GMM and present value tests of the C-CAPM under Transactions Costs: Evidence from the UK stock market

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Abstract

In this paper we test for the inclusion of the bid-ask spread in the consumption CAPM, in the UK stock market over the time period of 1980-2000. Two econometric models are used; first, Fisher's (1994) asset pricing model is estimated by GMM, and secondly, the VAR approach proposed by Campbell and Shiller is extended to include the bid-ask spread. Overall the statistical tests are unable to reject the bid-ask spread as an independent explanatory variable in the C-CAPM. This leads to the conclusion that transactions costs should be included in asset pricing models.

JEL Classification: G12, C22

Keywords: Asset Pricing, Bid-Ask spreads, GMM, VAR.

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1 Introduction

Traditionally, in the finance literature it is argued that risk is the principal determinant of differences in expected asset returns and that trading volume and transaction costs can be ignored in asset pricing. This view is well documented in the classical asset pricing papers such as, Sharpe (1964), Lintner (1965) and Mossin (1966) as well as the subsequent enrichments of that framework provided by Ross (1976) and Merton (1973). The traditional view is also at the heart of the more recent general equilibrium analyses of Lucas (1978) and Mehra and Prescott (1985).

Mehra and Prescott (1985), however, provide important evidence against the risk hypothesis. In a general equilibrium model calibrated to reflect the historic degree of consumption risk present in the US economy they generated an equity premium, defined as the difference between the return on risky equity and the return on a short-term riskless security, of less than 0.4%. This figure contrasts sharply with the historical US equity premium from 1889 to 1978 of about 6.2%. This finding has stimulated a great deal of research into what has become known as the "equity premium puzzle". The adopted framework of Mehra and Prescott (1985) assumes frictionless markets.[2]

[2] Labadie (1989), Rietz (1988) and Weil (1989) provide frictionless modifications to the basic model studied by Mehra and Prescott (1985). Aiyagari and Gertler (1991) study transaction costs and uninsurable risk.

Fisher (1994) developed an equilibrium asset-pricing model that attempts to explain the historical size of the US equity premium by distinguishing between gross and net returns accruing to agents. The model derived by Mehra and Prescott (1985) was augmented with a bid-ask spread, calibrated and simulated. Equity premia in the order of 3-4% were generated for plausible values of the transactions costs parameters. Estimates of the bid-ask spread, were obtained using Generalized Method of Moments (GMM) and tests of the overidentifying restrictions were not rejected for several lists of instrumental variables. Fisher therefore found that transactions costs explained a portion of the equity premium. The implication being that asset-pricing models should include market frictions, such as the bid-ask spread.

This paper investigates whether market frictions should be included in asset pricing models in the UK stock market. The investigation adopts a three-stage research strategy. First, we discuss the simple equilibrium transaction cost asset-pricing model that was derived by Fisher (1994). Second, the equilibrium asset pricing relations from the model are formally tested using Hansen's GMM estimation technique with historic returns and transactions costs data for the UK stock market using monthly data for the time period 1980 to 2000. Third, we estimate the C-CAPM that incorporates transactions costs using the Vector Autoregressive (VAR) approach proposed by Campbell and Shiller (1988a). Two models are estimated using different measures of transactions costs in order to establish the robustness of the econometric evidence.

It was shown (Hansen, 1982) that expanding the set of variables that included in the orthogonality condition cannot increase the covariance matrix of the estimator, but it is important to note that this is an asymptotic result. Tauchen (1986) has investigated the small sample properties of the GMM estimator with a different number of instruments. The overall conclusion is that the best performance of the GMM estimator is obtained with a limited number of instruments. Even if the quality of the instruments appears to be statistically satisfactory within the sample, we still have the problem of the fact that GMM estimation deals with unconditional moments in the model. So a long time series is required to deliver consistent estimates. Restricting the model to small samples will effect the precision of the estimates and tests of the overidentifying restrictions on the model.

Therefore, in order to establish that the influence of transactions-costs is not model dependent and that the results are robust we also estimate the C-CAPM with transactions-costs using the VAR methodology proposed by Campbell and Shiller (1988a) and compare the results.

The paper is organized in the following way. Section 2 discusses the Fisher asset-pricing model. Section 3 presents the empirical tests of the Fisher model using GMM estimation. Section 4 extends the C-CAPM model proposed by Campbell and Shiller (1988a) to include transactions costs. Section 5 presents empirical tests of the extended model using a VAR methodology. Section 6 provides a summary and conclusion of our main findings.

2 The Fisher Equilibrium Model of Expected Returns with a Bid-Ask Spread

An agent in an economy is assumed to maximize expected utility over random consumption paths of an infinite time horizon i.e.

$$Max_{c_t s_t b_t} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{1}$$

subject to:

$$c_t + P^a{}_t x_t + q_t b_t \le s_{t-1} d_t + \Omega_t s_{t-1} P^b{}_t + b_{t-1} + F_t \qquad \text{for all } t \tag{2}$$

$$s_t = (1-\Omega) s_{t-1} + x_t \qquad \text{for all } t \tag{3}$$

Where c_t is per capita consumption, s_t are per capita share holdings at date t, b_t are per capita bond holdings at date t, β is the subjective discount rate and E_0 the expectations operator at date 0. d_t is a stochastic dividend stream accruing to stock holders and the risk-less bond holdings, b_t,denotes the payoff of one unit of consumption one period ahead. Ω_t is the proportion of an agent's stock portfolio liquidated in the financial sector and x_t represents the re-investment of funds in the mutual fund. q_t, $P^b{}_t$, and $P^a{}_t$ represent the frictionless bond price, the bid price, and the ask price for the share portfolio which are announced by the financial sector, with $P^a{}_t \ge P^b{}_t$. F_t denotes the lump sum per capita transfer payment from the financial sector.

The budget constraint presented in equation (2) restricts the agent to the following behaviour. The agent enters the period with s_{t-1} shares of stock and instantly collects his dividend, d_t, plus his bond payoffs. The cash flow from the liquidation of an agent's stock portfolio by the financial sector at the bid price is given as $\Omega_t s_{t-1} P^b{}_t$. The right-hand side of the constraint represents the agent's free cash flow consisting of dividends, the liquidated value of his share portfolio, bond receipts, and the transfer from the financial sector, F_t. The agent next considers how to allocate his wealth between consumption, new riskless bond issues, and rebalancing his mutual fund holdings. New shares must be purchased at the ask price, $P^a{}_t$, while bonds can be purchased at q_t.

Equation (3) specifies the law of motion governing mutual fund holdings. Since $\Omega_t s_{t-1}$ units of stock are liquidated within the financial sector, $(1-\Omega_t)s_{t-1}$ units remain untraded in the agent's portfolio. The desired level of stock holdings to carry forward into the next period, s_t, is attainable with the re-investment of x_t units in the fund.

Equations (1) to (3) describe the agent's maximisation problem. The agent must choose consumption, bond holdings, and share holdings to maximise expected utility subject to his budget constraint in each period. Calculating efficiency conditions with respect to c_t, b_t, and s_t, optimal asset choice in this economy can be shown to result in the following system.

$$\beta E_t u'(c_{t+1}) - u'(c_t)q_t = 0 \tag{4a}$$

$$\beta E_t u'(c_{t+1})\{\Omega_{t+1} P^b{}_{t+1} + (1-\Omega_{t+1})P^a{}_{t+1} + d_{t+1}\} - u'(c_t)P^a t = 0 \tag{4b}$$

$$b_{t-1} + s_{t-1}d_t + s_{t-1}P^b{}_t + F_t - q_t b_t - c_t - s_t P^a{}_t = 0 \qquad \text{for all } t \tag{4c}$$

The determination of the bid and ask prices takes place in the financial sector. It is assumed that the financial sector calculates the bid and ask prices by applying a proportional transaction cost to equity trades. This per transaction service charge is added or subtracted from the market price so that the bid and ask prices can be represented as:

$$P^{b}{}_{t} = P_t(1-\alpha) \qquad (5a)$$

$$P^{a}{}_{t} = P_t(1+\alpha) \qquad (5b)$$

Where α is the proportional transaction costs. To close the model the financial sector is constrained to rebate its earnings to agents each period and so obeys the following budget constraint.[3]

$$F_t = x_t P^{a}{}_{t} - \Omega_t s_{t-1} P^{b}{}_{t} \qquad (6)$$

The right-hand side of equation (6) represents the net per capita cash flows of the financial sector generated in the stock and bond markets from the agents' trading activity.

The objective of this model is to derive expressions determining expected gross returns. Accordingly, substituting equations (5a), (5b) and (6) into equations (4a)-(4c) and imposing the asset market-clearing conditions that $s_{t-1} = s_t = 1$ and $b_{t-1} = b_t = 0$ provides an equilibrium pricing relation of the form.

$$u'(c_t)q_t = \beta E_t u'(c_{t+1}) \qquad (4a')$$

$$u'(c_t)P_t = \beta E_t u'(c_{t+1})\left(\frac{d_{t+1}}{1+\alpha} + \left[1 - \frac{2\alpha\Omega_{t+1}}{1+\alpha}\right]P_{t+1}\right) \qquad (4b')$$

$$c_t = d_t \qquad (4c')$$

Equation (4a′) is the familiar pricing equation for a security, which pays off one unit of consumption one period ahead under uncertainty. Equation (4c′) ensures

[3] This assumption has two purposes: (1) it ensures the existence of a suboptimal competitive equilibrium and (2) it simplifies the solution method used to simulate the model. Alternative rebating schemes will not affect the equilibrium provided they are not related to the investment decision.

that, consistent with equilibrium in an endowment economy, output is consumed each period.

Equation (4b′) deserves careful consideration. This expresses the equilibrium pricing relation in terms of the market price of the stock portfolio P_t. From the agents point of view this pricing equation has two crucial features. First, the capital gains and the dividend components of the expected gross return on the share portfolio are treated differently by the agent. To receive an additional unit of income in the form of dividends one period ahead an agent must purchase shares at the ‘ ask price’. However, since future dividends are paid to the shareholder directly, there are no transaction costs incurred on receiving a security’s payoffs in this form. Equation (4b′) shows that dividend income is discounted by the factor $(1/1+\alpha)$ to adjust for the marginal transaction cost of share purchases out of future dividends. The capital gains component of an agent’s cash flow, alternatively, is earned by liquidating securities in the secondary market.

From the agent’s viewpoint, the possibility of future liquidations of stock by mutual fund managers is a cost of holding equity in addition to the marginal cost of purchasing shares. Whilst the non-liquidated proportion of a stock portfolio may be carried forward into future periods. Thus the capital gains component of share ownership is discounted by the factor

$$\left[1-\frac{2\alpha\Omega_{t+1}}{1+\alpha}\right] \qquad \text{in equation (4b′).}^{4}$$

[4] In the certainty case, capital gains are discounted at a higher rate than dividends when $\Omega_{t+1} \succ 0.5$, reflecting the high costs of liquidating claims each period.

In the same equation it is apparent that prices also depend upon the expected future turnover rate (Ω_{t+1}). This is unlike most asset-pricing models that do exhibit such dependence upon a measure of turnover. The inclusion of this measure provides for the distinction between expected asset returns and implied asset trades. That is expected returns must reach a threshold for a trade to occur. Conventional models to do not allow for this thus implying 'too frequent' trading.

Equation (4b′) shows that when liquidating assets is costly, the proportion of a portfolio that is traded is an important determinant of the price of an asset in equilibrium. Expected returns should reflect the expected costs of trading assets. The pricing equation provides a link between turnover in financial markets and the price an optimizing agent is prepared to pay for an asset in the market. A model that breaks the separation between asset prices and trading volume is appealing once it is recognized that portfolio reallocation is costly. Higher asset turnover must necessarily generate higher transaction costs which agents should expect to be compensated for in the form of higher expected returns.

The effect of introducing the bid-ask spread and asset turnover into the agent's optimization problem will lead ceteris paribus to a higher expected return on risky equity to compare to the case of a zero bid-ask spread. The quantitative significance of the effects of these variables requires a formal estimation and this is the focus of the next section.

3 Generalized Method Of Moments Estimation

This section formally tests the Fisher model discussed is section 2 using the GMM technique set out in Hansen (1982). Section 3.1 expresses the gross equity premium as a function of the state variables and exogenous parameters in the model to deliver a reduced form amenable to testing the GMM. Section 3.2 describes the data employed to test the model. Section 3.3 presents the results of the tests.

3.1 Calculating the Equity Premium

Define $R_t = (P_{t+1} + d_{t+1}) / P_t$ and $R^f{}_t = (1/q_t)$. Then equations (4a') and (4c') combine to give:

$$1 = \beta E_t \frac{u'(d_{t+1})}{u'(d_t)} R^f{}_t \tag{7}$$

Similarly, equations (4b') and (4c') result in the following expression.

$$1 + \alpha = \beta E_t \frac{u'(d_{t+1})}{u' d_t} R_t + \beta E_t \left\{ \frac{u'(d_{t+1})}{u'(d_t)} \frac{P_{t+1}}{P_t} (\alpha - 2\alpha\Omega_{t+1}) \right\} \tag{8}$$

We assume that agents' utility is given by a time-separable, constant relative risk aversion utility function of the form.

$$u(c_t) = \frac{c_t^{1-\gamma} - 1}{1-\gamma} \qquad \gamma \succ 0, \quad \neq 1$$

$$= \log c_t \qquad \gamma = 1$$

The marginal rate of substitution can be represented as:

$$\frac{u'(d_{t+1})}{u'(d_t)} = \left(\frac{d_{t+1}}{d_t} \right)^{-\gamma}$$

Making this substitution and subtracting equation (7) from equation (8) gives the following solution:

$$\beta E_t\left(\frac{d_{t+1}}{d_t}\right)^{-\gamma}\left\{(R_t - R^f{}_t) + (\alpha - 2\alpha\Omega_{t+1})\frac{P_{t+1}}{P_t}\right\} - \alpha = 0 \quad (9)$$

Equation (9) is a nonlinear stochastic Euler equation of the form

$$E_t h(x_{t+1,}\lambda_0) = 0$$

Where:

$$x_{t+1} = \left[\frac{d_{t+1}}{d_t}, R_t - R^f{}_t, \frac{P_{t+1}}{P_t}, \Omega_{t+1}\right]$$

is a vector of variables observed at date $t+1$, while $\lambda_0 = [\alpha, \beta, \gamma]$ is an unknown parameter vector to be estimated. E_t is the expectations operator at date t conditioned on all variable information.

The estimation procedure described in Hansen and Singleton (1982) can be implemented in two steps using standard gradient methods for nonlinear least squares estimation. This is undertaken for the reduced form (9).

3.2 Data Description

Monthly data are collected for the FTSE All Share index for the time period 1980-2000.[5] Following Brown and Gibbons (1985), the variable $d_{t+1} - d_t$ is proxied using the growth rate of private consumption. UK stock returns on the FTSE All Share index and the returns on 3-month treasury bills are used to generate the series for $R_t - R^f{}_t$.

Following the methodology suggested by Fisher we obtain a measure of stock market price growth, P_{t+1}/P_t, using the FTSE transportation and industrial

[5] The problem with monthly data is that it may suffer from seasonal effects. In order to overcome this problem we collect seasonally adjusted data.

indices from 1980 to 2000. We calculate a composite FTSE index by combining the transportation and industrial indices with weights calculated to reflect the number of stocks represented by each index. [6]

Finally the data used to calculate the turnover rate are taken as a proxy for Ω_t and Ω_{t+1} in x_{t+1}. We calculate the turnover rate using data from the FTSE All Share index as

$$\Omega_t = \frac{(\text{Total number of shares traded})_t}{(\text{Total number of shares outstanding})_t}.$$

Theoretically, an infinite number of potential instruments are contained in individuals' information sets but these are not specified by the theory. This study follows Hansen and Singleton (1982) by using a constant, c, and lagged values of the state vector, so that $z_t = [c, x_1, x_{t-1}, \ldots\ldots\ldots, x_{t-n+1}]$ for n lags.[7] In practice the lag length is set at $n = 1$, 2, and 4.[8] The sensitivity of these results to the choice of instruments is investigated using the methods recommended by Pagan and Jung (1993) and Staiger and Stock (1993).[9]

3.3 Results

For the optimization algorithm to converge it is necessary to restrict one of the parameters. Since the subjective discount factor is the parameter of least interest in the present study, it is restricted to assume the value $\beta = 0.99$ in line with

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[6] Nominal stock prices and consumption are deflated by the implicit consumption deflator.

[7] The turnover rate is differenced in the instrument vector to ensure stationarity so that Ω_t and Ω_{t-1} appear in the z_t vector as $\Omega_t - \Omega_{t-1}$.

[8] This is the lag length that was suggested by Fisher (1994).

[9] We collect all the data with the use of Datastream.

economically acceptable values for this parameter.[10] The results therefore report estimates for $\hat{\lambda}_0$ conditional on one element being fixed. Restricting β places restrictions on other aspects of the model. Equation (4a') shows, for instance, that β is a major determinant of the level of the risk-free rate.

Table 1 shows the parameter estimates of the following model for the entire samples from 1980 to 2000.

Model: $$\beta E_t \left(\frac{d_{t+1}}{d_t} \right)^{-\gamma} \left\{ (R_t - R^f{}_t) + (\alpha - 2\alpha\Omega_{t+1}) \frac{P_{t+1}}{P_t} \right\} - \alpha = 0 \qquad (9)$$

Table 1 is arranged to report the values of $\hat{\alpha}$ and $\hat{\gamma}$ together with the chi-square statistic testing the overidentifying restrictions of the model and its p-value. The null for this test is that the overidentifying orthogonality restrictions are satisfied.

[INSERT TABLE 1 HERE]

For the full sample Table 1 reports estimates of $\hat{\alpha}$ which range between 0.013 and 0.076 depending upon the number of instruments selected. This corresponds to an estimated bid-ask spread of 1.3% to 7.6%. Estimates of the risk aversion parameter lie between 2.79 and 3.86. The Wald test of the overidentifying restrictions of the model never rejects the null hypothesis at the 5% level of significance.

The exact impact of the transactions costs embedded in the null hypothesis depends on the lag structure of the chosen instrument set. The test for the

[10] Mehra and Prescott (1985) calculate the discount rate to be equal to 0.99 using US historical data from 1900-1985.

overidentifying restrictions cannot be rejected at conventional levels of significance for the chosen information structures.

When the instrument set is reduced to the first lag, the first row of the Table reports estimates of $\hat{\alpha} = 0.013$ and $\hat{\gamma} = 2.79$. This implies a bid-ask spread of 1.3%, which seems low compared to Fisher's estimate of 9.4% and Stoll and Whaley's estimate of 2.79%.[11] The effect of adding more instruments is inconclusive. If two lags of the instrument set are employed, the p-value is 0.227, while the value of $\hat{\alpha} = 0.022$, which implies a bid-ask spread of 2.2%. If four lags of the information are used, the estimate $\hat{\alpha} = 0.076$ implies an implausibly large value of the bid-ask spread of 7.6%.

On the whole our results are realistic when we compare them to other studies. According to Stoll and Whaley the estimate for the bid-ask spread should be around 2.9%.

The most important result in our study are the significance of the transactions costs in the estimated equation. The parameter α is significantly different from zero based on t-statistics, indicating that transactions costs are important in asset pricing. We also find that γ is significantly different from zero irrespective of the chosen instrument set. Our estimates of the parameter appear very reasonable in terms of economic theory and close to ones estimated by Fisher for the US. The stability, the estimates, (vis-a-vis the information set) and statistical significance indicate that risk aversion is important and must be included in asset pricing. The

[11] Stoll and Whaley (1983) estimate the bid-ask spread on the NYSE between the time period of 1961-1981.

results are suggestive of strong support of the hypothesis that transactions costs are important determinants in asset pricing.

Since the GMM results are based on an instrumental variables estimation procedure, the credibility of the results depend on the quality of the instruments that are used. Pagan and Jung (1993) point out instances where the performance of the GMM estimator in small samples is poor due to poor instruments, and suggest diagnostic tests to evaluate the efficiency of the procedure. In the present context, the tests of the overidentifying restrictions and the parameter estimates reported in Table 1 might be misleading if the instruments are weakly correlated with the endogenous components of the stochastic Euler equations comprising the restrictions from the model.

Pagan and Jung (1993) suggest that (1) calculating the R-squared from regressing the derivatives of the Euler equations with respect to the estimated parameters against the instrument set and (2) an examination of the cross-correlations of these derivatives provide a check on the likely performance of the GMM estimator.[12] The results of these diagnostics are reported in Table 2 for the derivatives of the moment conditions with respect to α and γ for each set of instruments.

[INSERT TABLE 2 HERE]

Denoting R_{α}^{2} and R_{γ}^{2} as the R-squared from the regression of the derivatives against the bid-ask spread and the risk aversion parameter respectively, R_{α}^{2} peaks

[12] In applied work such as this, Pagan and Jung (1993) recommend that the derivatives be evaluated using the point estimates from the GMM estimator. In addition, the correlations of the derivatives with respect to each parameter will influence the performance of the estimator.

at 0.049 with 1 lag of the instrument set while $R_\gamma{}^2$ peaks at 0.042 with 2 lags of the instrument set. The correlations of the partial derivatives reported in the final column of Table 2 are quite strong at around an average of –0.72, so that it is difficult to make independent statements about the efficiency of each parameter estimate.

Based on the results of Table 2, the instrument sets employed in the GMM estimation do not diminish confidence in the estimation procedure. The diagnostics do suggest, however, that the estimate of the bid-ask spread parameter is relatively more efficient then that of the risk-aversion parameter.

The results so far are indicate that as we vary the instrument lag structure the estimates of the parameters (especially those of α) change. The resulting test statistics cannot provide us with an unambiguous choice of α . It appears to us the estimate of 2.2% is the one most consistent with observation.

4 Present Value Tests of the CCAPM including Transactions Costs

In the presence of the problem that was discussed above, Lund and Engsted (1996) suggest that the only way to obtain robust results of the C-CAPM is to estimate the model using both the GMM methodology and the VAR methodology proposed by Campbell and Shiller (1988a). The difference between the two methodologies is that the GMM methodology is based on the orthogonality condition given by the first order condition of the inter-temporal optimization problem (the Euler equation). The VAR approach is based on the linearised present value model that can be derived from the Euler equation. In other words, the difference between the two methodologies is the following:

The GMM uses information in order to derive an estimate of the bid-ask spread. From this estimate a t-statistic is calculated and the significance of the bid-ask spread in asset pricing models is evaluated. The GMM has two shortcomings; first, the results depend on the quality of the instruments that are used to proxy for the information, and second, the coefficient and the significance of the bid-ask spread variable are sensitive to the lag structure of the instruments.

However, the VAR approach differs from the GMM because it uses actual data on the bid-ask spread to calculate a test statistic to evaluate the significance of the bid-ask spread in asset pricing models. The VAR approach estimates the C-CAPM with the bid-ask spread included as an additional explanatory variable. The bid-ask spread is then tested for significance. The VAR approach provides further corroborative evidence, to the GMM based model, as the econometric results do not depend on instruments as are not required for the estimation.

In the next section of this paper we extend the VAR approach proposed by Campbell and Shiller (1988a) to include the bid-ask spread as an explanatory variable in the CCAPM. We then perform statistical tests to determine whether the bid-ask spread should be included in the CCAPM.

4.1 The Model

Following Lucas (1978) we assume the existence of a representative investor who chooses to consume and invest in a single asset (a stock index) so that at each time t she maximizes expected lifetime utility

$$MaxE_t\left[\sum_{\tau=0}^{\infty}\beta^{\tau}U\left(C_{t+\tau}\right)\right]; \qquad (10)$$

Subject to the budget constraint

$$C_t + S_t W_t = R_t S_t W_{t-1}; \qquad R_t = \frac{(P_t + D_t)}{P_{t-1}}, \tag{11}$$

Where:

W_t is the wealth invested in the stock index,

C_t is the real consumption,

P_t is the ex-dividend real stock price,

D_t is the real dividend received between time $t-1$ and t.

S_t is the bid-ask spread at time period t. [13]

We include transactions costs in the budget constraint of the investor. In this set-up the investor now has two budget constraints, the usual wealth constraint and the transactions costs that are incurred whenever she decides to trade the asset.

The first-order condition in this maximization problem is the stochastic Euler equation (Lucas, 1978).

$$E_t\left[\beta \frac{U'(C_{t+1})}{U'(C_t)} R_{t+1} S_{t+1} - 1\right] = 0 \tag{12}$$

Lucas (1978) considers a pure exchange economy with one perishable consumption good. This implies that we can ignore consumption decisions because by definition the representative investor must consume the entire income. However, with the utility function used below the equation above also obtains in

[13] Transactions costs are proxied by the bid-ask spread. Transactions costs are included as a single variable because both the Augmented Dickey Fuller (ADF) test (1981) and the Phillips Peron (PP) test (1988) suggest that the bid price minus the ask price follows a stationary process.

the general production economy of Breeden (1979) and Cox et al (1985) where consumption and investment decisions are made jointly.

In order to obtain testable implications we must specify a utility function for the representative investor. As in most other studies Campbell and Shiller (1988a) use the constant relative risk aversion (CRRA) utility function:

We can now go on and test the C-CAPM under transactions costs using the present value test suggested by Campbell and Shiller (1988a). We begin by defining h_{1t} as the logarithm of the utility-adjusted return, which is expressed by the following equation:

$$h_{1t} = \log\left(\frac{U'(C_{t+1})}{U'(C_t)} R_{t+1}(S_{t+1})\right) = \log\left(\left(\frac{P_{t+1}+D_{t+1}}{P_t}\right) - \left(\frac{S_{t+1}}{S_t}\right)\right) + \alpha\left(\frac{C_{t+1}}{C_t}\right) \quad (13)$$

$$= \log\left(\exp\left(\delta_t - \delta_{t+1}\right) + \exp\left(\delta_t\right)\right) + \Delta d_{t+1} - \Delta S_{t+1} + \alpha \Delta c_{t+1},$$

Where $\delta_t = \log\left(\frac{D_t}{P_t}\right)$, $d_t = \log(D_t)$ $S_t = \log(S_t)$ and $c_t = \log(C_t)$

Where:

δ_t is the natural logarithm of the dividend-price ratio at time period t.

d_t is the natural logarithm of dividends at time period t.

S_t is the natural logarithm of the bid-ask spread at time period t.[14]

Next, we linearize (13) around the point $\delta_t = \delta_{t+1} = \delta$:

$$h_{1t} \approx \delta_t - \rho\delta_{t+1} + \Delta d_{t+1} - \Delta S_{t+1} + \alpha \Delta c_{t+1} + K = \xi_{1,t}. \quad (14)$$

[14] The CCAPM is estimated in logarithms because of the excess skewness and kurtosis present in the raw data.

Here, $\rho = \frac{1}{(1+\exp(\delta))}$; while K is an inessential constant from the linearization.

Define variable ξ_1, by the sum of $\xi_{1,t}$ we have

$$\xi_1 = \sum_{j=0}^{i-1} \rho^j \xi_{1,t+j} = \delta_t - \rho^i \delta_{t+i} + \sum_{j=0}^{i-1} \rho^j \left(\Delta d_{t+j+1} - \Delta S_{t+j+1} + \alpha \Delta c_{t+j+1}\right) + \frac{1-\rho^i}{1-\rho} K. \quad (15)$$

We assume that $E_t(\xi_{1,y+j})$ is equal to a constant c for all $j \geq 0$. In equation (13) we expect $E_t(\xi_{1,y+j})$ to be close to $-\log(\beta)$, so the linearisation is a good approximation of (13). Given that this holds, we take conditional expectations on both sides of (15) and whilst we allow $i \to \infty$. After some manipulations we obtain the following equation.

$$\delta_t = -\sum_{j=0}^{\infty} \rho^j E_t \left(\Delta d_{t+j+1} - \Delta S_{t+j+1} + \alpha \Delta c_{t+j+1}\right) + \frac{c-K}{1-\rho}, \quad (16)$$

Since lim $\rho^i E_t(\delta_{t+i}) = 0$ as $i \to \infty$ (otherwise δ_t is non-stationary with an explosive root indicating that stock prices are driven by a rational bubble).[15]

With the use of equation (16) we can perform a statistical test to discover whether transactions costs should be incorporated in the C-CAPM. If transactions costs should be included, then the coefficient associated with ΔS_t will be statistically significant in the VAR model. We can employ this test by testing the cross-equation restrictions implied by the underlying theory on a VAR model. We define a limited information set H_t containing past and present values of

[15] Note that by letting the VAR model be formulated in terms of changes in dividends and consumption in terms of the dividend-price ratio instead of stock prices themselves, the results are robust to possible nonstationarity of p_t, d_t and c_t.

$x_t = (\delta_t, \Delta d_t, \Delta c_t, \Delta S_t)'$, and assume that expectations conditional on H_t are linear projections on the information set. This corresponds to the $\text{VAR}(p)$ specification:

$$z_{t+1} = Az_t + u_{t+1} \qquad (17)$$

Where $z_t = (x_t - E(x), ..., x_{t-p+1} - E(x))$ is a $4p \times 1$ vector and A is the $4p \times 4p$ companion matrix of the $\text{VAR}(p)$ system. See Campbell and Shiller (1987, 1988a) for details.

4.2 Data Description

For this application we are using monthly seasonally adjusted aggregate consumption expenditure data for the UK. For the dividend to price ratio we collect data on the price and dividend yield of the FTSE All Share index. The bid-ask spread is calculated by collecting data on the bid price and the ask price on the FTSE All Share index. Nominal stock prices, dividends and consumption are deflated by the implicit consumption deflator. The time period of the data is between 1980-2000. All the data are collected from Datastream.

4.3 Results

We can test if transactions costs are statistically significant in the C-CAPM by employing the Granger-Causality tests proposed by Lund and Engsted (1996).[16] If past values of the changes in the bid-ask spread predict the dividend-price ratio, one can say that bid-ask spreads "Granger-cause" dividend yield. This would imply that transactions costs are statistically significant in the C-CAPM, which would therefore suggest that transactions costs are important in asset pricing

[16] For more details on Granger-Causality tests see Granger (1969).

models. We test this hypothesis in the context of the following system, included in the previously defined VAR.

$$\delta_t = \sum_{k=1}^{2} \alpha_k Z_{t-k} - \sum_{k=1}^{2} \beta_k \Delta S_{t-k} + u_t \qquad (18)$$

where Z denotes all other information

Where k represents the number of months each variable is lagged and the variables are as defined earlier and test the restriction:

$$\sum \beta_k = 0, \ \forall k$$

In this case both p and q are equal to 2[17]. The test results in an F-statistic of 5.04 with a p-value of 0.02. The null hypothesis is rejected, implying that past values of the bid-ask spread do affect the dividend yield. This leads us to conclude that transactions costs should be included in the C-CAPM.[18]

We can obtain a measure of transactions costs by solving equation (16) for the expectation of δ with respect to ΔS, given the fact that ΔS is stationary. From equation (16) we find that when δ is equal to 4.02% p.a.(the sample mean), transactions costs are equal to 2.64% p.a.. This estimate mimics closely our previously obtained value of 2.2% (under GMM with two lags).

It is very encouraging that using two different methodologies result in almost identical estimates regarding the importance of transactions costs in the UK equity market. Transactions costs were shown to have an independent influence in

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[17] Both the Akaike information criterion and the Schwartz Bayesian criterion suggest that the optimal lag for δ and ΔS is equal to 2.

[18] The estimation of the entire equation (16) accompanied with diagnostic tests can be seen in Table 3, which can be found in the appendix.

the presence of other market information and their inclusion in the pricing model does not depend neither upon the chosen functional form nor the chosen estimator.

5 Discussions and Conclusions

In this paper we tested if transactions costs should be included in asset pricing models. We tested this hypothesis by using two different methodologies. First, we apply the equilibrium asset-pricing model proposed by Fisher (1994). The Fisher model is unique from other asset pricing models because it includes the bid-ask spread as a variable that influences excess returns, whereas the more traditional asset-pricing models include only the level of risk as the factor that influences excess returns.

We estimate the Fisher model with the GMM estimation technique of Hansen (1982) using seasonally adjusted monthly data for the UK stock market over the time period of 1980-2000. The formal GMM tests of the model yield economically plausible values of the unknown parameters, and tests of the overidentifying restrictions of the model could not be rejected.

The model appears to perform relatively well when confronted with data. The parameter associated with our chosen proxy for the bid-ask spread as well as the risk-aversion parameter was found to be significant for all the different instrument sets that we tested. This lead us to conclude that both transactions costs and risk should be included in asset pricing models, due to our evidence from the UK stock market. Fisher has also found similar evidence of the importance of transactions costs in asset pricing with respect to the US stock market.

As the GMM estimation procedure relies heavily on the quality of the instruments, we therefore tested the quality of the instruments of our model using the diagnostic tests proposed by Pagan and Jung (1993). We found the instruments to be adequate, a result that for our estimation period and sample overcame a fundamental problem with the GMM estimation technique. In order to establish the robustness of our results we tested the same hypothesis in the context of the C-CAPM under transactions costs using a different methodology that does not suffer from the problems of the GMM.

We provide further evidence of the relationship between the dividend to price ratio and the bid-ask spread. We found that when transactions costs are equal to 2.64%. pa. the resulting dividend yield equals to 4.02% pa. (for any given total return).

Our findings would appear to have important implications for the many empirical studies that use some version of the CAPM to adjust for risk. Databases such as those compiled by the Center of Research in Security Prices only report gross returns on a daily, monthly or annual basis. This means that researchers using gross returns data to adjust for risk without specifying structural assumptions on the transactions technology may bias their results. We very rarely see such assumptions being stated explicitly.[19]

The many studies that document anomalies in financial markets as well as the many that are consistent with market efficiency may have been predicted upon data that do not fit the specifications of the hypotheses being tested. The empirical

[19] Deechow (1990) makes this point with reference to the detection of accounting anomalies.

findings in this paper give evidence to suggest that this criticism cannot be dismissed.

In terms of further work one could look at extending the Fisher Asset pricing model and the VAR model which includes the spread, with the use of a decreasing absolute risk aversion utility (DARA) function instead of the CRRA utility function that is currently used. The rationale for doing this would be to see how an investor's attitude to risk influences the relationship between transactions costs and asset pricing.

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Appendix The Estimation of Equation (16)

TABLE 3. Present value Tests of the CCAPM including Transactions Costs, estimated for the UK stock market for the period 1980-2000.

Lag	N	Δd_t	ΔS_t	Δc_t	$\overline{R}^2$
1	239	0.423 (2.21)*	-0.029 (-2.89)*	0.343 (2.45)*	0.372
2	238	0.315 (2.54)*	-0.026 (-2.72)*	0.294 (2.36)*	0.351

Notes:

The t statistics are shown in brackets and * indicates significance at the 5% level.
All the variables in the above equation are expressed as natural logarithms.

Diagnostic results (p-values)

Period	Serial Correlation Test	Heteroscedasticity Test	Normality Test	Functional Form Test
1980-2000	0.48	0.27	0.35	0.75

Notes:

All the diagnostic statistics that are reported are based on the F statistic.
The heteroscedasticity test is based on the test proposed by White (1980).
The serial correlation test is based on the test proposed by Godfrey (1978a, 1978b).
The normality test is based on the test proposed by Jacque and Bera (1987).
The functional form test is based on the Ramsey (1969) test.

Tables

TABLE 1. GMM estimates for the period 1980-2000

NLAG	NOBS	$\hat{\alpha}$	$\hat{\gamma}$	χ^2	Degrees of freedom	p-value
1	239	0.013 (2.10)*	2.79 (2.62)*	2.61	3	0.106[a]
2	238	0.022 (2.09)*	2.997 (2.89)*	4.33	7	0.227[a]
4	236	0.076 (2.42)*	3.68 (2.49)*	4.96	15	0.664[a]

Notes:

γ is the estimate of the coefficient of relative risk aversion.

The subjective discount factor is restricted to assume the value of $\beta=0.99$.

White consistent adjusted t-statistics shown in brackets () and * indicates significance at 5% level.

Instrumental variables used are [z_t = constant, $x_t,\ldots,x_{t-n+1}$] for $n=n$ lag and

$$x_t=\left[\frac{d_{t+1}}{d_t}, R_t-R^f{}_t, \frac{P_{t+1}}{P_t}, \Omega_t-\Omega_{t-1}\right]$$

representing, respectively, output growth, the equity premium, FTSE price index growth, and the difference of the FTSE turnover rate and its lag.

[a] Do not reject the hypothesis that the overidentifying restrictions are orthogonal to the errors at the 5% level of significance.

TABLE 2. Diagnostic tests examining the efficiency of the instruments employed in the GMM estimates of the model

Instrument set NLAG	${R_\alpha}^2$	${R_\gamma}^2$	$Cor\left(\frac{\partial h_t}{\partial \alpha}, \frac{\partial h_t}{\partial \gamma}\right)$
1	0.049	0.007	-0.69
2	0.016	0.042	-0.71
4	0.011	0.002	-0.77

Notes:

${R_\alpha}^2$ denotes the R-squared from the regression of $\frac{\partial h_t}{\partial \alpha}$, against the instrument set.

${R_\gamma}^2$ denotes the R-squared from the regression of $\frac{\partial h_t}{\partial \gamma}$, against the instrument set.

$Cor\left(\frac{\partial h_t}{\partial \alpha}, \frac{\partial h_t}{\partial \gamma}\right)$ is the correlation of the partial derivatives evaluated at the point estimates from each instrument set.