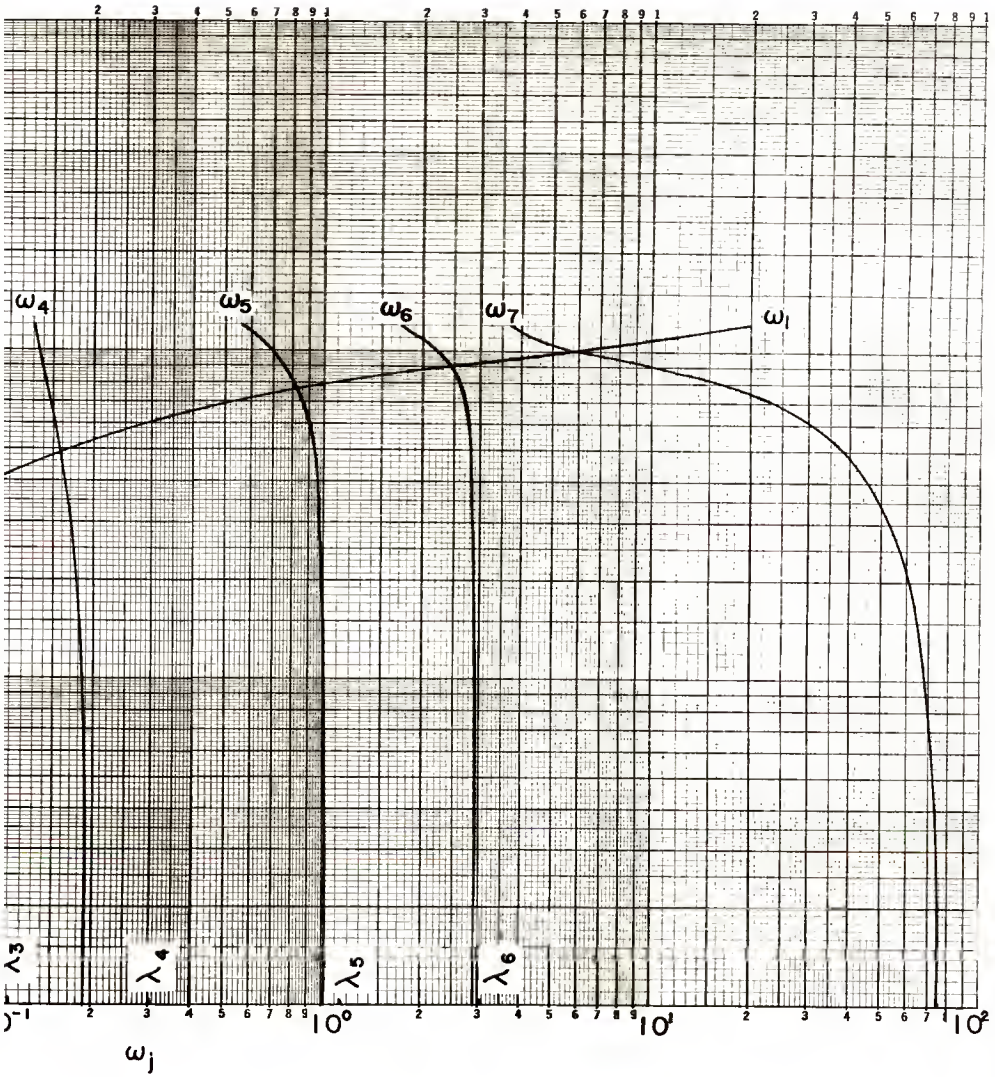
The prompt neutron lifetime, $\ell = 8.0 \times 10^{-5}$ seconds.

The λ's are the decay constants for the neutron precursor groups.

Keepin's delayed neutron parameters.

Data computed from Eq. (9).


ω_2
ω_3
POSITIVE ROOT
λ_1
λ_2
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-2}
ρ


ω4
ω5
ω6
ω7
ω1
λ3
λ4
λ5
λ6
10⁰
10¹
10²
ωj

EXPLANATION OF PLATE XVIII

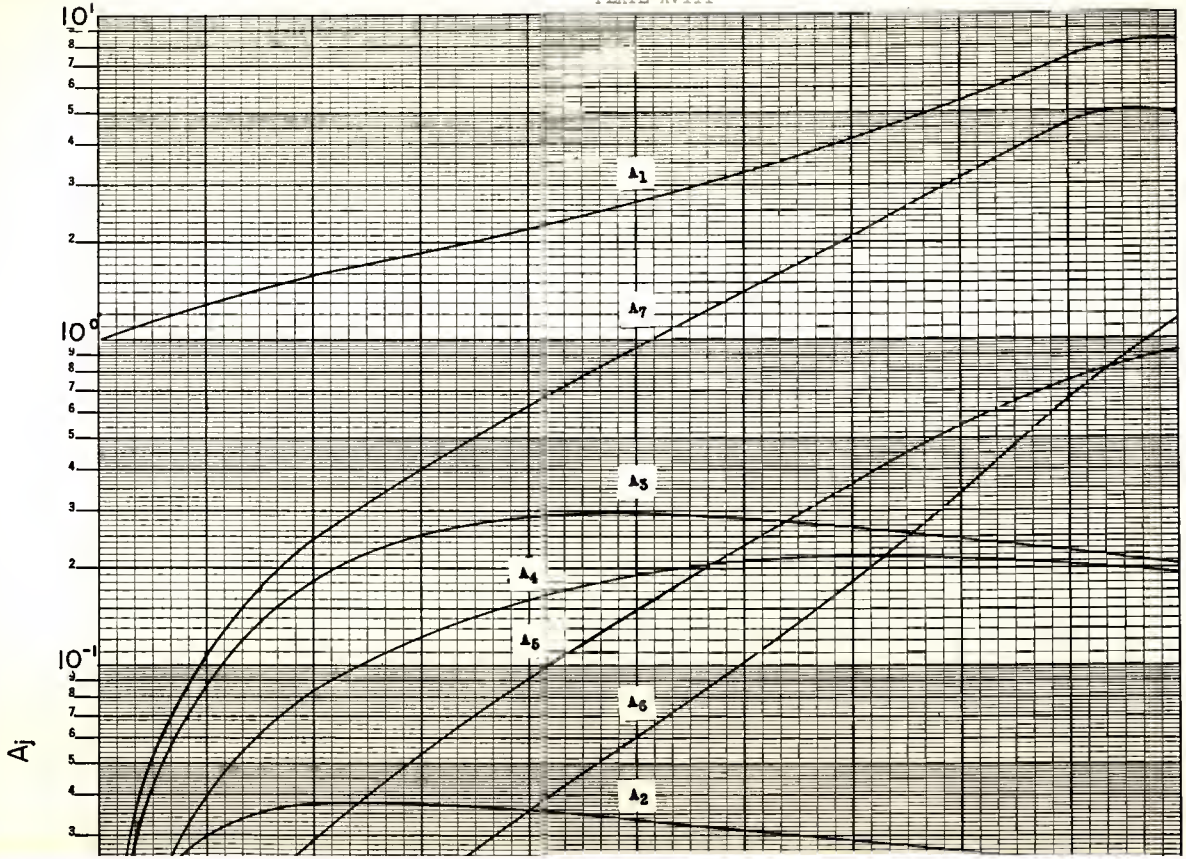
Dimensionless coefficients, A_j's, of the flux equation vs. negative reactivity, ρ (dollars), for step changes in reactivity.

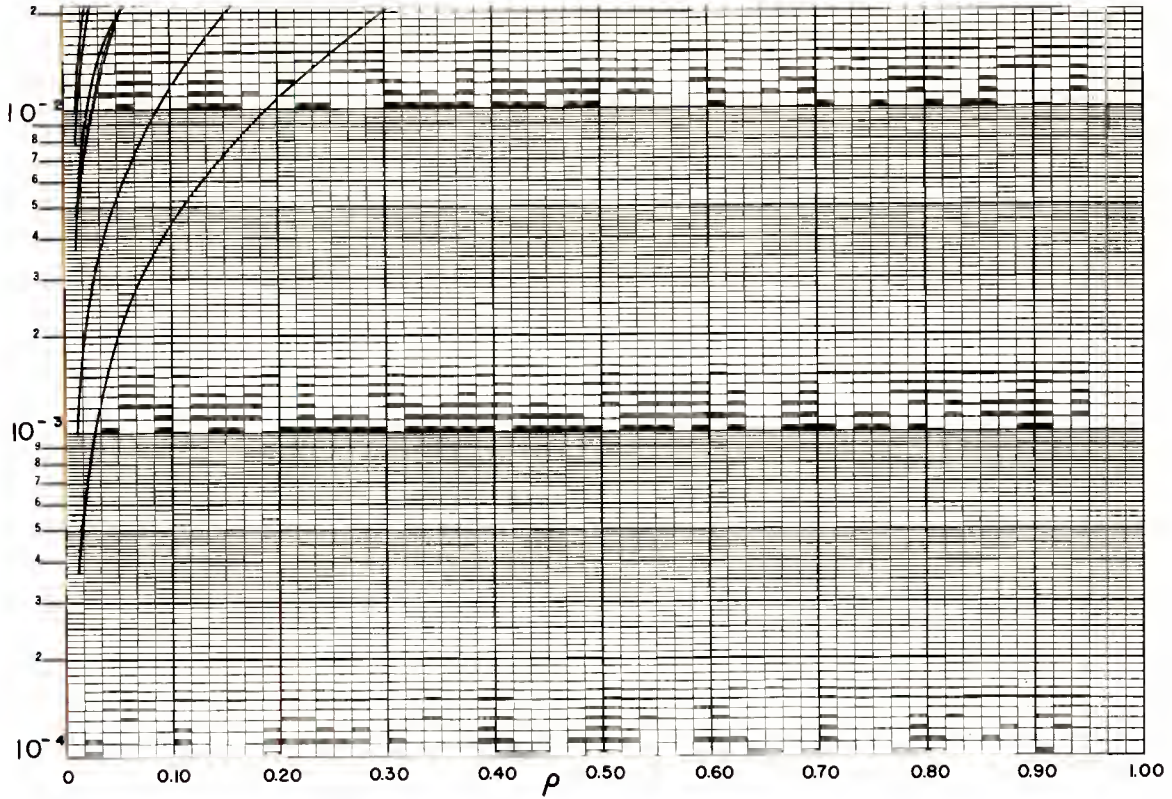
Prompt neutron lifetime, $\ell = 8.0 \times 10^{-5}$ seconds.

Keepin's delayed neutron parameters.

Data computed from Eq. (12).

PLATE XVIII




10^{-2}
10^{-3}
10^{-4}
0
0.10
0.20
0.30
0.40
ρ
0.50
0.60
0.70
0.80
0.90
1.00

EXPLANATION OF PLATE XIX

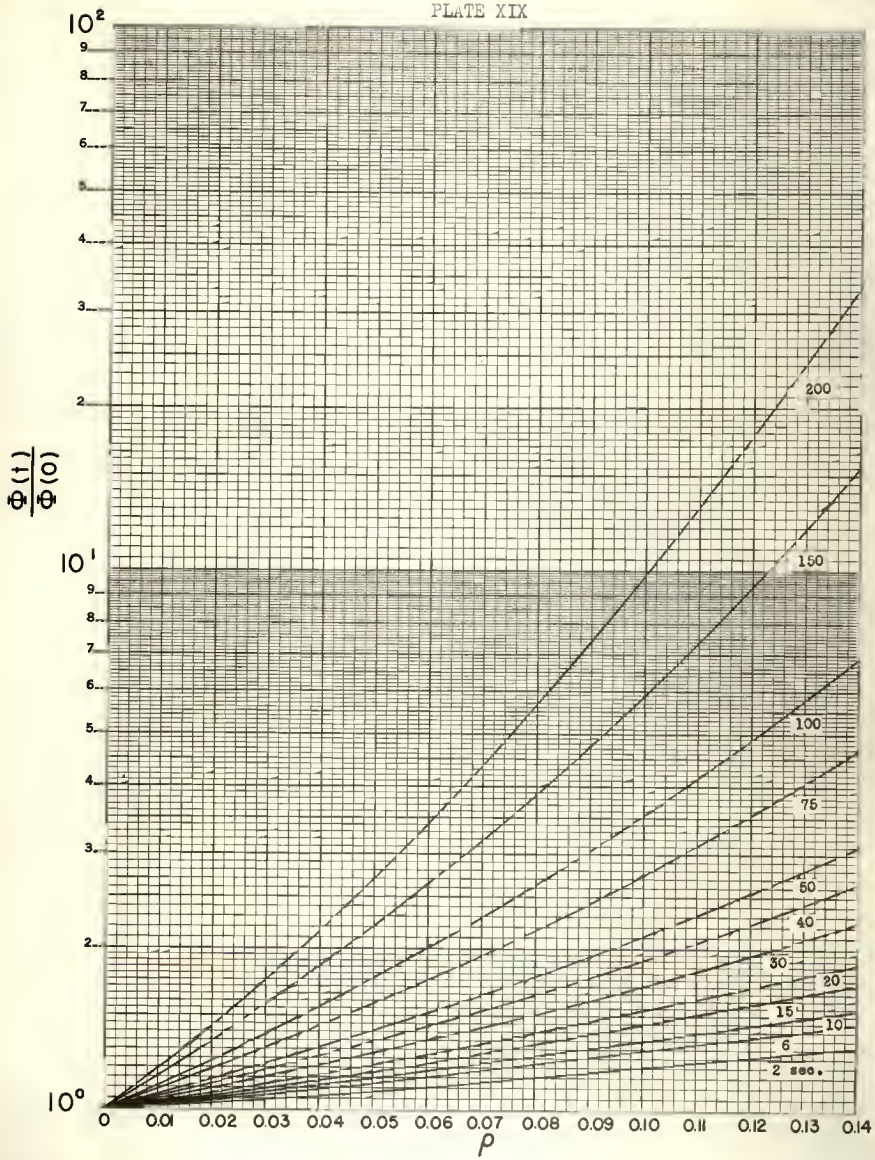
Flux ratio, $\phi(t)/\phi(0)$, vs. step changes of positive reactivity, ρ (dollars), for various times, t (seconds), after the reactivity step.

The prompt neutron lifetime, $\ell = 8.0 \times 10^{-5}$ seconds.

Keepin's delayed neutron data.

Data computed from Eq. (11).

PLATE XIX


10^2
9
8
7
6
5
4
3
2
10^1
9
8
7
6
5
4
3
2
10^0
$\frac{\Phi(t)}{\Phi(0)}$
200
150
100
75
50
40
30
20
15
10
6
2 sec.
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.10
0.11
0.12
0.13
0.14
ρ

Beta Parameter

The total fraction of delayed neutrons has been shown to vary for different fissionable materials. Keepin (12) published a complete list of the various fissionable materials and their respective fractions of delayed neutrons. Even with the same fissionable material, the effectiveness of the delayed neutrons to promote fission has been found to differ from reactor to reactor, depending upon the size and configuration of each reactor.

Since effective delayed neutron fractions are generally difficult to obtain, it was considered important to evaluate the dependence of the kinetic equations upon the total fraction of delayed neutrons. The ratio, $\phi(t)/\phi(0)$, was calculated for various times after a step insertion of reactivity using different total fractions of delayed neutrons. The total fractions of delayed neutrons, β's, used in the calculations varied from 0.005 to 0.008, therefore the β's used cover the range of effective fractions used in most thermal reactors that utilize U^{235} as the primary fuel. The study included several negative and positive reactivities (see Appendix - F).

The same computations, as described above, were recalculated using a constant reactivity of -0.0748 per cent. A plot of the ratio, $\phi(t)/\phi(0)$, vs. the total fraction of delayed neutrons for two different times was made (see PLATE XX). Also shown on the same plot are the flux ratios as a function of β when the reactivity insertion was -\$0.10. It was found that the computed flux ratios for a particular reactivity (\$) insertion

EXPLANATION OF PLATE XX

Flux ratio, $\phi(t)/\phi(o)$, vs. the total fraction of delayed neutrons, β.

Flux ratio evaluated at two time points, 10 seconds and 75 seconds after a step change in reactivity.

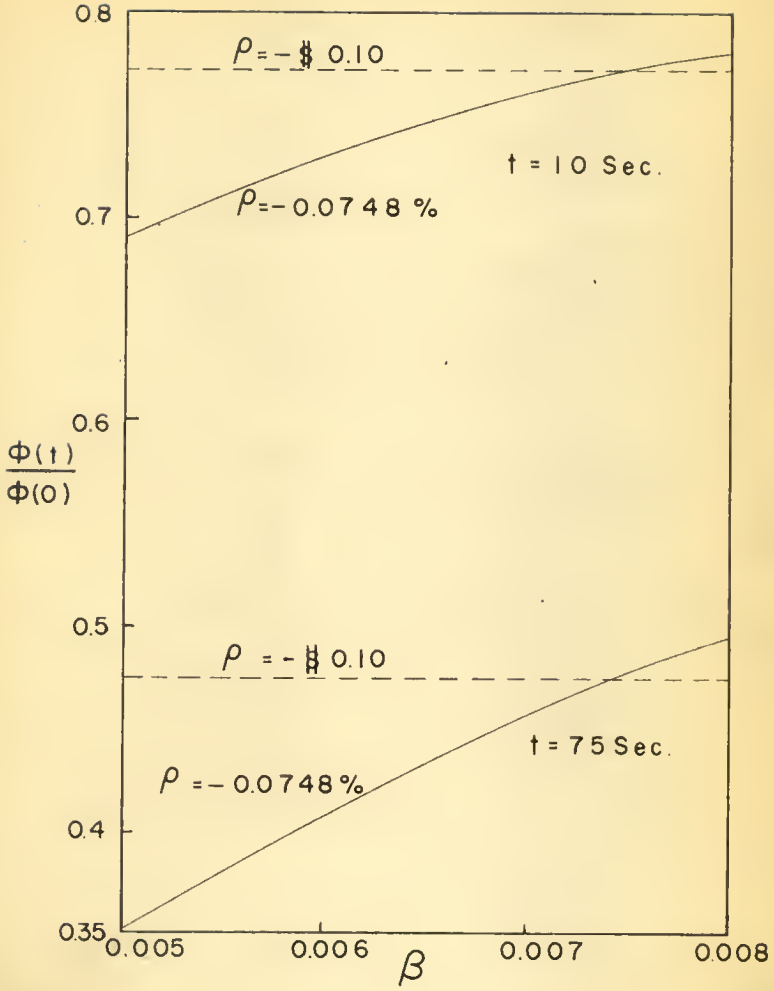
—— Reactivity was -0.0748 per cent.

---Reactivity was -$0.10, as defined by each β.

Hughes' delayed neutron parameters.

Prompt neutron lifetime, $\ell = 8.0 \times 10^{-5}$ sec.

PLATE


0.8
0.7
0.6
0.5
0.4
0.35
ρ = - $ 0.10
t = 10 Sec.
ρ = - 0.0748 %
ρ = - $ 0.10
t = 75 Sec.
ρ = - 0.0748 %
Φ(t) / Φ(0)
0.005
0.006
0.007
0.008
β

was essentially independent of β , whereas the same was not true when the reactivity insertion was given in units of per cent.

Decay Constant Parameter

The decay constants of the neutron precursor groups were measured experimentally by Keepin, who quoted the uncertainty of each of the decay constants. In this paper an evaluation was to determine the effect of varying each decay constant within the experimental uncertainty as reported by Keepin. A set of computed data was made consisting of the ratio, $\phi(t)/\phi(0)$, for various times after step insertions of reactivity using each decay constant modified to its highest value of uncertainty (see Appendix - F).

Prompt Neutron Lifetime Parameter

Since the prompt neutron lifetime varies considerably from reactor to reactor, it is important to compute the effectiveness of the lifetime on the kinetic equations. For thermal reactors, lifetimes are known to vary from approximately 3.0×10^{-5} seconds for certain swimming pool type reactors to approximately 1.0×10^{-3} seconds for large graphite piles.

The flux ratio equation was solved for both positive insertions of reactivity and negative insertions of reactivity using various values of the prompt neutron lifetime. The range of lifetimes investigated was from 1×10^{-5} seconds to 1×10^{-3}

seconds. Plots were made of the flux ratio as a function of time after a step insertion of reactivity for several reactivities and several lifetimes (see PLATES XXI and XXII). The plots indicated that the lifetime parameter is not an important parameter of the kinetic equations except when considering large step changes of positive reactivity, $\rho > \$0.30$.

Three Groups of Delayed Neutrons

The reactor kinetic equations were solved using three groups of delayed neutrons. The resulting characteristic equation was programmed and the roots were computed by the IBM-650. It was demonstrated that the six delayed neutron group kinetic equations could be reasonably approximated with three groups of delayed neutrons.

It is difficult to obtain a set of three group constants analytically, either experimentally or theoretically, which will reasonably approximate the kinetic equations using six delayed neutron groups. Therefore, a trial and error method was used to determine the three group constants. Flux ratios had been computed for step reactivity insertions using the six group constants. The three group constants were then obtained such that the flux ratios as given by the three group constants agreed with those as given by the six group constants (see PLATES XXIII and XXIV). A set of neutron constants determined in this manner is given in Table 4.

EXPLANATION OF PLATE XXI

Flux ratio, $\phi(t)/\phi(o)$, vs. time (seconds) after a negative step change of reactivity, (dollars).

Hughes' delayed neutron parameters.

Two different lifetimes, $\ell = 1 \times 10^{-5}$ seconds.

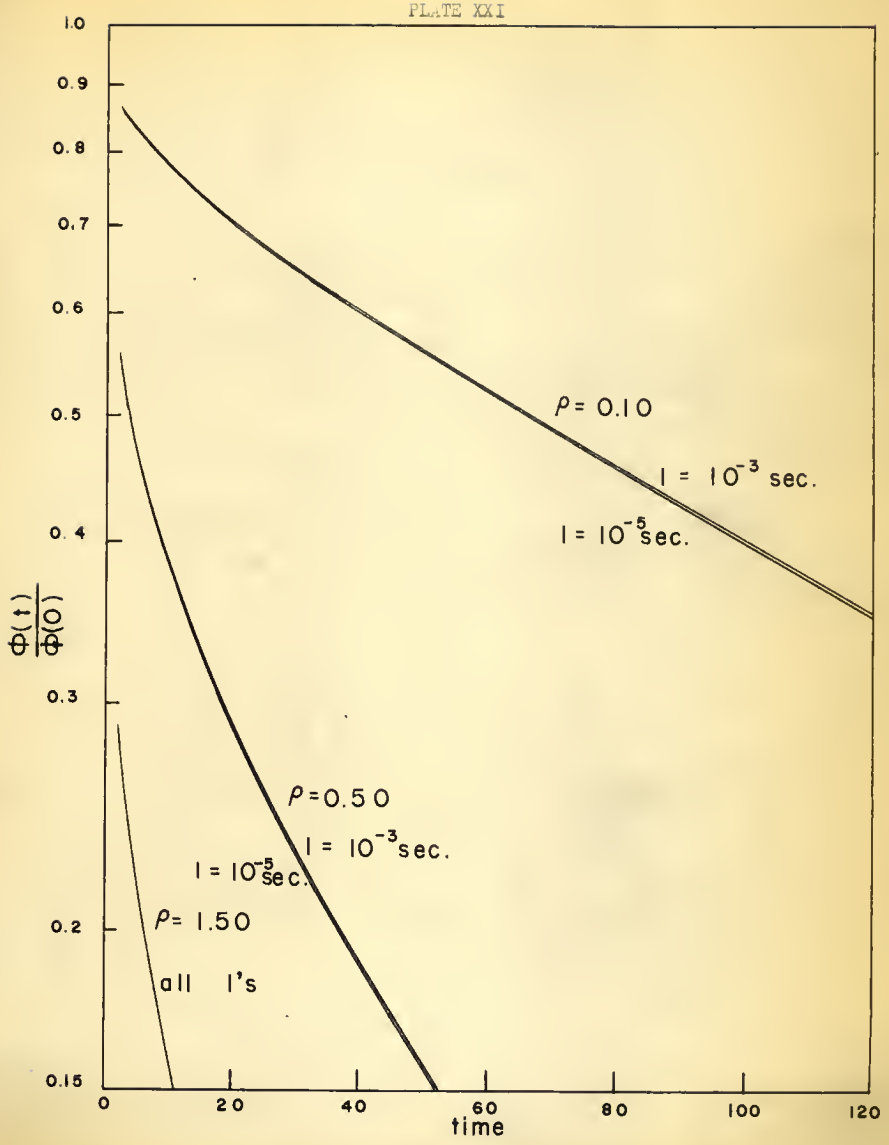
$\ell = 1 \times 10^{-3}$ seconds.

Three different reactivities, $\rho = -\$0.10$

$\rho = -\$0.50$

$\rho = -\$1.50$

PLATE XXI


1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.15
$\frac{\phi(t)}{\phi(0)}$
ρ = 0.10
l = 10⁻³ sec.
l = 10⁻⁵ sec.
ρ = 0.50
l = 10⁻³ sec.
l = 10⁻⁵ sec.
ρ = 1.50
all l's
0
20
40
60
80
100
120
time

EXPLANATION OF PLATE XXII

Flux ratio, $\phi(t)/\phi(0)$, vs. time (seconds) after a positive step change in reactivity, ρ (dollars).

Hughes' delayed neutron parameters.

Two different lifetimes are shown, $\ell = 1 \times 10^{-5}$ seconds.

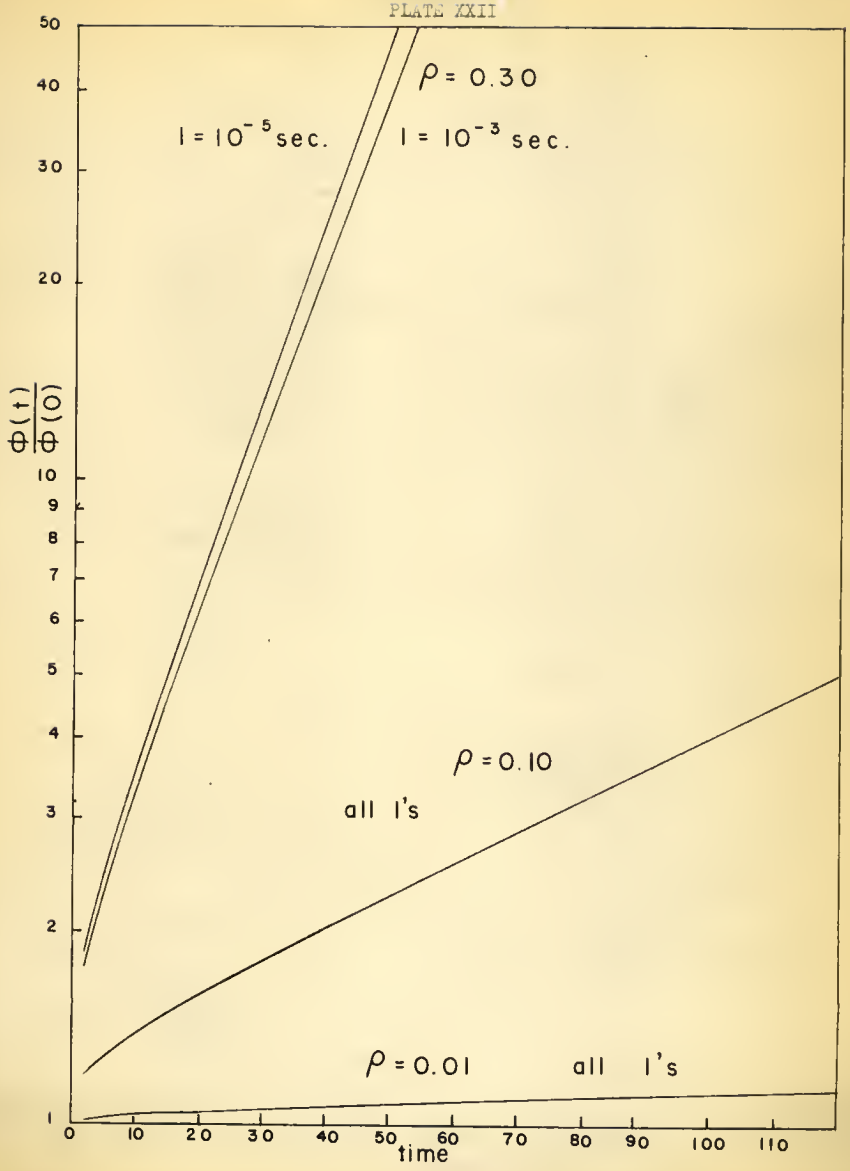
$\ell = 1 \times 10^{-3}$ seconds.

Three different reactivities, $\rho = \$0.01$

$\rho = \$0.10$

$\rho = \$0.30$

PLATE XXII


ρ = 0.30
l = 10⁻⁵ sec.
l = 10⁻³ sec.
ρ = 0.10
all l's
ρ = 0.01
all l's
φ(t)/φ(0)
time
50
40
30
20
10
9
8
7
6
5
4
3
2
1
0
10
20
30
40
50
60
70
80
90
100
110

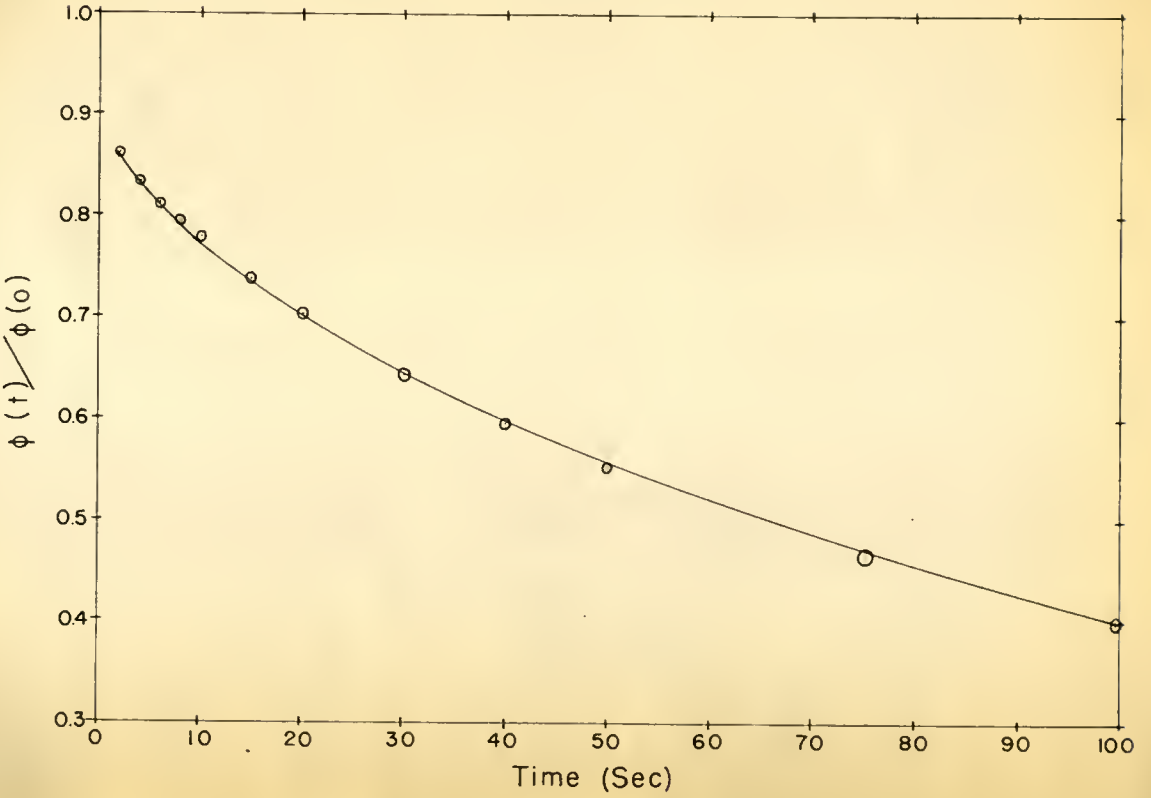
EXPLANATION OF PLATE XXIII

Flux ratio, $\phi(t)/\phi(0)$, vs. time (seconds) after a negative step change in reactivity of -$0.10.

——— Theoretically predicted flux ratio using the six group kinetics model.
Hughes' delayed neutron parameters.

⊙ Theoretically predicted flux ratio using the three group kinetics model.
Three group parameters are given in Table 4.

PLATE XXIII



EXPLANATION OF PLATE XXIV

Flux ratio, $\phi(t)/\phi(o)$, vs. time (seconds) after a negative step change in reactivity of -$1.42.

——— Theoretically predicted flux ratio using the six group kinetics model.

⊙ Theoretically predicted flux ratio using the three group kinetics model.

Hughes' delayed neutron parameters.

Three group parameters are given in Table 4.

PLATE XXIV

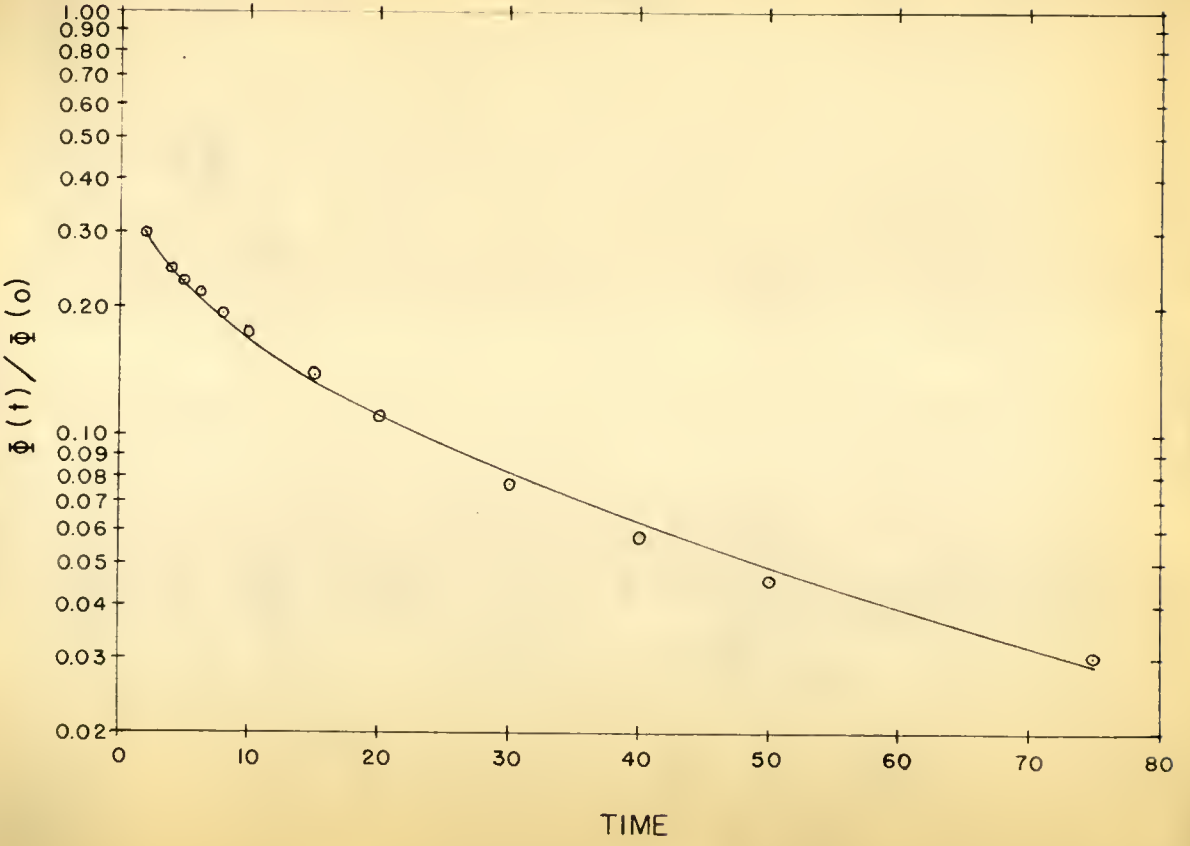

1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
Φ (t) / Φ (o)
0
10
20
30
40
50
60
70
80
TIME

Table 4. Empirically determined three delayed neutron group constants.

Group	λ_i (Sec.$^{-1}$)	β_i
1	0.79	0.0033
2	0.091	0.00349
3	0.0117	0.00069

For each reactivity investigated using the three group analysis, the comparisons with the six group analysis gave similar flux ratio deviations as a function of time. At two seconds after the step change in reactivity, the two models, the three group and the six group, gave almost the same flux ratio. For times between six seconds and 20 seconds, the three group model gave slightly higher flux ratios. For times between 20 seconds and 100 seconds, the six group model gave slightly higher flux ratios. For times larger than 100 seconds, the three groups model gave higher flux ratios. The difference between the flux ratios as given by the two models differed significantly for times larger than 150 seconds. As the step input of reactivity increased, the difference in the flux ratios, as given by the two models, increased. The largest reactivity used was -$1.42 (see PLATE XXIV).

The six group code required approximately 100 seconds of IBM-650 time to solve the characteristic equation and calculate flux ratios for 15 time points. The three group code required approximately 25 seconds to solve the characteristic equation

and calculate the flux ratios for 15 time points. Thus, if it becomes necessary to perform a kinetic analysis using certain additional reactivity parameters, the three group analysis will greatly simplify the calculations.

EXPERIMENTAL MEASUREMENTS

General

The experimental data were obtained with the TRIGA reactor located at the John Jay Hopkins Laboratory, San Diego, California and with the Argonaut reactor located at Argonne National Laboratory. The TRIGA data were obtained by the author and the Argonaut data were forwarded to this author by Dr. Bill Sturm, Manager of the Argonaut reactor.

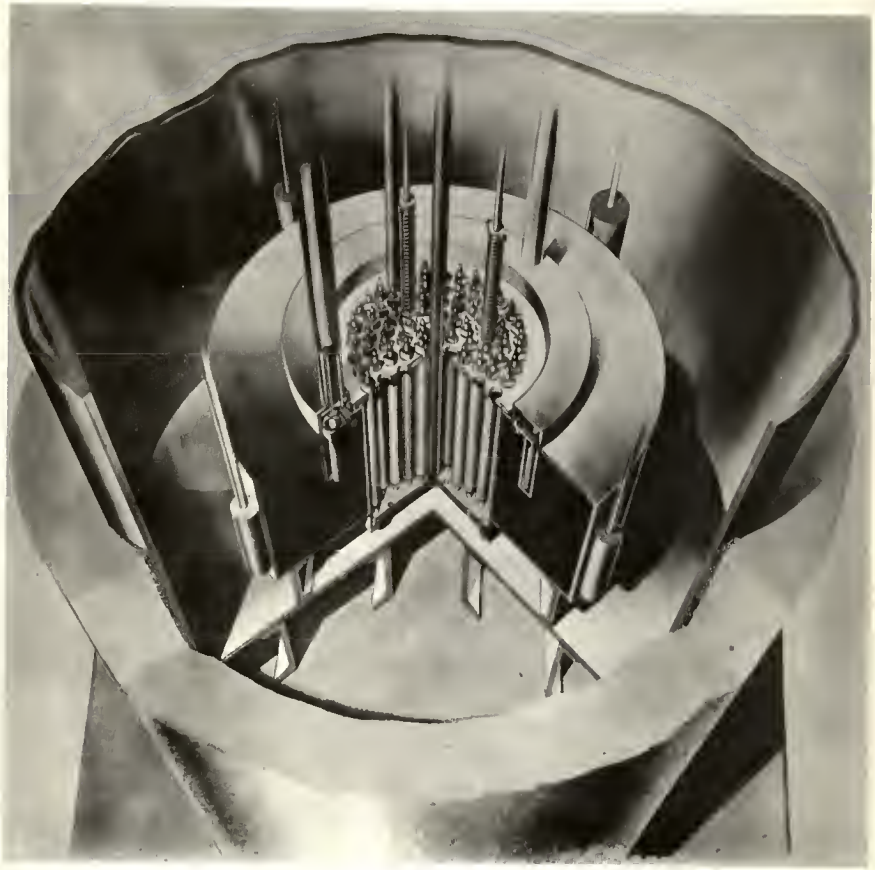
Experimental System

Briefly, the TRIGA reactor consisted of a cylindrical core containing a lattice of cylindrical fuel elements surrounded by a graphite reflector and located at the bottom of a 20 foot aluminum tank (see PLATES I and XXV). The fuel elements contained fissionable material in the form of an alloy of uranium-zirconium hydride composed of eight wt. per cent uranium enriched to 20 per cent in U^{235}. The reactor contained four control rods. The control rods used in the reactivity measurements were the shim rod and the regulating rod. The positions of the rods are shown in PLATE XXVII. More descriptive information of the TRIGA reactor may be found in the references (19), (21), and

EXPLANATION OF PLATE XXV

View of the core of the TRIGA reactor showing the positions of the ionization chambers located just outside the core reflector.

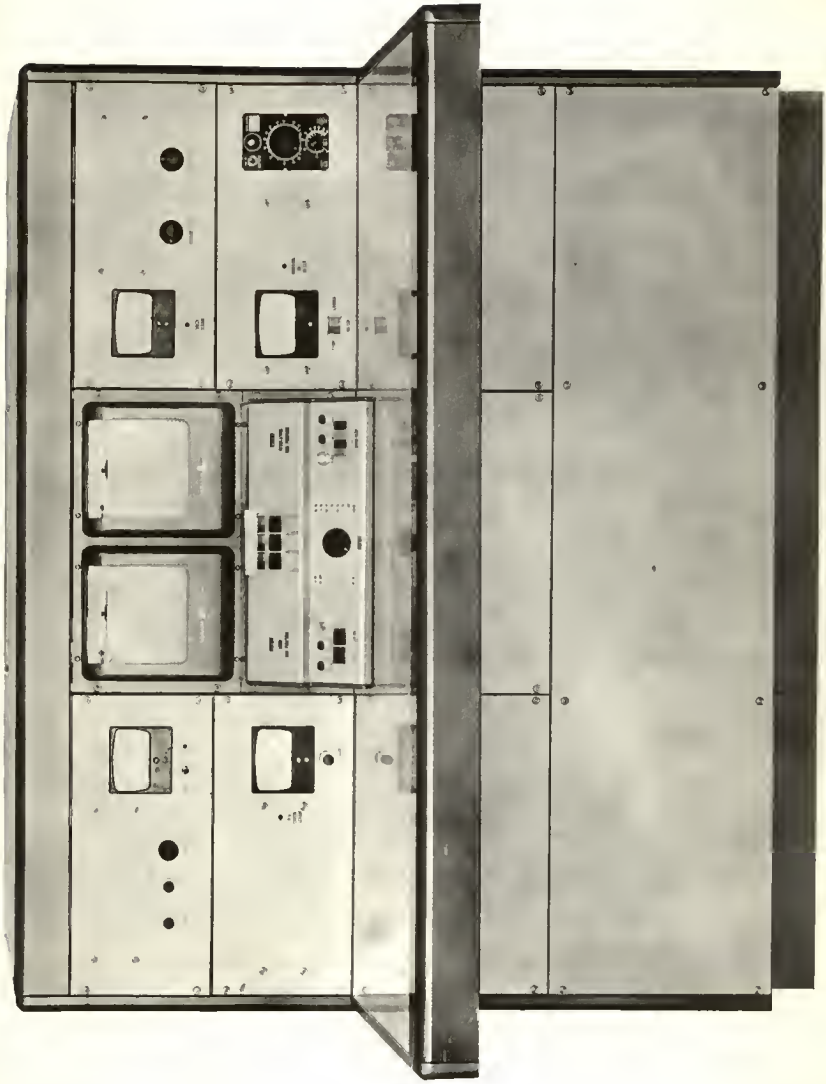
PLATE XXV



EXPLANATION OF PLATE XXVI

View of a TRIGA reactor consolo.

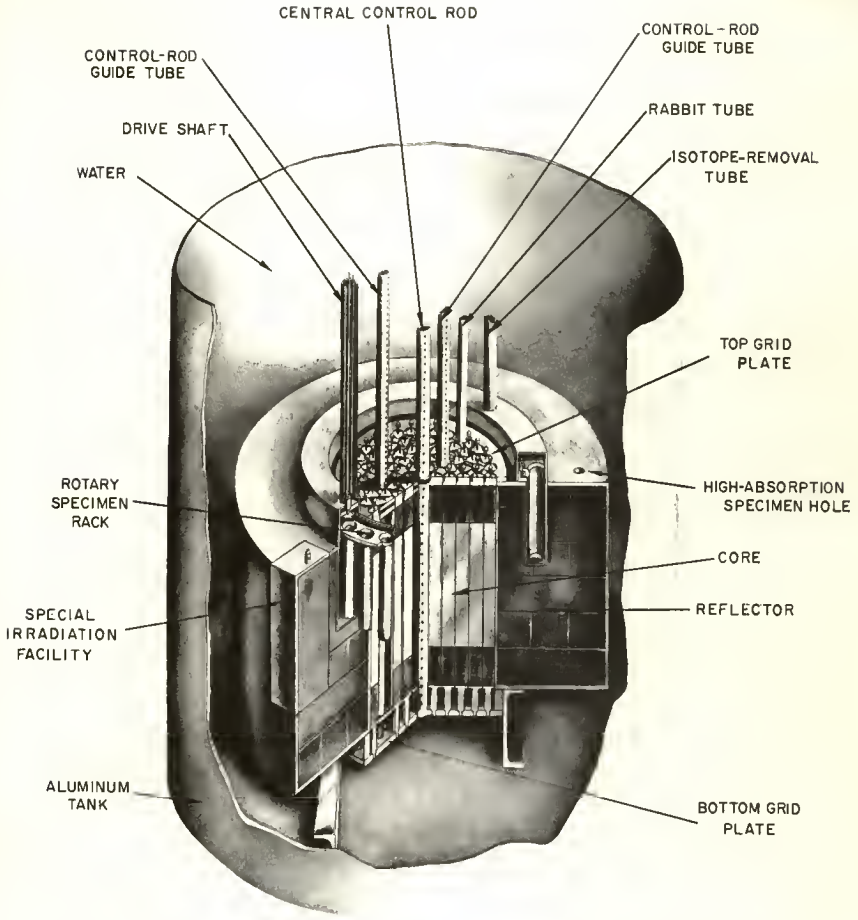
PLATE XXVI



EXPLANATION OF PLATE XXVII

A sketch of the TRIGA reactor showing the basic core components.

PLATE XXVII



REACTOR CORE AND REFLECTOR ASSEMBLY

(22).

The reactivity insertion was made by changing the position of one of the two control rods. The control rods were composed of boron-carbide canned in aluminum tubes. The control guide tubes may be seen in PLATE XXVIII. The rods had a 15½ inch travel from their "full out" position to "full in". During the rod drop experiments or during a scram, the rods fell under the force of gravity. The time required for the rods to fall from "full out" to "full in" was approximately 0.35 seconds.

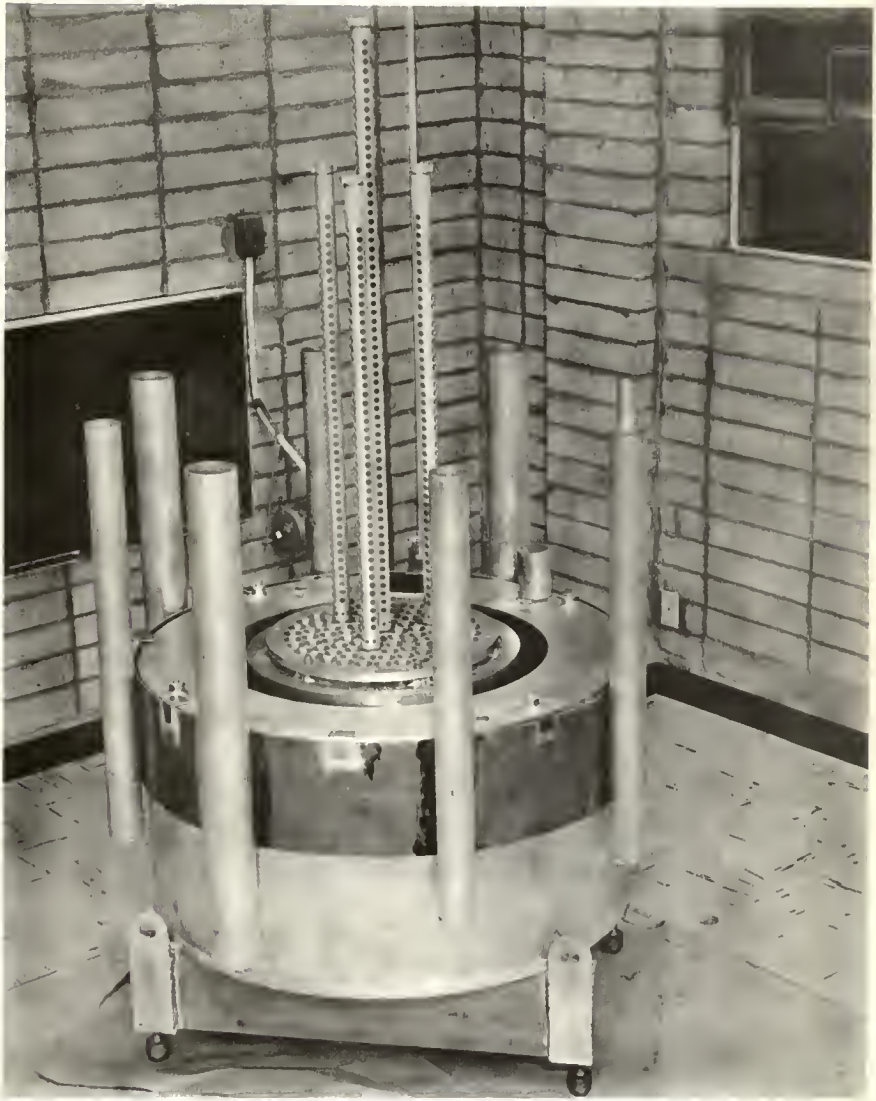
The neutron flux level was measured with the use of two Westinghouse WL-6377 compensated ion chambers. The ion chamber sensitivity was 4×10^{-4} amps/nv and were electrically compensated having a compensated gamma sensitivity of about 3×10^{-13} amps/roentgen/hr. The compensated ion chambers are shown in PLATE I.

The current from the compensated ion chambers was monitored with two Keithly micro-micro ammeters. The ammeters are shown in PLATE XXIX just above the reactor operator's head. The signal from the micro-micro ammeters was recorded by a Midwestern model 608 direct-recording, oscillograph. (See PLATE XXX).

EXPLANATION OF PLATE XXVIII

View of the TRIGA core before positioned into the reactor tank.
Metal tubes at the periphery will hold the ionization chambers.
The perforated tubes are the control rod guide tubes.

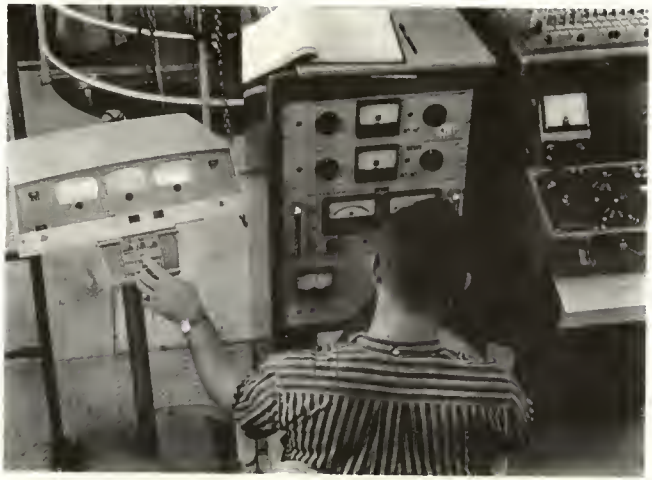
PLATE XXVIII



EXPLANATION OF PLATE XXIX

View of the simplified TRIGA reactor console (at left).
The two Keithly micro-micro ammeters are located just above the head of the reactor operator (at right).

PLATE XXIX



EXPLANATION OF PLATE XXX

General layout of the TRIGA building.
Midwestern direct-recording oscillograph is
located on the table (center) and is being
operated by the man who is kneeling beside it.

PLATE XXX

EXPERIMENTAL PROCEDURE

General

Two experimental methods were used to measure the reactivity worth of a control rod. The first method described is by far more commonly used in the nuclear field for reactivity measurement, and this procedure is known as the positive period method. The second method is known as the rod drop method.

The positive period method consisted of several well defined procedural steps. First, the reactor was brought to a critical position at a low power.[1] The reactor was maintained at the critical position for approximately eight minutes to insure that the criterion for delayed criticality[2] was satisfied. A positive reactivity change was made by partially withdrawing one of the control rods. The neutron population in the core increased as a function of time. One hundred seconds after the change in reactivity the time function of the neutron density was approximately represented by a single exponential function. The time constant of the exponential function was the reciprocal of the stable reactor period. The time required for the neutron density to increase by a factor of "e" was measured and that

---

[1]The TRIGA reactor has a significant temperature coefficient of reactivity and thus any temperature change during reactivity measurements would have surely invalidated the measurements. Other than the heating effect, power was not a factor in the reactivity measurements.

[2]Delayed criticality is defined as the state of a nuclear reactor in which all the neutrons resulting from fission, both delayed and prompt, are required to maintain a chain reaction.

time was the stable reactor period for that particular reactivity change. From an analytical relationship between the stable reactor period and the magnitude of the step change in reactivity, the reactivity was established for the particular experiment.

Instrumentation Calibration

Frequent checks were made to insure that the instruments used were calibrated and functioning properly. The linearity of the compensated ionization chambers was checked numerous times to insure that the current output of the chambers was proportional to the reactor power level. The micro-micro ammeters were checked several times during each run to insure that the proper zero position was being used. The Midwestern recorder was calibrated several times to insure that the proper calibration was maintained.

It was assumed, in the TRIGA reactor, that the thermal power level was proportional to the neutron density at the position of the detector. The power level calibration was obtained by measuring the average reactor tank water temperature as a function of time while the reactor was operating at a steady power. The temperature measurements were made at several power levels. The calibration indicated that the current output of the ionization chamber was truely proportional to the power level regardless of the detector position.

During a particular run, the micro-micro ammeter measured neutron flux levels over several decades of power. Each decade of the micro-micro ammeter required a slightly different zero

position, thus when the micro-micro ammeter was switched from one decade to a lower value a new zero position was made by an adjustment on the ammeter.

The Midwestern recorder operated on a galvanometer principle, i.e., the current supplied to the recorder galvanometer was proportional to the current fed to the ammeter. The galvanometer reflected ultraviolet light onto a moving photosensitive recording paper. The recorder was calibrated by imposing a false signal, of known value, from the micro-micro ammeter to the galvanometer. The deflection of the false signal line on the recorder paper was then measured as a function of the magnitude of the false signal (see PLATE XXXII).

Positive Period Measurements

The two control rods that were calibrated with the positive period method were the shim rod and the regulating rod (see PLATE XXXI). The safety rod calibration was furnished by General Atomic personnel. The shim and regulating rod calibrations were made using this safety rod calibration.

The reactor was brought to a delayed critical position with the safety rod out of the core, the shim rod completely in the core, and the regulating rod at a position necessary for criticality.[1] The positions of the control rods at the critical position were recorded in the reactor log book. The reactor was

---

[1]The power level at the critical position was generally around 20 watts.

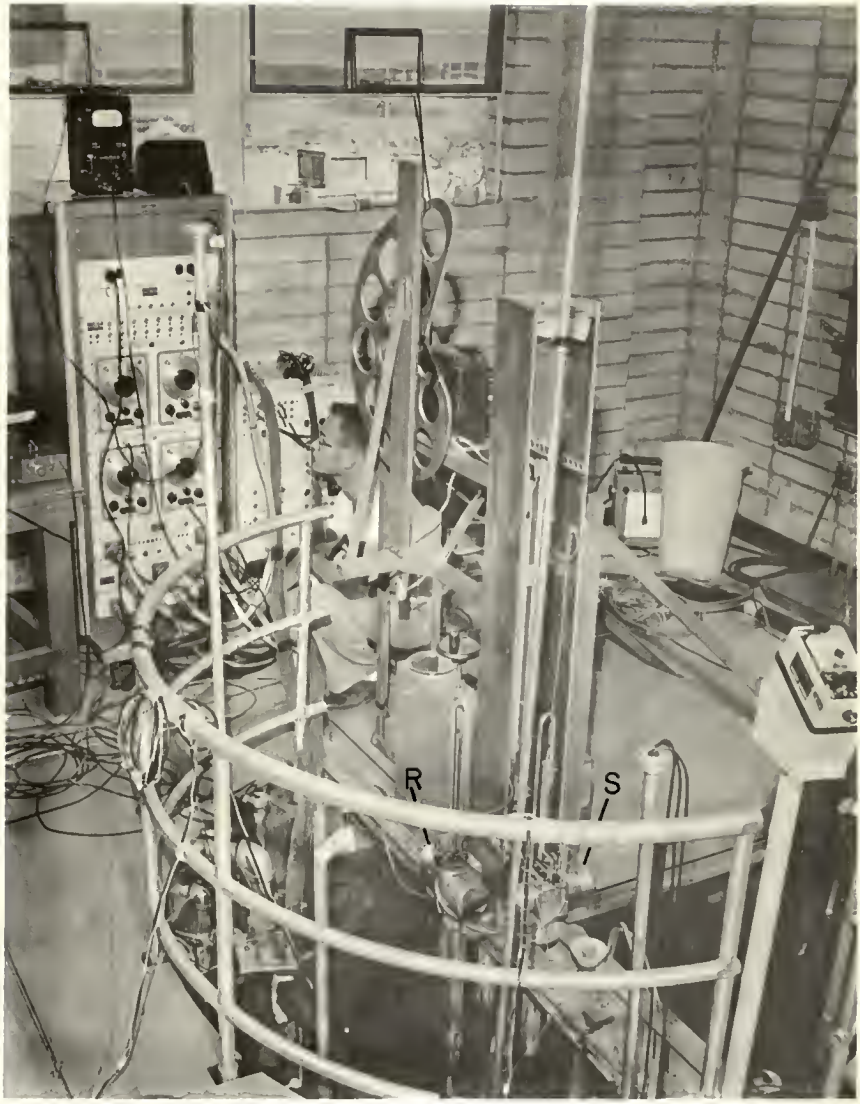
EXPLANATION OF PLATE XXXI

View of the top of the TRIGA reactor showing the control rod drive motors and the transient rod mechanism.

S - shim rod drive mechanism.

R - regulating rod drive mechanism.

PLATE XXXI



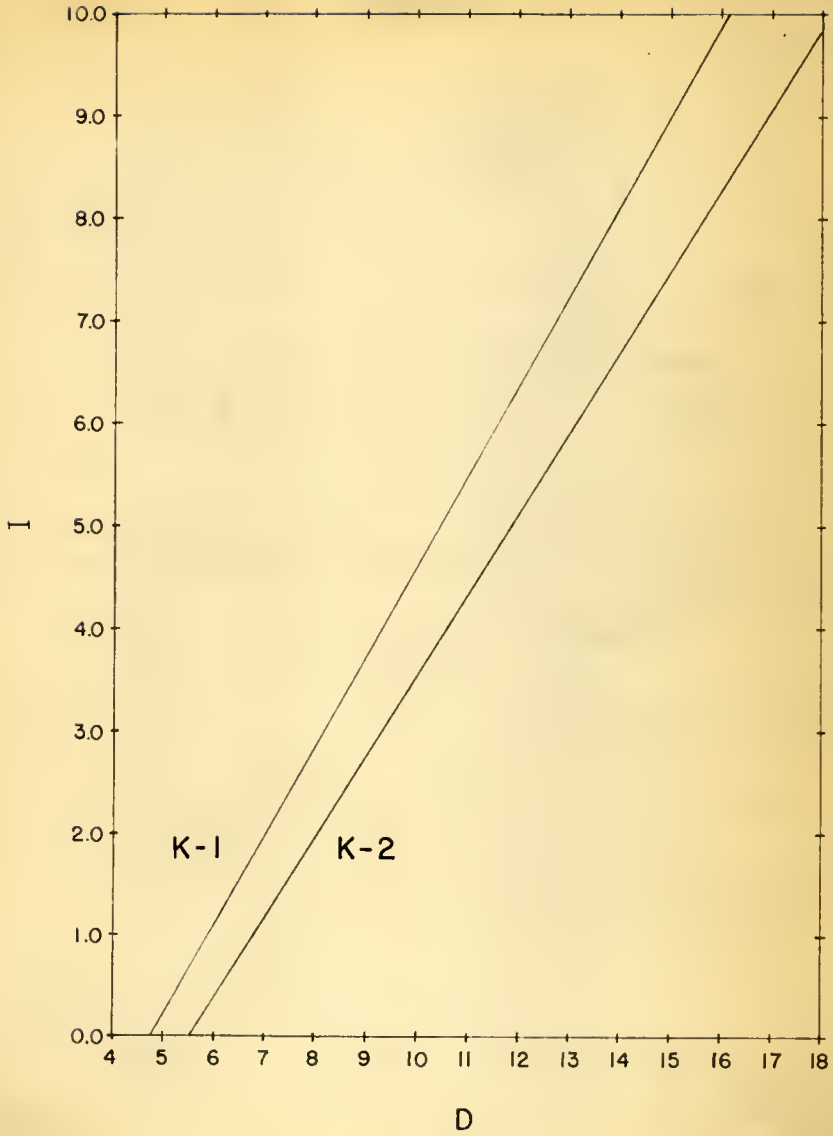
EXPLANATION OF PLATE XXXII

Calibration curve for the Midwestern Model 608 direct-recording oscillograph.

Current, I (arbitrary units) vs. deflection, D (cm.), of signal line away from the reference line.

K-1 and K-2 are recording channels.

PLATE XXXII


10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
I
K-1
K-2
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
D

left in the critical state for approximately eight minutes to insure that a delayed criticality had been obtained. The shim rod was then withdrawn from the core a short distance to make the reactor super-critical. The new position of the shim rod was recorded in the reactor log book. The reactor was left in the supercritical state for at least 100 seconds to insure that the increase in power as a function of time was exponential. A stop watch was used to measure the time required for the reactor power to increase by a factor of 1.5. The time measured was then divided by the natural logarithm of 1.5, 0.4055, to obtain the stable reactor period. Several such measurements were made with each reactivity change. The reactor was brought back to the critical position at the original power level by means of inserting the regulating rod into the core. A technique was used to initially overcompensate the reactivity of the reactor with the regulating rod. It consisted of inserting the regulating rod into the core by an amount greater than necessary to bring the reactor to a critical position at the original power. The power level was allowed to undershoot the original power and then the regulating rod was withdrawn from the core in small steps to obtain the desired criticality. The purpose of the undershoot in power level was to "balance out" the effects of the delayed neutrons such as to obtain a delayed criticality in a much shorter time. The new position of the regulating rod was recorded in the reactor log book. The procedure was repeated by withdrawing the shim rod another short distance. The stepwise

measurements were continued until the regulating rod was almost completely inserted into the reactor core. At that point, the safety rod was slowly inserted into the core while the regulating rod was being withdrawn, to insure that near criticality conditions were being maintained. After the safety rod was fully inserted into the core, the new position of the regulating rod was recorded. The stepwise reactivity measurements were again continued until the shim rod and the regulating rod were completely calibrated.

A theoretical relationship between the reactivity change and the resulting stable period was used to evaluate the reactivity of each of the rod movements. The relationship (see PLATE XXXIII) was derived by General Atomic (19). The reactivity changes were produced by the shim rod, thus the reactivity of each of the rod movements was summed to obtain the integral worth of the shim rod (see PLATE XXXIV). Since the shim rod reactivities were compensated with regulating rod, similar calibration curves were plotted for the regulating rod (see PLATE XXXV).

Rod Drop Measurements

Rod drop measurements were made with two control rods, the shim and the regulating rod. When available, two linear power level channels were used for neutron density measurements. At other times, the second compensated ionization chamber was too far away from the core to accurately measure the low power used in the rod drop measurements.

EXPLANATION OF PLATE XXXIII

Positive reactivity, ρ (dollars), vs. asymptotic period, τ (seconds). Analytical data calculated by General Atomic (19) using Keepin's delayed neutron data. Total effective fraction of delayed neutrons, 0.0078.

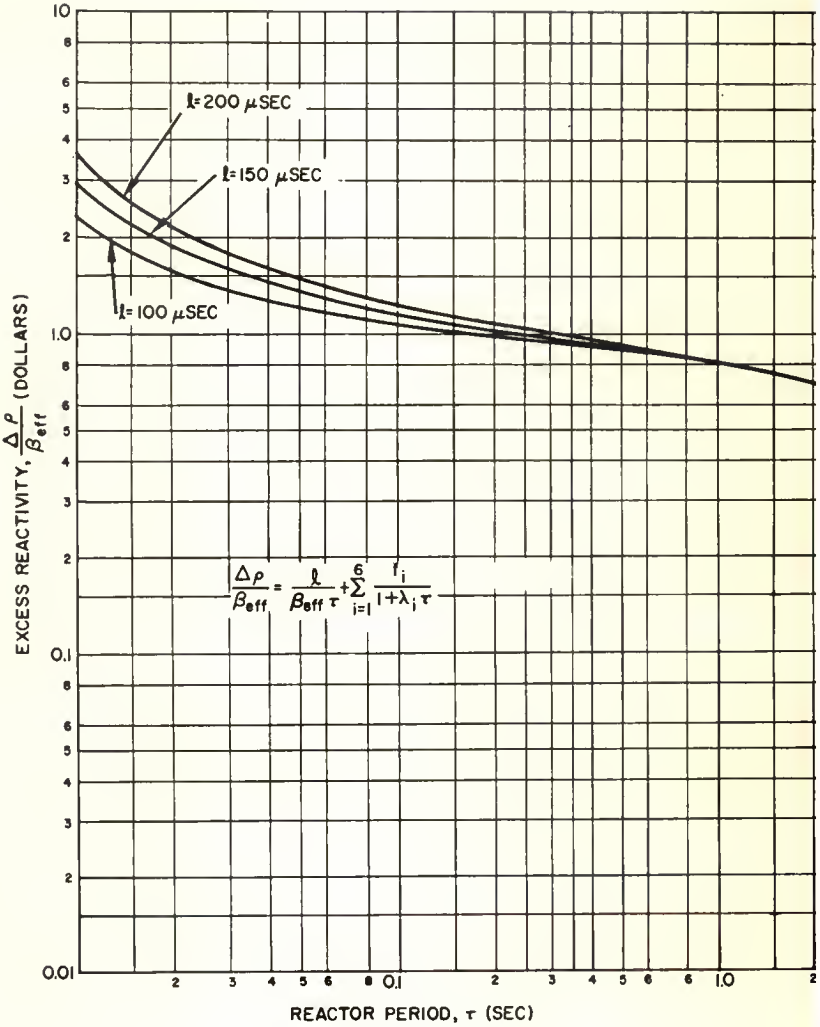
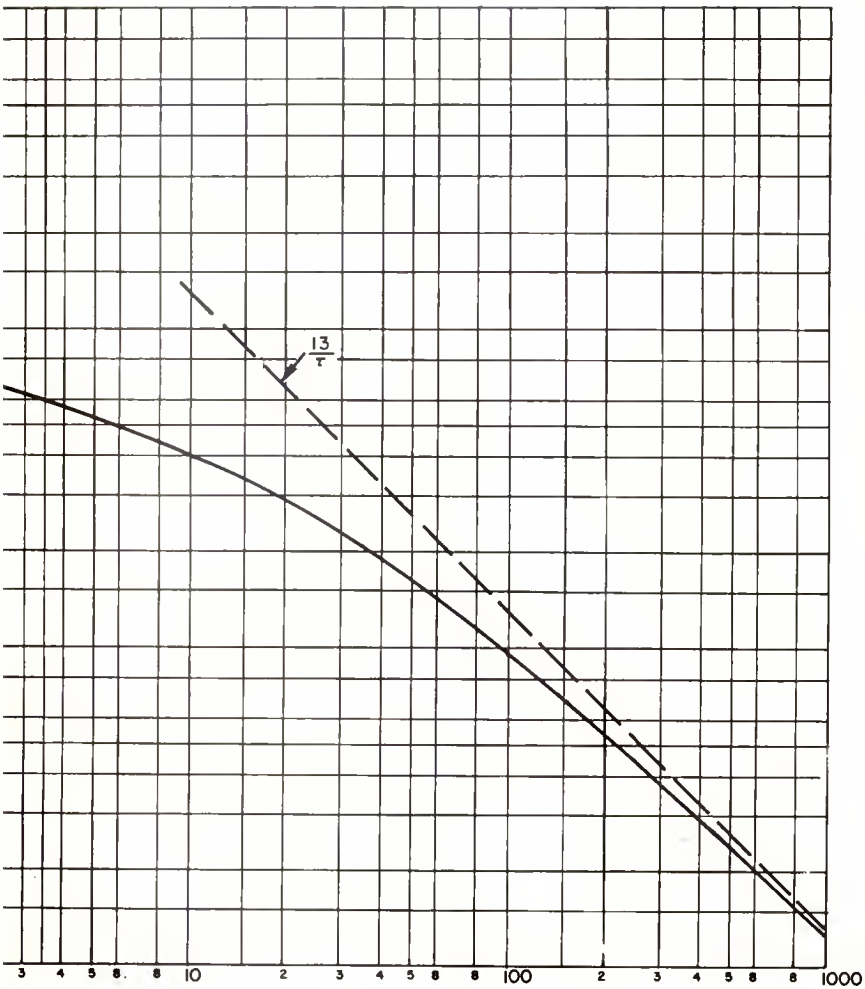

l=200 μSEC
l=150 μSEC
l=100 μSEC
$\frac{\Delta\rho}{\beta_{eff}} = \frac{l}{\beta_{eff}\,\tau} + \sum_{i=1}^{6} \frac{f_i}{1+\lambda_i \tau}$
EXCESS REACTIVITY, $\frac{\Delta\rho}{\beta_{eff}}$ (DOLLARS)
REACTOR PERIOD, τ (SEC)
10
8
6
5
4
3
2
1.0
0.1
0.01
0.1
1.0

PLATE XXVIII



EXPLANATION OF PLATE XXXIV

Shim control rod calibration for the TRIGA reactor.

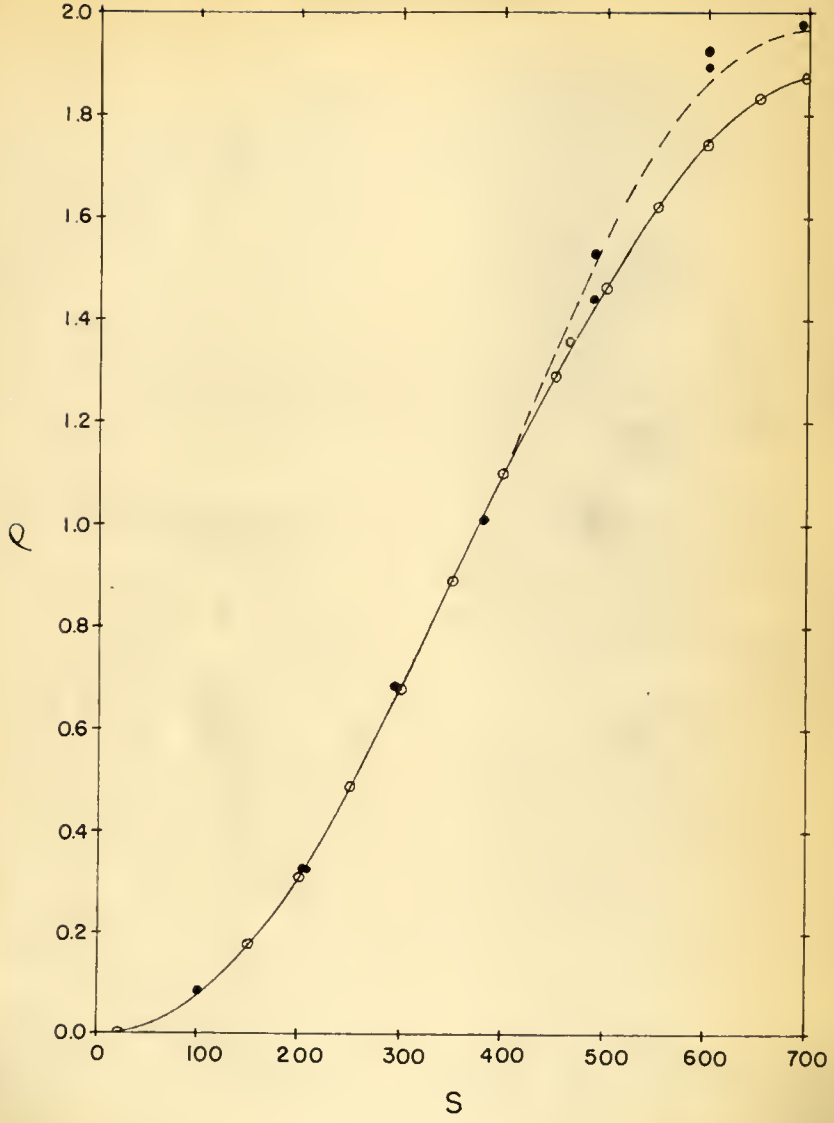
Reactivity, ρ (dollars), vs. control rod position, S (units).

---•--- Measured by dropping the shim rod from various critical positions to "full in".

—o— Measured by the positive period method.

Keepin's delayed neutron parameters.

PLATE XXXIV



EXPLANATION OF PLATE XXXV

Regulating control rod calibration for the TRIGA REACTOR.

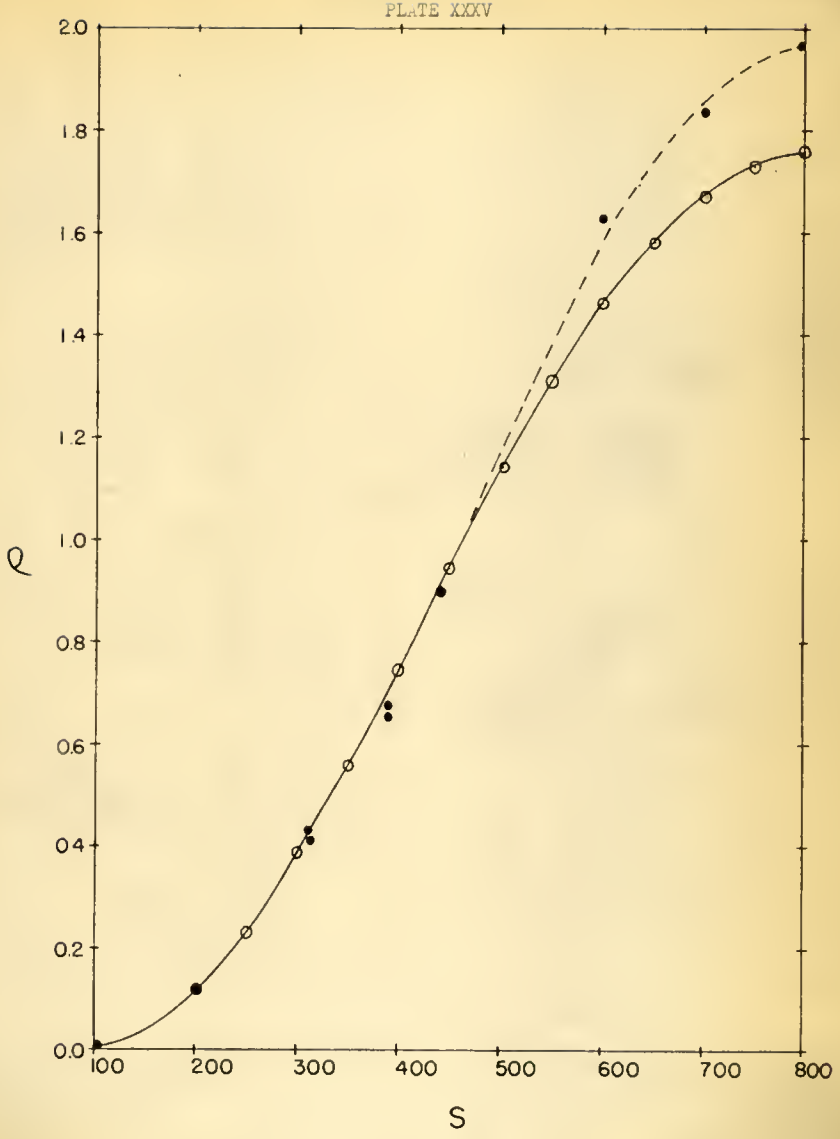
Reactivity, ρ (dollars), vs. control rod position, S (units).

---•--- Measured by dropping the regulating rod from various critical positions to "full in".

—⊙— Measured by the positive period method using increments of reactivity.

Keepin's delayed neutron data.

PLATE XXXV



The first step of the rod drop measurements was to bring the reactor to a critical configuration with the rod to be calibrated at the position desired and with the other rods at positions necessary for criticality. The reactor was allowed to remain at the critical positions for a length of time necessary to insure that the criterion for delayed criticality had been satisfied. The Midwestern recorder was turned on to allow for sufficient warm-up time. A zero adjust check was made on the Keithly micro-micro ammeter. The magnet holding the control rod was then de-energized and the rod fell freely into the core. Since there was no means available to stop the rod between the critical position and the "full in" position, all of the rod drop measurements were made from the critical position to the "full in" position.

The scales of the Keithly micro-micro ammeter were 1×10^{-n} and 3×10^{-n}. Each time that the current from the ion chamber became too low, the decade setting on the ammeter was switched to a lower value, therefore a new zero adjust was required. The zero adjust operation was performed quickly and very little recording signal was lost during the adjusting period.

A signal proportional to the power level of the reactor was recorded by the Midwestern recorder. The power level was proportional to the deflection of the recorder signal line from a reference line. Thus, the deflection of the signal line was measured and from the calibration graph (see PLATE XXXII), the power level as a function of time was obtained. The measured power level was then normalized to unity at the critical power

level just before the rod drop, thus the normalized power level gave the ratio, $\phi(t)/\phi(o)$. From the theoretical plots (see PLATES II and III) the reactivity of the rod drop was determined. Since the value of the ratio, $\phi(t)/\phi(o)$, was measured for the various times, two seconds, four seconds, etc., it was possible to obtain the reactivity at the various times. The rod drops were analyzed with two sets of neutron data, Keepin's (13) and Hughes' (10).

After each rod drop was completed, the reactor was brought back to the initial critical power level, thus all of the rod drop measurements started with the same critical power level. A critical configuration was obtained with the rod to be calibrated in a new position and then the rod drop procedure was repeated. Rod drops were made from various critical positions of the rod that was being calibrated, until the reactivity worth of the complete rod was determined. Both the shim and the regulating rods were calibrated in the same manner as described above (see PLATES XXXIV and XXXV).

EXPERIMENTAL ANALYSIS

TRIGA Data

As outlined in the procedure, two methods of reactivity measurements were used to calibrate two control rods of the TRIGA reactor. The control rods that were calibrated were the shim rod and the regulating rod.

When the positive period method was used to measure the

reactivity worth of the shim rod, the rod was calculated to have a complete worth of $1.87, according to the General Atomic (19) analytical data (see PLATE XXXIV). In these data General Atomic used Keepin's (13) delayed neutron data but instead of the total fraction of delayed neutron as given by Keepin, they calculated a total effective fraction of delayed neutrons. General Atomic used as a total fraction the value of 0.0078 as compared to that given by Keepin of 0.0064, but as was shown in the analytical data of this paper (see Appendix - F), there was no significant difference between the stable period relationship and reactivity when either 0.0064 or 0.0078 was used for the total fraction of delayed neutrons.

Because of the difference between the two sets of neutron constants as given by Keepin as compared to those given by Hughes, there was found to be a slightly lower complete reactivity worth of the control rod when the rod was calibrated by the positive period method using Hughes' delayed neutron data (see PLATE XXXVI). The difference between the two delayed neutron models was most significantly demonstrated when the two plots of reactivity ρ, vs. the stable period, τ, were compared (see PLATE XXXVII). The total worth of the shim rod was calculated to be $1.75 when Hughes' delayed neutron data were used.

Similarly, the regulating rod was calibrated by the positive period method using Keepin's delayed neutron data (see PLATE XXXV) and Hughes' delayed neutron data (see PLATE XXXVIII). The regulating rod complete reactivity worth was calculated to be $1.76 and $1.64 using Keepin's and Hughes' data respectively.

EXPLANATION OF PLATE XXXVI

Shim control rod calibration for the TRIGA reactor.

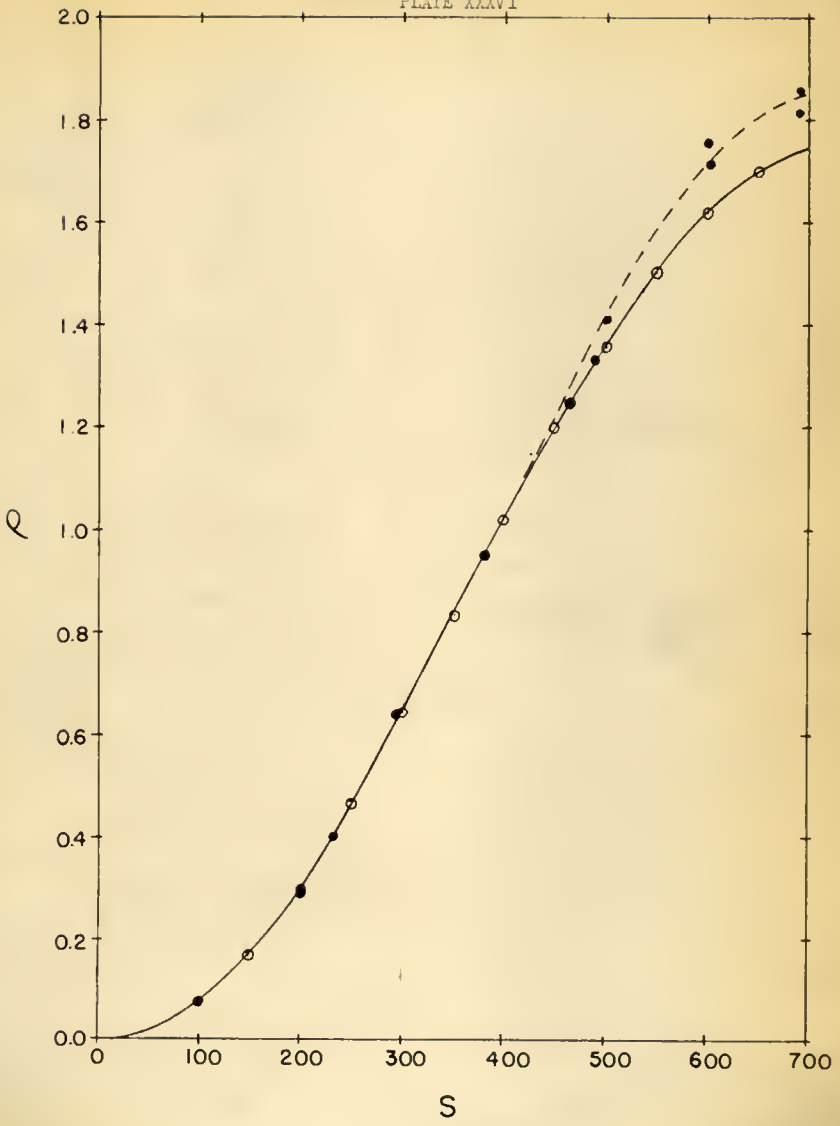
Reactivity, ρ (dollars), vs. control rod position, S (units).

---•--- Measured by dropping the shim rod from various critical positions to "full in".

—⊙— Measured by the positive period method using increments of reactivity.

Hughes' delayed neutron data.

PLATE XXXVI



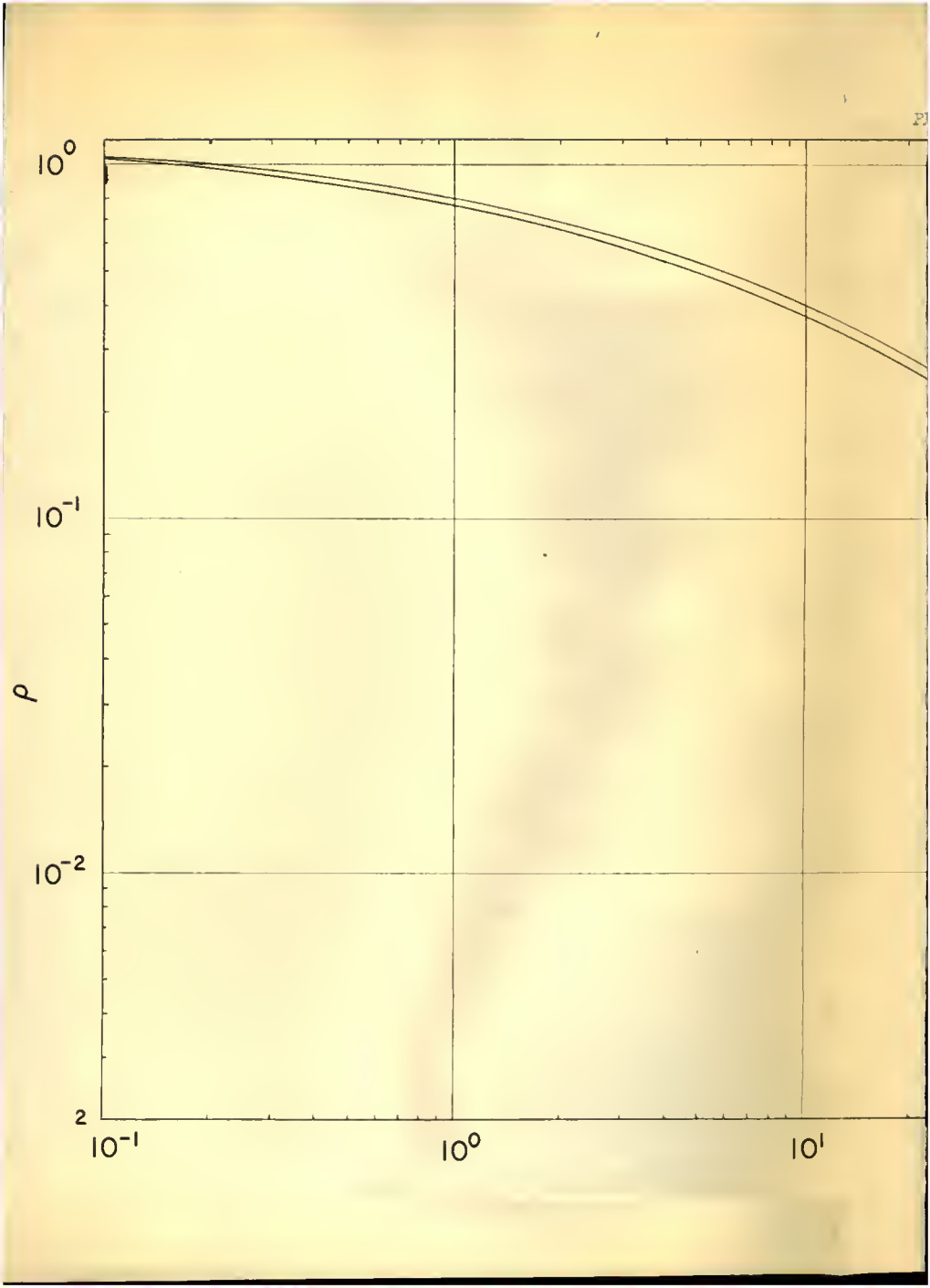
EXPLANATION OF PLATE XXXVII

Positive reactivity, ρ (dollars), vs. positive asymptotic period, τ (seconds).

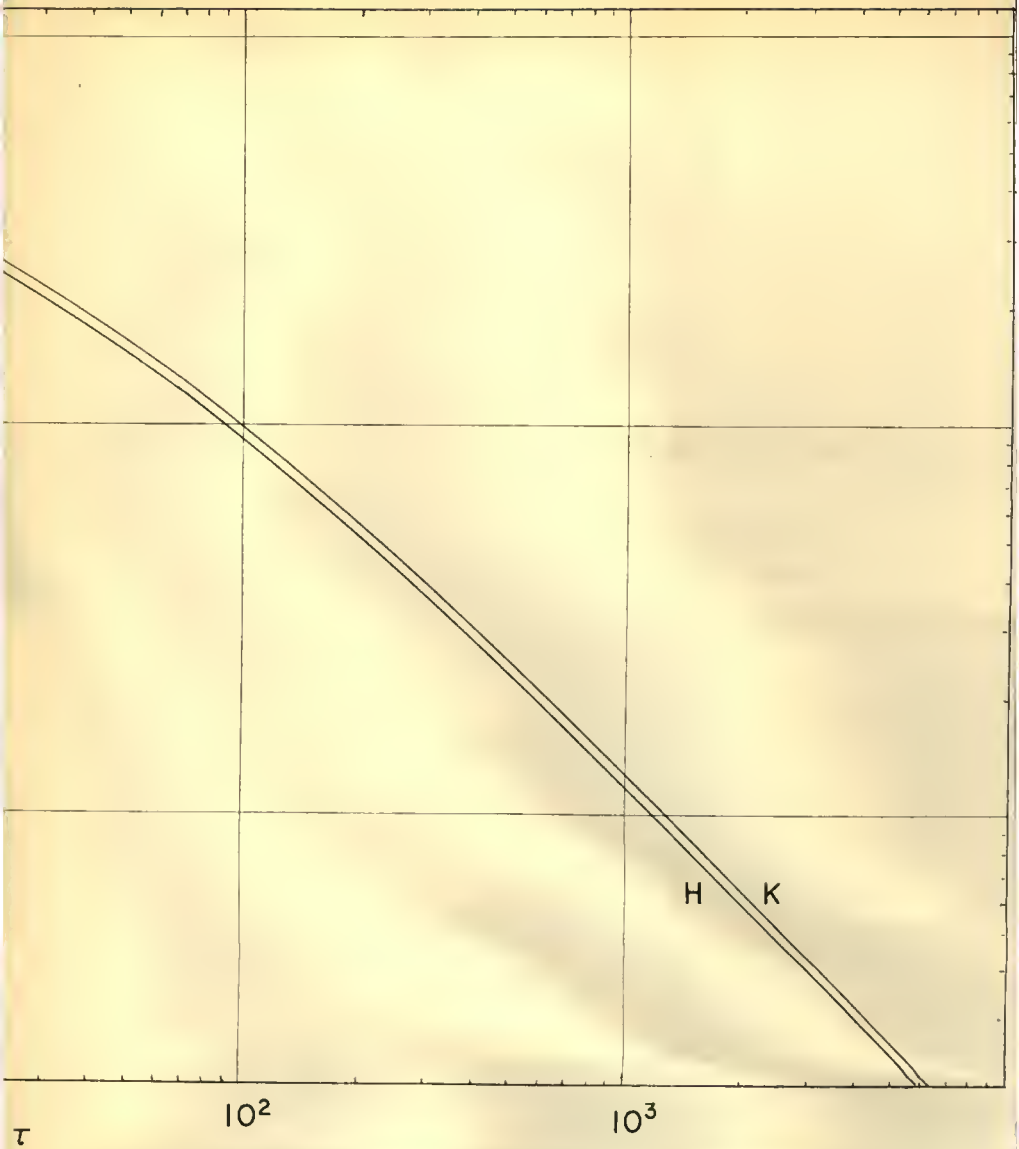
K curve: Keepin's delayed neutron parameters.

H curve: Hughes' delayed neutron parameters.

Neutron lifetime, $\ell = 8.0 \times 10^{-5}$ seconds.


10^0
10^{-1}
ρ
10^{-2}
2
10^{-1}
10^0
10^1

TE XXXVII



EXPLANATION OF PLATE XXXVIII

Regulating control rod calibration for the TRIGA reactor.

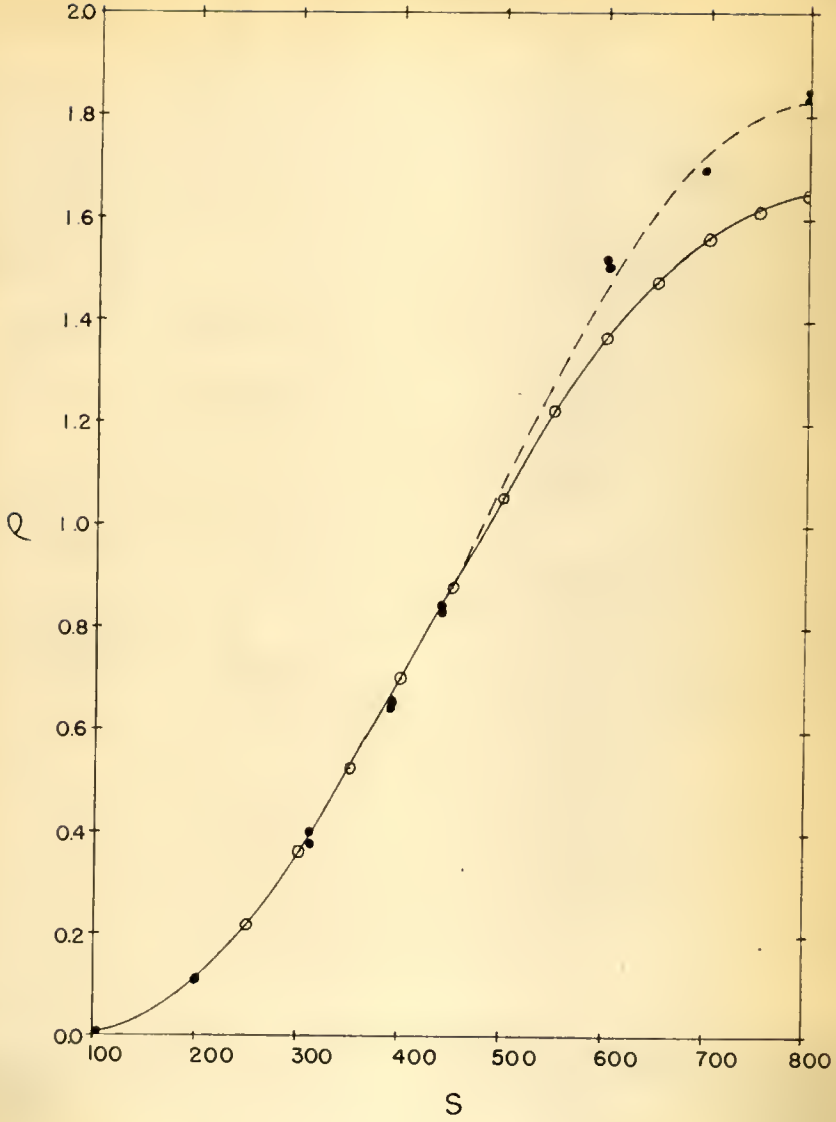
Reactivity, ρ (dollars), vs. control rod positions, S (units).

---•--- Measured by dropping the regulating rod from various critical positions to "full in".

—⊙— Measured by the positive period method using increments of reactivity.

Hughes' delayed neutron parameters.

PLATE XXXVIII



An interesting point was demonstrated in the control rod calibration by the positive period method. During the process of positive period measurements, the safety rod, a control rod of almost equal magnitude reactivity worth as the shim rod or the regulating rod, was moved from its "full out" position to its "full in" position. Thus, the safety rod was moved from one extreme of its reactivity worth position to the other extreme. The point of interest is that the position of the safety rod had no effect on the calibration of either the shim rod or the regulating rod. If the position of the safety rod had created another parameter, "shadowing effect", in the calibration procedure, a shift in the reactivity calibration curve would have been observed. The shift would have been in the form of a finite jump in the calibration curve.

Forty-three rod drop experiments were performed to measure the reactivity worth of the control rods. The control rods used in the rod drop experiments were the shim rod and the regulating rod. In each case the control rod was dropped from its critical position to its "full in" position; thus the reactivity that was calculated was the difference in reactivity of the two positions. It was assumed that the rod drop represented a negative step change in reactivity. The time required for each control rod to travel from its "full out" to its "full in" position was approximately 0.35 seconds. Thus, at times large compared to 0.35 seconds, the reactivity insertion appeared as a negative step change.

Because of the finite reactivity insertion, it was found

that for times less than eight seconds after the rod drop was initiated the measured neutron flux was higher than given by the theoretically predicted flux. Since a step change in reactivity was assumed in the theory, this observed effect was expected. The discrepancy between the analytically determined flux and the experimentally measured flux was greater, for short times after the rod drop, when larger reactivity insertions were made (see PLATE XLII and PLATE XLIII). Since the reactivities were determined from the ratio, $\phi(t)/\phi(0)$, at various times after the change in reactivity, the apparent reactivity, measured at times just after the rod drop was made, were smaller in magnitude than the true value of the reactivity change. In each case, the apparent reactivity as a function of time would reach a fairly stable value for times larger than ten seconds and then would vary somewhat around the stable value. There was found to be no uniform shift in the reactivity values as given at large times after the rod drop, although the reactivities did tend to drift away from the stable value of reactivity for times larger than 75 seconds. In each run, the reactivities were averaged for times greater than ten seconds and less than 100 seconds and the mean value was reported as the measured reactivity of the rod drop.

Integral control rod calibrations were made for the shim rod and the regulating rod (see PLATES XXXVI and XXXVIII). The total worth of the shim rod was calculated to be $1.97 and $1.84 using Keepin's and Hughes' delayed neutron data respectively. The rod drop method gave reactivity worths which were larger in

magnitude for large reactivities than did the positive period method (see PLATE XXXVIII). The total worth of the shim rod was found to be 5.1 per cent higher when measured with the rod drop method than when measured with the positive period method using either Keepin's or Hughes' delayed neutron data. (See PLATES XXXIV and XXXVI). This discrepancy is attributed primarily to the fact that the total reactivity worth of the control rod was measured in one reactivity step when the rod drop method was used whereas when the positive period method was used, the total reactivity worth was determined by 12 incremental steps of reactivity.

Argonaut Data

Through the courtesy of the Argonaut reactor personnel at the International School of Nuclear Science and Engineering, Argonne National Laboratory, experimental data were obtained from the Argonaut reactor. The data consisted of several rod drop experiments with both a slab and an annular core loading.

An analysis of the data gave similarly behaved results as that given by the TRIGA. When the rod drop experiments were analyzed using Keepin's data the evaluated reactivity was always higher than that given by Hughes' data. The same type of disagreement was found between reactivity as measured by the positive period method and reactivity as measured by the rod drop method. At small reactivities, the two methods gave substantially the same results but at larger reactivities, the rod drop method gave higher values. The fine rod of the Argonaut reactor

was calibrated both by dropping the rod from various critical positions to "full in" and then by dropping the rod from a critical position with the fine rod in its "full out" positions to various lower positions. The two methods of rod drops gave approximately the same results (see PLATES XLI and XLII).

Reactivity as a Function of Time

General Atomic (19) measured the time required for a typical TRIGA control rod to fall from its "full out" position to its "full in" position. The time required was approximately 0.35 seconds. The fall time was certainly finite when considering times less than ten seconds after initiating the rod drop. It was considered important to study the problem of predicting the reactivity insertion rate and the resulting time dependence of the neutron flux.

Since the TRIGA core was located in a water bath, the control rods fell in a fairly resistive medium during rod drop measurements (see Experimental System). It was assumed that the control rod motion could be treated as a falling body in a viscous medium where the resistive force is proportional to velocity (3). Therefore, the force, F, acting on the control rod, was assumed to be of the form

$$F = - mg - r\frac{dy}{dt} = m\frac{d^2y}{dt^2} . \qquad (14)$$

Where r is the proportionality constant, m is the mass of the control rod, g is the acceleration due to gravity, and $\frac{dy}{dt}$

EXPLANATION OF PLATE XXXIX

Calibration curve for the fine control rod of the Argonaut reactor.

Reactivity, ρ (dollars), vs. control rod position, S.

Upper set of curves: Keepin's delayed neutron data.

—⊙— Measured by dropping rod from various critical positions to full in.

⊡ Measured by dropping rod from a critical position at 100 to various lower positions.

--•-- Measured by positive period method.

Lower set of curves: Hughes' delayed neutron data.

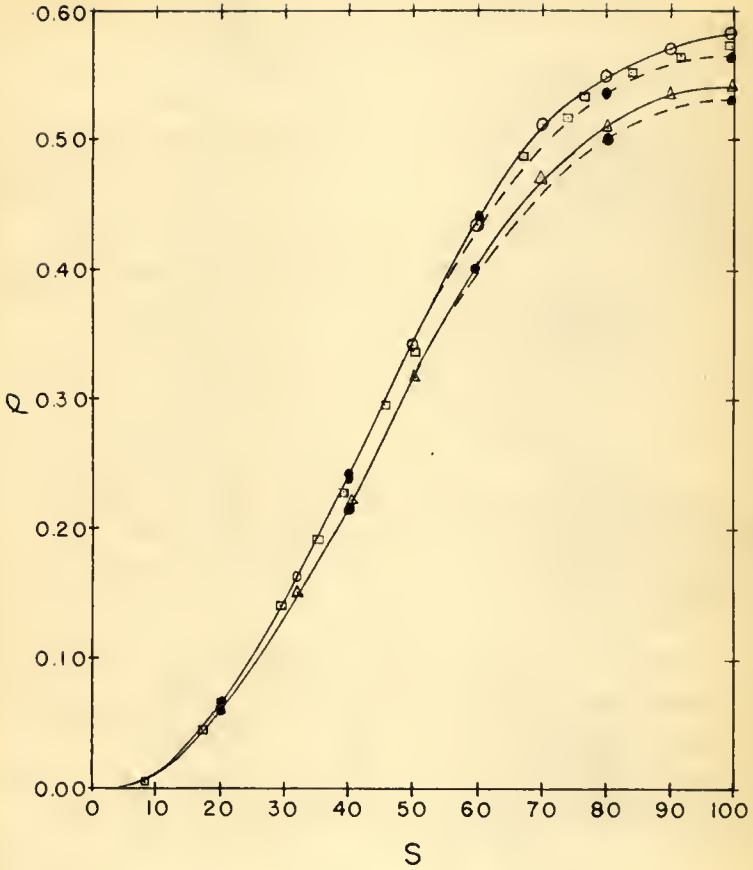
—△— Measured by dropping rod from various critical positions to full in.

--•-- Measured by positive period method.

The prompt neutron lifetime, $= 2.0 \times 10^{-4}$ seconds.

Slab core loading.

PLATE XXXIX



EXPLANATION OF PLATE XL

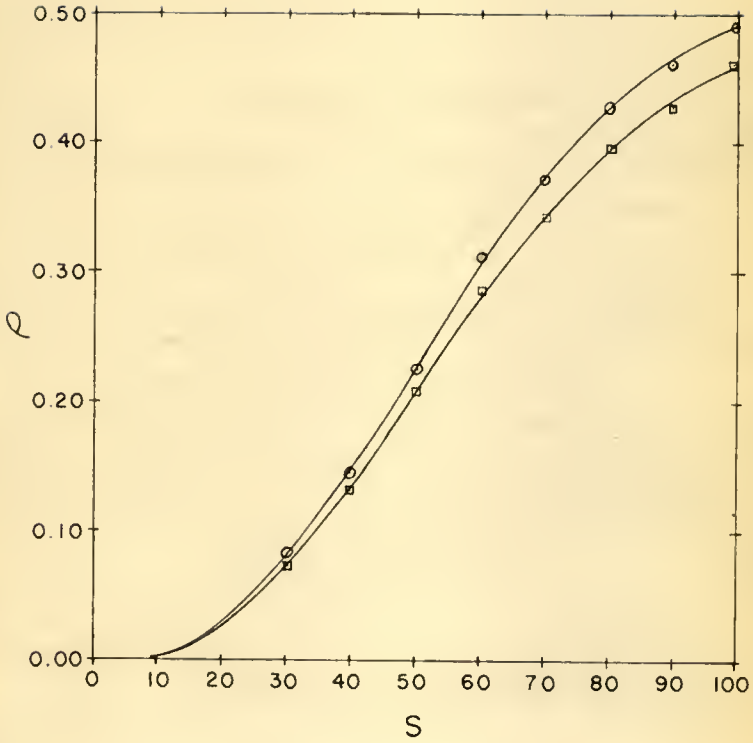
Shim control rod calibration for the Argonaut reactor, annular loading.

Reactivity, ρ (dollars), vs. control rod position, S (units).

—⊕— Measured by the rod drop method using Keepin's delayed neutron parameters.

—⊡— Measured by the rod drop method using Hughes' delayed neutron parameters.

PLATE XL



is the velocity of the control rod, positive upward. Eq. (14) was solved for the rod displacements, y , as a function of time, i.e.,

$$y = - mgt/r + \frac{m}{r}\left[\frac{dy}{dt}(0) + \frac{mg}{r}\right](1 - e^{-\frac{r}{m}t}) \ . \qquad (15)$$

Since the control rods were dropped from rest, $\frac{dy}{dt}(0)$ was equal to zero. All of the TRIGA control rods contained approximately the same amount of mass, therefore, the ratio, m/r, was considered to be a constant, c . The rod position as a function of time could then be written as

$$y = cgt - c^2g(1 - e^{-ct}) \ . \qquad (16)$$

The boundary conditions were

$$y(0) = 0$$

and $y(0.35 \text{ Sec.}) = -15.25$ inches.

Eq. (16) was solved for the constant, $c = 1.0298$ seconds, by trial and error.

The rod position was then plotted as a function of time (see PLATE XLI). From the rod calibration curve and the rod position curve, a reactivity as a function of time relationship was obtained (see PLATE XLI). It was desirable to obtain an analytic function for reactivity as a function of time because most methods for solving the kinetic equations, where $k_{eff}(t)$ is a function of time, demand well behaved time functions of reactivity (1). The form

$$k_{eff}(t) = 1 - A(1 - e^{-bt^n}) \ , \qquad t \geq 0 \qquad (17)$$

EXPLANATION OF PLATE XLI

S curve: Fall of a control rod in the TRIGA reactor, Eq. (16).

The position of a control rod, S (inches), vs. time (seconds).

The zero position of the control rod is the "full out" position.

——— Reactivity curve: The reactivity insertion due to a falling control rod in the TRIGA reactor as predicted by

$$\rho \doteq \frac{A}{\beta}\left(1 - e^{-bt^n}\right) .$$

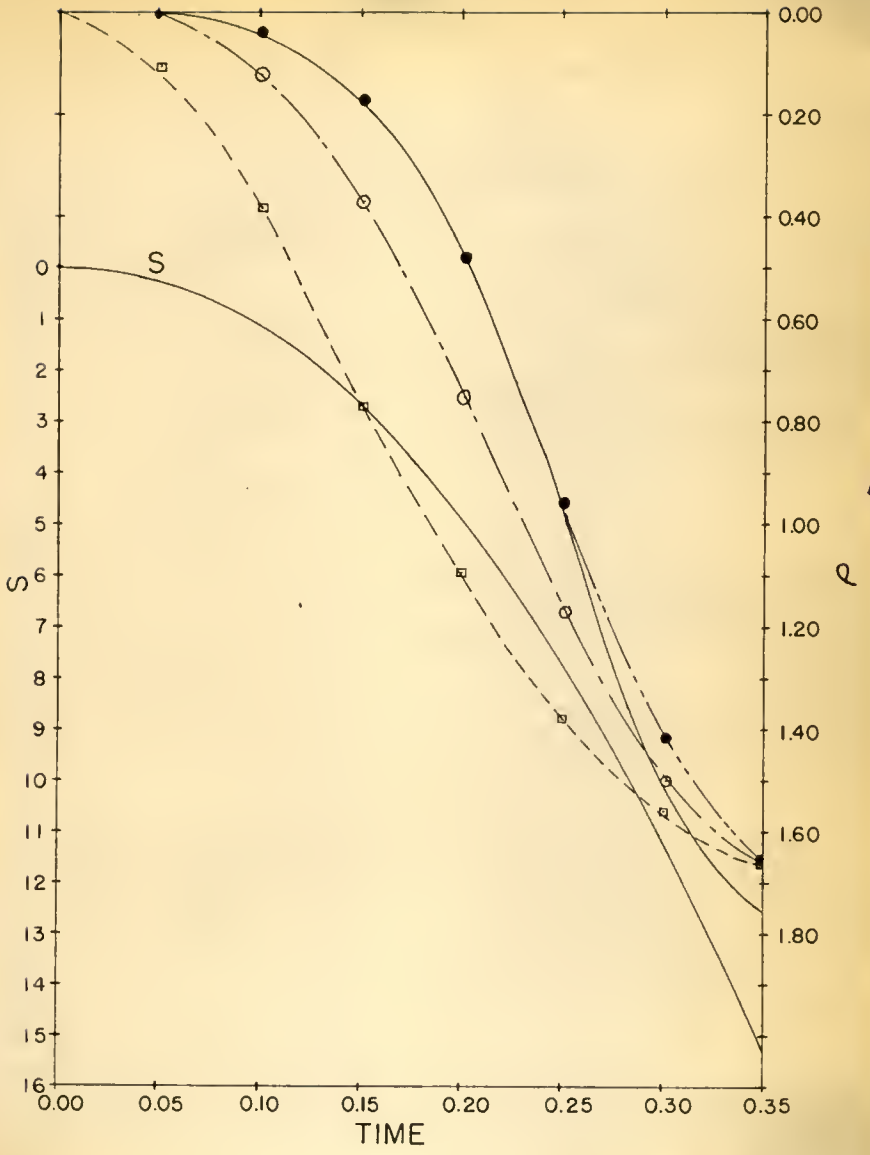
Reactivity, ρ (dollars) vs. time (seconds).

--⊡-- where n is equal to 2.0.

—·⊙·— where n is equal to 3.0.

—··•·— where n is equal to 4.0.

PLATE XLI


S
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
S
0.00
0.20
0.40
0.60
0.80
1.00
1.20
1.40
1.60
1.80
ρ
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
TIME

is used here. Note that $k_{eff}(0)$ is equal to unity and $k_{eff}(t)$ is approximately equal to (1.0 - A) at large times.

In order to solve for b in Eq. (17), an arbitrary boundary condition was imposed upon Eq. (17) such that the time dependent term of $k_{eff}(t)$ be 0.95 of its final value at time equal to 0.35 seconds, i.e., $k_{eff}(0.35) = 1.00 - 0.95A$. Corresponding to various assumed values of n the following values for b were calculated:

$$n = 2 \; ; \; b = 24.5$$
$$n = 3 \; ; \; b = 69.9$$
$$n = 4 \; ; \; b = 300$$

Since reactivity is

$$\rho = \frac{k_{eff} - 1}{k_{eff}} = \frac{k_{ex}}{k_{eff}} = k_{ex} \qquad (18)$$

and $$\rho(\$) = \frac{k_{ex}}{\beta}$$

then

$$\rho(\$) = \frac{A}{\beta}(1 - e^{-bt^n}) \; . \qquad (19)$$

Reactivity, ρ, from Eq. (19) was plotted as a function of time (see PLATE XLI) for the various values of n. It was found that n equal to 4.0 gave the best empirical fit to the reactivity curve.

$k_{eff}(t)$ from Eq. (17) was used in the reactor kinetic equation (see Appendix - B). Several computations were made and the resulting predictions of neutron flux as a function of time

after a change in reactivity were compared to experimentally measured values. The kinetic equations used for the calculations were of the form

$$n(t) = n(o)\left[\frac{a_2 + (\beta-1)}{a_2 + (\beta-1)e^{-bt^n}}\right] e^{-\int_o^t \frac{\lambda dt'}{a_2 + (\beta-1)e^{-bt'^n}} + \int_o^t \frac{\lambda dt'}{a_2 e^{-bt'^n} + (\beta-1)}} \qquad (20)$$

where the decay constant, λ, represents the single delayed neutron group. It was found that the value of n equal to 4.0 gave the best comparison with the experimental data. Even then, although the predicted flux levels from the approximated reactivity equation gave better comparisons for very short times after the initial rod drop than those using the six group step input analysis, the results were in general quite poor. It was then decided to let both b and λ be empirical parameters. The best fit to experimental measurements was obtained when (see PLATE XLII)

$$b = 10.0 \text{ Sec.}^{-1}$$
$$\lambda = 0.20 \text{ Sec.}^{-1}$$

These parameters gave good results over a fairly wide range of reactivities. The largest reactivity insertion used was -\$1.42 (see PLATE XLII). The finite reactivity insertion method gave decisively better predictions of the time behavior of the neutron flux for small times, $t <$ four seconds. At larger times the model gave increasingly poorer results due to the assumption of one group of delayed neutrons.

EXPLANATION OF PLATE XLII

Flux ratio, $\phi(t)/\phi(0)$, vs. time (seconds) after a negative change in reactivity of - $0.40.

⊙ Experimentally measured flux ratios in the TRIGA reactor after dropping the regulating rod from position 310 to 100.

——·—— Theoretically predicted flux ratios using six groups of delayed neutrons and assuming a step change in reactivity.

Hughes' delayed neutron parameters.

——— Theoretically predicted flux ratios using the reactivity as a function of time, Eq. (16), and assuming one group of delayed neutrons.

b was equal to 10.0 and λ was equal to 0.20 seconds^{-1}.

* In this case λ was equal to 0.22 seconds^{-1}.

——— Total rod drop time.

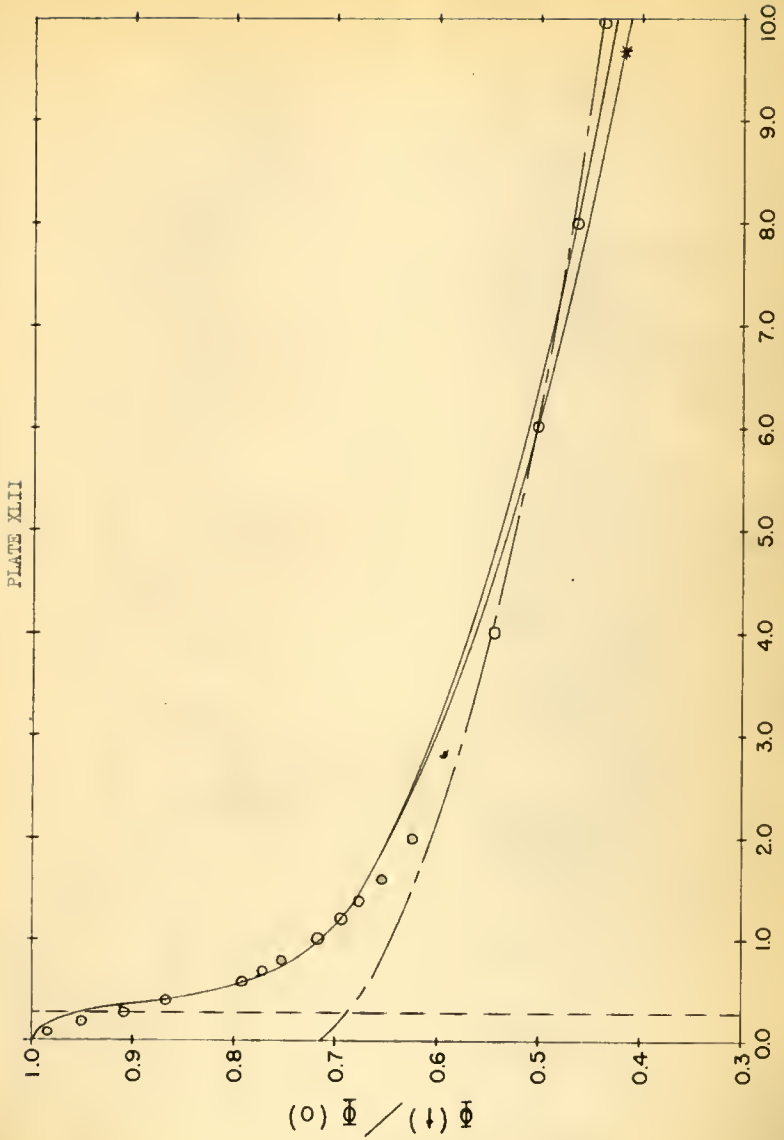


PLATE XLII
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0
TIME
Φ (t) / Φ (0)

EXPLANATION OF PLATE XLIII

Flux ratio, $\phi(t)/\phi(o)$, vs. time (seconds) after a negative change in reactivity of - $1.42.

⊙ Experimentally measured flux ratios in the TRIGA reactor after dropping the shim rod from position 500 to 29.

——-—- Theoretically predicted flux ratios using six groups of delayed neutrons and assuming a step change in reactivity.

Hughes' delayed neutron parameters.

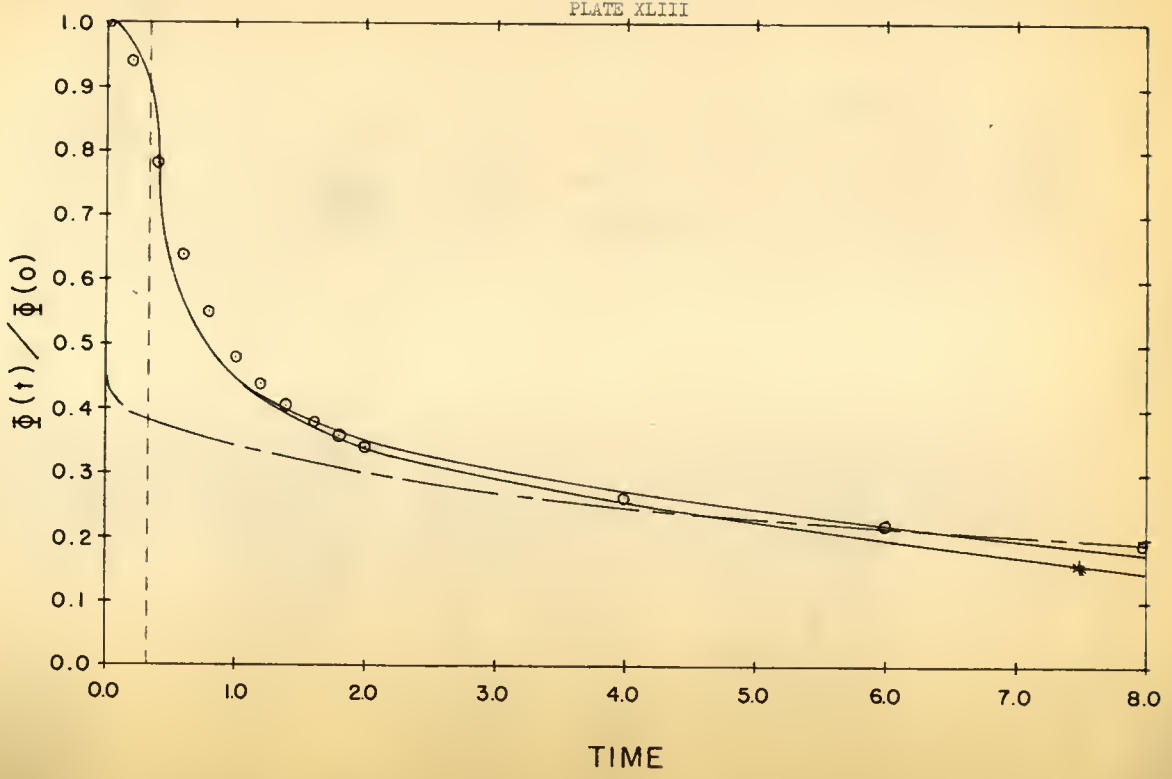
——— Theoretically predicted flux ratios using the reactivity as a function of time, Eq. (16), and assuming one group of delayed neutrons.

b was equal to 10.0 and λ was equal to 0.20 seconds^{-1}.

* In this case λ was equal to 0.22 seconds^{-1}.

——— Total rod drop time.

PLATE XLIII



CONCLUSIONS

The reactor kinetic theory used in this paper is quite satisfactory for use with the TRIGA reactor, at least for reactor operation at low power levels. The conclusion above is based on the agreement between reactivity determination by the method of positive period measurements and by the method of rod drop measurements. The rod drop method depended upon the theoretical prediction of the flux ratio over wide ranges of time. Good agreement was found between the predicted flux ratios and the experimentally measured flux ratios for Keepin's and Hughes' delayed neutron parameters. The agreements between predicted and measured flux ratios were poorer for large than for small reactivities. Agreement was unacceptable at reactivities greater than $2.00. For reactivity measurements greater than $1.00, there was observed a slight scatter of the experimentally determined flux ratios. It is not certain whether the scatter of the observed data was due to the inadequacy of the kinetics model or to the failure of the experimental equipment to accurately measure the rapidly falling flux levels. A possibility that the kinetics model cannot be extended to cover large negative changes in reactivity was indicated by the consistent prediction of larger values of reactivity by the rod drop method as compared with the positive period method. It must be remembered that all of the positive period measurements were composed of small positive changes in reactivity. It is recommended that when using the rod drop method, for evaluating the reactivity

worth of a control rod, the measurements be made using moderately small changes, i.e., that the total reactivity worth of a control rod be measured by a series of incremental rod drops and not be dropping the control rod from "full out" to "full in".

It was found that the two delayed neutron parameter models, Hughes' and Keepin's, gave similar theoretical predictions of the kinetic behavior of a reactor. Keepin's model consistently gave larger evaluations of reactivity, approximately eight per cent, when both the positive period and the rod drop methods were used to calibrate the control rods of the TRIGA reactor and the Argonaut reactor. The two delayed neutron models gave similarly shaped control calibrations for both the TRIGA reactor and the Argonaut reactor, that is, when evaluating large negative changes in reactivity the disagreement between the positive period and the rod drop method evaluations were similar for both reactors and for both delayed neutron models. No conclusion could be made as to which model fit the experimental measurements best, because both models were consistent, within themselves, when considering small changes in reactivity and both models gave similar disagreements for large reactivities. But it is the opinion of this author that either Keepin's or Hughes' model may be used in reactivity evaluation.

It was found, in the Beta Parameter analysis, that if reactivities are quoted in units of dollars there is no confusion as to the response of a reactor to a particular change in reactivity ($), that is, regardless of the total fraction of delayed neutrons, the same reactivity ($) input gave approximately the

same predicted flux ratios.

The kinetic equation parameter analysis revealed that the only significant parameter is reactivity. The other parameters may be varied to any reasonable values and the resulting theoretical change of the kinetic behavior of a reactor will be slight. The parameter study confirms a generalized study by Henry (9).

The three group kinetic equations used in this paper confirm work done by Skinner and Cohen (16), in that the six group equations may be presented by the reduced group equations. The real usefulness of the three group equations probably does not lie in their ability to predict the reactivity worth of reactor control rods, but rather, to predict the general nature of the time dependency of the neutron flux in a reactor where a series of reactor calculations must be made considering many parameters in addition to the usual parameters. The computer code developed for this paper may be altered to consider a wide class of kinetic problems. These computer calculations are more rapid and easier to analyze than the calculations obtained with the six group equations.

When considering rod drop experiments or other reactivity insertion problems, where the reactivity insertion is rapid but still finite, the six group kinetic equations will predict the time dependency of the neutron density when evaluated at times much larger in magnitude than the total reactivity insertion time. But, when neutron densities are evaluated at times comparable to the total insertion time, it is necessary to consider

reactivity as a function of time instead of to assume the step function. A set of equations developed in this paper approximated the reactivity insertion rate of a control rod drop and the resulting neutron flux in the TRICA reactor. The predicted time dependency of the neutron flux gave values which were in good agreement with the experimentally measured neutron flux. The approximated reactivity equations gave better predictions of the neutron flux at times less than four seconds, after initiating the reactivity change, than did the six group equations using a step input of reactivity. For times greater than four seconds, the neutron flux was predicted to a good degree of accuracy by the kinetic equations using six delayed groups and a step input. The reason for the poor results, using the approximated reactivity equations at large times was, no doubt, due to the one delayed neutron group assumption.

ACKNOWLEDGMENTS

The author takes this opportunity to express his appreciation to the following people whose advice, encouragement, and technical guidance have made the completion of this project possible:

Dr. William R. Kimel, Head of the Department, for suggesting this study and for his excellent technical judgment which greatly aided the progression of this thesis.

Richard C. Bailie, for his excellent guidance and supervision in the use of the IBM-650 computer and for his valuable time that he spent in writing portions of a computer program.

General Atomic, for the use of their TRIGA reactor and the excellent cooperation of the TRIGA reactor staff.

Dr. William Sturm, of Argonne National Laboratory, for the experimental data forwarded to this author.

The author also wishes to express his gratitude to the Kansas State University Engineering Experiment Station for their financial support and to the Atomic Energy Commission for the opportunity to attend graduate school with the aid of an A.E.C. Fellowship.

BIBLIOGRAPHY

1. Ash, M.
   Solutions of the reactor kinetics equations for time varying reactivities. J. Appl. Phys. 27:1030-1031. Sept., 1956.

2. Bloomfield, M., and Bennett, F. C.
   Reactor kinetics, a bibliography. NAA-SR-3808, 1959.

3. Constant, F. W.
   Theoretical physics. Reading: Addison-Wesley, 1954.

4. Ewen, R. L., and Wick, R. S.
   Delayed neutrons in reactor kinetics calculations. WAPD-BT-13.

5. Friedman, L.
   Few group constants for delayed neutrons. WAPD-TM-6.

6. Garabedian, H. L., and Leffert, C. B.
   A time-dependent analysis of spatial flux distribution. Nuclear Sci. and Eng. 6:26-33. July, 1959.

7. Glasstone, S.
   Principles of nuclear reactor engineering. Princeton: Van Nostrand, 1952.

8. Glasstone, S., and Edlund, M. C.
   The elements of nuclear reactor theory. Princeton: Van Nostrand, 1952.

9. Henry, A. F.
   Computation of parameters appearing in the reactor kinetics equations. WAPD-142, 1955.

10. Hughes, D. J., Dalbs, J., Cahn, A., and Hall, D.
    Delayed neutrons from fission of U^{235}. Phy. Rev. 73:111-124. Jan. 15, 1948.

11. Hurwitz, H., Jr.
    Derivation and integration of the pile-kinetic equations. Nucleonics. 5:61-67. July, 1949.

12. Keepin, G. R.
    Delayed neutrons. Progress in nuclear energy. Series I. Physics and mathematics. Chapt. 7. New York: Pergamon Press, 1956.

13. Keepin, G. R., Wimett, T. F., and Zeigler, R. K.
    Delayed neutrons from fissionable isotopes of uranium, plutonium, and thorium. LA-2118, 1957.

14. Kimel, W. R., Carey, W. E., Prohammer, F. G., and Baldwin, G. C.
Determination of time behavior of neutron density and of reactivity in the Argonaut reactor. Nuclear Sci. and Eng. 6:233-237. Sept., 1959.

15. MacPhee, J.
Approximate solutions of the reactor kinetic equations for ramp inputs. Nuclear Sci. and Eng. 7:33-44. Jan., 1960.

16. Skinner, R. E., and Cohen, E. R.
Reduced delayed neutron group representation. Nuclear Sci. and Eng. 5:291-298. May, 1959.

17. Smets, H. B.
Exact solution of the reactor kinetics equations for linear, exponential, or reciprocal reactivity dependence on time. Bulletin de la Classe Des Science. Academie royal de Belgique. 45:256-271. March, 1959.

18. Soodak, H.
Pile kinetics in the science and engineering of nuclear power. Vol. II. Chapt. 8. Reading: Addison-Wesley, 1949.

19. Technical foundations of TRIGA. GA-471, 1958.

20. Toppel, B. J.
Source of error in reactivity determination by means of asymptotic period measurements. Nuclear Sci. and Eng. 5:88-99. Feb., 1959.

21. TRIGA reactor description. GA-436. Aug., 1958.

22. TRIGA Mark II reactor description. GA-485. Aug., 1958.

23. Wylie, C. R., Jr.
Advanced engineering mathematics. New York: McGraw-Hill, 1950.

APPENDIX

Part - A

Theoretical Development of the Kinetic Equations

The system considered was a bare, homogeneous, thermal reactor. A thermal neutron balance on a volume element of the reactor core gives (8)

$$D\nabla^2\Phi(r,t) - \Sigma_a\Phi(r,t) + S(r,t) = \frac{\partial n(r,t)}{\partial t} = \frac{1}{v}\frac{\partial \Phi(r,t)}{\partial t} \qquad (20)$$

The Fermi age equation is used as the source of neutrons for the thermal diffusion equation. The source term for prompt neutrons is

$$S(r,t)_{Prompt} = (1-\beta)k\Sigma_a\Phi(r,t)e^{-B^2\tau} \qquad (21)$$

The rate equation for the precursor density is

$$\frac{\partial C_i(r,t)}{\partial t} = -\lambda_i C_i(r,t) + \frac{k\beta_i\Sigma_a\Phi(r,t)}{p} \qquad (22)$$

where $-\lambda_i C_i$ gives the loss of neutron precursors due to radioactive decay and $\frac{k}{p}\beta_i\Sigma_a\Phi(r,t)$ gives the production of precursors from the fission process. Since delayed neutrons are produced following the beta decay of the precursors, the total production rate of all delayed neutrons is $\sum_{i=1}^{m}\lambda_i C_i$. Since there are six groups of delayed neutrons, m is equal to six in the above summation. Delayed neutrons are born with energies between 0.2 and 0.8 Mev., thus the production rate of delayed neutron term must be multiplied by the nonleakage probability

during slowing down and by the resonance escape probability.

$$S(\underline{r},t)_{\text{Delayed}} = \sum_{i=1}^{6} p_i e^{-B^2\tau_i} \lambda_i C_i(\underline{r},t) \tag{23}$$

The source term for both the prompt neutrons and the delayed neutrons is substituted into the thermal diffusion equations and the resulting equation is

$$D\nabla^2\Phi(\underline{r},t) - \Sigma_a\Phi(\underline{r},t) + (1-\beta)k\Sigma_a\Phi(\underline{r},t)e^{-B^2\tau} + \sum_{i=1}^{6} p_i e^{-B^2\tau_i} \lambda_i C_i(\underline{r},t) = \frac{1}{v} \cdot \frac{\partial\Phi(\underline{r},t)}{\partial t} \tag{24}$$

The following, well known, relations were substituted into Eq. (24)

$$L^2 = D/\Sigma_a \tag{25}$$

$$l_o = 1/\Sigma_a v \tag{26}$$

Eq. (24) was then divided by Σ_a, which gives

$$L^2\nabla^2\Phi(\underline{r},t) + \left[(1-\beta)ke^{-B^2\tau} - 1\right]\Phi(\underline{r},t) \frac{1}{\Sigma_a}\sum_{i=1}^{6} p_i e^{-B^2\tau} \lambda_i C_i(\underline{r},t) = l_o \frac{\partial\Phi(\underline{r},t)}{\partial t} \tag{27}$$

The space and time variables of the thermal diffusion and the precursor rate equations were assumed to be separable, that is

$$\Phi(r,t) = F(r)\,\phi(t) \tag{28}$$

and

$$C_i(r,t) = G_i(r)\,H_i(t) . \tag{29}$$

It is assumed that the spatial distribution of the thermal neutrons can be given by the fundamental mode of the wave equation,

$$\nabla^2 F(r) + B^2 F(r) = 0 . \tag{30}$$

Since ∇^2 is a spatial operator, $B^2 F(r)$ is substituted into Eq. (27) after the separation of variables, thus

$$-L^2 \phi(t) B^2 F(r) + \left[(1-\beta) k e^{-B^2\tau} - 1\right] F(r)\,\phi(t) + \sum_{i=1}^{6} \frac{p_i e^{-B^2\tau_i}}{\Sigma_a} \lambda_i G_i(r) H_i(t) = l_0 F(r) \frac{d\phi(t)}{dt} . \tag{31}$$

The definition of the effective multiplication factor and the finite prompt neutron lifetime is used to simplify Eq. (32)

$$k_{eff} \equiv \frac{k e^{-B^2\tau}}{1 + L^2 B^2} = \text{effective multiplication factor} \tag{33}$$

$$l \equiv \frac{l_0}{1 + L^2 B^2} = \text{finite prompt neutron lifetime} \tag{34}$$

Eq. (32) is divided by $1+L^2B^2$ and then Eq. (33) and Eq. (34) is substituted into Eq. (32).

Eq. (31) is then divided by the spatial flux function to give

$$\frac{\ell\, d\phi(t)}{dt} = -\phi(t) + (1-\beta)\, k_{eff}\, \phi(t) + \frac{1}{\Sigma_a (1+L^2B^2)\, F(\underline{r})} \cdot \sum_{i=1}^{6} p_i e^{-B^2\tau_i} \lambda_i C_i(\underline{r}) H_i(t) \quad (35)$$

Eq. (35) is divided by the neutron velocity, v, and the expression

$$\frac{\phi(t)}{v} = n(t)$$

is substituted into Eq. (35) giving

$$\ell \frac{dn(t)}{dt} = -n(t) + (1-\beta) k_{eff}\, n(t) + \frac{1}{\Sigma_a v\, F(\underline{r})(1+L^2B^2)} \cdot \sum_{i=1}^{6} p_i e^{-B^2\tau_i} \lambda_i C_i(\underline{r}) H_i(t). \quad (36)$$

The relationship

$$\ell = \frac{1}{\Sigma_a v (1+L^2B^2)} = \text{finite prompt neutron lifetime}$$

is substituted into Eq. (36) to give

$$\left[(1-\beta) k_{eff} - 1\right] \frac{n(t)}{\ell} + \frac{1}{F(\underline{r})} \sum_{i=1}^{6} p_i e^{-B^2\tau_i} \lambda_i C_i(\underline{r}) H_i(t) = \frac{dn(t)}{dt} \quad (37)$$

$\frac{\rho_i e^{-B^2\tau_i} G_i(r)}{F(r)}$ is defined as the space independent constant, ϵ_i.

The procursor rate equation is then written in the separated variable form as

$$\frac{dH_i(t)}{dt} G_i(r) = -\lambda_i G_i(r) H_i(t) + \frac{k\beta_i \Sigma_a F(r) \phi(t)}{\rho} \quad (38)$$

The resulting equation, (39), contains functions of time alone on the left side and functions of space alone on the right side.

$$\frac{dH_i(t)}{dt} \frac{1}{\phi(t)} + \frac{\lambda_i H_i(t)}{\phi(t)} = \frac{k\beta_i \Sigma_a F(r)}{\rho G_i(r)} \quad (39)$$

Therefore ϵ_i is a time independent parameter.

The space independent neutron density equation is then written as

$$\left[(1-\beta) k_{eff} - 1\right] \frac{n(t)}{\ell} + \sum_{1}^{6} \epsilon_i \lambda_i H_i(t) = \frac{dn(t)}{dt} \quad (40)$$

The space independent precursor equation is then obtained by dividing Eq. (40) by $G_i(r)$, which gives

$$\frac{dH_i(t)}{dt} = -\lambda_i H_i(t) + \frac{k\beta_i \Sigma_a \phi(t) F(r)}{\rho G_i(r)} \quad (41)$$

The last term on the right hand side of Eq. (41) is then multiplied and divided by $\upsilon e^{-B^2\tau}(1+L^2B^2)$ to give

$$\frac{dH_i(t)}{dt} = -\lambda_i H_i(t) + \frac{ke^{-B^2\tau}}{1+L^2B^2} \cdot \frac{\phi(t)}{v} \beta_i \frac{\xi_2 v (1+L^2B^2) F(\underline{r})}{G_i(\underline{r}) p e^{-B^2\tau}} \quad (42)$$

ϵ_i' then defined as $\frac{F(\underline{r})}{G_i(\underline{r}) p e^{-B^2\tau}}$, thus ϵ_i' is similar to ϵ of Eq. (35) and likewise time independent. The resultant space independent precursor rate equation is obtained as

$$\frac{dH_i(t)}{dt} = -\lambda_i H_i(t) + \frac{k_{eff}\, n(t) \beta_i \epsilon_i'}{\ell} . \quad (43)$$

The usual definition of reactivity, ρ, is

$$\rho = \frac{k_{eff} - 1}{k_{eff}} . \quad (44)$$

Thus

$$k_{eff} = \frac{1}{1-\rho} . \quad (45)$$

Eq. (45) is substituted in the space independent kinetic equation to give

$$\frac{\rho - \beta}{\ell(1-\rho)} n(t) + \sum_{i=1}^{6} \epsilon_i \lambda_i H_i(t) = \frac{dn(t)}{dt} . \quad (46)$$

The same substitution, as above, into Eq. (43) results in

$$\frac{dH_i(t)}{dt} = -\lambda_i H_i(t) + \frac{\beta_i n(t) \epsilon_i'}{\ell(1-\rho)} . \quad (47)$$

Eq.'s (46) and (47) constitute a set of seven linear differential equations, therefore solutions of the form

$$n(t) = A' e^{\omega t} \tag{48}$$

$$H_i(t) = B_i' e^{\omega t} \tag{49}$$

may be assumed.

Substitutions of Eq.'s (48) and (49) into Eq.'s (46) and (47) give

$$\left(\omega - \frac{\rho - \beta}{\ell(1-\rho)}\right) A' + \sum_{i=1}^{6} \epsilon_i \lambda_i B_i' = 0 \tag{50}$$

and

$$\frac{\epsilon_i' \beta_i A'}{\ell(1-\rho)} - (\omega + \lambda_i) B_i' = 0 \tag{51}$$

A solution for the coefficient A' is displayed by Cramer's rule as follows:

$$
A = \begin{vmatrix}
0 & \lambda_1\epsilon_1 & \lambda_2\epsilon_2 & \lambda_3\epsilon_3 & \lambda_4\epsilon_4 & \lambda_5\epsilon_5 & \lambda_6\epsilon_6 \\
0 & -(\omega+\lambda_1) & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -(\omega+\lambda_2) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -(\omega+\lambda_3) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -(\omega+\lambda_4) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -(\omega+\lambda_5) & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -(\omega+\lambda_6)
\end{vmatrix}
$$

$$
= \begin{vmatrix}
\omega-\left(\frac{\ell-\beta}{1-\ell}\right)\frac{1}{\ell} & \lambda_1\epsilon_1 & \lambda_2\epsilon_2 & \lambda_3\epsilon_3 & \lambda_4\epsilon_4 & \lambda_5\epsilon_5 & \lambda_6\epsilon_6 \\
\frac{\beta_1\epsilon_1'}{\ell(1-\ell)} & -(\omega+\lambda_1) & 0 & 0 & 0 & 0 & 0 \\
\frac{\beta_2\epsilon_2'}{\ell(1-\ell)} & 0 & -(\omega+\lambda_2) & 0 & 0 & 0 & 0 \\
\frac{\beta_3\epsilon_3'}{\ell(1-\ell)} & 0 & 0 & -(\omega+\lambda_3) & 0 & 0 & 0 \\
\frac{\beta_4\epsilon_4'}{\ell(1-\ell)} & 0 & 0 & 0 & -(\omega+\lambda_4) & 0 & 0 \\
\frac{\beta_5\epsilon_5'}{\ell(1-\ell)} & 0 & 0 & 0 & 0 & -(\omega+\lambda_5) & 0 \\
\frac{\beta_6\epsilon_6'}{\ell(1-\ell)} & 0 & 0 & 0 & 0 & 0 & -(\omega+\lambda_6)
\end{vmatrix} = 0 \qquad (52)
$$

Since A' is not identically zero except for the trivial solution, the determinant of the coefficients must be identically equal to zero. The determinant of the coefficients was simplified by multiplying the following columns by the indicated products:

column 1 by $(\omega+\lambda_1)(\omega+\lambda_2)\cdots(\omega+\lambda_6)$

column 2 by $(\omega+\lambda_2)(\omega+\lambda_3)\cdots(\omega+\lambda_6)\beta_1\epsilon_1'/\ell(1-\rho)$

column 3 by $(\omega+\lambda_1)(\omega+\lambda_3)\cdots(\omega+\lambda_6)\beta_2\epsilon_2'/\ell(1-\rho)$

column 4 by $(\omega+\lambda_1)(\omega+\lambda_2)(\omega+\lambda_4)\cdots(\omega+\lambda_6)\beta_3\epsilon_3'/\ell(1-\rho)$

column 5 by $(\omega+\lambda_1)(\omega+\lambda_2)(\omega+\lambda_3)(\omega+\lambda_5)(\omega+\lambda_6)\beta_4\epsilon_4'/\ell(1-\rho)$

column 6 by $(\omega+\lambda_1)(\omega+\lambda_2)\cdots(\omega+\lambda_4)(\omega+\lambda_6)\beta_5\epsilon_5'/\ell(1-\rho)$

column 7 by $(\omega+\lambda_1)(\omega+\lambda_2)\cdots(\omega+\lambda_5)\beta_6\epsilon_6'/\ell(1-\rho)$

Each of the columns, except column 1, was added to column 1, thus the determinant is cast into the following form.

$$\begin{vmatrix} A & B & C & D & E & F & G \\ 0 & I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & J & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & K & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & L & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & M & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & N \end{vmatrix} = 0 \qquad (53)$$

The product, AIJKLMN, is set equal to zero. Since the terms, I through N, are product terms of the reactor parameters, which are finite, they cannot be equal to zero, hence the A

term must be identically zero. The term, A, in expandod form is

$$A = \left[\omega - \frac{1}{\ell}\left(\frac{\rho-\beta}{1-\rho}\right)\right]\left[\Pi_i(\omega+\lambda_i)\right]$$

$$- \frac{1}{\ell(1-\rho)} \sum_{i=1}^{6} \frac{\Pi_i(\omega+\lambda_i)}{(\omega+\lambda_i)} \cdot \lambda_i \beta_i \epsilon_i \epsilon_i' \qquad (54)$$

Equation (54) is divided by $\Pi_i(\omega+\lambda_i)$ which gives

$$\ell(1-\rho)\left[\omega + \frac{1}{\ell}\left(\frac{\rho-\beta}{1-\rho}\right)\right] = \sum_{i=1}^{6} \frac{\epsilon_i \epsilon_i' \lambda_i \beta_i}{(\omega+\lambda_i)} \qquad (55)$$

Since

then, Eq. (55) becomes

$$\ell\omega - \rho(\ell\omega+1) = \sum_{i=1}^{6} \epsilon_i \epsilon_i' \left[\frac{\lambda_i \beta_i}{\omega+\lambda_i} - \frac{\beta_i \omega + \beta_i \lambda_i}{\omega+\lambda_i}\right] \qquad (56)$$

and

$$\rho = \frac{\ell\omega}{\ell\omega+1} + \frac{1}{\ell\omega+1}\sum_{i=1}^{6}\frac{\epsilon_i\epsilon_i'\beta_i\,\omega}{\omega+\lambda_i} \qquad (57)$$

$\epsilon_i\epsilon_i'\beta_i$ from Eq. (56) is defined as the effective fraction of delayed neutrons, i.e. $\epsilon_i\epsilon_i'$ gives the relative effectiveness of a delayed neutron for producing a succeeding fission as compared to a prompt neutron. $\epsilon_i\epsilon_i'$ is the ratio of the product of the resonance escape probability and the nonleakage probability during slowing down for each of the precursor neutron groups to the product of the resonance escape probability and the nonleakage probability during slowing down for fission spectrum neutrons, that is

$$\epsilon_i\epsilon_i' = \frac{p_i e^{-B^2\tau_i}}{p\,e^{-B^2\tau}} \qquad (58)$$

thus $\epsilon_i\epsilon_i'\beta_i = \beta_{effi}$.

β_{eff_i} was substituted into Eq. (51) and the resulting equation is the characteristic equation for the kinetic differential equations,

$$\rho = \frac{\ell\omega}{\ell\omega+1} + \frac{1}{\ell\omega+1}\sum_{i=1}^{6}\frac{\beta_{eff_i}\,\omega}{\omega+\lambda_i} \qquad (59)$$

Equation (59) is a seventh degree polynomial equation in ω and thus there are seven roots of Eq. (59). The roots are the

eigen-values, $\omega_1, \omega_2, \ldots, \omega_7$, of the characteristic equation. The solution for the neutron density as a function of time is then expressed as

$$n(t) = \sum_{j=1}^{7} A_j e^{\omega_j t} \tag{60}$$

and the precursor density is expressed as

$$H_i(t) = \sum_{j=1}^{7} B_{ij} e^{\omega_j t} \tag{61}$$

The two coefficients, A_j and B_{ij}, are not independent but may be related by the substitution of Eq.'s (60) and (61) into the rate equation giving

$$B_{ij}(\omega_j + \lambda_i) = \frac{A_j \beta_i \epsilon_i'}{(1-\rho)\ell}$$

Therefore, (62)

$$B_{ij} = \frac{\epsilon_i' A_j}{(1-\rho)(\omega_j + \lambda_i)}$$

The precursor density equations may then be expressed as

$$H_i(t) = \frac{\beta_i \epsilon_i'}{(1-\rho)} \sum_{j=1}^{7} \frac{A_j e^{\omega_j t}}{(\omega_j + \lambda_i)} \tag{63}$$

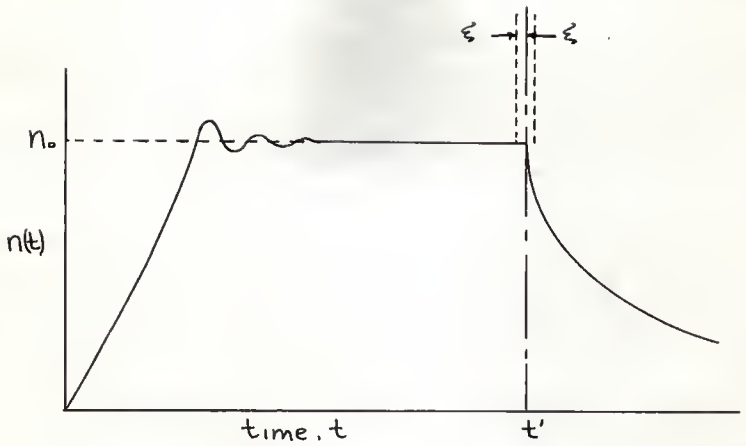


Fig. 1

The neutron and precursor densities are evaluated immediately before and immediately after a step change in reactivity for the following hypothetical case. The reactor is brought to a critical configuration and the neutron density distribution of time is considered as shown in Figure 1. A step change in reactivity occurs at time, t'. At time, $t'-\xi$, the reactivity, ρ, is equal to zero and at time, $t'+\xi$, the reactivity is equal to a constant.

Before the change in reactivity, the neutron density is expressed as

$$n_o = A_{o_1} e^{\omega_{o_1} t} + A_{o_2} e^{\omega_{o_2} t} + \cdots + A_{o_7} e^{\omega_{o_7} t} \qquad (64)$$

When the reactivity is zero, one of the roots, ω_{o_1}, is zero. The fact that one of the roots is exactly equal to zero may be seen by writing the characteristic equation for zero reactivity.

$$\rho = 0 = \frac{\ell\omega}{\ell\omega+1} + \frac{1}{\ell\omega+1}\sum_{i=1}^{6}\frac{\beta_{eff_i}\,\omega}{\omega+\lambda_i} \qquad (65)$$

The term, $\ell\omega+1$, may be cancelled from Eq. (65) and the resulting equation is of sixth degree in ω. The remaining roots, all negative, have been called the "natural vibration rates" of the pile by Zoodak (18). Since the reactor is assumed to be critical in the delayed sense, the terms of Eq. (64), $A_{o_2}e^{\omega_{o_2}t}, A_{o_3}e^{\omega_{o_3}t}, \cdots, A_{o_7}e^{\omega_{o_7}t}$, truly vanish leaving only the first term, A_{o_1}. The neutron density is then a single constant at time, $t'-\xi$.

$$n(t'-\xi) = A_{o_1} = n_o \qquad (66)$$

After the change in reactivity, the neutron density is expressed as

$$n(t'+\xi) = \sum_{\gamma=1}^{7} A_\gamma e^{\omega_\gamma (t'+\xi)} \qquad (67)$$

The precursor density before the change in reactivity is evaluated from Eq. (62) and Eq. (63) to be

$$H_i(t'-\xi) = \frac{\beta_i A_{o_1}\epsilon_i'}{\lambda_i} = \frac{\beta_i n(t'-\xi)\epsilon_i'}{\lambda_i} = \frac{\beta_i \epsilon_i' n_o}{\lambda_i} \qquad (68)$$

Since $n(t)$ must be continuous at t', the limit as $\xi \longrightarrow 0$ of Eq. (67) is

$$n(t') = \sum_{\gamma=1}^{7} A_\gamma e^{\omega_\gamma t'} = n_o \qquad (69)$$

$$H_i(t') = \frac{\beta_i \epsilon_i'}{(1-\rho)} \sum_{\gamma=1}^{7} \frac{A_\gamma e^{\omega_\gamma t'}}{\omega_\gamma + \lambda_i} = \frac{\beta_i n_o \epsilon_i'}{\lambda} \quad . \tag{70}$$

Since t' is an arbitrary time, a more convenient time was chosen for the step change in reactivity to occur, i.e., $t' = 0$, then Eq. (70) may be written as

$$n(0) = \sum_{\gamma=1}^{7} A_\gamma = n_o \quad . \tag{71}$$

The equilibrium precursor density is then

$$H_i(0) = \frac{\beta_i \epsilon_i}{\lambda_i} \quad n(0) \quad . \tag{72}$$

The Laplace transform of the spatially independent kinetic equation, Eq. (40), is

$$s\bar{n}(s) - n(0+) = \frac{1}{\ell} \left(\frac{\rho - \beta}{1-\rho} \right) \bar{n}(s) + \sum_{i=1}^{6} \epsilon_i \lambda_i \bar{H}(s) \quad . \tag{73}$$

Also, the Laplace transform of the precursor equation is

$$s\bar{H}_i(s) - H_i(0+) = -\lambda_i \bar{H}_i(s) + \frac{\beta_i \epsilon_i' \bar{n}(s)}{\ell (1-\rho)} \quad . \tag{74}$$

Eq. (74) is rewritten as

$$\bar{H}_i(s)(s+\lambda_i) = H_i(0+) + \frac{\beta_i \bar{n}(s) \epsilon_i'}{\ell (1-\beta)} \quad . \tag{75}$$

$\bar{H}_i(s)$ from Eq. (75) is substituted into Eq. (67) and the result is

$$s\bar{n}(s) - n(0+) = \frac{1}{\ell}\left(\frac{\ell-\beta}{1-\ell}\right)\bar{n}(s) + \sum_{i=1}^{6} \epsilon_i \lambda_i \left(\frac{H_i(0+) + \frac{\beta_i \bar{n}(s)\epsilon_i}{\ell(1-\ell)}}{s+\lambda_i} \right) \quad (76)$$

Since $\beta = \sum_{i=1}^{6} \beta_i \quad ; \quad H_i(0+) = \frac{\beta_i n(0) \epsilon_i'}{\lambda_i}$

and $n(0+) = n_0$

then Eq. (76) may be written as

$$\bar{n}(s)\left[s + \frac{1}{\ell}\left(\frac{\ell-\beta}{1-\ell}\right) - \sum_{i=1}^{6} \frac{\epsilon_i \epsilon_i' \lambda_i \beta_i n(0)}{(s+\lambda_i)} \right] =$$

$$n(0) + \sum_{i=1}^{6} \frac{\epsilon_i \epsilon_i' \beta_i n(0)}{(s+\lambda_i)} \quad (77)$$

Eq. (77) may be solved for $\bar{n}(s)$,

$$\bar{n}(s) = \frac{n(0) + n(0) \sum_{i=1}^{6} \frac{\epsilon_i \epsilon_i' \beta_i}{s+\lambda_i}}{s - \frac{1}{\ell}\left(\frac{\ell-\beta}{1-\ell}\right) - \sum_{i=1}^{6} \frac{\epsilon_i \epsilon_i' \lambda_i \beta_i}{\ell(1-\ell)(s+\lambda_i)}} \quad (78)$$

The characteristic equation may be rewritten as

$$\omega - \frac{\ell-\beta}{\ell(1-\ell)} - \sum_{i=1}^{6} \frac{\epsilon_i \epsilon_i' \beta_i \lambda_i}{\ell(1-\ell)(\omega+\lambda_i)} = 0 \quad (79)$$

The denominator of Eq. (78) may be recognized as exactly the same form as Eq. (79) and since the roots, ω, satisfy the characteristic equation the denominator of Eq. (78) is zero. Therefore, an inverse transform of Eq. (78) is of the form

$$\mathcal{L}^{-1}\left[\bar{n}(s)\right] = n(t) = \sum_{j=1}^{7} A_j(s_j)\, e^{s_j t} = \sum_{j=1}^{7} A_j(\omega_j)\, e^{\omega_j t} \tag{80}$$

A method, as given by Wylie (33), predicts the coefficients, $A_j(\omega_j)$'s, by using the following form of the inverse transform

$$\mathcal{L}^{-1}\left[\bar{n}(s)\right] = \sum_{j=1}^{7} \frac{p(\omega_j)}{q'(\omega_j)}\, e^{\omega_j t} \tag{81}$$

where $q'(s)$ is given by the equation

$$q'(s)\Big|_{s=\omega_j} = \frac{d}{ds}\left\{ s - \frac{1}{\ell}\left(\frac{\ell-\beta}{1-\ell}\right) - \sum_{i=1}^{6} \frac{\epsilon_i \epsilon_i' \lambda_i \beta_i}{\ell(1-\ell)(s+\lambda_i)} \right\}\Bigg|_{s=\omega_j}$$

$$= 1 + \frac{1}{\ell(1-\ell)} \sum_{i=1}^{6} \frac{\epsilon_i \epsilon_i' \lambda_i \beta_i}{(\omega_j+\lambda_i)^2} \tag{82}$$

Using Eq. (82), the coefficients are determined as

$$A_j(\omega_j) = n(0)(1-\varrho)\left[\frac{\ell + \sum_{i=1}^{6} \frac{\beta_{eff\,i}}{(\omega_j + \lambda_i)}}{\ell(1-\varrho) + \sum_{i=1}^{6} \frac{\beta_{eff_i}\lambda_i}{(\omega_j + \lambda_i)^2}}\right] \quad (83)$$

Therefore, the neutron density equation is

$$\frac{n(t)}{n(0)} = \frac{\phi(t)}{\phi(0)} = \sum_{j=1}^{7} (1-\varrho)\left[\frac{\ell + \sum_{i=1}^{6} \frac{\beta_{eff\,i}}{(\omega_j + \lambda_i)}}{\ell(1-\varrho) + \sum_{i=1}^{6} \frac{\beta_{eff_i}\lambda_i}{(\omega_j + \lambda_i)^2}}\right] e^{\omega_j t} \quad (84)$$

which is called the neutron density ratio or flux ratio.

Part - B

Reactivity as a Function of Time

When the coefficient, k_{eff}, in Eq.'s (40) and (43), remains a constant or is a step function of time, the kinetic differential equations are easily solved. When k_{eff} has a time dependence, the kinetic differential equations become difficult to solve exactly (1). The degree of difficulty depends upon the time function which describes $k_{eff}(t)$ and on the number of delayed neutron groups used in the model.

Glasstone (7) presents an approximate solution of the spatially independent kinetic equations where k_{eff} is a linear function of time. One group of delayed neutrons was assumed in

Glasstone's solution.

A solution to the spatially independent kinetic equations is given in this paper in which $k_{eff}(t)$ is given by Eq. (14). Eq. (14) is an approximation to the reactivity insertion rate for a rod drop in the TRIGA reactor. For simplicity of solution, it is assumed that there is only one group of delayed neutrons. The usual assumption connected with the simplified kinetics model are made (see Appendix - A). Also, in accordance with an assumption made by McPhee (15), in a similar development, the prompt neutron lifetime was assumed to be approximately zero.

The spatially independent kinetic equations are the same as Eq. (40) and Eq. (43), except that here there is assumed to be only one group of delayed neutrons. Thus Eq.'s (40) and (43) become

$$\left[(1-\beta)\, k_{eff}(t) - 1\right] \frac{n(t)}{\ell} + \epsilon \lambda H(t) = \frac{dn(t)}{dt} \qquad (85)$$

and

$$\frac{dH(t)}{dt} = -\lambda H(t) + \frac{k_{eff}(t)\, n(t)\, \beta \epsilon'}{\ell} \,. \qquad (86)$$

$\lambda H(t)$ is substituted from Eq. (86) into Eq. (85) to obtain

$$\left[(1-\beta)\, k_{eff}(t) - 1\right] \frac{n(t)}{\ell} + \epsilon \left[\frac{k_{eff}(t)\, n(t)\, \beta \epsilon'}{\ell} - \frac{dH(t)}{dt}\right] = \frac{dn(t)}{dt} \,. \qquad (87)$$

It was also assumed that the effective fraction of delayed neutron constant, $\epsilon\epsilon'$, is equal to unity. Thus, the two terms cancel in Eq. (87).

$$(k_{eff}(t) - 1)\frac{n(t)}{\ell} - \epsilon\frac{dH(t)}{dt} = \frac{dn(t)}{dt} \qquad (88)$$

Eq. (86) is then differentiated with respect to time to give

$$\frac{d^2n(t)}{dt^2} = (k_{eff}(t) - 1)\frac{1}{\ell}\cdot\frac{dn(t)}{dt} + \frac{n(t)}{\ell}\frac{dk_{eff}(t)}{dt} - \epsilon\frac{d^2H(t)}{dt^2} \qquad (89)$$

Differentiating Eq. (86) gives

$$\frac{d^2H(t)}{dt^2} = -\lambda\frac{dH(t)}{dt} + \frac{k_{eff}(t)\beta\epsilon'}{\ell}\cdot\frac{dn(t)}{dt} \qquad (90)$$

Eq. (88) is then solved for $\frac{dH(t)}{dt}$.

$$\frac{dH(t)}{dt} = \left[k_{eff}(t) - 1\right]\frac{n(t)}{\ell\epsilon} - \frac{dn(t)}{dt}\cdot\frac{1}{\epsilon} \qquad (91)$$

$\frac{dH(t)}{dt}$ is substituted from Eq.(91) into Eq. (90) to give

$$\frac{d^2H(t)}{dt^2} = -\lambda\left[(k_{eff}(t) - 1\right]\frac{n(t)}{\ell\epsilon} - \frac{dn(t)}{dt}\cdot\frac{1}{\epsilon} \qquad (92)$$

Then, from Eq.'s (89) and (92), $\frac{d^2n}{dt^2}$ is found to be

$$\frac{d^2n(t)}{dt^2} = \left(k_{eff}(t)-1\right)\frac{1}{\ell}\cdot\frac{dn(t)}{dt} + \frac{n(t)}{\ell}\frac{dk_{eff}(t)}{dt} + \lambda\left(\frac{k_{eff}(t)-1}{\ell}\right)n(t) - \lambda\frac{dn(t)}{dt} \quad . \tag{93}$$

Eq. (93) is simplified into

$$\frac{d^2n(t)}{dt^2} = \frac{dn(t)}{dt}\cdot\frac{1}{\ell}\left[k_{eff}(t)-1-\lambda\ell-k_{eff}(t)\beta\right] + \frac{n(t)}{\ell}\left[\frac{dk_{eff}(t)}{dt} + \lambda\left(k_{eff}(t)-1\right) - \frac{\beta\, dk_{eff}(t)}{dt}\right]. \tag{94}$$

Following McPhee, it is assumed that the products, $\frac{d^2n(t)}{dt^2}\ell$ and $\lambda\ell$ are approximately zero, therefore Eq. (94) becomes

$$\left[(1-\beta)k_{eff}(t)-1\right]\frac{dn(t)}{dt} \doteq -n(t)\left[\lambda\left(k_{eff}(t)-1\right)+(1-\beta)\frac{dk_{eff}(t)}{dt}\right] \tag{95}$$

Since $$k_{eff}(t) \doteq 1 - A\left(1-e^{-bt^4}\right), \tag{96}$$

therefore $$\frac{dk_{eff}(t)}{dt} = -4bAt^3e^{-bt^4}. \tag{97}$$

Eq. (95) becomes

$$\left[(1-\beta)\left(1-A\left\{1-e^{-bt^4}\right\}\right)-1\right]\frac{dn(t)}{dt} = -n(t)\left[\lambda\left(1-A\left\{1-e^{-bt^4}\right\}\right)-\lambda+(1-\beta)\left(-4bAt^3e^{-bt^4}\right)\right]. \tag{98}$$

Terms in n(t) are collected to give

$$\frac{1}{n(t)} \cdot \frac{dn(t)}{dt} = \frac{\left[-(1-e^{-bt^4})\lambda + (1-\beta)(-4bt^3 e^{-bt^4})\right]}{1-\beta+\frac{\beta}{A}-(1-\beta)e^{-bt^4}} \quad (99)$$

Eq. (99) is integrated from zero time to time t.

$$\int_{n(0)}^{n(t)} \frac{1}{n(t)}\, dn(t) = -\int_0^t \frac{\lambda dt'}{a_2+(\beta-1)e^{-bt'^4}} - \int_0^t \frac{(\beta-1)(-4bt'^3)e^{-bt'^4}dt'}{a_2+(\beta-1)e^{-bt'^4}}$$

$$+ \int_0^t \frac{\lambda dt'}{a_2 e^{+bt'^4}+(\beta-1)} \quad (100)$$

Where a_2 is

$$a_2 = 1-\beta+\frac{\beta}{A} \quad (101)$$

The integration is performed directly with one of the terms of Eq. (100) to give

$$\int_0^t \frac{(\beta-1)(-4bt'^3)e^{-bt'^4}dt'}{a_2+(\beta-1)e^{-bt'^4}} = \ln\left[\frac{a_2+(\beta-1)e^{-bt^4}}{a_2+(\beta-1)}\right]$$

The last two terms of Eq. (100) are integrated numerically with the IBM-650 using Simpson's Rule. The neutron flux is then represented by

$$n(t) = n(0)\left[\frac{a_2 + (\beta-1)}{a_2 + (\beta-1)e^{-bt}}\right] e^{-\int_0^t \frac{\lambda dt'}{a_2 + (\beta-1)e^{-bt'}} + \int_0^t \frac{\lambda dt'}{a_2 e^{+bt'} + (\beta-1)}} \qquad (102)$$

Part - C

Computer Program for the Kinetic Equations With Six Groups of Delayed Neutrons

General. This program solves the reactor kinetic equations for a bare homogeneous thermal reactor assuming that there are six delayed neutron groups (Appendix, Part - A). The code was written in SOAP-II for the IBM-650 computer. The output data from the code produces the following quantities:

1) Coefficients of the characteristic polynomial.
2) Roots of the characteristic equation.
3) Coefficients of the neutron flux ratio equation.
4) The ratio, $\phi(t)/\phi(0)$, for various times after a step insertion of reactivity.

The Coefficients of the Characteristic Equations. The expansion of the characteristic equation may be written as

$$\left[\rho(\ell\omega+1) - \ell\omega\right] = \frac{\beta_1\omega}{\omega+\lambda_1} + \frac{\beta_2\omega}{\omega+\lambda_2} + \cdots + \frac{\beta_6\omega}{\omega+\lambda_6} \qquad (103)$$

By multiplying both sides of Eq. (97) by $\prod_i (\omega+\lambda_i)$ and collecting terms in powers of ω, the following type equation results

$$\sum_{j=0}^{7} H_j \omega^j = 0 \qquad (104)$$

Roots of the Characteristic Equation. Once the reactor parameters, ℓ, ρ, $\lambda_i' s$, and $\beta_i' s$ are known, the numerical coefficients of the characteristic equation may be calculated and thus the particular characteristic equation is determined. Roots of such an equation may be determined in many different fashions. Since all of the roots of the equation are real and the function, $f(\omega)$, is differentiable, the Newton-Raphson method may be used. The Newton-Raphson method is an iterative procedure and convergence to a root of the polynomial equation is fast if the equation is well behaved.

Educated guesses for the roots are aided by the fact that the characteristic equation has six poles, located at the negative value of each of the decay constants of the precursor groups. Test for proper convergence is simple since the roots must lie in well defined positions. One root must be algebraically larger than the negative value of the smallest decay constant and one root must be algebraically smaller than the negative value of the largest decay constant. Each of the other roots lie between respective poles.

For the roots to lie between two poles, one of the poles is tried for a guess of the root. If proper convergence is not obtained, the other pole is tried as a guess for the root. If proper convergence is still not obtained, the midpoint between the two poles is tried as a guess. From experience with the program, the three guesses are sufficient to extract the desired

root when the root lies between two poles. Since the largest negative root is strongly dependent upon the magnitude of the step change in reactivity, a series of guesses are offered. First, the largest negative pole is tried as a guess and if convergence is not obtained to the proper value, a multiple of the pole magnitude is tried. The largest algebraic root is always algebraically larger than the negative value of the smallest decay constant. If the reactivity insertion is negative, this root will be negative and if the reactivity insertion is positive, this root will be positive.

If the reactivity insertion is negative several guesses are tried, i.e., zero, the largest pole, and the midpoint between zero and the largest pole. If the reactivity is positive an approximation, as given by Glasstone (7), is used as a guess for the root.

$$\omega \doteq \frac{\lambda_{av}\,\rho}{\beta - \rho} \tag{105}$$

Where λ_{av} is the average decay constant and is defined as the inverse of average mean life of the delayed neutrons, i.e.,

$$\lambda_{av} \equiv \frac{\beta}{\sum_{i=1}^{6} \beta_i \frac{1}{\lambda_i}} \tag{106}$$

If convergence to the desired root is not obtained, a multiple of the approximation is used.

Coefficients of the Flux Equations. The coefficients, A_j, of the flux equation

$$\frac{\phi(t)}{\phi(0)} = \sum_{j=1}^{7} A_j e^{\omega_j t} \qquad (107)$$

as defined by Eq. (94) are computed in a straight forward manner. Since this is very simple, a discussion of the procedure is not given.

Flux Ratios. Since all of the ω's and A_j's have been computed, it is a simple manner to calculate the flux ratios.

Program Operation. If one wishes to compute continuously a set of varying parameters, such as reactivity, the following two cards are added to the program just before the parameter data.

Symbolic				Instruction	Op	Data	Address
	NZC	CON65	CON80	1683	48	1633	1534
CON80	NOP	0000	8000	1543	00	0000	8000

The position, 1534, will then be removed from the table of availability. The various parameter changes may be added to the end of the data and a transfer card positioned between each data card.

The computer will tend to overflow when larger positive reactivities are used, $\rho > \$1.00$. The overflow is due to the unruly nature of the characteristic polynomial at larger reactivities and no special programming precaution has been used to prevent the overflow in this case. In order to prevent large exponential values from causing the computer to overflow, all values "e" to the power $\pm$ 135.0 and larger are omitted. At larger reactivities, $\rho > \$0.60$, the computer will give erroneous results for large times after the step change in reactivity.

This is no real handicap since $\frac{\phi(t)}{\phi(0)} \geq 10^{27}$ has no practical value for the usual reactor operation.

Input Data. The nuclear parameters are placed in the code in the following positions.

Position	Symbol	Parameter	
1384	6	λ_6	(smallest decay constant)
1385	5	λ_5	
1386	4	λ_4	
1387	3	λ_3	
1388	2	λ_2	
1389	1	λ_1	
1374	BETA6	β_6	(the fraction associated with λ_6)
1375	BETA5	β_5	
1376	BETA4	β_4	
1377	BETA3	β_3	
1378	BETA2	β_2	
1379	BETA1	β_1	
0148	BETAT	β	
0239	LAMAV	λ_{av}	
0191	RHO	ρ	
0078	L	Prompt Neutron lifetime, ℓ	

Output DATA. The output data, ω'_Δ, H's, A_j's, and $\frac{\phi(t)}{\phi(0)}$'s are given in the following order:

First line of data - all of the decay constants and the average decay constant.

Second line - All of the delayed neutron fractions and the total fraction.

Third line - Reactivity and the prompt neutron lifetime.

Fourth line - The coefficients of the characteristic polynomial.

Fifth line - Roots of the characteristic equation.

Sixth line - Coefficients of the flux equation.

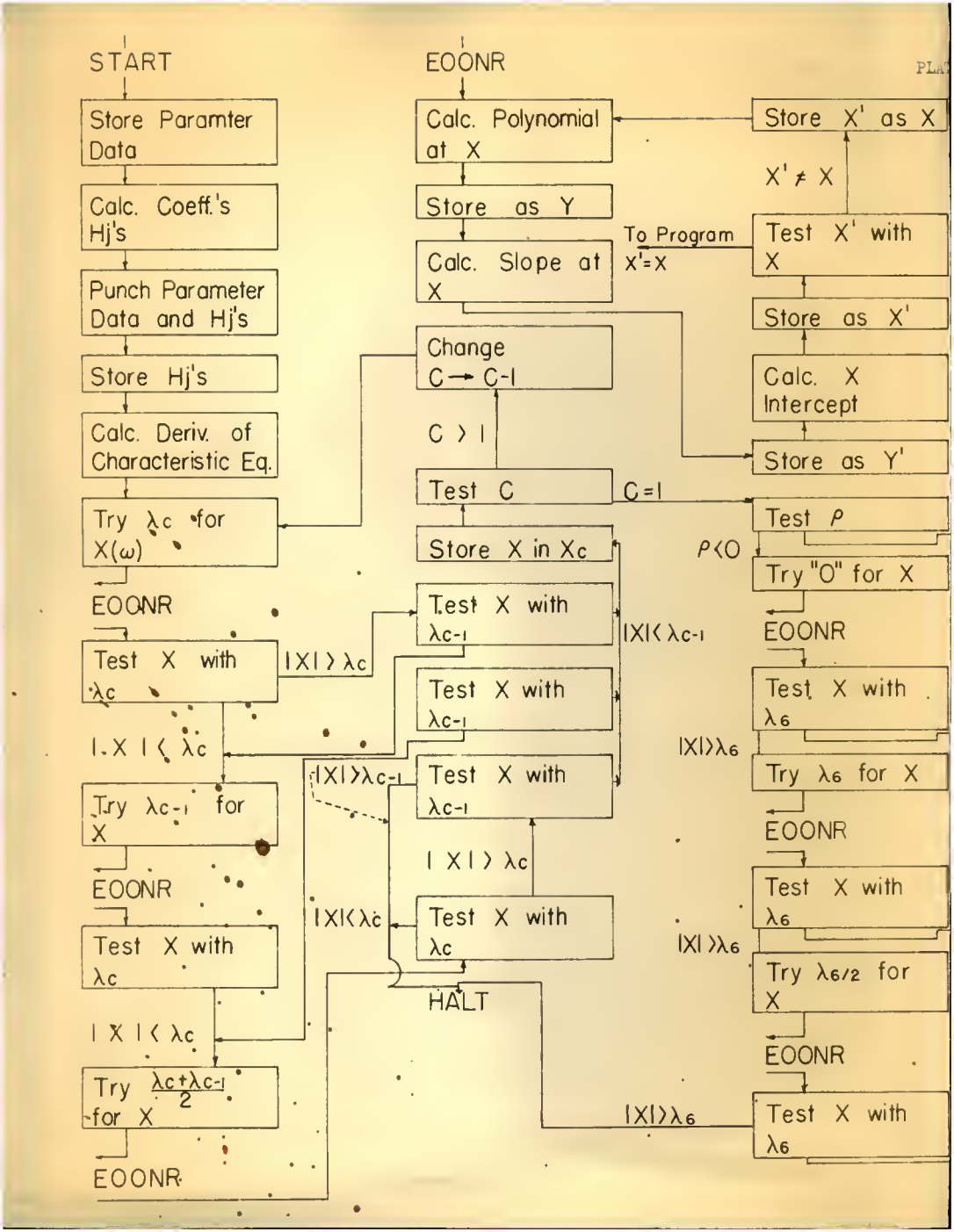
Lines seven through 22 - Flux ratios at the various times.

After the data cards have been punched and positioned in the program deck, the following console settings are used.

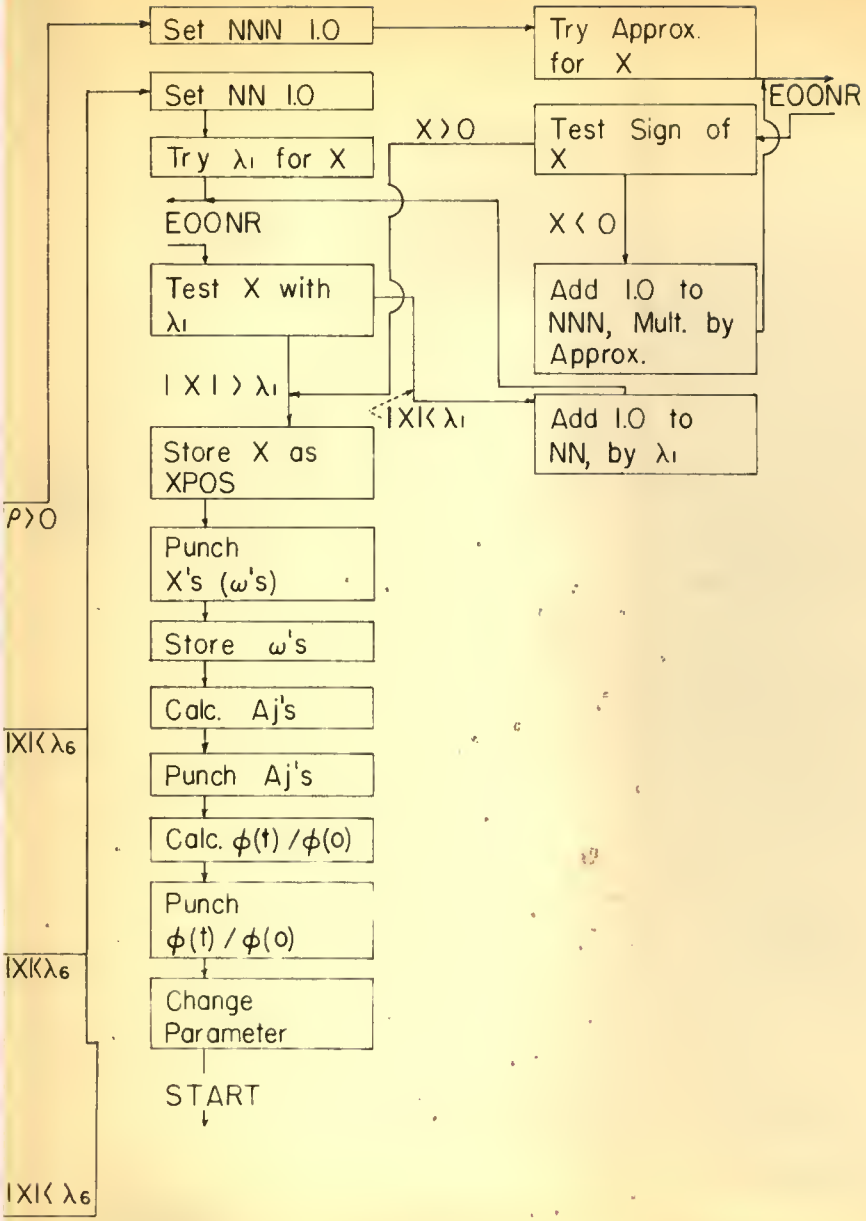
Storage Entry Switches	70 1951 1999 plus .
Programmed Stop	STOP
Half Cycle	RUN
Control	RUN
Address Selection	Anything
Display	Anything
Overflow	SENSE
ERROR	STOP

EXPLANATION OF PLATE XLIV

Flow diagram of the computer program for the solution of the reactor kinetic equations using six groups of delayed neutrons.


START
Store Paramter Data
Calc. Coeff.'s Hj's
Punch Parameter Data and Hj's
Store Hj's
Calc. Deriv. of Characteristic Eq.
Try λc for X(ω)
EOONR
Test X with λc
|X| > λc
|X| < λc
Try λc-1 for X
EOONR
Test X with λc
|X| < λc
Try (λc+λc-1)/2 for X
EOONR
EOONR
Calc. Polynomial at X
Store as Y
Calc. Slope at X
Change C → C-1
C > 1
Test C
C=1
Store X in Xc
Test X with λc-1
Test X with λc-1
Test X with λc-1
|X| < λc-1
|X| > λc-1
|X| > λc
Test X with λc
HALT
To Program X'=X
Store X' as X
X' ≠ X
Test X' with X
Store as X'
Calc. X Intercept
Store as Y'
Test ρ
ρ<0
Try "0" for X
EOONR
Test X with λ6
|X| > λ6
Try λ6 for X
EOONR
Test X with λ6
|X| > λ6
Try λ6/2 for X
EOONR
Test X with λ6
|X| > λ6

E XLIV


Set NNN I.O
Try Approx. for X
Set NN I.O
EOONR
X > O
Test Sign of X
Try λ1 for X
EOONR
X < O
Test X with λ1
Add I.O to NNN, Mult. by Approx.
I X I > λ1
I X I < λ1
Add I.O to NN, by λ1
Store X as XPOS
P > O
Punch X's (ω's)
Store ω's
Calc. Aj's
I X I < λ6
Punch Aj's
Calc. φ(t) / φ(0)
Punch φ(t) / φ(0)
I X I < λ6
Change Parameter
START
I X I < λ6

SIX
DELAYED
NEUTRON
GROUP
REACTOR
KINETIC
EQUATIONS

```
                BLR   0440          0499
                BLR   0500          0999
                BLR   0800          0899
                BLR   1300          1400
                BLR   1951          1960
                BLR   1977          1986
                SYN   A             0500
                SYN   AP            0490
                SYN   X             1000
                SYN   T             1350
                SYN   BPJ           1360
                SYN   BJ            1370
                SYN   BETA          1380
                SYN   LAMDA         1390
                SYN   OMEGA         1400
                SYN   START         1999
                SYN   HLT           1111
EQOCL           STO   ZZZ11         CONT1
CONT1           LDO   ZZZ10
                STO   1977
                STO   1978
                STO   1979
                STO   1980
                STD   1981
                STO   1982
                STO   1983
                STO   1984          ZZZ1
EQOHR           STO   NRM           CONT2
CONT2           RSA   0007
                RAU   A             A CONT3
CONT3           FMP   X
                AXA   0001
                NZA   LOOP2         CONT4
```

CORRECT
TRACE

READ
PRINT

READ
OUT
DATA
ZERO
SUBROUTINE

1	0000	00	0000	0000
2	0000	00	0000	0000
3	0000	00	0000	0000
4	0000	00	0000	0000
5	0000	00	0000	0000
6	0000	00	0000	0000
7	0000	00	0000	0000
8	0000	00	0000	0000
9	0000	00	0000	0000
10	0000			
11	0000			
12	0000	00	0000	0000
13	0000	00	0000	0000
14	0000	00	0000	0000
15	0000	00	0000	0000
16	0000	00	0000	0000
17	0000	00	0000	0000
18	0000	24	0003	0006
19	0006	69	0009	0012
20	0012	24	1977	0030
21	0030	24	1978	0031
22	0031	24	1979	0032
23	0032	24	1980	0033
24	0033	24	1981	0034
25	0034	24	1982	0035
26	0035	24	1983	0036
27	0036	24	1984	0003
28	0050	24	0053	0056
29	0056	81	0007	0062
30	0062	60	2500	0005
31	0005	39	1000	0100
32	0100	50	0001	0106
33	0106	40	0059	0010

```
LOOP2   FAD  A          A  CONT3
CONT4   FAD  A          A
        STU  Y
        RBR   0006
        RAU  AP         B  CONT6
CONT6   FMP  X
        AXR   0001
        NZB  LOOP3         CONT5
LOOP3   FAD  AP         B  CONT6
CONT5   FAD  AP
        STU  YP
        RSU  Y
        FOV  YP
        FAD  X
        STU  XP
        RAU  X
        FOV  PREC
        STU  OIFF
        RAU  X
        FOV  PREC
        STU  OIFF
        RAU  X
        FSB  XP
        RAM   8003
        RAU   8002
        FBM  OIFF
        BMI  FINIS      ITER
ITER    LOD  XP
        STO  X             CONT2
FINIS   RAL  XP            NRM
EODEA   STO  AAA1                  E TO THE X
        STL  AAA2                  SUBROUTINE
        RAM  AAA2
        STL  AAA3
        RAU  AAA3
        FMP  AAA16
        FAD  AAA15
        FMP  AAA3
        FAD  AAA14
        FMP  AAA3
        FAD  AAA13
        FMP  AAA3
        FAD  AAA12
        FMP  AAA3
        FAD  AAA11
        FMP  AAA3
        FAD  AAA10
        STU  AAA4
        FMP  AAA4
        STU  AAA4
        FMP  AAA4
        STU  AAA4
        RAU  AAA2
        BMI  AAA5          AAA6
AAA5    RAU  AAA10
        FOV  AAA4
        STU  AAA4          AAA6
AAA6    RAL  AAA4          AAA1
START   LOD   1384                 LAMBDA 6
        STO  6
        LOD   1385                 LAMBDA 5
        STO  5
        LOD   1386                 LAMBDA 4
        STO  4
        LOD   1387                 LAMBDA 3
        STO  3
        LOD   1388                 LAMBDA 2
        STO  2
        LOD   1389                 LAMBDA 1
        STO  1
        LOD   1374
        STO  BETA6
        LOD   1375
        STO  BETA5
        LOD   1376
        STO  BETA4
        LOD   1377
        STO  BETA3
        LOD   1378
        STO  BETA2
        LOD   1379
        STO  BETA1
        RAU  RHO
        FSB  ONE
        FMP  L
        STU  H7                    H 7
        RBR   0006
        RAU  LAMDA A  CON13
CON13   AXR   0001
        NZA  CON11    CON12
CON11   FAD  LAMDA A  CON13
CON12   STU  AO
        RAU  RHO
        FSB  BETAT
        STU  ROMI8
        RAU  H7
        FMP  AO
        FAO  ROMI8
        STU  H6                    H 6
        RAU  ZERO
        STU  SML81
        RBR   0006         CON14
CON14   RAU  LAMDA A
        FMP  BETA  A
        FAO  SML81
        STU  SML81
        AXR   0001
        NZA  CON14         CON15
CON15   RAU  ZERO
        STU  SUML2
        RBR   0006         CON16
CON16   RAU  LAMDA A
        FMP  LAMDA A
        FAO  SUML2
        STU  SUML2
        AXR   0001
        NZA  CON16         CON17
CON17   RAU  AO
        FMP  AO
        FSB  SUML2
        FOV  TWO
        STU  AO2
        RAU  H7
        FMP  AO2
        STU  H5
        RAU  AO
        FMP  ROMI8
        FAO  SML81
        FAO  H5
        STU  H5
        RAU  ZERO                  H 5
        STU  SUML3
```

```
34  0059  32 2500  0005
35  0010  32 2500  0027
36  0027  21 0082  0085
37  0085  83 0006  0041
38  0041  60 4490  0045
39  0045  39 1000  0150
40  0150  52 0001  0156
41  0156  42 0109  0060
42  0109  32 4490  0045
43  0060  32 0490  0017
44  0017  21 0022  0025
45  0025  61 0082  0037
46  0037  34 0022  0072
47  0072  32 1000  0077
48  0077  21 0132  0135
49  0135  60 1000  0055
50  0055  34 0008  0058
51  0058  21 0112  0015
52  0015  60 1000  0105
53  0105  34 0008  0108
54  0108  21 0112  0065
55  0065  60 1000  0155
56  0155  33 0132  0159
57  0159  67 8003  0067
58  0067  60 8002  0075
59  0075  38 0112  0039
60  0039  46 0042  0043
61  0043  69 0132  0185
62  0185  84 1000  0056
63  0042  65 0132  0053
64  0200  24 0103  0206
65  0206  30 0011  0014
66  0014  67 0011  0115
67  0115  30 0019  0122
68  0122  60 0019  0023
69  0023  39 0026  0076
70  0076  32 0029  0205
71  0205  39 0019  0069
72  0069  32 0172  0049
73  0049  39 0019  0119
74  0119  32 0222  0099
75  0099  39 0019  0169
76  0169  32 0272  0149
77  0149  39 0019  0219
78  0219  32 0322  0199
79  0199  39 0019  0269
80  0269  32 0372  0249
81  0249  21 0004  0007
82  0007  39 0004  0054
83  0054  21 0004  0057
84  0057  39 0004  0104
85  0104  21 0004  0107
86  0107  60 0011  0165
87  0165  46 0018  0319
88  0018  60 0372  0127
89  0127  34 0004  0154
90  0154  31 0004  0319
91  0319  65 0004  0103
92  1999  69 1384  0087
93  0087  24 0040  0093
94  0093  69 1385  0038
95  0038  24 0091  0044
96  0044  69 1386  0089
97  0089  24 0092  0095
98  0095  69 1387  0090
99  0090  24 0143  0146
100 0046  69 1388  0141
101 0141  24 0094  0047
102 0047  69 1389  0092
103 0142  24 0145  0048
104 0048  69 1374  0177
105 0177  24 0080  0083
106 0023  69 1375  0088
107 0088  24 0081  0084
108 0084  69 1376  0079
109 0079  24 0182  0235
110 0235  69 1377  0130
111 0130  24 0133  0086
112 0086  69 1378  0131
113 0131  24 0134  0137
114 0137  69 1379  0232
115 0232  24 0285  0088
116 0088  60 0191  0195
117 0195  33 0098  0185
118 0125  39 0078  0185
119 0128  21 0288  0335
120 0335  81 0006  0241
121 0241  60 3390  0245
122 0245  50 0001  0001
123 0001  40 0304  0255
124 0204  32 3390  0245
125 0255  21 0110  0013
126 0013  60 0191  0295
127 0295  33 0148  0175
128 0175  31 0180  0183
129 0183  60 0288  0187
130 0187  39 0110  0160
131 0160  32 0180  0157
132 0157  21 0163  0215
133 0215  60 0048  0073
134 0073  21 0178  0181
135 0181  81 0006  0237
136 0237  60 3390  0345
137 0345  39 3380  0230
138 0230  32 0178  0305
139 0305  21 0178  0231
140 0231  50 0001  0287
141 0287  40 0237  0291
142 0291  60 0068  0123
143 0123  21 0228  0281
144 0281  81 0006  0337
145 0337  60 3390  0395
146 0395  39 3390  0140
147 0140  32 0228  0355
148 0355  21 0228  0331
149 0331  50 0001  0387
150 0387  40 0337  0341
151 0341  60 0110  0265
152 0265  39 0110  0210
153 0210  33 0228  0405
154 0405  34 0158  0208
155 0208  21 0212  0315
156 0315  60 0282  0437
157 0437  39 0312  0262
158 0262  21 0016  0369
159 0369  60 0110  0365
160 0365  39 0180  0280
161 0280  32 0178  0505
162 0505  32 0016  0193
163 0193  21 0016  0419
164 0419  60 0064  0173
165 0173  21 0278  0381
```

```
                RSA    0006      CO418
CON18           RAU    LAMOA   A
                FMP    LAMOA   A
                FMP    LAMOA   A
                FAD    SUML3
                STU    SUML3
                AXA    0001
                NZA    CON18     CON19
CON19           RAU    ZERO
                BTU    SMLB2
                RSA    0006      CON20
CON20           RAU    LAMOA   A
                FMP    LAMOA   A
                FMP    BETA    A
                FAO    SMLB2
                BTU    SMLB2
                AXA    0001
                NZA    CON20     CON21
CON21           RAU    AO2
                FBB    BUML2
                FMP    AO
                FAO    SUML3
                FOV    THREE
                BTU    AO3
                RAU    H7
                FMP    AO3
                STU    H4
                RAU    AO2
                FMP    ROMI8
                FAO    H4
                BTU    H4
                RAU    AO
                FMP    BMLB1
                FAD    H4
                FBB    SMLB2
                STU    H4                          H 4
                RAU    ZERO
                BTU    SUML4
                RSA    0006      CON22
CON22           RAU    LAMOA   A
                FMP    LAMOA   A
                FMP    LAMOA   A
                FMP    LAMOA   A
                FAO    SUML4
                STU    SUML4
                AXA    0001
                NZA    CON22     CON23
CON23           RAU    ZERO
                BTU    SMLB3
                RSA    0006      CON24
CON24           RAU    LAMOA   A
                FMP    LAMOA   A
                FMP    LAMOA   A
                FMP    BETA    A
                FAO    BMLB3
                BTU    BMLB3
                AXA    0001
                NZA    CON24     CON25
CON25           RAU    AO3
                FAO    BUML3
                FMP    AO
                FBB    BUML4
                STU    AO4
                RBU    AO2
                FMP    BUML2
                FAO    AO4
                FDV    FOUR
                STU    AO4
                RAU    H7
                FMP    AO4
                FAO    BMLB3
                BTU    H3
                RAU    AO2
                FMP    ROMI8
                FAD    H3
                STU    H3
                RBU    AO
                FMP    BMLB2
                FAO    H3
                STU    H3
                RAU    AO2
                FMP    BMLB1
                FAD    H3
                STU    H3                          H 3
                RAU    ZERO
                STU    SUML5
                RSA    0006      CON26
CON26           RAU    LAMOA   A
                FMP    LAMOA   A
                FMP    LAMOA   A
                FMP    LAMOA   A
                FMP    LAMOA   A
                FAO    SUML5
                BTU    SUML5
                AXA    0001
                NZA    CON26     CON27
CON27           RAU    ZERO
                STU    SMLB4
                RSA    0006      CON28
CON28           RAU    LAMOA   A
                FMP    LAMOA   A
                FMP    LAMOA   A
                FMP    LAMOA   A
                FMP    BETA    A
                FAD    SMLB4
                BTU    SMLB4
                AXA    0001
                NZA    CON28     CON29
CON29           RAU    AO4
                FBB    SUML4
                FMP    AO
                FAO    SUML5
                STU    EO
                RBU    AO3
                FMP    BUML2
                FAO    EO
                BTU    EO
                RAU    AO2
                FMP    BUML3
                FAO    EO
                FOV    FIVE
                STU    EO
                RAU    H7
                FMP    EO
                FBB    BMLB4
                STU    H2
                RAU    AO4
                FMP    ROMI8
                FAO    H2
                STU    H2
                RAU    AO3
                FMP    SMLB1
```

```
166  0331  61  0006  0537
167  0537  60  3390  0545
168  0545  39  3390  0190
169  0190  39  3390  0240
170  0240  32  0278  0555
171  0555  21  0278  0431
172  0431  60  0001  0567
173  0367  40  0537  0391
174  0391  60  0068  0883
175  0883  21  0388  0531
176  0331  61  0006  0637
177  0637  60  3390  0595
178  0595  39  3390  0190
179  0890  39  3380  0330
180  0330  32  0328  0605
181  0605  21  0388  0581
182  0581  50  0001  0687
183  0687  40  0637  0541
184  0541  60  0212  0117
185  0117  33  0288  0655
186  0655  39  0110  0260
187  0260  32  0278  0705
188  0705  34  0258  0308
189  0308  21  0312  0415
190  0415  60  0282  0737
191  0737  39  0312  0362
192  0362  21  0066  0519
193  0519  60  0212  0167
194  0167  39  0180  0380
195  0380  32  0066  0243
196  0243  21  0066  0569
197  0569  60  0110  0515
198  0515  39  0178  0378
199  0378  32  0066  0893
200  0293  33  0328  0755
201  0755  21  0066  0619
202  0619  60  0068  0873
203  0873  21  0428  0631
204  0631  61  0006  0787
205  0787  60  3390  0645
206  0645  39  3390  0340
207  0340  39  3390  0390
208  0390  39  3390  0540
209  0540  32  0428  1005
210  1005  21  0428  0681
211  0681  50  0001  1037
212  1037  40  0787  0591
213  0591  60  0068  0323
214  0323  21  0528  0731
215  0731  61  0006  1087
216  1087  60  3390  0695
217  0695  39  3390  0590
218  0590  39  3390  0640
219  0640  39  3380  0430
220  0430  32  0528  1055
221  1055  21  0528  0781
222  0781  50  0001  1137
223  1137  40  1087  0641
224  0641  60  0312  0217
225  0217  32  0278  1105
226  1105  39  0110  0310
227  0310  33  0428  1155
228  1155  21  0360  0063
229  0063  61  0212  0267
230  0267  39  0228  0378
231  0578  32  0360  1187
232  1187  34  0690  0740
233  0740  21  0360  0113
234  0113  60  0288  1237
235  1237  39  0360  0410
236  0410  32  0528  1205
237  1205  21  0510  0163
238  0163  60  0312  0317
239  0317  39  0180  0530
240  0530  32  0510  1287
241  1287  21  0510  0213
242  0213  61  0110  0565
243  0565  39  0328  0628
244  0628  32  0510  1437
245  1437  21  0510  0263
246  0263  60  0212  0367
247  0367  39  0178  0678
248  0678  32  0510  1487
249  1487  21  0510  0313
250  0313  60  0066  0373
251  0373  21  0720  1031
252  1031  61  0006  1537
253  1537  60  3390  0745
254  0745  39  3390  0790
255  0790  39  3390  1040
256  1040  39  3390  1090
257  1090  39  3390  1240
258  1140  32  0728  1255
259  1255  21  0728  1083
260  1081  50  0001  1587
261  1587  40  1537  0691
262  0691  60  0068  0423
263  0423  21  0778  1131
264  1131  61  0006  1637
265  1637  60  3390  0795
266  0795  39  3390  1190
267  1190  39  3390  1240
268  1240  39  3390  1290
269  1290  39  3380  0580
270  0580  32  0778  1405
271  1405  21  0778  1181
272  1181  50  0001  1687
273  1687  40  1637  0741
274  0741  60  0360  0615
275  0615  33  0428  1455
276  1455  39  0110  0560
277  0560  32  0728  1505
278  1505  21  0610  0363
279  0363  61  0312  0417
280  0417  39  0228  1028
281  1028  32  0610  1737
282  1737  21  0610  0413
283  0413  60  0212  0517
284  0517  39  0278  1078
285  1078  32  0610  1787
286  1787  34  1440  1490
287  1490  21  0610  0513
288  0513  60  0282  1837
289  1837  39  0610  0660
290  0660  33  0778  1585
291  1585  21  0710  0563
292  0563  60  0360  0665
293  0665  39  0180  0630
294  0630  32  0710  1887
295  1887  21  0710  0613
296  0613  60  0312  0567
297  0567  39  0178  1138
```

```
FAD  H2
STO  H2
RSU  A02
FMP  SML82
FAD  H2
STO  H2
RAU  A0
FMP  SML83
FAD  H2
STO  H2                    H 2
RAU  1
FMP  2
FMP  3
FMP  4
FMP  5
FMP  6
STO  H10
RAU  H10
FDV  1
STO  H11
RAU  H10
FDV  2
STO  H12
RAU  H10
FDV  3
STO  H13
RAU  H10
FDV  4
STO  H14
RAU  H10
FDV  5
STO  H15
RAU  H10
FDV  6
STO  H16
RAU  BETA1
FMP  H11
STO  BH11
RAU  BETA2
FMP  H12
STO  BH12
RAU  BETA3
FMP  H13
STO  BH13
RAU  BETA4
FMP  H14
STO  BH14
RAU  BETA5
FMP  H15
STO  BH15
RAU  BETA6
FMP  H16
STO  BH16
RAU  H7
FMP  H10
FSB  BH11
FSB  BH12
FSB  BH13
FSB  BH14
FSB  BH15
FSB  BH16
STO  H1
RAU  H11
FAD  H12
FAD  H13
FAD  H14
FAD  H15
FAD  H16
FMP  RHO
FAD  H1
STO  H1
RAU  RHO
FMP  H10
STO  HO
LDD  D1                    H O
STD   1977                 PUNCH
                           LAMBDAS
LDD   1                    AND
STD   1978                 LAMBDA
LDD   2                    AVERAGE
STD   1979
LDD   3
STO   1980
LDD   4
STD   1981
LDD   5
STD   1982
LDD   6
STD   1983
LDD  LAMAV
STO   1984
PCH   1977
LDD  D2                    PUNCH
STD   1977                 BETAS
LDD  BETA1                 AND
STO   1978                 BETA TOTAL
LDD  BETA2
STO   1979
LDD  BETA3
STD   1980
LDD  BETA4
STO   1981
LDD  BETA5
STD   1982
LDD  BETA6
STO   1983
LDD  BETAT
STD   1984
PCH   1977
LDD                  EOOCL PUNCH
LDD  D3                    RHO AND L
STD   1977
LDD  RHO
STD   1978
LDD  L
STD   1979
PCH   1977
LDD  H7                    PUNCH
STO   1977                 COEFF
LDD  H6                    OF
STO   1978                 OMEGA
LDD  H5
STO   1979
LDD  H4
STD   1980
LDD  H3
STD   1981
LDD  H2
STD   1982
LDD  H1
STD   1983
LDD  HO
STO   1984
```

```
298  1128   32 0710 1937
299  1937   21 0710 0663
300  0663   61 0212 0617
301  0617   39 0328 1178
302  1178   32 0710 1987
303  1987   21 0710 0713
304  0713   60 0110 0715
305  0715   39 0528 1228
306  1228   32 0710 0138
307  0138   21 0710 0763
308  0763   60 0145 0299
309  0299   39 0094 0144
310  0144   39 0143 0343
311  0343   39 0092 0198
312  0192   39 0091 0791
313  0791   39 0040 1540
314  1540   21 0194 0097
315  0097   60 0194 0349
316  0349   34 0145 1045
317  1045   21 0850 0153
318  0153   60 0194 0399
319  0399   34 0094 0244
320  0244   21 0198 0051
321  0051   60 0194 0549
322  0549   34 0143 0393
323  0393   21 0248 0101
324  0101   60 0194 0599
325  0599   34 0098 0242
326  0242   21 0096 0649
327  0649   60 0194 0699
328  0699   34 0091 1041
329  1041   21 0146 0749
330  0749   60 0194 0799
331  0799   34 0040 1590
332  1590   21 0294 0147
333  0147   60 0285 0139
334  0139   39 0250 0300
335  0300   21 0254 0207
336  0207   60 0134 0189
337  0189   39 0198 0298
338  0298   21 0002 1605
339  1605   60 0133 0188
340  0188   39 0248 0348
341  0348   21 0052 1655
342  1655   60 0182 0238
343  0238   39 0096 0196
344  0196   21 0350 0303
345  0203   60 0081 0385
346  0385   39 0146 0246
347  0246   21 0400 0253
348  0253   60 0080 0135
349  0435   39 0294 0344
350  0344   21 0398 0151
351  0151   60 0282 0288
352  0288   39 0194 0394
353  0394   33 0254 1231
354  1231   33 0002 0129
355  0129   33 0052 0179
356  0179   33 0350 0227
357  0227   33 0400 0277
358  0277   33 0398 0225
359  0225   21 0680 0233
360  0233   60 0250 1705
361  1705   32 0198 0275
362  0275   32 0248 0325
363  0325   32 0096 0523
364  0523   32 0146 0573
365  0573   32 0294 0021
366  0021   39 0191 1091
367  1091   32 0680 0257
368  0257   21 0680 0283
369  0283   60 0191 1095
370  1095   39 0194 0544
371  0544   21 0548 0201
372  0201   69 0304 0307
373  0307   24 1977 0730
374  0730   69 0145 0598
375  0598   24 1978 1281
376  1281   69 0094 0197
377  0197   24 1979 0332
378  0332   69 0143 0296
379  0296   24 1980 0333
380  0333   69 0092 1145
381  1145   24 1981 0184
382  0184   69 0091 0594
383  0594   24 1982 0535
384  0535   69 0040 0543
385  0543   24 1983 0135
386  0136   69 0239 0292
387  0292   24 1984 0338
388  0338   71 1977 0387
389  0327   69 0780 0383
390  0383   24 1977 1030
391  1030   69 0285 0388
392  0388   24 1978 1431
393  1431   69 0134 0438
394  0438   24 1979 0382
395  0382   69 0133 0186
396  0186   24 1980 0433
397  0433   69 0182 0585
398  0585   24 1981 0234
399  0234   69 0081 0284
400  0284   24 1982 0635
401  0635   69 0080 0533
402  0533   24 1983 0236
403  0236   69 0148 0251
404  0251   24 1984 0538
405  0538   71 1977 0377
406  0377   69 1060 0000
407  1080   69 0583 0286
408  0286   24 1977 1130
409  1130   69 0191 0644
410  0644   24 1978 1481
411  1481   69 0078 1531
412  1531   24 1979 0432
413  0432   71 1977 0487
414  0427   69 0282 0685
415  0685   24 1977 1180
416  1180   69 0162 0765
417  0765   24 1978 1581
418  1581   69 0016 0669
419  0669   24 1979 0332
420  0332   69 0066 0719
421  0719   24 1980 0633
422  0633   69 0310 1013
423  1013   24 1981 0334
424  0334   69 0710 1063
425  1063   24 1982 0735
426  0735   69 0680 0683
427  0683   24 1983 0336
428  0336   69 0348 0301
429  0301   24 1984 0588
```

```
        PCH   1977                                 430   0568   71 1977 0527
        LOD  H7                    STORE  H        431   0527   69 0282 0785
        STO   0493                 IN  A           432   0785   24 0493 0346
        LDD  H6                                    433   0346   69 0162 1015
        STO   0494                                 434   1015   24 0494 0247
        LDD  H5                                    435   0247   69 0016 0769
        STO   0495                                 436   0769   24 0495 0648
        LDD  H4                                    437   0648   69 0066 1019
        STO   0496                                 438   1019   24 0496 1049
        LDD  H3                                    439   1049   69 0510 1113
        STO   0497                                 440   1113   24 0497 0550
        LDD  H2                                    441   0550   69 0710 1163
        STO   0498                                 442   1163   24 0498 0351
        LDD  H1                                    443   0351   69 0680 0733
        STO   0499                                 444   0733   24 0499 0102
        LDD  HO                                    445   0102   69 0548 0401
        STO   0500                                 446   0401   24 0500 0303
        RSA   0007                 TAKE  DERIV     447   0303   81 0007 0209
        RSB   0006                                 448   0209   83 0006 1065
        RAU  SEVEN                 OF  OMEGA       449   1065   60 0118 0623
        STU  N            CON30                    450   0623   21 1278 1631
CON30   FMP  A       A             FUNCTION        451   1631   39 2500 0600
        STU  AP      B                             452   0600   21 4490 0593
        AXA   0001                                 453   0593   50 0001 1099
        NZA  LOOP5        CON31                    454   1099   40 0152 0353
LOOP5   AXB   0001                                 455   0152   52 0001 0358
        RAU  N                                     456   0358   60 1278 0783
        FSB  ONE                                   457   0783   33 0098 0375
        STU  N            CON30                    458   0375   21 1278 1631
CON31   RBC   0006        CON32                    459   0353   89 0006 0259
CON32   RBL  LAMDA   C             TRY  LAMDA      460   0259   66 7390 1195
        STL  X                     FOR  OMEGA      461   1195   20 1000 0403
        LDD               EDDNR                    462   0403   69 0256 0050
        STL  X       C                             463   0256   20 7000 0503
        RAM  LAMDA   C                             464   0503   67 7390 1245
        RAU   8002                                 465   1245   60 8002 0553
        FSM  X       C                             466   0553   38 7000 0577
        BMI  CON33        CON34                    467   0577   46 1230 1681
CON33   AXC   0001                                 468   1230   58 0001 0386
        RAM  LAMDA   C                             469   0386   67 7390 1295
        RAU   8002                                 470   1295   60 8002 0603
        SXC   0001                                 471   0603   59 0001 0309
        FSM  X       C                             472   0309   38 7000 0627
        BMI  CON34        CON35                    473   0627   46 1681 1731
CON34   AXC   0001                                 474   1681   58 0001 0638
        RBL  LAMDA   C                             475   0638   66 7390 1445
        STL  X                                     476   1445   20 1000 0653
        LDD               EDDNR                    477   0653   69 0306 0050
        SXC   0001                                 478   0306   59 0001 0412
        STL  X       C                             479   0412   20 7000 0703
        RAM  LAMDA   C                             480   0703   67 7390 1495
        RAU   8002                                 481   1495   60 8002 0753
        FSM  X       C                             482   0753   38 7000 0677
        BMI  CON36        CON37                    483   0677   46 1280 1781
CON36   AXC   0001                                 484   1280   58 0001 0436
        RAM  LAMDA   C                             485   0436   67 7390 1545
        RAU   8002                                 486   1545   60 8002 1003
        SXC   0001                                 487   1003   59 0001 0359
        FSM  X       C                             488   0359   38 7000 0727
        BMI  CON37        CON35                    489   0727   46 1781 1731
CON37   RAU  LAMDA   C                             490   1781   60 7390 1595
        AXC   0001                                 491   1595   58 0001 0501
        FAD  LAMDA   C                             492   0501   32 7390 0667
        FDV  TWO                                   493   0667   34 0158 0408
        STU  Q                                     494   0408   21 0512 1115
        RSL  Q                                     495   1115   66 0512 0717
        STL  X                                     496   0717   20 1000 1053
        LDD               EDDNR                    497   1053   69 0356 0050
        SXC   0001                                 498   0356   59 0001 0562
        STL  X       C                             499   0562   20 7000 1103
        RAM  LAMDA   C                             500   1103   67 7390 1645
        RAU   8002                                 501   1645   60 8002 1153
        FSM  X       C                             502   1153   38 7000 0777
        BMI  CON38        CON39                    503   0777   46 1430 1831
CON38   AXC   0001                                 504   1430   58 0001 0536
        RAM  LAMDA   C                             505   0536   67 7390 1695
        RAU   8002                                 506   1695   60 8002 1203
        SXC   0001                                 507   1203   59 0001 0409
        FSM  X       C                             508   0409   38 7000 1027
        BMI  CON40        CON35                    509   1027   46 1480 1731
CON40   NOP               CON39                    510   1480   00 1530 1831
CON35   AXC   0002                                 511   1731   58 0002 0688
        BMC  CON41        CON42                    512   0688   49 1141 0342
CON41   SXC   0001        CON32                    513   1141   59 0001 0259
CON42   RAU  RHO                                   514   0342   60 0191 1745
        BMI  CON43        CON44                    515   1745   46 0698 1149
CON43   RAL  ZZZ10                 CALC  OMEGA     516   0698   65 0009 1213
        STL  X                     7               517   1213   20 1000 1253
        LDD               EDDNR                    518   1253   69 0406 0050
        STL  XPOS                                  519   0406   20 0061 0064
        RAU  6                                     520   0064   60 0040 1795
        FAD  XPOS                                  521   1795   32 0061 0738
        BMI  CON45        CON46                    522   0738   46 1191 0392
CON45   RBL  6                                     523   1191   66 0040 1845
        STL  X                                     524   1845   20 1000 1403
        LDD               EDDNR                    525   1403   69 0506 0050
        STL  XPOS                                  526   0506   20 0061 0114
        RAU  6                                     527   0114   60 0040 1895
        FAD  XPOS                                  528   1895   32 0061 0788
        BMI  CON47        CON46                    529   0788   46 1241 0392
CON47   RSU  6                                     530   1241   61 0040 1945
        FDV  TWO                                   531   1945   34 0158 0508
        STU  X                                     532   0508   21 1000 1453
        RAL  X                                     533   1453   65 1000 1755
        STL  X                                     534   1755   20 1000 1503
        LDD               EDDNR                    535   1503   69 0556 0050
        STL  XPOS                                  536   0556   20 0061 0164
        RAU  6                                     537   0164   60 0040 1995
        FAD  XPOS                                  538   1995   32 0061 1038
        BMI  CON39        CON46                    539   1038   46 1831 0392
CON44   LDD  ONE                   CALC  POS       540   1149   69 0098 0551
        STD  NNN                   OMEGA           541   0551   24 0354 0357
        STD  NN                                    542   0357   24 0760 1263
        RAU  LAMAV                                 543   1263   60 0239 0643
        FMP  RHO                                   544   0643   39 0191 1291
        STU  T                                     545   1291   21 1350 1553
        RAU  BETAT                                 546   1553   60 0148 1603
        FSB  RHO                                   547   1603   33 0191 0767
        STU  R                                     548   0767   21 0422 0425
        RAU  T                                     549   0425   60 1350 1805
        FDV  R                                     550   1805   34 0422 0522
        STU  S            CON48                    551   0522   21 0126 0229
CON48   RAL  S                                     552   0229   65 0126 1881
        STL  X                                     553   1881   20 1000 1653
        LDD               EDDNR                    554   1653   69 0606 0050
        STL  XPOS                                  555   0606   20 0061 0214
        RAU  XPOS                                  556   0214   60 0061 1165
        BMI  CON49        CON46                    557   1165   46 0168 0392
CON49   RAU  NNN                                   558   0168   60 0354 0509
        FAD  ONE                                   559   0509   32 0098 0525
        STU  NNN                                   560   0525   21 0354 0407
        FMP  S                                     561   0407   39 0126 0176
```

```
         FMP  NNN                                    562   0176    39  0354  0404
         STU  S          CON48                       563   0404    21  0126  0229
CON46    RSC  0001                                   564   0392    69  0001  0748
         RSU  LAMDA  C                               565   0748    01  7390  0396
         BTU  X          CON5D                       566   0396    21  1000  1703
CON50    LOD              EDONR                      567   1703    69  0656  0050
         STL  X      C                               568   0656    20  7000  1753
         RAM  LAMDA  C                               569   1753    67  7390  0546
         RAU  BDD2                                   570   0546    60  8002  1855
         FSM  X      C                               571   1855    38  7000  1077
         BMI  CON51      CON52                       572   1077    46  1580  1931
CON52    RAU  ONE                                    573   1931    60  0098  1803
         FAD  NN                                     574   1803    32  0760  1088
         STU  NN                                     575   1088    21  0760  1413
         FMP  LAMDA  C                               576   1413    39  7390  1640
         FMP  NN                                     577   1640    39  0760  1010
         RSL  BOO3                                   578   1010    66  8003  1017
         STL  X          CON50                       579   1017    20  1000  1703
CON51    LOD              EDOCL                      580   1580    69  1033  0000
         LDP  04          CON54                      581   1D33    69  0586  0289
CON54    STD  1977                                   582   0289    24  1977  1630
         RSC  0006                   ROOTS           583   1630    89  0006  0636
         LOD  X      C               F               584   0636    69  7000  1853
         STO  1978                   CHARACTER       585   1853    24  1978  0582
         STD  1393                   1STIC           586   0582    24  1393  0596
         AXC  0001                   EQUATION        587   0596    58  0001  0202
         LOD  X      C               .               588   0202    69  7000  1903
         STD  1979                   .               589   1903    24  1979  0632
         STO  1394                   .               590   0632    24  1394  0897
         AXC  0001                   .               591   0897    58  0001  0504
         LDD  X      C               .               592   0504    69  7000  0554
         STD  1980                   .               593   0554    24  1980  1083
         STO  1395                   .               594   1083    24  1395  0798
         AXC  0001                   .               595   0798    58  0001  0604
         LOD  X      C               .               596   0604    69  7000  0654
         STD  1981                   .               597   0654    24  1981  0384
         STD  1396                   .               598   0384    24  1396  1199
         AXC  0001                   .               599   1199    58  0001  1905
         LOD  X      C               .               600   1905    69  7000  0704
         STO  1982                   .               601   0704    24  1982  1035
         STO  1397                   .               602   1035    24  1397  0650
         AXC  0001                   .               603   0650    58  0001  0706
         LDD  X      C               .               604   0706    69  7000  0754
         STO  1983                   .               605   0754    24  1983  0686
         STO  1398                   .               606   0686    24  1398  0601
         LOD  XP03                   .               607   0601    69  0061  0264
         STO  1984                   .               608   0264    24  1984  1138
         STO  1399                   .               609   1138    24  1399  0252
         PCH  1977                   .               610   0252    71  1977  1127
         RSB  0007       CON55       CALC            611   1127    83  0007  1133
CON55    LOD  ZERO                   COEFF           612   1133    69  0068  0071
         STO  SUM1                O  F               613   0071    24  0024  1177
         STO  SUM2                   FLUX            614   1177    24  1680  1183
         RSA  0006       CON56       EQUATION        615   1183    81  0006  0339
CON56    RAU  OMEGA  B               .               616   0339    60  8400  0756
         FAD  LAMDA  A               .               617   0756    32  3390  1067
         STU  TEMP1                  .               618   1067    21  0572  0575
         RAU  BETA   A               .               619   0575    60  3380  1085
         FDV  TEMP1                  .               620   1085    34  0572  0628
         FAD  SUM1                   .               621   0622    32  0024  0651
         STU  SUM1                   .               622   0651    21  0024  1227
         RAU  BETA   A               .               623   1227    60  3380  1135
         FDV  TEMP1                  .               624   1135    34  0572  0672
         FDV  TEMP1                  .               625   0672    34  0572  0722
         FMP  LAMDA  A               .               626   0722    39  3390  1690
         FAD  SUM2                   .               627   1690    32  1680  0507
         STU  SUM2                   .               628   0507    21  1680  1233
         AXA  0001                   .               629   1233    50  0001  0389
         NZA  CON56      CON57       .               630   0389    40  0339  0693
CON57    RAU  ONE                    .               631   0693    60  0098  1004
         FSB  RHO                    .               632   1004    33  0191  1117
         STU  TEMP1                  .               633   1117    21  0572  0625
         FMP  L                      .               634   0625    39  0078  1428
         FAD  SUM2                   .               635   1428    32  1680  0557
         STU  TEMP2                  .               636   0557    21  0612  1215
         RAU  L                      .               637   1215    60  0078  1283
         FAD  SUM1                   .               638   1283    32  0024  0701
         FMP  TEMP1                  .               639   0701    39  0572  0772
         FDV  TEMP2                  .               640   0772    34  0612  0668
         STU  BPJ    B               .               641   0668    21  3360  1463
         AXB  0001                   .               642   1463    52  0001  1069
         NZB  CON55      CON58       .               643   1069    42  1133  0673
CON58    LOD  ZERO                   .               644   0673    69  0068  0121
         STO  SUMBJ                  .               645   0121    24  0074  1277
         RSA  0007       CON59       .               646   1277    81  0007  1433
CON59    RAU  SUMBJ                  .               647   1433    60  0074  0279
         FAD  BPJ    A               .               648   0279    32  3360  1188
         STU  SUMBJ                  .               649   1188    21  0074  1427
         AXA  0001                   .               650   1427    50  0001  1483
         NZA  CON59      CON60       .               651   1483    40  1433  1238
CON60    RBA  0007       CON61       .               652   1238    81  0007  0694
CON61    RAU  BPJ    A               .               653   0694    60  3360  1265
         FDV  SUMBJ                  .               654   1265    34  0074  0124
         STU  BJ     A               .               655   0124    21  3370  0783
         AXA  0001                   .               656   0783    50  0001  1339
         NZA  CON61      CON62       .               657   0329    40  0694  1533
CON62    LOD              EDOCL      .               658   1533    69  0736  0000
         LDP  05          CON63      PUNCH           659   0736    69  0439  0542
CON63    STO  1977                   COEFF           660   0542    24  1977  1730
         RSA  0007                   .               661   1730    81  0007  0786
         LDD  BJ     A               .               662   0786    69  3370  0773
         STO  1978                   .               663   0773    24  1978  0682
         AXA  0001                   .               664   0682    50  0001  1288
         LDD  BJ     A               .               665   1288    69  3370  1083
         STO  1979                   .               666   1023    24  1979  0732
         AXA  0001                   .               667   0732    50  0001  1438
         LDD  BJ     A               .               668   1438    69  3370  1073
         STO  1980                   .               669   1073    24  1980  0583
         AXA  0001                   .               670   1583    50  0001  0539
         LDD  BJ     A               .               671   0539    69  3370  1123
         STO  1981                   .               672   1123    24  1981  0434
         AXA  0001                   .               673   0434    50  0001  1740
         LDD  BJ     A               .               674   1740    69  3370  1173
         STO  1982                   .               675   1173    24  1982  1185
         AXA  0001                   .               676   1185    50  0001  1441
         LDD  BJ     A               .               677   1441    69  3370  1223
         STO  1983                   .               678   1223    24  1983  1036
         AXA  0001                   .               679   1036    50  0001  0592
         LDD  BJ     A               .               680   0592    69  3370  1273
         STD  1984                   .               681   1273    24  1984  1488
         PCH  1977       CON64       .               682   1488    71  1977  1477
CON64    RSC  0015       CON65       CALC            683   1477    89  0015  1633
CON65    RSA  0007                   FLUX            684   1633    81  0007  0589
         LDD  ZERO                   AS              685   0589    69  0068  0171
         STO  SUMFX      CON66       A               686   0171    24  0174  1527
CON66    RAU  OMEGA  A               FUNCTION        687   1527    60  3400  1006
         FMP  T      C               OF              688   1006    39  7350  0700
         BTU  W                      TIME            689   0700    81  1054  0607
         RAM  H003                                   690   0607    67  8003  1415
         RAU  H008                                   691   1415    60  8008  1283
         FSM  135                                    692   1483    38  0226  1104
         BMI  CON70      CON71                       693   1104    46  0657  0559
```

```
CON70   RAL  W
        LOD                 E00EA
        RAU    8002
        FMP  BJ      A
        FAD  SUMFX
        STU  SUMFX          CON71
CON71   AXA    0001
        NZA  CON66          CON67
CON67   LOD                 E00CL
        LOD  T       C
        STO    1977
        LOD  SUMFX
        STO    1978
        PCH    1977
        AXC    0001
        NZC  CON65          CON39
CON39   HLT
ZERO      00    0000        0051
ONE       10    0000        0051
TWO       20    0000        0051
THREE     30    0000        0051
FOUR      40    0000        0051
FIVE      50    0000        0051
SIX       60    0000        0051
SEVEN     70    0000        0051
01        01    0000        0000
02        02    0000        0000
03        03    0000        0000
04        04    0000        0000
05        05    0000        0000
PREC      10    0000        0053
ZZZ10     00    0000        0000
AAA10     10    0000        0051
AAA11     24    9998        6850
AAA12     31    8575        8349
AAA13     25    9137        1248
AAA14     17    1562        0047
AAA15     54    3020        0045
AAA16     69    0600        0044
135       13    5000        0053
```

```
694  0657  65  1054  0559
695  0559  69  0712  0200
696  0712  60  8002  0221
697  0221  39  3370  0020
698  0020  32  0174  0751
699  0751  21  0174  0558
700  0558  50  0001  0314
701  0314  40  1527  0218
702  0218  69  0271  0000
703  0271  69  7350  1154
704  1154  24  1977  1780
705  1780  69  0174  1577
706  1577  24  1978  0782
707  0782  71  1977  1627
708  1627  58  0001  1683
709  1683  48  1633  1831
710  1831  01  1038  1032
711  0068  00  0000  0051
712  0098  10  0000  0051
713  0158  20  0000  0051
714  0258  30  0000  0051
715  0690  40  0000  0051
716  1440  50  0000  0051
717  0750  60  0000  0051
718  0118  70  0000  0051
719  0304  01  0000  0000
720  0780  02  0000  0000
721  0583  03  0000  0000
722  0586  04  0000  0000
723  0439  05  0000  0000
724  0008  10  0000  0053
725  0009  00  0000  0000
726  0372  10  0000  0051
727  0322  24  9998  6850
728  0272  31  2575  8349
729  0222  25  9137  1248
730  0172  17  1562  0047
731  0029  54  3020  0045
732  0026  69  0600  0044
733  0226  13  5000  0053
     1335  20  0000  0053
     1336  15  0000  0053
     1337  10  0000  0053
     1338  75  0000  0052
     1339  50  0000  0052
     1340  40  0000  0052
     1341  30  0000  0052
     1342  20  0000  0052
     1343  15  0000  0052
     1344  10  0000  0052
     1345  80  0000  0051
     1346  60  0000  0051
     1347  50  0000  0051
     1348  40  0000  0051
     1349  20  0000  0051
```

Part - D

Three Delayed Neutron Group Program

General. The basic principles of the computer program used to solve the three group kinetic equations were the same as those used in solving the equations for six delayed neutron groups. The program was simplified considerably since the coefficients of the characteristic polynomial were greatly simplified. Since the roots of the characteristic equation were farther apart, in magnitude, additional trials for the roots between the poles of the equation were required in order that the Newton-Raphson method would converge to the proper root.

Input Data. The required input data is:

Position	Parameter
1387	λ_3 (smallest decay constant)
1388	λ_2
1389	λ_1
1377	β_3 (Associated with the smallest decay constant)
1378	β_2
1379	β_1
0141	Reactivity, ℓ
0028	Prompt neutron lifetime, l
0236	Average decay constant, λ_{av}
0020	Total fraction of delayed neutrons, β

Output DATA. The output data appears in the same order as in the six group program.

THREE

OELAYEO

NEUTRON

GROUP

REACTOR

KINETIC

EOUATIONS

```
         BLR  0440     0499                  1   0000   00 0000 0000
         BLR  0900     0999     CORRECT      2   0000   00 0000 0000
         BLR  0800     0899     TRACE        3   0000   00 0000 0000
         BLR  1300     1400                  4   0000   00 0000 0000
         BLR  1951     1960     REAO         5   0000   00 0000 0000
         BLR  1977     1986     PRINT        6   0000   00 0000 0000
         SYN  A        0500                  7   0000   00 0000 0000
         SYN  AP       0490                  8   0000   00 0000 0000
         SYN  X        1000                  9   0000   00 0000 0000
         SYN  T        1350                 10   0000
         SYN  BPJ      1360                 11   0000   00 0000 0000
         SYN  BJ       1370                 12   0000   00 0000 0000
         SYN  BETA     1380                 13   0000   00 0000 0000
         SYN  LAMOA    1390                 14   0000   00 0000 0000
         SYN  OMEGA    1400                 15   0000   00 0000 0000
         SYN  START    1999                 16   0000   00 0000 0000
         SYN  HLT      1111                 17   0000   00 0000 0000
EOOCL    STO  ZZZ1     CONT1    REAO        18   0000   24 0003 0006
CONT1    LDO  ZZZ10             OUT         19   0006   69 0009 0012
         STO  1977              OATA        20   0012   24 1977 0030
         STO  1978              ZERO        21   0030   24 1978 0031
         STO  1979              SUBROUTINE  22   0031   24 1979 0032
         STO  1980                          23   0032   24 1980 0033
         STO  1981                          24   0033   24 1981 0034
         STO  1982                          25   0034   24 1982 0035
         STO  1983                          26   0035   24 1983 0036
         STO  1984     ZZZ1                 27   0036   24 1984 0003
EOOEA    STO  AAA1              E TO THE X  28   0050   24 0053 0056
         STL  AAA2              SUBROUTINE  29   0056   20 0011 0014
         RAM  AAA2                          30   0014   67 0011 0015
         STL  AAA3                          31   0015   20 0019 0022
         RAU  AAA3                          32   0022   60 0019 0023
         FMP  AAA16                         33   0023   39 0026 0076
         FAO  AAA15                         34   0076   32 0029 0005
         FMP  AAA3                          35   0005   39 0019 0069
         FAO  AAA14                         36   0069   32 0072 0049
         FMP  AAA3                          37   0049   39 0019 0119
         FAO  AAA13                         38   0119   32 0122 0099
         FMP  AAA3                          39   0099   39 0019 0169
         FAO  AAA12                         40   0169   32 0172 0149
         FMP  AAA3                          41   0149   39 0019 0219
         FAO  AAA11                         42   0219   32 0222 0199
         FMP  AAA3                          43   0199   39 0019 0269
         FAO  AAA10                         44   0269   32 0272 0249
         STU  AAA4                          45   0249   21 0004 0007
         FMP  AAA4                          46   0007   39 0004 0054
         STU  AAA4                          47   0054   21 0004 0057
         FMP  AAA4                          48   0057   39 0004 0104
         STU  AAA4                          49   0104   21 0004 0107
         RAU  AAA2                          50   0107   60 0011 0065
         BMI  AAA5     AAA6                 51   0065   46 0018 0319
AAA5     RAU  AAA10                         52   0018   60 0272 0027
         FOV  AAA4                          53   0027   34 0004 0154
         STU  AAA4     AAA6                 54   0154   21 0004 0319
AAA6     RAL  AAA4     AAA1                 55   0319   65 0004 0053
EOONR    STO  NRM      CONT2                56   0100   24 0103 0106
CONT2    RSA   0004                         57   0106   81 0004 0062
         RAU  A      A CONT3                58   0062   60 2500 0055
CONT3    FMP  X                             59   0055   39 1000 0150
         AXA   0001                         60   0150   50 0001 0156
         NZA  LOOP2    CONT4                61   0156   40 0059 0010
LOOP2    FAO  A      A CONT3                62   0059   32 2500 0055
CONT4    FAO  A      A                      63   0010   32 2500 0077
         STU  Y                             64   0077   21 0082 0085
         RSB   0003                         65   0085   83 0003 0041
         RAU  AP     B CONT6                66   0041   60 4490 0045
CONT6    FMP  X                             67   0045   39 1000 0200
         AXB   0001                         68   0200   52 0001 0206
         NZB  LOOP3    CONT5                69   0206   42 0109 0060
LOOP3    FAO  AP     B CONT6                70   0109   32 4490 0045
CONT5    FAO  AP                            71   0060   32 0490 0017
         STU  YP                            72   0017   21 0322 0025
         RSU  Y                             73   0025   61 0082 0037
         FOV  YP                            74   0037   34 0322 0372
         FAO  X                             75   0372   32 1000 0127
         STU  XP                            76   0127   21 0132 0135
         RAU  X                             77   0135   60 1000 0105
         FOV  PREC                          78   0105   34 0008 0058
         STU  OIFF                          79   0058   21 0112 0115
         RAU  X                             80   0115   60 1000 0155
         FOV  PREC                          81   0155   34 0008 0108
         STU  OIFF                          82   0108   21 0112 0165
         RAU  X                             83   0165   60 1000 0205
         FSB  XP                            84   0205   33 0132 0159
         RAM   8003                         85   0159   67 8003 0067
         RAU   8002                         86   0067   60 8002 0075
         FSM  OIFF                          87   0075   38 0112 0039
         BMI  FINIS    ITER                 88   0039   46 0042 0043
ITER     LOO  XP                            89   0043   69 0132 0185
         STO  X        CONT2                90   0185   24 1000 0106
FINIS    RAL  XP       NRM                  91   0042   65 0132 0103
START    LOO   1387             LAMBDA3     92   1999   69 1387 0040
         STO  3                             93   0040   24 0093 0046
         LOO   1388             LAMBDA2     94   0046   69 1388 0091
         STO  2                             95   0091   24 0044 0047
         LOO   1389             LAMBDA1     96   0047   69 1389 0092
         STO  1                             97   0092   24 0095 0048
         LOO   1377                         98   0048   69 1377 0080
```

```
STO  BETA3                              99  0080   24 0083 0086
LDD    1378                            100  0086   69 1378 0081
STO  BETA2                             101  0081   24 0084 0087
LOD    1379                            102  0087   69 1379 0182
STO  BETA1                             103  0182   24 0835 0038
RAU  RHO                               104  0038   60 0141 0145
FSB  ONE                               105  0145   33 0098 0125
FMP  L                                 106  0125   39 0028 0078
STU  H4                        H4      107  0078   21 0832 0285
RAU  1                                 108  0285   60 0095 0299
FAD  2                                 109  0299   32 0044 0021
FAD  3                                 110  0021   32 0093 0369
STU  AD                                111  0369   21 0084 0177
RAU  AD                                112  0177   60 0084 0079
FMP  H4                                113  0079   39 0832 0282
FAD  RHO                               114  0282   32 0141 0117
FSB  BETA1                             115  0117   33 0020 0097
STU  H3                        H3      116  0097   21 0002 0355
RAU  1                                 117  0255   60 0095 0349
FMP  2                                 118  0349   39 0044 0094
STU  12                                119  0094   21 0148 0001
RAU  1                                 120  0001   60 0095 0399
FMP  3                                 121  0399   39 0093 0143
STU  13                                122  0143   21 0198 0051
RAU  2                                 123  0051   60 0044 0549
FMP  3                                 124  0549   39 0093 0193
STU  23                                125  0193   21 0248 0101
RAU  12                                126  0101   60 0148 0153
FAD  13                                127  0153   32 0198 0175
FAD  23                                128  0175   32 0248 0285
STU  A1                                129  0285   21 0130 0133
RAU  BETA1                             130  0133   60 0235 0089
FMP  2                                 131  0089   39 0044 0144
STU  B12                               132  0144   21 0298 0151
RAU  BETA2                             133  0151   60 0084 0139
FMP  1                                 134  0139   39 0095 0195
STU  B21                               135  0195   21 0250 0203
RAU  BETA3                             136  0203   60 0083 0137
FMP  1                                 137  0137   39 0095 0245
STU  B31                               138  0245   21 0300 0253
RAU  BETA1                             139  0253   60 0235 0189
FMP  3                                 140  0189   39 0093 0243
STU  B13                               141  0243   21 0348 0201
RAU  BETA2                             142  0201   60 0084 0239
FMP  3                                 143  0239   39 0093 0293
STU  B23                               144  0293   21 0398 0251
RAU  BETA3                             145  0251   60 0083 0187
FMP  2                                 146  0187   39 0044 0194
STU  B32                               147  0194   21 0548 0301
RAU  A1                                148  0301   60 0130 0335
FMP  H4                                149  0335   39 0832 0332
STU  H2                                150  0332   21 0136 0289
RAU  AD                                151  0289   60 0084 0129
FMP  RHO                               152  0129   39 0141 0191
FAD  H2                                153  0191   32 0136 0013
FSB  B12                               154  0013   33 0298 0275
FSB  B21                               155  0275   33 0250 0227
FSB  B31                               156  0227   33 0300 0277
FSB  B13                               157  0277   33 0348 0325
FSB  B23                               158  0325   33 0398 0375
FSB  B32                               159  0375   33 0548 0425
STU  H2                        H2      160  0425   21 0136 0339
RAU  B12                               161  0339   60 0298 0303
FMP  3                                 162  0303   39 0093 0343
STU  B123                              163  0343   21 0598 0351
RAU  B21                               164  0351   60 0250 0305
FMP  3                                 165  0305   39 0093 0393
STU  B213                              166  0393   21 0648 0401
RAU  B31                               167  0401   60 0300 0355
FMP  2                                 168  0355   39 0044 0844
STU  B312                              169  0844   21 0698 0501
RAU  1                                 170  0501   60 0095 0599
FMP  2                                 171  0599   39 0044 0894
FMP  3                                 172  0894   39 0093 0543
STU  A2                                173  0543   21 0748 0551
RAU  A2                                174  0551   60 0748 0353
FMP  H4                                175  0353   39 0238 0382
STU  H1                                176  0382   21 0186 0389
RAU  A1                                177  0389   60 0130 0385
FMP  RHO                               178  0385   39 0141 0241
FAD  H1                                179  0241   32 0186 0063
FSB  B123                              180  0063   33 0598 0525
FSB  B213                              181  0525   33 0648 0575
FSB  B312                              182  0575   33 0698 0625
STU  H1                        H1      183  0625   21 0186 0439
RAU  A2                                184  0439   60 0748 0403
FMP  RHO                               185  0403   39 0141 0291
STU  HO                        HO.     186  0291   21 0096 0649
LDD    EOOCL                           187  0649   69 0052 0000
LDD  O1                        PUNCH   188  0052   69 0405 0158
STO    1977                            189  0158   24 1977 0180
LDD  1                                 190  0180   69 0095 0798
STO    1978                            191  0798   24 1978 0131
LDD  2                                 192  0131   69 0044 0147
STO    1979                            193  0147   24 1979 0432
LDD  3                                 194  0432   69 0093 0146
STO    1980                            195  0146   24 1980 0183
LDD  LAWAY                             196  0183   69 0236 0539
STO    1981                            197  0539   24 1981 0134
PCH    1977                            198  0134   71 1977 0327
LDD    EOOCL                           199  0327   69 0830 0000
LDD  O2                        PUNCH   200  0230   69 0833 0286
STO    1977                            201  0286   24 1977 0280
LDD  BETA1                             202  0280   69 0235 0088
STO    1978                            203  0088   24 1978 0161
LDD  BETA2                             204  0181   69 0084 0837
STO    1979                            205  0237   24 1979 0532
LDD  BETA3                             206  0532   69 0083 0336
STO    1980                            207  0336   24 1980 0383
LDD  BETAT                             208  0283   69 0020 0073
STO    1981                            209  0073   24 1981 0184
PCH    1977                            210  0184   71 1977 0377
LDD    EOOCL                           211  0377   69 0330 0000
LDD  D3                        RHO AND L 212  0330   69 0333 0386
STO    1977                            213  0386   24 1977 0380
LDD  RHO                               214  0380   69 0141 0344
STO    1978                            215  0344   24 1978 0231
LOD  L                                 216  0231   69 0028 0881
STO    1979                            217  0281   24 1979 0582
PCH    1977                            218  0582   71 1977 0427
LDD    EOOCL                           219  0427   69 0430 0000
LOO  O4                                220  0430   69 0383 0436
STO    1977                            221  0436   24 1977 0530
LDD  H4                                222  0530   69 0832 0435
STO    1978                            223  0435   24 1978 0331
LOO  H3                                224  0331   69 0002 0505
STO    1979                            225  0505   24 1979 0632
LOD  H2                                226  0632   69 0136 0589
STO    1980                            227  0589   24 1980 0433
LOO  H1                                228  0433   69 0186 0639
STO    1981                            229  0639   24 1981 0834
LOO  HO                                230  0234   69 0096 0699
```

```
                STO   1982                                  231   0699    24 1982 0535
                PCH   1977                                  232   0535    71 1977 0527
                LOD   H4                                    233   0527    69 0232 0585
                STO   0496                                  234   0585    24 0496 0749
                LOD   H3                                    235   0749    69 0002 0555
                STO   0497                                  236   0555    24 0497 0350
                LOD   H2                                    237   0350    69 0136 0689
                STO   0498                                  238   0689    24 0498 0601
                LOD   H1                                    239   0601    69 0186 0719
                STO   0499                                  240   0719    24 0499 0102
                LOD   H0                                    241   0102    69 0096 0799
                BTO   0500                                  242   0799    24 0500 0503
                RBA   0004                                  243   0503    81 0004 0209
                RBB   0003                                  244   0209    83 0003 0215
                RAU   FOUR                                  245   0215    60 0068 0183
                STU   N          CON30                      246   0183    21 0128 0381
CON30           FMP   A          A       FUNCTION           247   0381    39 2500 0400
                STU   AP         B   *                      248   0400    21 4490 0593
                AXC   0001                                  249   0593    50 0001 1049
                NZA   LOOP5      CON31                      250   1049    40 0152 0553
LOOP5           AXB   0001                                  251   0152    52 0001 0208
                RAU   N                                     252   0208    60 0128 0533
                FSB   ONE                                   253   0533    33 0098 0675
                STU   N          CON30                      254   0675    21 0128 0381
CON31           RSC   0003       CON32                      255   0553    89 0003 0259
CON32           RBL   LAMOA  C                 TRY LAMRDA   256   0259    66 7390 0295
                STL   X                         FOR OMEGA  257   0295    20 1000 0603
                LOD                EOONR                    258   0603    69 0256 0100
                STL   X          C                          259   0256    20 7000 0653
                RAM   LAMDA    C                            260   0653    67 7390 0345
                RAU   8002                                  261   0345    60 8002 0703
                FSM   X          C                          262   0703    38 7000 0577
                BMI   CON33      CON34                      263   0577    46 0580 0431
CON33           AXC   0001                                  264   0580    58 0001 0536
                RAM   LAMOA    C                            265   0536    67 7390 0395
                RAU   8002                                  266   0395    60 8002 0753
                SXC   0001                                  267   0753    59 0001 0309
                FSM   X          C                          268   0309    38 7000 0627
                BMI   CON34      CON35                      269   0627    46 0431 0531
CON34           AXC   0001                                  270   0431    58 0001 0287
                RBL   LAMOA    C                            271   0287    66 7390 0545
                STL   X                                     272   0545    20 1000 1003
                LOD                EOONR                    273   1003    69 0306 0100
                SXC   0001                                  274   0306    59 0001 0168
                STL   X          C                          275   0168    20 7000 1053
                RAM   LAMOA    C                            276   1053    67 7390 0595
                RAU   8002                                  277   0595    60 8002 1103
                FSM   X          C                          278   1103    38 7000 0677
                BMI   CON36      CON37                      279   0677    46 0630 0581
CON36           AXC   0001                                  280   0630    58 0001 0586
                RAM   LAMDA    C                            281   0586    67 7390 0645
                RAU   8002                                  282   0645    60 8002 1153
                SXC   0001                                  283   0153    59 0001 0359
                FSM   X          C                          284   0359    38 7000 0727
                BMI   CON37      CON35                      285   0727    46 0581 0531
CON37           RAU   LAMOA    C                            286   0581    60 7390 0295
                AXC   0001                                  287   0695    58 0001 0651
                FAO   LAMOA    C                            288   0651    32 7390 0167
                FOV   TWO                                   289   0167    34 0070 0120
                STU   D                                     290   0120    21 0074 0777
                RSL   Q                                     291   0777    66 0074 0179
                STL   X                                     292   0179    20 1000 1203
                LOD                EOONR                    293   1203    69 0356 0100
                SXC   0001                                  294   0356    59 0001 0818
                STL   X          C                          295   0218    20 7000 1153
                RAM   LAMDA    C                            296   1253    67 7390 0745
                RAU   8002                                  297   0745    60 8002 1403
                FSM   X          C                          298   1403    38 7000 0827
                BMI   CON38      CON70                      299   1027    46 0680 0631
CON38           AXC   0001                                  300   0680    58 0001 0636
                RAM   LAMDA    C                            301   0636    67 7390 0795
                RAU   8002                                  302   0795    60 8002 1453
                SXC   0001                                  303   1453    59 0001 0409
                FSM   X          C                          304   0409    38 7000 0777
                BMI   CON70      CON35                      305   1077    46 0631 0531
CON70           AXC   0001                                  306   0631    58 0001 0337
                RAU   LAMOA    C                            307   0337    60 7390 1045
                FAO   O                                     308   1045    32 0074 0701
                FOV   TWO                                   309   0701    34 0070 0170
                STU   K                                     310   0170    21 0124 1127
                RSL   K                                     311   1127    66 0124 0229
                STL   X                                     312   0229    20 1000 1503
                LOD                EOONR                    313   1503    69 0406 0100
                SXC   0001                                  314   0406    59 0001 0262
                STL   X          C                          315   0262    20 7000 1553
                RAM   LAMOA    C                            316   1553    67 7390 1095
                RAU   8002                                  317   1095    60 8002 1603
                FSM   X          C                          318   1603    38 7000 1177
                BMI   CON71      CON72                      319   1177    46 0730 0681
CON71           AXC   0001                                  320   0730    58 0001 0686
                RAM   LAMOA    C                            321   0686    67 7390 1145
                RAU   8002                                  322   1145    60 8002 1653
                SXC   0001                                  323   1653    59 0001 0509
                FSM   X          C                          324   0509    38 7000 1227
                BMI   CON72      CON35                      325   1227    46 0681 0531
CON72           RAU   LAMDA    C                            326   0681    60 7390 1195
                FAO   O                                     327   1195    32 0074 0751
                FOV   TWO                                   328   0751    34 0070 0220
                STU   M                                     329   0220    21 0174 1277
                RSL   M                                     330   1277    66 0174 0279
                STL   X                                     331   0279    20 1000 1703
                LOD                EOONR                    332   1703    69 0506 0100
                STL   X          C                          333   0506    20 7000 1753
                RAM   LAMDA    C                            334   1753    67 7390 1245
                RAU   8002                                  335   1245    60 8002 1803
                FSM   X          C                          336   1803    38 7000 1427
                BMI   CON73      CON39                      337   1427    46 0780 0731
CON73           AXC   0001                                  338   0780    58 0001 0736
                RAM   LAMOA    C                            339   0736    67 7390 1295
                RAU   8002                                  340   1295    60 8002 1853
                SXC   0001                                  341   1853    59 0001 0559
                FSM   X          C                          342   0559    38 7000 1477
                BMI   CON39      CON35                      343   1477    46 0731 0531
CON35           LOD                EOOCL                    344   0531    69 0284 0000
                LOD   X          C                          345   0284    69 7000 1903
                STO   1977                                  346   1903    24 1977 1030
                PCH   1977                                  347   1030    71 1977 1527
                AXC   0002                                  348   1527    58 0002 0783
                BMC   CON41      CON42                      349   0583    49 0786 0387
CON41           SXC   0001       CON32                      350   0786    59 0001 0259
CON42           RAU   RHO                                   351   0387    60 0114 1445
                BMI   CON43      CON44                      352   1445    46 1048 1099
CON43           RAL   ZZZ10                   CALC OMEGA   353   1048    85 0009 0113
                STL   X                         7          354   0113    20 1000 0204
                LOD                EOONR                    355   0204    69 0157 0100
                STL   XPOS                                  356   0157    20 0061 0064
                RAU   3                                     357   0064    60 0093 0197
                FAO   XPOS                                  358   0197    32 0061 0437
                BMI   CON45      CON46                      359   0437    46 0090 0341
CON45           RSL   3                                     360   0090    66 0093 0147
                STL   X                                     361   0247    20 1000 0254
                LOD                EOONR                    362   0254    69 0207 0100
```

```
                STL   XPOS
                RAU   3
                FAD   XPOS
                SMI   CON47     CON46
CON47           RSU   3
                FOV   TWO
                STU   X
                RAL   X
                STL   X
                LOD             EOONR
                STL   XPOS
                RAU   3
                FAD   XPOS
                SMI   WELL      CON46
WELL            RAU   POINT
                STU   N         TRY
TRY             RSU   3
                FOV   N
                STU   X
                RAL   X
                LOD             EOONR
                STL   XPOS
                RAU   3
                FAD   XPOS
                SMI   SAO       CON46
SAO             RAU   N
                FAD   ONE
                STU   N         TRY
CON44           LOD   ONE                       CALC POS
                STO   NNN                       OMEGA
                STO   NN
                RAU   LAMAV
                FMP   RHO
                STU   T
                RAU   BETAT
                FSB   RHO
                STU   R
                RAU   T
                FOV   R
                STU   8         CON48
CON48           RAL   8
                STL   X
                LOD             EOONR
                STL   XPOS
                RAU   XPOS
                SMI   CON49     CON46
CON49           RAU   NNN
                FAD   ONE
                STU   NNN
                FMP   8
                FMP   NNN
                STU   8         CON48
CON46           LOD             EOOCL
                LOD   XPOS
                STO    1977
                PCH   1977
                RSC    0001
                RSU   LAMOA  C
                STU   X         CON50
CON50           LOD             EOONR
                STL   X    • C
                RAM   LAMDA  C
                RAU    8002
                F8M   X      C
                SMI   CON51     CON52
CON52           RAU   ONE
                FAD   NN
                STU   NN
                FMP   LAMOA  C
                FMP   NN
                RBL    8003
                STL   X         CON50
CON51           LOD             EOOCL
                LOD   04        CON54
CON54           STO    1977
                RSC    0003
                LOD   X      C
                STO    1981
                STO    1396
                AXC    0001
                LOD   X      C
                STO    1982
                STO    1397
                AXC    0001
                LOD   X      C
                STO    1983
                STO    1398
                LOD   XPOS
                STO    1984
                STO    1399
                PCH    1977
                RSB    0004     CON55
CON55           LOD   ZERO                      COEFF
                STO   SUM1              D       F
                STO   SUM2                      FLUX
                RSA    0003     CON56
CON56           RAU   ONE    8
                FAD   LAMDA  A
                STU   TEMP1
                RAU   BETA   A
                FOV   TEMP1
                FAD   SUM1
                STU   SUM1
                RAU   BETA   A
                FOV   TEMP1
                FOV   TEMP2
                FMP   LAMDA  A
                FAD   SUM2
                STU   SUM2
                AXA    0001
                NZA   CON56     CON57
CON57           RAU   ONE
                FSB   RHO
                STU   TEMP1
                FMP   L
                FAD   SUM2
                STU   TEMP2
                RAU   L
                FAD   SUM1
                FMP   TEMP1
                FOV   TEMP2
                STU   8PJ    8
                AXB    0001
                NZB   CON55     CON58
CON58           LOD   ZERO
                STO   SUM8J
                RBA    0004     CON59
CON59           RAU   SUM8J
                FAD   8PJ    A
                STU   SUM8J
                AXA    0001
                NZA   CON59     CON60

363   0207   20 0061 0114
364   0114   60 0093 0297
365   0297   32 0061 0537
366   0537   46 0140 0341
367   0140   61 0093 0347
368   0347   34 0070 0370
369   0270   21 1000 0304
370   0304   65 1000 0605
371   0605   20 1000 0354
372   0354   69 0357 0100
373   0257   20 0061 0164
374   0164   60 0093 0397
375   0397   32 0061 0587
376   0587   46 0190 0341
377   0190   60 0643 0547
378   0547   21 0128 0781
379   0781   61 0093 0597
380   0597   34 0128 0178
381   0178   21 1000 0404
382   0404   65 1000 0655
383   0655   69 0858 0100
384   0858   20 0061 0814
385   0814   60 0093 0647
386   0647   32 0061 0637
387   0637   46 0240 0341
388   0240   60 0128 0633
389   0633   32 0098 0785
390   0785   21 0128 0781
391   1099   69 0098 1001
392   1001   24 0504 0307
393   0307   24 0110 0163
394   0163   60 0236 0391
395   0391   39 0141 0541
396   0541   21 1330 0594
397   0554   60 0220 0775
398   0775   33 0141 0217
399   0217   21 0428 1085
400   1085   60 1330 0705
401   0705   34 0422 0522
402   0522   21 0126 0329
403   0329   65 0126 1031
404   1031   20 1000 0604
405   0604   69 0357 0100
406   0357   20 0061 0264
407   0264   60 0061 0265
408   0265   46 0118 0341
409   0118   60 0504 0609
410   0609   32 0098 1075
411   1075   21 0504 0407
412   0407   39 0126 0176
413   1176   39 0504 0654
414   0654   21 0126 0329
415   0341   69 0394 0000
416   0394   69 0061 0314
417   0314   24 1977 1080
418   1080   73 1977 1577
419   1577   89 0003 1683
420   0683   61 7390 1495
421   1495   21 1000 0704
422   0704   69 0587 0100
423   0507   20 7000 0754
424   0754   67 7390 1545
425   1545   60 8002 1004
426   1004   36 7000 1627
427   1687   46 1130 1081
428   1081   60 0098 0654
429   1054   32 0110 0687
430   0687   21 0110 0213
431   0213   39 7390 0290
432   0290   39 0110 0160
433   0160   66 8003 0267
434   0267   20 1000 0704
435   1130   69 0733 0000
436   0733   69 0383 1036
437   1036   24 1977 1180
438   1180   89 0003 1086
439   1086   69 7000 1104
440   1104   24 1981 0334
441   0334   24 1396 1149
442   1149   58 0001 0755
443   0755   69 7000 1154
444   1154   24 1982 0635
445   0635   24 1397 0550
446   0550   58 0001 0556
447   0556   69 7000 1204
448   1204   24 1983 1136
449   1136   24 1398 1051
450   1051   69 0061 0364
451   0364   24 1984 0737
452   0737   24 1399 0202
453   0202   71 1977 1677
454   1677   83 0004 0783
455   0783   69 1186 0789
456   0789   24 0148 1195
457   1595   24 1098 1101
458   1101   81 0003 0557
459   0557   60 5400 1005
460   1005   32 3390 0317
461   0317   21 0572 1125
462   1125   60 3380 0685
463   0685   34 0572 0622
464   0622   32 0142 0419
465   0419   21 0142 1645
466   1645   60 3380 0735
467   0735   34 0572 0672
468   0672   34 0572 0722
469   0722   39 3390 0340
470   0340   38 1098 1175
471   1175   21 1098 1151
478   1151   50 0001 0607
473   0607   40 0557 0111
474   0111   60 0198 1454
475   1254   33 0141 0367
476   0367   21 0572 1225
477   1225   39 0082 0228
478   0228   32 1098 1275
479   1275   21 1230 1033
480   1033   60 0082 1083
481   1083   32 0142 0519
482   0519   39 0572 0772
483   0772   34 1230 1288
484   1288   21 5360 0233
485   0263   52 0001 0569
486   0569   42 0783 1173
487   0173   69 1186 1039
488   1039   84 0198 1695
489   1695   81 0004 1201
490   1201   60 0198 0697
491   0697   32 3360 0787
492   0787   21 0192 1745
493   1745   50 0001 1251
494   1251   40 1201 1055
```

```
CON60   R8A   0004        CON61
CON61   RAU   BPJ     A
        FOV   SUMBJ
        STU   8J      A
        AXA   0001
        NZA   CON61       CON62
CON62   LOO               EOOCL
        LOO   05          CON63*      PUNCH
CON63   STO   1977                    COEFF
        R8A   0004
        LOO   8J      A
        STO   1981
        AXA   0001
        LOO   8J      A
        STO   1982
        AXA   0001
        LOO   8J      A
        STO   1983
        AXA   0001
        LOO   8J      A
        STO   1984
        PCH   1977        CON64
CON64   RBC   0015        CON65       CALC
CON65   RSC   0004
        LOO   ZERO                    AB
        STO   8UMFX       CON66       A
CON66   RAU   OMECA   A               FUNCTION
        FMP   T       C               OF
        STU   W                       TIME
        RAM   8003
        RAU   8002
        FSM   135
        BMI   CON75       CON76
CON75   RAL   W
        LOO               EOOEA
        RAU   8002
        FMP   8J      A
        FAO   8UMFX
        STU   8UMFX       CON76
CON76   AXA   0001
        NZA   CON66       CON67
CON67   LOO               EOOCL
        LOO   T       C
        STO   1977
        LOO   8UMFX
        STO   1978
        PCH   1977
        AXC   0001
        NZC   CON65       CON99
CON99   NOP               8000
CON39   HLT
01      01    0000        0000
02      02    0000        0000
03      03    0000        0000
04      04    0000        0000
05      05    0000        0000
PREC    10    0000        0054
ZERO    00    0000        0051
ONE     10    0000        0051
TWO     20    0000        0051
THREE   30    0000        0051
FOUR    40    0000        0051
FIVE    50    0000        0051
SIX     60    0000        0051
SEVEN   70    0000        0051
135     13    5000        0053
POINT   11    0000        0051
AAA10   10    0000        0051
AAA11   24    9998        6850
AAA12   31    2575        8349
AAA13   25    9137        1248
AAA14   17    1562        0047
AAA15   54    3020        0045
AAA16   69    0600        0044
ZZZ10   00    0000        0000
```

```
495   1055   81 0004 0161
496   0161   60 3360 0315
497   0315   34 0192 0242
498   0242   21 3370 0323
499   0323   50 0001 0379
500   0379   40 0161 1133
501   1133   69 1236 0000
502   1236   69 1089 0292
503   0292   24 1977 1430
504   1430   81 0004 1286
505   1286   69 3370 0273
506   0273   24 1981 0384
507   0384   50 0001 0390
508   0390   69 3370 0323
509   0323   24 1982 0785
510   0785   50 0001 0591
511   0591   69 3370 0373
512   0373   24 1983 1436
513   1436   50 0001 0342
514   0342   69 3370 0443
515   0443   24 1984 1037
516   1037   71 1977 1727
517   1727   89 0015 1183
518   1183   81 0004 1139
519   1139   69 1186 1189
520   1189   24 0392 1795
521   1795   60 3400 1105
522   1105   39 7350 0600
523   0600   21 1404 0657
524   0657   67 8003 0365
525   0365   60 8002 0523
526   0523   38 0226 1454
527   1454   46 0707 0308
528   0707   65 1404 0659
529   0659   69 0312 0050
530   0312   60 8002 0071
531   0071   39 3370 0320
532   0320   32 0392 0619
533   0619   21 0392 0308
534   0308   50 0001 0414
535   0414   40 1795 0168
536   0168   69 0121 0000
537   0121   69 7350 1504
538   1504   24 1977 1480
539   1480   69 0392 1845
540   1845   24 1978 1131
541   1131   71 1977 1777
542   1777   58 0001 1833
543   1233   48 1183 1087
544   1027   08 1137 8000
545   0731   01 1181 1181
546   0405   01 0000 0000
547   0233   02 0000 0000
548   0333   03 0000 0000
549   0383   04 0000 0000
550   1099   05 0000 0000
551   0008   10 0000 0054
552   1186   00 0000 0051
553   0098   10 0000 0051
554   0070   20 0000 0051
555   0650   30 0000 0051
556   0068   40 0000 0051
557   0700   50 0000 0051
558   0750   60 0000 0051
559   1050   70 0000 0051
560   0226   13 5000 0053
561   0643   11 0000 0051
562   0272   10 0000 0051
563   0232   24 9998 6850
564   0172   31 2575 8349
565   0122   25 9137 1248
566   0078   17 1562 0047
567   0039   54 3020 0045
568   0026   69 0600 0044
569   0009   00 0000 0000
```

Part - E

Reactivity as a Function of Time

The equation that required a numerical integration was

$$n(t) = n(0)\left[\frac{a_2+(\beta-1)}{a_2+(\beta-1)e^{-bt'}}\right]e^{-\int_0^t \frac{\lambda dt'}{a_2+(\beta-1)e^{-bt'}} + \int_0^t \frac{\lambda dt'}{a_2 e^{+bt'}+(\beta-1)}} \tag{108}$$

The two integrals of Eq. (108) were solved numerically by use of Simpson's Rule,

$$\int_0^t \frac{\lambda dt'}{a_2+(\beta-1)e^{-bt'}} = \int_0^t y(t')dt' \doteq \frac{h}{3}\left[(y_0+y_{2m}) + 4(y_1+y_3+\cdots+y_{2m-1}) + 2(y_2+y_4+\cdots+y_{2m-2})\right] \tag{109}$$

where the interval from 0 $\longrightarrow$ t was divided into 2m increments. Thus

$$h = \frac{t-0}{20} = \Delta t \tag{110}$$

The two integrals of Eq. (108) are almost alike, thus the same part of the program was used to evaluate the two integrals. The two integrals were identified by index register A. Since the integrals were evaluated at various times after the reactivity change, time was indexed by register C. The integrals were evaluated between time (C) and time (C - 1) and then added to the value of the integral from time (0) to time (C).

A continuous looping system was devised such that the computer kept track of the point being calculated to determine whether it was an odd or an even time point. If the point was odd, the value of the function at that point was multiplied by four and if the point was even, the function evaluated at that point was multiplied by two, as indicated by Eq. (109). After the function was evaluated at a particular time, the next time point was obtained by adding to the last time point. The two end points y_0 and y_{2m} were determined by testing the time to see if it corresponded to the end point times.

Since the exponential function grew very large or very small as time increased, a test was used to see whether the exponential was larger or smaller than 135.0. An overflow by the computer would have destroyed the results of the computation.

The following symbolic terms were used in the program:

SUM - $\int_{T_c}^{T_{c+1}} y(t)_A \, dt$

PART - $y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \cdots + y_{2m}$

SUM_A - $\sum_{c=0}^{c=n} \int_{T_c}^{T_{c+1}} y(t)_A \, dt$

NOFT - Neutron density as a function of time.

T - Time.

A2 - $1 - \beta + \frac{\theta}{A}$

A - Magnitude of the final value of k_{ex}.

DELTA - $\frac{T_{c+1} - T_c}{20} = H$

TWETY - 20.0.

BMONE - $(\beta - 1)$

ETOFL - $e^{-\int_0^t y(t)_1 dt + \int_0^t y(t)_2 dt}$

Input Data. The data that is required is listed below:

Position	Description
0086	β , total fraction of delayed neutrons.
0106	b, constant of the reactivity term.
0091	$(\beta - 1)$
0044	Absolute reactivity. Positive value for negative insertion of reactivity, A.
0136	λ
0601 to 0621	The value of time at which the flux is to be calculated, T_c.

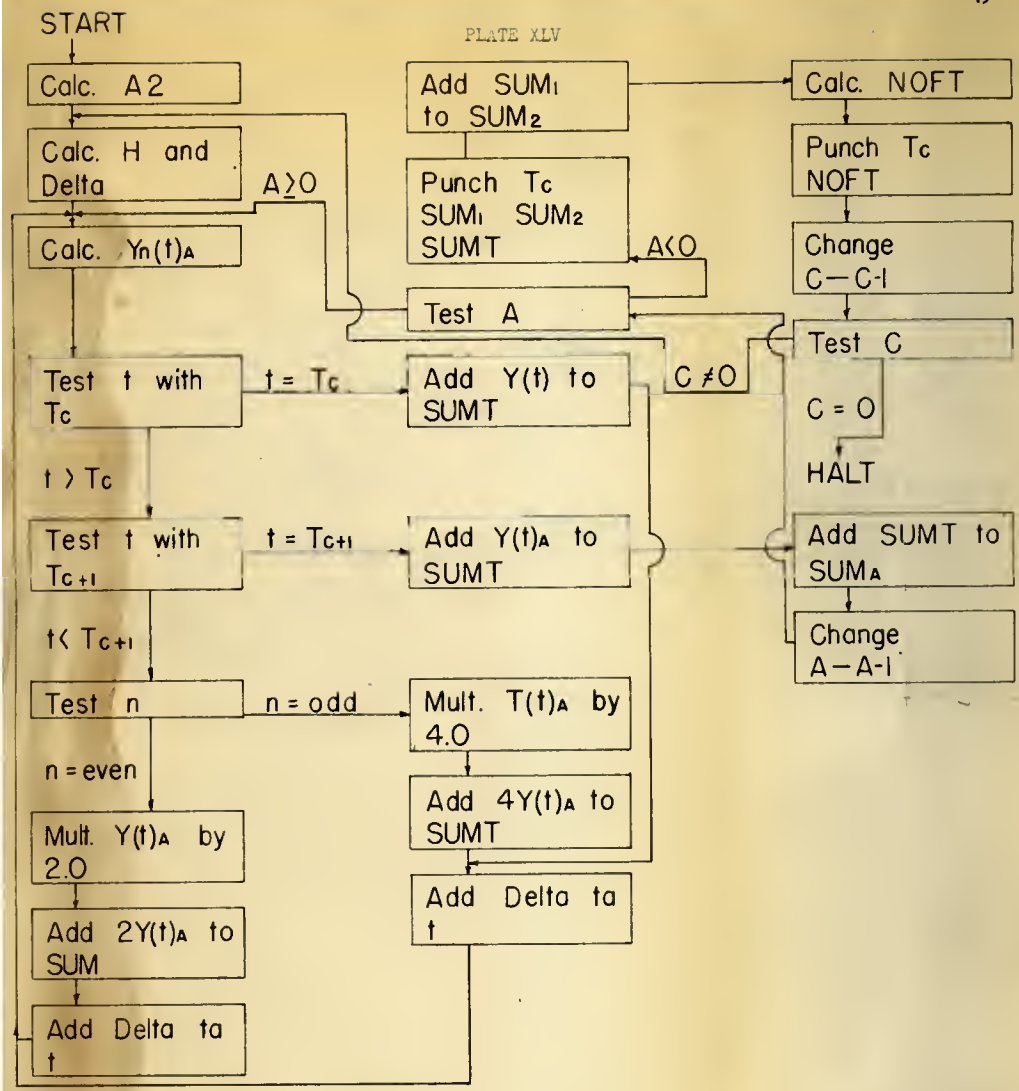
Output Data. Two output cards are punched for each time point. The data will appear in the following form.

Print position.	1977	1978	1979	1980	1981	1982
First line.	T(C)	SUMT	$SUM_{A=1}$	$SUM_{A=2}$		
Second line.					T(C)	NOFT

EXPLANATION OF PLATE XLV

Flow diagram of the computer program for the solution of the reactor kinetic equations using one group of delayed neutrons and when reactivity is a function of time.

PLATE XLV


START
Calc. A2
Calc. H and Delta
Calc. Yn(t)A
Test t with Tc
t = Tc
t > Tc
Test t with Tc+1
t = Tc+1
t < Tc+1
Test n
n = odd
n = even
Mult. Y(t)A by 2.0
Add 2Y(t)A to SUM
Add Delta ta t
Add Y(t) to SUMT
Add Y(t)A to SUMT
Mult. T(t)A by 4.0
Add 4Y(t)A to SUMT
Add Delta ta t
Add SUM1 to SUM2
Punch Tc SUM1 SUM2 SUMT
Test A
A≥0
A<0
C≠0
Calc. NOFT
Punch Tc NOFT
Change C—C-1
Test C
C = 0
HALT
Add SUMT to SUMA
Change A—A-1

REACTIVITY
AS A
FUNCTION
OF TIME

```
         BLR  0600      0700                         1   0000    00 0000 0000
         BLR  0700      0800    CORRECTION           2   0000    00 0000 0000
         BLR  0800      0900    TRACE                3   0000    00 0000 0000
         BLR  1951      1960    REAO                 4   0000    00 0000 0000
         BLR  1977      1984    PRINT                5   0000    00 0000 0000
         SYN  T         0600                         6   0000    00 0000 0000
         SYN  SUM       0630                         7   0000    00 0000 0000
         SYN  PART      0640                         8   0000    00 0000 0000
         SYN  START     1999                         9   0000    00 0000 0000
EOOCL    STO  ZZZ1      CONT1   REAO                10   0000    24 0003 0006
CONT1    LOO  ZZZ10             OUT                 11   0006    69 0009 0012
         STO  1977              OATA                12   0012    24 1977 0030
         STO  1978              ZERO                13   0030    24 1978 0031
         STO  1979              SUBROUTINE          14   0031    24 1979 0032
         STO  1980                                  15   0032    24 1980 0033
         STO  1981                                  16   0033    24 1981 0034
         STO  1982                                  17   0034    24 1982 0035
         STO  1983                                  18   0035    24 1983 0036
         STO  1984      ZZZ1                        19   0036    24 1984 0003
EOOEA    STO  AAA1              E TO THE X          20   0050    24 0053 0056
         STL  AAA2              SUBROUTINE          21   0056    20 0011 0014
         RAM  AAA2                                  22   0014    67 0011 0015
         STL  AAA3                                  23   0015    20 0019 0022
         RAU  AAA3                                  24   0022    60 0019 0023
         FMP  AAA16                                 25   0023    39 0026 0076
         FAO  AAA15                                 26   0076    32 0029 0005
         FMP  AAA3                                  27   0005    39 0019 0069
         FAO  AAA14                                 28   0069    32 0072 0049
         FMP  AAA3                                  29   0049    39 0019 0119
         FAO  AAA13                                 30   0119    32 0122 0099
         FMP  AAA3                                  31   0099    39 0019 0169
         FAO  AAA12                                 32   0169    32 0172 0149
         FMP  AAA3                                  33   0149    39 0019 0219
         FAO  AAA11                                 34   0219    32 0222 0199
         FMP  AAA3                                  35   0199    39 0019 0269
         FAO  AAA10                                 36   0269    32 0272 0249
         STU  AAA4                                  37   0249    21 0004 0007
         FMP  AAA4                                  38   0007    39 0004 0054
         STU  AAA4                                  39   0054    21 0004 0057
         FMP  AAA4                                  40   0057    39 0004 0104
         STU  AAA4                                  41   0104    21 0004 0107
         RAU  AAA2                                  42   0107    60 0011 0065
         BMI  AAA5      AAA6                        43   0065    46 0018 0319
AAA5     RAU  AAA10                                 44   0018    60 0272 0027
         FOV  AAA4                                  45   0027    34 0004 0154
         STU  AAA4      AAA6                        46   0154    21 0004 0319
AAA6     RAL  AAA4      AAA1                        47   0319    65 0004 0053
START    RAC  0020                                  48   1999    88 0020 0055
         RAA  0001                                  49   0055    80 0001 0061
         RAU  ZERO                                  50   0061    60 0064 0369
         STU  SUM     A                             51   0369    21 2630 0083
         SXA  0001                                  52   0023    51 0001 0039
         STU  SUM     A                             53   0039    21 2630 0133
         RAU  BETA                                  54   0133    60 0086 0041
         FOV  A                                     55   0041    34 0044 0094
         FSB  BETA                                  56   0094    33 0086 0013
         FAO  ONE                                   57   0013    32 0016 0043
         STU  A2        CON40                       58   0043    21 0048 0001
CON40    RAU  T       C                             59   0001    60 6600 0105
         AXC  0001                                  60   0105    58 0001 0111
         FSB  T       C                             61   0111    33 6600 0077
         FOV  TWETY                                 62   0077    34 0080 0130
         STU  H                                     63   0130    21 0084 0037
         STU  OELTA                                 64   0037    21 0042 0045
         RAA  0001      CON13                       65   0045    80 0001 0051
CON13    RSB  0001                                  66   0051    83 0001 0157
         RAU  ZERO                                  67   0157    60 0064 0419
         STU  SUMT                                  68   0419    21 0024 0127
         RAU  T       C CONT9                       69   0127    60 6600 0155
CONT9    STU  T                                     70   0155    21 0600 0103
         FMP  B                                     71   0103    39 0106 0156
         FMP  T                                     72   0156    39 0600 0100
         FMP  T                                     73   0100    39 0600 0150
         FMP  T                                     74   0150    39 0600 0200
         STU  B                                     75   0200    21 0204 0207
         RAM  8003                                  76   0207    67 8003 0115
         RAU  8002                                  77   0115    60 8002 0073
         FSM  ONE35                                 78   0073    38 0126 0153
         BMI  CON30     CON31                       79   0153    46 0206 0257
CON30    NZA  CON14     CON15                       80   0206    40 0059 0010
CON14    RSL  B         CON16                       81   0059    66 0204 0109
CON16    STL  ARG                                   82   0109    20 0063 0066
         LOO            EOOEA                       83   0066    69 0469 0050
         RAU  8002                                  84   0469    60 8002 0177
         STU  ETOB                                  85   0177    21 0082 0085
         NZA  CON17     CON18                       86   0085    40 0038 0089
CON17    FMP  BMONE             BETAMINUS1          87   0038    39 0091 0141
         FAO  A2                                    88   0141    32 0048 0025
         STU  BONE                                  89   0025    21 0180 0183
         RAU  L                                     90   0183    60 0136 0191
         FDV  BONE                                  91   0191    34 0180 0230
         STU  PART    A CON20                       92   0230    21 2640 0093
CON31    NZA  CON50     CON51                       93   0257    40 0060 0161
CON50    RAU  L                                     94   0060    60 0136 0241
         FOV  A2                                    95   0241    34 0048 0098
         STU  PART    A                             96   0098    21 2640 0143
         RAU  A2                                    97   0143    60 0048 0203
         STU  BONE      CON20                       98   0203    21 0180 0093
CON51    RAU  ZERO                                  99   0161    60 0064 0519
```

```
                STU  PART    A  CON20
CON18           FMP  A2
                FAD  SMONE
                STU  AZERO
                RAU  L
                FQV  AZERO
                STU  PART    A  CON20
CON15           RAL  S          CON18
CON20           RAU  T       C
                FSB  T
                NZU  CONT3      CONT4
CONT3           SXC  0001       CONT6
                RAU  T       C
                FSB  T
                NZU  CONT5      CONT6
CONT5           AXC  0001
                AXB  0001
                NZE  CONT7      CONT8
CONT7           RAU  PART    A
                FMP  TWO
                FAD  SUMT
                STU  SUMT
                RAU  T
CONT8           RAU  PART    A  CONT9
                FMP  FOUR
                FAD  SUMT
                STU  SUMT
                SXB  0002       CON10
CON10           RAU  T          CONT9
                FAD  DELTA
CONT4           RAU  PART    A
                FAD  SUMT
                STU  SUMT       CON10
CONT6           RAU  PART    A
                FAD  SUMT
                STU  SUMT
                FMP  M
                FDV  THREE
                STU  SUMT
                FAD  SUM     A
                STU  SUM     A
                SXA  0001
                SMA  CON12      CON63
CON63           AXC  0001       CON13
CON12           LOD  SUMT       E00CL
                LOD  T       C
                STD  1977
                LOD  SUMT    A
                STD  1978
                RAA  0001
                LOD  SUM     A
                STO  1979
                SXA  0001
                LOD  SUM     A
                STD  1980
                PCH  1977
                RAU  SUM     A
                AXA  0001    A
                FSB  SUM     A
                STU  EPDNN
                RAL  EPDNN
                LDD                  E00EA
                RAU  8002
                STU  ETOFL
                RAU  SMONE
                FAD  A2
                FQV  SONE
                FMP  ETOFL
                STU  NOFT
                LDD                  E00CL
                LOO  T       C
                STO  1981
                LDD  NOFT
                STO  1982
                PCH  1977
                SXC  0001
                NZC  CON40      CON41
CON41           NZE  0000       6000
ZERO            00   0000       0000
ONE             10   0000       0051
TWO             30   0000       0051
THREE           30   0000       0051
FOUR            40   0000       0051
TWETY           20   0000       0052
ONE35           13   5000       0053
ZZZ10           00   0000       0000
AAA10           10   0000       0051
AAA11           24   9998       6850
AAA12           31   8575       8349
AAA13           25   9137       1248
AAA14           17   1562       0047
AAA15           54   3030       0045
AAA16           69   0600       0044
   621          00   0000       0051
   620          50   0000       0049
   619          10   0000       0050
   618          20   0000       0050
   617          40   0000       0050
   616          60   0000       0050
   615          80   0000       0050
   614          10   0000       0051
   613          12   0000       0051
   612          14   0000       0051
   611          16   0000       0051
   610          18   0000       0051
   609          20   0000       0051
   608          25   0000       0051
   607          30   0000       0051
   606          40   0000       0051
   605          50   0000       0051
   604          60   0000       0051
   603          70   0000       0051
   602          80   0000       0051
   601          10   0000       0052
```

```
100   0519   21 3640 0093
101   0089   39 0048 0148
102   0148   32 0091 0017
103   0017   21 0322 0075
104   0075   60 0136 0291
105   0291   34 0322 0372
106   0372   21 2640 0093
107   0010   65 0204 0109
108   0093   60 6600 0205
109   0205   33 0600 0227
110   0227   44 0081 0132
111   0081   59 0001 0087
112   0087   60 6600 0255
113   0255   33 0600 0277
114   0277   44 0131 0182
115   0131   58 0001 0137
116   0137   52 0001 0193
117   0193   42 0046 0047
118   0046   60 2640 0095
119   0095   39 0198 0248
120   0248   32 0024 0101
121   0101   21 0024 0327
122   0327   60 0600 0305
123   0305   32 0024 0155
124   0047   60 2640 0145
125   0145   39 0398 0348
126   0348   32 0024 0251
127   0151   21 0024 0377
128   0377   53 0002 0233
129   0233   60 0600 0355
130   0355   32 0042 0155
131   0138   60 2640 0195
132   0195   32 0024 0201
133   0201   21 0024 0233
134   0182   60 2640 0245
135   0245   32 0024 0351
136   0251   21 0024 0427
137   0427   39 0084 0134
138   0134   34 0187 0237
139   0237   21 0024 0477
140   0477   32 2630 0307
141   0307   21 2630 0383
142   0283   51 0001 0139
143   0139   41 0092 0243
144   0243   58 0001 0051
145   0092   69 0295 0000
146   0295   69 6600 0253
147   0253   24 1977 0280
148   0280   69 0024 0027
149   0527   24 1978 0181
150   0181   80 0001 0287
151   0287   69 2630 0333
152   0333   24 1979 0238
153   0238   51 0001 0088
154   0088   69 2630 0383
155   0383   24 1980 0433
156   0433   71 1977 0377
157   0577   60 2630 0135
158   0135   50 0001 0341
159   0341   33 2630 0387
160   0357   81 0062 0165
161   0165   65 0062 0067
162   0067   69 0020 0080
163   0020   60 8002 0079
164   0079   21 0184 0337
165   0337   60 0091 0345
166   0345   32 0048 0125
167   0125   34 0180 0330
168   0330   39 0184 0234
169   0234   21 0138 0301
170   0391   69 0144 0000
171   0144   69 6600 0303
172   0303   24 1981 0284
173   0284   69 0138 0441
174   0441   24 1982 0185
175   0185   71 1977 0927
176   0927   59 0001 0483
177   0483   48 0001 0387
178   0387   00 0437 0491
179   0064   00 0000 0000
180   0016   10 0000 0051
181   0198   20 0000 0051
182   0187   10 0000 0051
183   0398   40 0000 0051
184   0080   20 0000 0052
185   0136   13 5000 0053
186   0009   00 0000 0000
187   0272   10 0000 0051
188   0222   24 9998 6850
189   0172   31 8575 8349
190   0122   25 9137 1248
191   0072   17 1562 0047
192   0029   54 3030 0045
193   0026   69 0600 0044
194   0621   00 0000 0051
195   0620   50 0000 0049
196   0619   10 0000 0050
197   0618   20 0000 0050
198   0617   40 0000 0050
199   0616   60 0000 0050
200   0615   80 0000 0050
201   0614   10 0000 0051
202   0613   12 0000 0051
203   0612   14 0000 0051
204   0611   16 0000 0051
205   0610   18 0000 0051
206   0609   20 0000 0051
207   0608   25 0000 0051
208   0607   30 0000 0051
209   0606   40 0000 0051
210   0605   50 0000 0051
211   0604   60 0000 0051
212   0603   70 0000 0051
213   0602   80 0000 0051
214   0601   10 0000 0052
```

Part - F

Tabulated Data, Analytical

Table 5. Flux ratio as a function of the value of the decay constant.

Group	Value Used For λ ** (Sec.$^{-1}$)	Reactivity ($)	Flux Ratios at Time (Sec.) 2	10	40	75
1	0.01216	-0.10	0.8698	0.7852	0.6194	0.4931
2	0.02958	-0.10	0.8699	0.7856	0.6216	0.4968
3	0.10747	-0.10	0.8670	0.7861	0.6209	0.4943
4	0.2900	-0.10	0.8705	0.7867	0.6205	0.4940
5	1.0002	-0.10	0.8705	0.7856	0.6195	0.4936
*		-0.10	0.8698	0.7852	0.6192	0.4927
1	0.01216	-0.50	0.5683	0.4019	0.2018	0.1105
2	0.02958	-0.50	0.5684	0.4026	0.2043	0.1131
3	0.10747	-0.50	0.5687	0.4034	0.2029	0.1108
4	0.2900	-0.50	0.5699	0.4042	0.2023	0.1106
5	1.002	-0.50	0.5698	0.4023	0.2018	0.1103
*		-0.50	0.5683	0.4019	0.2016	0.1102
1	0.01216	-1.50	0.3023	0.1712	0.0653	0.0298
2	0.02958	-1.50	0.3024	0.1718	0.0665	0.0307
3	0.10747	-1.50	0.3027	0.1723	0.0656	0.0297
4	0.2900	-1.50	0.3039	0.1726	0.0654	0.0297
5	1.0002	-1.50	0.3036	0.1713	0.0652	0.0296
*		-1.50	0.3023	0.1712	0.0652	0.0296
1	0.01216	0.01	1.0152	1.0273	1.0578	1.0890
2	0.02958	0.01	1.0152	1.0272	1.072	1.0572
3	0.10747	0.01	1.0152	1.0271	1.0573	1.0886
4	0.2900	0.01	1.0151	1.0270	1.0574	1.0887
5	1.0002	0.01	1.0151	1.0272	1.0576	1.0890
*		0.01	1.0152	1.0273	1.0577	1.0891
1	0.01216	0.10	1.1734	1.3407	1.9008	2.7335
2	0.02958	0.10	1.1736	1.3401	1.8924	2.7074
3	0.10747	0.10	1.1734	1.3391	1.8929	2.7169
4	0.2900	0.10	1.1725	1.3374	1.8926	2.7174
5	1.0002	0.10	1.1725	1.3398	1.8987	2.7305
*		0.10	1.1737	1.3410	1.9014	2.7360

Keepin's delayed neutron parameters.
Prompt neutron lifetime, $\ell = 8.0 \times 10^{-5}$ seconds.

* Here, all of the accepted values of the decay constants were used.

** Here, the value for the particular decayed constant was the accepted value for the i-th group plus the experimental uncertainty.

Table 6. Flux ration as a function of the total fraction of delayed neutrons.

β	Reactivity ($)	Flux Ratios at Times (Sec.) 2	10	40	100	200
0.006	0.01	1.0160	1.0292	1.0618	1.1194	1.2112
0.007	0.01	1.0184	1.0338	1.0717	1.1391	1.2571
0.008	0.01	1.0161	1.0295	1.0624	1.1204	1.2210
0.006	0.10	1.1863	1.3736	2.0246	4.0070	12.306
0.007	0.10	1.1894	1.3844	2.0478	4.1021	12.852
0.008	0.10	1.1865	1.3780	2.0260	4.0128	12.338
0.006	0.30	1.8493	3.5377	25.615	1194.5	6.873×10^{-5}
0.007	0.30	1.8558	3.5643	26.181	1256.8	7.577×10^{-5}
0.008	0.30	1.8514	3.5442	25.738	1207.3	7.016×10^{-5}
0.006	-0.10	0.8622	0.7701	0.5977	0.3969	0.2096
0.007	-0.10	0.8642	0.7722	0.6022	0.4029	0.2147
0.008	-0.10	0.8623	0.7703	0.5980	0.3981	0.2099
0.006	-0.50	0.5513	0.3795	0.1870	0.0672	0.0155
0.007	-0.50	0.5526	0.3808	0.1880	0.0676	0.0156
0.008	-0.50	0.5517	0.3799	0.1873	0.0673	0.0155

Hughes' delayed neutron parameters.
Prompt neutron lifetime, $\ell = 8.0 \times 10^{-5}$ seconds.

Part - G

Tabulated Data, Experimental

Table 7. Experimental rod drop No. A-2

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($), Keepin
2	0.580	0.445	0.481
4	0.528	0.435	0.465
6	0.479	0.445	0.471
8	0.455	0.420	0.456
10	0.417	0.435	0.472
15	0.364	0.425	0.463
20	0.319	0.435	0.468
30	0.259	0.435	0.468
40	0.214	0.415	0.464
50	0.181	0.430	0.466
75	0.121	0.430	0.461
100	0.083	0.425	0.458

Reactor: Argonaut Critical rod positions: Shim - 90
Rod dropped: Shim

Table 8. Experimental rod drop No. A-3.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($), Keepin
2	0.521	0.560	0.603
4	0.458	0.575	0.589
6	0.424	0.550	0.585
8	0.394	0.535	0.574
10	0.363	0.535	0.580
15	0.315	0.525	0.564
20	0.274	0.530	0.568
30	0.219	0.525	0.563
40	0.179	0.525	0.560
50	0.149	0.520	0.562
75	0.099	0.515	0.544
100	0.069	0.495	0.527

Reactor: Argonaut Critical rod positions: Shim - 29.5
Rod dropped: Shim and Fine Fine - 100

Table 9. Experimental rod drop No. A-4.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($), Keepin
2	0.956	0.029	0.037
4	0.939	0.033	0.035
6	0.927	0.033	0.036
8	0.921	0.033	0.038
10	0.909	0.034	0.038
15	0.888	0.035	0.038
20	0.872	0.035	0.038
30	0.841	0.036	0.039
40	0.808	0.038	0.039
50	0.796	0.035	0.038
75	0.739	0.036	0.039
100	0.691	0.035	0.038

Reactor: Argonaut Critical rod positions: Fine - 20
Rod dropped: Fine

Table 10. Experimental rod drop No. A-5.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($), Keepin
2	0.594	0.420	0.454
4	0.541	0.410	0.444
6	0.500	0.410	0.432
8	0.470	0.400	0.433
10	0.438	0.400	0.432
15	0.386	0.390	0.427
20	0.341	0.395	0.428
30	0.282	0.395	0.426
40	0.234	0.390	0.424
50	0.198	0.390	0.424
75	0.134	0.395	0.422
100	0.093	0.390	0.422

Reactor: Argonaut Critical rod positions: Shim - 80
Rod dropped: Shim

Table 11. Experimental rod drop No. A-6.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($), Keepin
2	0.628	0.370	0.393
4	0.583	0.345	0.376
6	0.547	0.340	0.364
8	0.516	0.335	0.364
10	0.477	0.345	0.373
15	0.421	0.340	0.376
20	0.373	0.345	0.378
30	0.311	0.345	0.378
40	0.262	0.345	0.376
50	0.223	0.345	0.376
75	0.155	0.345	0.372
100	0.109	0.340	0.372

Reactor: Argonaut Critical rod positions: Shim - 70
Rod dropped: Shim

Table 12. Experimental rod drop No. A-7.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($), Keepin
2	0.683	0.285	0.306
4	0.618	0.299	0.325
6	0.583	0.292	0.318
8	0.556	0.283	0.312
10	0.529	0.281	0.319
15	0.471	0.283	0.312
20	0.422	0.290	0.312
30	0.354	0.294	0.316
40	0.306	0.292	0.313
50	0.267	0.287	0.307
75	0.188	0.288	0.313
100	0.140	0.282	0.304

Reactor: Argonaut Critical rod positions: Shim - 60
Rod dropped: Shim

Table 13. Experimental rod drop No. A-8.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($), Keepin
2	0.738	0.221	0.233
4	0.696	0.212	0.235
6	0.661	0.210	0.230
8	0.643	0.199	0.219
10	0.608	0.206	0.228
15	0.578	0.195	0.218
20	0.513	0.217	0.226
30	0.444	0.209	0.226
40	0.389	0.211	0.228
50	0.348	0.207	0.225
75	0.263	0.208	0.224
100	0.201	0.217	0.224

Reactor: Argonaut
Rod dropped: Shim

Critical rod positions: Shim - 50

Table 14. Experimental rod drop No. A-9.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($), Keepin
2	0.823	0.133	0.141
4	0.790	0.130	0.141
6	0.768	0.125	0.136
8	0.737	0.131	0.141
10	0.714	0.131	0.145
15	0.666	0.132	0.145
20	0.632	0.131	0.144
30	0.562	0.135	0.145
40	0.514	0.133	0.146
50	0.469	0.135	0.146
75	0.382	0.135	0.145
100	0.306	0.138	0.147

Reactor: Argonaut
Rod dropped: Shim

Critical rod positions: Shim - 40

Table 15. Experimental rod drop No. A-10.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($), Keepin
2	0.892	0.075	0.081
4	0.872	0.073	0.080
6	0.853	0.071	0.080
8	0.842	0.068	0.077
10	0.822	0.072	0.080
15	0.790	0.072	0.080
20	0.759	0.074	0.081
30	0.711	0.074	0.081
40	0.670	0.075	0.081
50	0.634	0.074	0.081
75	0.549	0.076	0.083
100	0.480	0.076	0.083

Reactor: Argonaut
Rod dropped: Shim

Critical rod positions: Shim - 30

Table 16. Experimental rod drop No. A-11.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($) Keepin
2	0.968	0.021	0.022
4	0.952	0.024	0.027
6	0.946	0.023	0.027
8	0.940	0.023	0.026
10	0.931	0.024	0.028
15	0.914	0.026	0.029
20	0.899	0.026	0.029
30	0.871	0.028	0.031
40	0.849	0.029	0.031
50	0.830	0.028	0.031
75	0.778	0.030	0.032
100	0.737	0.029	0.032

Reactor: Argonaut
Rod dropped: Shim

Critical rod positions: Shim - 20

Table 17. Experimental rod drop No. A-12.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($), Keepin
2			
4			
6			
8			
10	0.988	0.004	0.004
15			
20	0.983	0.006	0.005
30			
40			
50	0.974	0.004	0.004
75			
100	0.948	0.005	0.005

Reactor: Argonaut
Rod dropped: Shim

Critical rod positions: Shim - 10

Table 18. Experimental rod drop No. A-13.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($), Keepin
2	0.625	0.376	0.394
4	0.564	0.375	0.405
6	0.516	0.385	0.410
8	0.483	0.375	0.414
10	0.440	0.390	0.424
15	0.386	0.380	0.426
20	0.340	0.400	0.430
30	0.277	0.400	0.430
40	0.228	0.405	0.434
50	0.195	0.400	0.432
75	0.130	0.405	0.433
100	0.090	0.400	0.433

Reactor: Argonaut
Rod dropped: Fine

Critical rod positions: Fine - 100

Table 19. Experimental rod drop No. A-14.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($) Keepin
2	0.604	0.400	0.435
4	0.553	0.390	0.422
6	0.515	0.385	0.410
8	0.475	0.390	0.426
10	0.439	0.400	0.430
15	0.384	0.395	0.428
20	0.341	0.395	0.429
30	0.275	0.405	0.434
40	0.229	0.405	0.432
50	0.194	0.400	0.433
75	0.131	0.405	0.431
100	0.091	0.405	0.426

Reactor: Argonaut
Rod dropped: Fine

Critical rod positions: Fine - 90

Table 20. Experimental rod drop No. A-15.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($) Keepin
2			
4			
6			
8			
10	0.683	0.150	0.166
15			
20	0.598	0.150	0.164
30	0.536	0.149	0.163
40	0.482	0.150	0.165
50	0.438	0.150	0.163
75			
100			

Reactor: Argonaut
Rod dropped: Fine

Critical rod positions: 32.1

Table 21. Experimental rod drop No. A-16.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($) Keepin
2	.	.	.
4	.	.	.
6	.	.	.
8	.	.	.
10	0.593	0.218	0.243
15	.	.	.
20	0.494	0.221	0.243
30	0.425	0.224	0.242
40	0.374	0.224	0.241
50	0.332	0.221	0.239
75	.	.	.
100	.	.	.

Reactor: Argonaut Critical rod positions: Fine - 40
Rod dropped: Fine

Table 22. Experimental rod drop No. A-17.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($) Keepin
2			
4			
6			
8			
10	0.500	0.315	0.341
15			
20	0.398	0.316	0.344
30	0.334	0.316	0.340
40	0.280	0.319	0.346
50	0.244	0.315	0.341
75			
100			

Reactor: Argonaut Critical rod positions: Fine - 50
Rod dropped: Fine

Table 23. Experimental rod drop No. A-18.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($) Keepin
2			
4			
6			
8			
10	0.436	0.415	0.434
15			
20	0.337	0.405	0.433
30	0.274	0.405	0.436
40	0.229	0.400	0.433
50	0.194	0.400	0.433
75			
100			

Reactor: Argonaut
Rod dropped: Fine

Critical rod positions: Fine - 60

Table 24. Experimental rod drop No. A-19.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($) Keepin
2			
4			
6			
8			
10	0.387	0.490	0.525
15	0.297	0.480	0.513
20	0.237	0.490	0.513
30	0.196	0.480	0.510
40	0.165	0.475	0.508
50			
75			
100			

Reactor: Argonaut
Rod dropped: Fine

Critical rod positions: Fine - 70

Table 25. Experimental rod drop No. A-20.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($) Keepin
2			
4			
6			
8			
10	0.372	0.520	0.560
15			
20	0.280	0.515	0.553
30	0.225	0.510	0.551
40	0.185	0.505	0.540
50	0.154	0.500	0.453
75			
100			

Reactor: Argonaut
Rod dropped: Fine

Critical rod positions: Fine - 80

Table 26. Experimental rod drop No. A-21.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($) Keepin
2			
4			
6			
8			
10	0.365	0.540	0.575
15			
20	0.271	0.535	0.577
30	0.217	0.535	0.570
40	0.178	0.530	0.563
50	0.148	0.530	0.562
75			
100			

Reactor: Argonaut
Rod dropped: Fine

Critical rod positions: Fine - 90

Table 27. Experimental rod drop No. A-22.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($) Keepin
2			
4			
6			
8			
10	0.362	0.540	0.583
15			
20	0.267	0.550	0.585
30	0.213	0.540	0.583
40	0.175	0.540	0.584
50	0.144	0.540	0.580
75			
100			

Reactor: Argonaut
Rod dropped: Fine

Critical rod positions: Fine - 100

Table 28. Experimental rod drop No. A-23.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($) Keepin
2			
4			
6			
8			
10	0.980	0.007	0.007
15			
20	0.972	0.007	0.008
30	0.965	0.007	0.008
40	0.959	0.007	0.008
50	0.945	0.007	0.008
75			
100			

Reactor: Argonaut
Rod dropped: Fine

Critical rod positions: Fine - 100
Final rod position: -91.1

Table 29. Experimental rod drop No. A-24.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($) Keepin
2			
4			
6			
8			
10	0.985	0.014	0.016
15			
20	0.938	0.016	0.018
30	0.925	0.016	0.017
40	0.915	0.015	0.017
50	0.903	0.015	0.017
75			
100			

Reactor: Argonaut
Rod dropped: Fine
Critical rod positions: Fine - 100
Final rod position: 84.2

Table 30. Experimental rod drop No. A-25.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($) Keepin
2			
4			
6			
8			
10	0.908	0.034	0.038
15	0.875		
20	0.875	0.033	0.038
30	0.843	0.035	0.039
40	0.820	0.035	0.038
50	0.800	0.034	0.037
75			
100			

Reactor: Argonaut
Rod dropped: Fine
Critical rod positions: Fine - 100
Final rod position: 76.5

Table 31. Experimental rod drop No. A-26.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($) Keepin
2			
4			
6			
8			
10	0.870	0.050	0.055
15	0.828	0.049	0.054
20	0.828	0.049	0.054
30	0.792	0.049	0.054
40	0.760	0.050	0.054
50	0.732	0.048	0.053
75			
100			

Reactor: Argonaut
Rod dropped: Fine

Critical rod positions: Fine - 100
Final rod positions: 73.0

Table 32. Experimental rod drop No. A-27.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($) Keepin
2			
4			
6			
8			
10	0.818	0.075	0.083
15			
20	0.754	0.075	0.083
30	0.702	0.076	0.085
40	0.662	0.076	0.084
50	0.622	0.076	0.085
75			
100			

Reactor: Argonaut
Rod dropped: Fine

Critical rod positions: Fine - 100
Final rod position: 67.1

Table 33. Experimental rod drop No. A-28.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($) Keepin
2			
4			
6			
8			
10	0.818	0.215	0.238
15			
20	0.502	0.215	0.235
30	0.436	0.215	0.234
40	0.385	0.214	0.233
50	0.341	0.214	0.231
75			
100			

Reactor: Argonaut
Rod dropped: Fine

Critical rod positions: Fine - 100
Final rod position: 50.0

Table 34. Experimental rod drop No. A-29.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($) Keepin
2			
4			
6			
8			
10	0.558	0.252	0.277
15			
20	0.456	0.255	0.277
30	0.390	0.255	0.277
40	0.338	0.255	0.276
50	0.296	0.254	0.274
75			
100			

Reactor: Argonaut
Rod dropped: Fine

Critical rod positions: Fine - 100
Final rod position: 45.3

Table 35. Experimental rod drop No. A-30.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($) Keepin
2			
4			
6			
8			
10	0.496	0.318	0.347
15			
20	0.400	0.314	0.342
30	0.335	0.316	0.340
40	0.286	0.311	0.340
50	0.245	0.314	0.340
75			
100			

Reactor: Argonaut
Rod dropped: Fine

Critical rod positions: Fine - 100
Final rod position: 39.3

Table 36. Experimental rod drop No. A-31.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($) Keepin
2			
4			
6			
8			
10	0.476	0.350	0.385
15			
20	0.375	0.345	0.375
30	0.310	0.345	0.375
40	0.263	0.340	0.376
50	0.225	0.345	0.374
75			
100			

Reactor: Argonaut
Rod dropped: Fine

Critical rod positions: Fine - 100
Final rod position: 35.2

Table 37. Experimental rod drop No. A-32.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($) Keepin
2			
4			
6			
8			
10	0.440	0.400	0.429
15	0.340	0.400	
20	0.340	0.400	0.430
30	0.278	0.400	0.430
40	0.234	0.399	0.425
50	0.196	0.398	0.429
75			
100			

Reactor: Argonaut
Rod dropped: Fine

Critical rod positions: Fine - 100
Final rod position: 29.2

Table 38. Experimental rod drop No. A-33.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($) Keepin
2			
4			
6			
8			
10	0.370	0.525	0.564
15			
20	0.274	0.530	0.567
30	0.219	0.530	0.563
40	0.179	0.520	0.558
50	0.151	0.520	0.556
75			
100			

Reactor: Argonaut
Rod dropped: Fine

Critical rod positions: Fine - 100
Final rod position: 8.6

Table 39. Experimental rod drop No. A-34.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($) Keepin
2			
4			
6			
8			
10	0.372	0.520	0.560
15			
20	0.279	0.515	0.554
30	0.224	0.515	0.550
40	0.185	0.505	0.540
50	0.154	0.505	0.543
75			
100			

Reactor: Argonaut
Rod dropped: Fine

Critical rod positions: Fine - 100
Final rod position: 10.8

Table 40. Experimental rod drop No. A-35.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($) Keepin
2			
4			
6			
8			
10	0.383	0.495	0.537
15			
20	0.289	0.495	0.532
30	0.232	0.500	0.530
40	0.191	0.490	0.524
50	0.161	0.485	0.521
75			
100			

Reactor: Argonaut
Rod dropped: Fine

Critical rod positions: Fine - 100
Final rod position: 17.0

Table 41. Experimental rod drop No. 7.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($) Keepin
2			
4			
6	0.178	1.75	1.89
8	0.154	1.78	1.92
10	0.135	1.79	1.95
15	0.106	1.82	1.97
20	0.086	1.83	1.99
30	0.062	1.85	2.00
40			
50			
75			
100			

Reactor: TRIGA
Rod dropped: Shim

Critical rod positions: Shim - up
Reg. - 299
Safety - up

Table 42. Experimental rod drop No. 8.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($) Keepin
2	0.541	0.516	0.560
4	0.480	0.515	0.562
6	0.428	0.532	0.578
8	0.396	0.526	0.574
10	0.368	0.524	0.572
15	0.442	0.350	0.387
20	0.390	0.356	0.386
30	0.311	0.344	0.375
40	0.254	0.355	0.387
50	0.212	0.368	0.397
75	0.127	0.410	0.44
100	0.100	0.345	0.40

Reactor: TRIGA
Rod dropped: Regulating

Critical rod positions: Shim - up
Reg. - 308
Safety - up

Table 43. Experimental rod drop No. 9.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($) Keepin
2	0.258	1.91	2.02
4	0.195	1.89	2.01
6			
8			
10			
15	0.107	1.80	1.96
20	0.089	1.79	1.95
30	0.066	1.77	1.92
40			
50	0.037	1.87	1.99
75	0.022	1.82	1.95
100	0.014	1.76	1.89

Reactor: TRIGA
Rod dropped: Shim

Critical rod positions: Shim - up
Reg. - 299
Safety - up

Table 44. Experimental rod drop No. 11.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($) Keepin
2	0.281	1.56	1.65
4	0.208	1.75	1.88
6	0.169	1.84	1.99
8	0.145	1.85	2.00
10	0.135	1.80	1.95
15	0.108	1.78	1.94
20	0.090	1.76	1.91
30	0.062	1.83	1.99
40	0.047	1.83	1.99
50	0.040	1.77	1.90
75	0.023	1.77	1.90
100			

Reactor: TRIGA
Rod dropped: Shim

Critical rod positions: Shim - up
Reg. - 311
Safety - up

Table 45. Experimental rod drop No. 13.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($), Keepin
2	0.625	0.365	0.422
4	0.552	0.387	0.423
6	0.511	0.384	0.443
8	0.463	0.406	0.443
10	0.437	0.404	0.433
15	0.381	0.400	0.432
20	0.339	0.399	0.427
30	0.274	0.400	0.436
40	0.224	0.413	0.439
50	0.194	0.403	0.434
75	0.125	0.416	0.447
100	0.087	0.413	0.440

Reactor: TRIGA
Rod dropped: Regulating

Critical rod positions: Shim - up
Reg. - 310
Safety - up

Table 46. Experimental rod drop No. 17.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($), Keepin
2	0.418	0.85	0.91
4	0.338	0.91	1.00
6	0.300	0.90	0.98
8	0.258	0.94	1.03
10	0.236	0.94	1.03
15	0.196	0.93	1.01
20	0.164	0.94	1.01
30	0.127	0.94	1.01
40	0.101	0.93	1.00
50	0.082	0.92	0.98
75	0.050	0.90	0.96
100	0.033	0.89	0.92

Reactor: TRIGA
Rod dropped: Shim

Critical rod positions: Shim - 380
Reg. - 577
Safety - up

Table 47. Experimental rod drop No. 20.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($), Keepin
2	0.289	1.50	1.61
4	0.218	1.65	1.78
6	0.187	1.65	1.80
8			
10			
15	0.113	1.70	1.80
20	0.095	1.67	1.80
30	0.069	1.68	1.83
40	0.053	1.70	1.82
50	0.041	1.73	1.81
75	0.023	1.74	1.83

Reactor: TRIGA
Rod dropped: Regulating

Critical rod positions: Shim - up
Reg. - 689
Safety - up

Table 48. Experimental rod drop No. 21.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($), Keepin
2	0.438	0.79	0.84
4	0.361	0.83	0.90
6	0.315	0.84	0.92
8	0.285	0.83	0.91
10			
15	0.229	0.80	0.85
20	0.187	0.82	0.89
30	0.142	0.83	0.90
40	0.112	0.84	0.91
50	0.088	0.85	0.91
75	0.055	0.83	0.90
100	0.034	0.84	0.89

Reactor: TRIGA
Rod dropped: Regulating

Critical rod positions: Shim - up
Reg. - 438
Safety - up

Table 49. Experimental rod drop No. 16.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($), Keepin
2	0.356	1.10	1.18
4	0.283	1.17	1.27
6	0.242	1.20	1.31
8	0.207	1.25	1.36
10			
15	0.150	1.25	1.35
20	0.126	1.25	1.34
30	0.097	1.21	1.36
40	0.074	1.24	1.34
50			
75	0.040	1.26	1.36
100			

Reactor: TRIGA
Rod dropped: Shim

Critical rod positions: Shim - 466
Reg. - 474
Safety - up

Table 50. Experimental rod drop No. 19.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($), Keepin
2			
4			
6			
8			
10			
15	0.184	1.06	1.09
20	0.157	0.99	1.06
30	0.123	0.96	1.03
40	0.099	0.98	1.04
50	0.088	0.92	1.00
75	0.051	0.94	1.02
100	0.036	0.96	0.98

Reactor: TRIGA
Rod dropped: Regulating

Critical rod positions: Shim - up
Reg. - 519
Safety - up

Table 51. Experimental rod drop No. 22.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($), Keepin
2	0.443	0.78	0.82
4	0.367	0.81	0.82
6	0.318	0.83	0.91
8	0.290	0.82	0.89
10	0.261	0.83	0.91
15	0.213	0.84	0.92
20	0.183	0.84	0.91
30	0.140	0.84	0.91
40	0.109	0.85	0.92
50	0.086	0.82	0.93
75	0.053	0.86	0.91
100	0.034	0.84	0.90

Reactor: TRIGA
Rod dropped: Regulating

Critical rod positions: Shim - up
Reg. - 438
Safety - up

Table 52. Experimental rod drop No. 24.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($), Keepin
2	0.526	0.55	0.60
4	0.448	0.59	0.63
6	0.400	0.59	0.65
8	0.357	0.61	0.67
10	0.330	0.61	0.66
15	0.276	0.62	0.67
20	0.237	0.62	0.68
30	0.179	0.65	0.70
40	0.142	0.66	0.71
50	0.116	0.66	0.71
75	0.071	0.66	0.71
100			

Reactor: TRIGA
Rod dropped: Shim

Critical rod positions: Shim - 296
Reg. - down
Safety - 3/4

Table 53. Experimental rod drop No. 26.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($), Keepin
2	0.339	1.23	1.26
4	"	"	"
6	"	"	"
8	"	"	"
10	0.168	1.40	1.51
15	0.133	1.42	1.54
20	"	"	"
30	0.082	1.43	1.52
40	0.063	1.43	1.54
50	0.050	1.43	1.54
75	0.029	1.42	1.53
100	"	"	"

Reactor: TRIGA
Rod dropped: Shim

Critical rod positions: Shim - 500
Reg. - down
Safety - 1/2

Table 54. Experimental rod drop No. 29-1.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($), Keepin
2	0.283	1.56	1.65
4	"	"	"
6	"	"	"
8	"	"	"
10	"	"	"
15	0.104	1.85	1.98
20	0.085	1.85	2.00
30	0.063	1.85	2.00
40	0.048	1.87	2.01
50	0.038	1.86	2.00
75	0.022	1.85	1.99
100	0.013	1.84	1.97

Reactor: TRIGA
Rod dropped: Regulating

Critical rod positions: Shim - 338
Reg. - 805
Safety - down

Table 55. Experimental rod drop No. 29-2.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($), Keepin
2	0.283	1.56	1.65
4			
6	0.171	1.84	2.00
8	0.149	1.85	2.00
10	0.131	1.85	2.00
15	0.104	1.85	1.98
20	0.085	1.85	2.00
30	0.063	1.85	2.00
40	0.048	1.87	2.01
50	0.036	1.86	2.00
75	0.022	1.85	1.99
100	0.013	1.84	1.97

Reactor: TRIGA
Rod dropped: Regulating

Critical rod positions: Shim - 338
Reg. - 805
Safety - down

Table 56. Experimental rod drop No. 37-1.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($), Keepin
2	0.326	1.27	1.31
4	0.257	1.34	1.45
6	0.218	1.37	1.48
8	0.192	1.38	1.50
10			
15	0.130	1.48	1.60
20			
30	0.076	1.53	1.66
40	0.058	1.56	1.67
50	0.045	1.56	1.68
75	0.025	1.57	1.69
100	0.016	1.56	1.68

Reactor: TRIGA
Rod dropped: Regulating

Critical rod positions: Shim - 423
Reg. - 598
Safety - down

Table 57. Experimental rod drop No. 37-2.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($), Keepin
2	0.331	1.23	1.32
4	0.265	1.29	1.40
6	.	.	.
8	.	.	.
10	0.164	1.39	1.52
15	.	.	.
20	0.104	1.52	1.65
30	0.077	1.50	1.64
40	0.058	1.56	1.67
50	0.045	1.57	1.69
75	0.026	1.55	1.66
100	0.012	1.55	1.63

Reactor: TRIGA
Rod dropped: Regulating

Critical rod positions: Shim - 423
Reg. - 598
Safety - down

Table 58. Experimental rod drop No. 39.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($), Keepin
2	0.360	1.09	1.17
4	0.286	1.15	1.18
6	0.245	1.17	1.26
8	0.216	1.19	1.27
10	0.197	1.16	1.30
15	0.154	1.20	1.24
20	0.131	1.20	1.30
30	0.098	1.19	1.30
40	0.078	1.17	1.26
50	0.062	1.17	1.25
75	.	.	.
100	.	.	.

Reactor: TRIGA
Rod dropped: Regulating

Critical rod positions: Shim - 495
Reg. - 520
Safety - down

Table 59. Experimental rod drop No. 27.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($), Keepin
2	0.282	1.56	1.65
4	0.231	1.57	1.66
6	0.195	1.58	1.70
8	0.170	1.59	1.71
10	0.145	1.65	1.80
15	0.107	1.80	1.94
20	0.089	1.78	1.93
30	0.066	1.79	1.94
40	0.050	1.80	1.94
50	0.040	1.79	1.92
75	0.021	1.83	1.99
100	0.012	1.84	1.99

Reactor: TRIGA
Rod dropped: Shim

Critical rod positions: Shim - 600
Reg. - down

Table 60. Experimental rod drop No. 28.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($), Keepin
2	0.292	1.49	1.57
4			
6			
8	0.150	1.81	1.98
10	0.136	1.82	1.95
15	0.107	1.83	1.96
20	0.085	1.83	1.97
30	0.064	1.86	1.98
40	0.049	1.88	1.96
50	0.037	1.87	1.99
75	0.022	1.86	1.98
100			

Reactor: TRIGA
Rod dropped: Shim

Critical rod positions: Shim - 693
Reg. - down
Safety - 1/4

Table 61. Experimental rod drop No. 36.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($), Keepin
2	0.528	0.55	0.59
4	0.451	0.58	0.62
6	0.404	0.58	0.64
8	0.372	0.58	0.63
10	0.372	0.59	0.63
15	0.292	0.66	0.62
20	0.256	0.67	0.66
30	0.189	0.61	0.68
40	0.148	0.63	0.67
50	0.122	0.64	0.67
75	0.077	0.62	0.66
100	0.051	0.62	0.64

Reactor: TRIGA
Rod dropped: Regulating

Critical rod positions: Shim - 389
Reg. - 389
Safety - down

Table 62. Experimental rod drop No. A-1.

Time after rod drop (Sec.)	Flux Ratio	Reactivity worth of rod drop ($), Hughes	Reactivity worth of rod drop ($), Keepin
2	0.549	0.505	0.544
4	0.510	0.465	0.501
6	0.469	0.465	0.490
8	0.437	0.455	0.490
10	0.402	0.460	0.497
15	0.347	0.460	0.496
20	0.309	0.455	0.490
30	0.250	0.455	0.487
40	0.208	0.450	0.476
50	0.174	0.450	0.484
75			
100			

Reactor: Argonaut
Rod dropped: Shim

Critical rod positions: Shim - 100

THEORETICAL REACTOR KINETIC MODELS AND EXPERIMENTAL VERIFICATION

by

MARVIN KEITH DRAKE

B. S., Kansas State University, 1959

AN ABSTRACT OF A THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Nuclear Engineering

KANSAS STATE UNIVERSITY
OF AGRICULTURE AND APPLIED SCIENCE

1960

An analytical study was made of the reactor kinetic equations for a bare homogeneous thermal reactor assuming step changes of reactivity. The study considered two sets of delayed neutron parameters, Hughes' and Keepin's, and the results of the study are given in graphical form. Experimental measurements of the reactivity worth of the control rods in the Torry Pines TRIGA reactor were made. The reactivity values obtained with positive period measurements were compared to those obtained by means of rod drop measurements. Agreement of the two methods indicated that the kinetics theory was valid when the reactor was operating at low power levels and reactivity changes were not extremely large ($\rho < \$2.00$).

A parameter analysis of the kinetic equations was made. The reactor parameters, prompt neutron lifetime, total fraction of delayed neutrons, and decay constants of the delayed neutrons, were evaluated over a relatively wide range of values. The importance of each parameter in the kinetic equations is given.

An approximate solution to the kinetic equations was obtained using three groups of delayed neutrons instead of the usual six groups. The delayed neutron parameters were empirically chosen to fit the computed data obtained with six delayed neutron groups.

Kinetics theory for a finite reactivity insertion rate was developed. The kinetics model included the assumption that the delayed neutrons could be approximated by one group of delayed

neutrons. The reactivity insertion rate was approximated by a function, of the form $A(1 - e^{-bt^n})$, to fit the reactivity insertion rate of the TRIGA reactor. The finite insertion rate equations predicted the neutron flux to a greater degree of accuracy for short times after the reactivity change, than the neutron flux as predicted by the usual six group kinetic equations with step input reactivities.

Three IBM-650 programs are given, solution of the kinetic equations for six groups of delayed neutrons, solution of the kinetic equations for three groups of delayed neutrons, and solution of the one delayed neutron group kinetic equations when reactivity is a function of time.