November 29, 2011

Lecture 25

Impedance Matching

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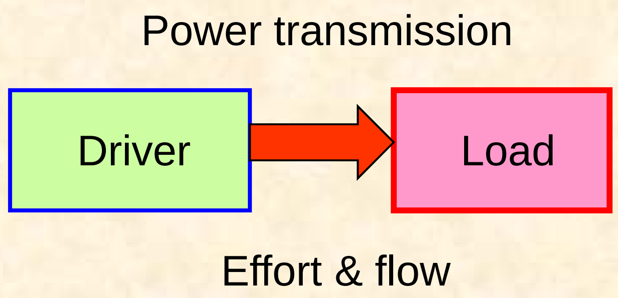
Note: You will not learn the following material in an engineering course. However, it is the most important technical material your lecturer learned & practiced in the last 30 years.

Impedance matching

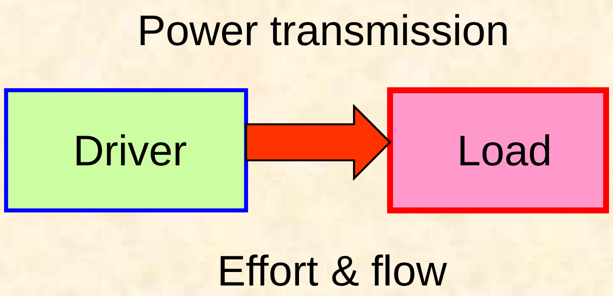


Take a driver and connect it to a load. Assume the system operates at a **steady-state condition** (time invariant)

Drivers are power supplies, batteries and generators, motors, turbines, IC engines, bike rider, etc. A few **loads** are electrical appliances (ovens, lights), PCs, pumps, compressors, fans, electrical generators, road conditions, etc.

The aim is to match the driver to the load to transmit power in the best & most efficient manner

Efforts and flows

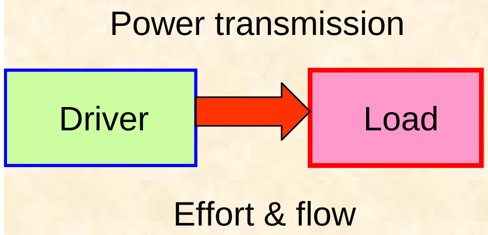


The driver delivers an **effort (*e*),** typically a function of its **flow (*f*).**

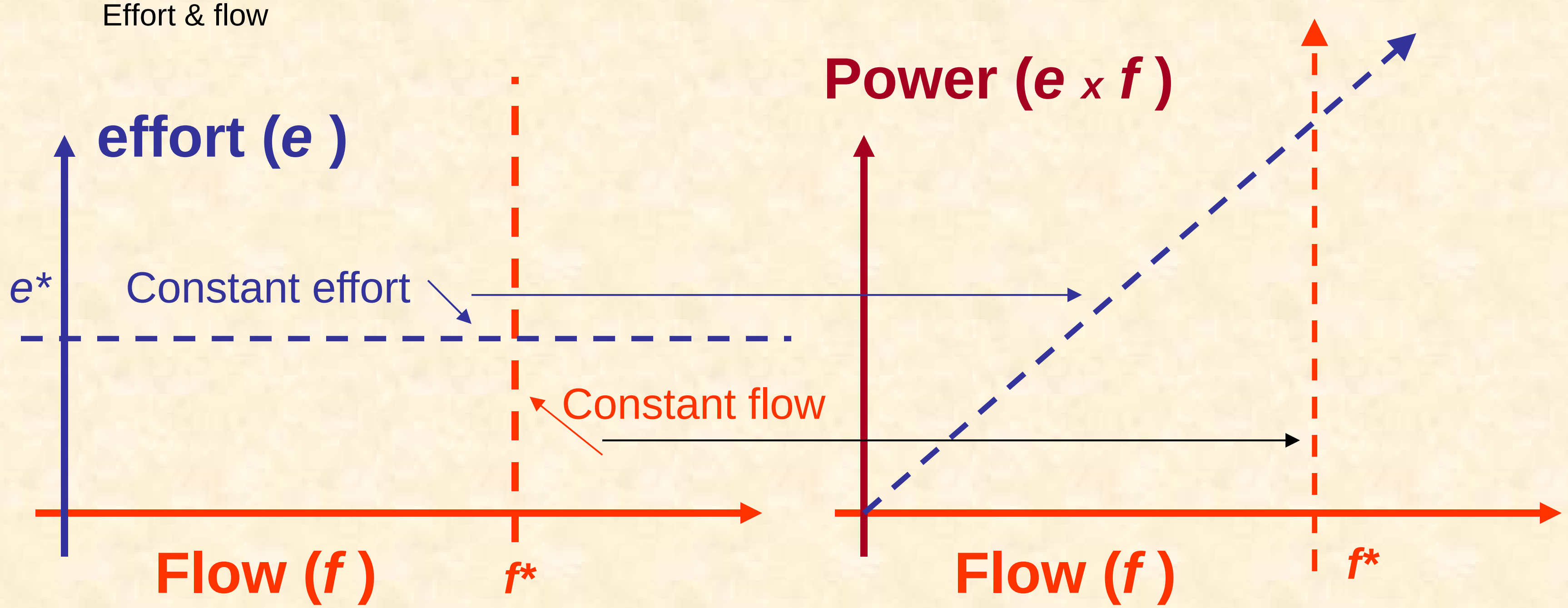
Power (*P*) = $e \times f$ (W)

System type	***effort***	***flow***
Mechanical translation	***F*** : Force (N)	***v*** : Velocity (m/s)
Mechanical rotational	***T*** : Torque (N.m)	ω: Angular speed (rad/s)
Electrical	***V*** : Voltage (V)	***I*** : Current (A)
Fluidic	ΔP: Pressure drop, (N/m^2)	***Q***: Flow rate (m^3/s)
Thermal	ΔT:Temperature, (°C)	***q***: Heat flow (W)

Ideal sources of effort and flow



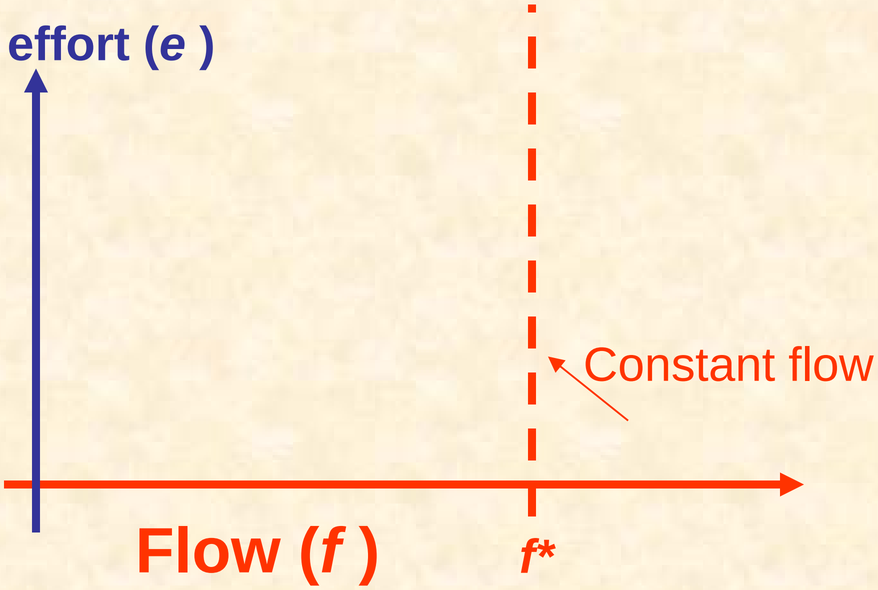
Power (*P*) = *e* x *f* (W)



Ideal sources provide as much power as needed by load. Examples?

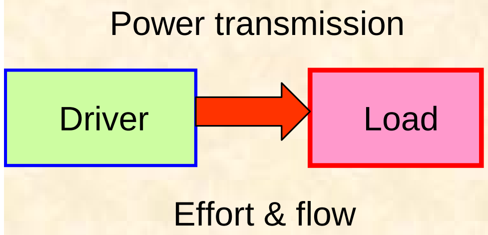
Ideal source of flow: a river

How is a river an ideal source (*f =invariant)? Wouldn't flow increase with the pressure difference or height ?**

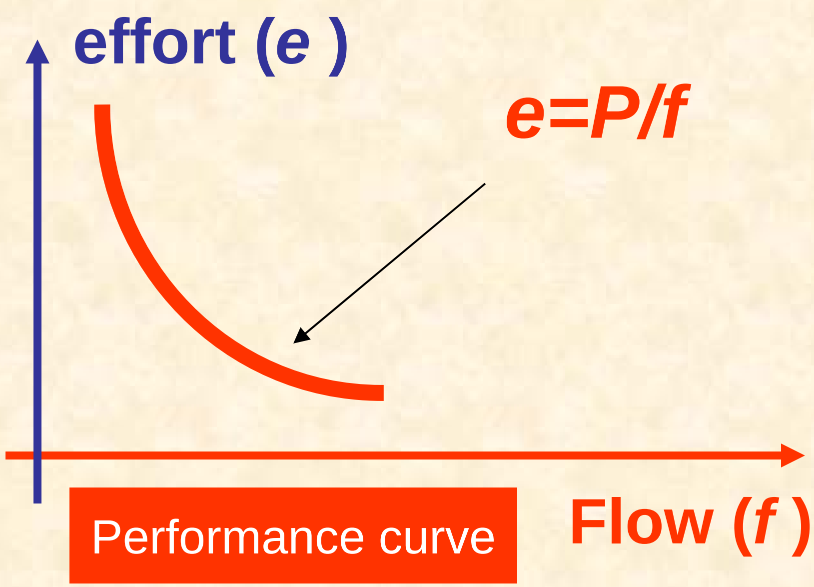


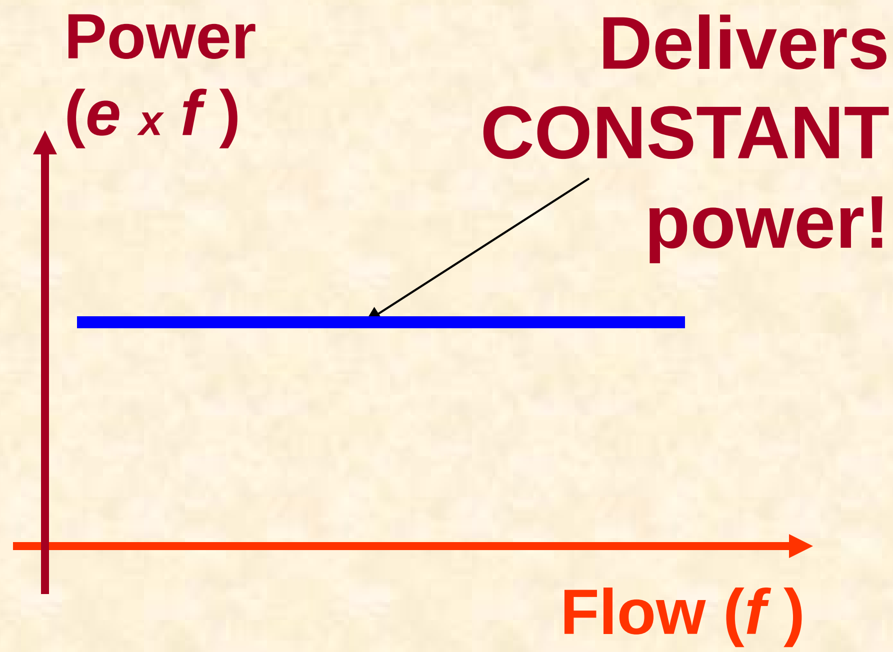
Flow variation is seasonal. However, for operating purposes, flow is NOT affected by the load. That is, upper stream condition is NOT disturbed by what happens downstream.

Most Ideal driver



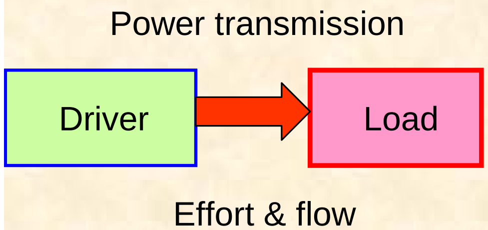
Power *(P)* = *e* x *f* (W)



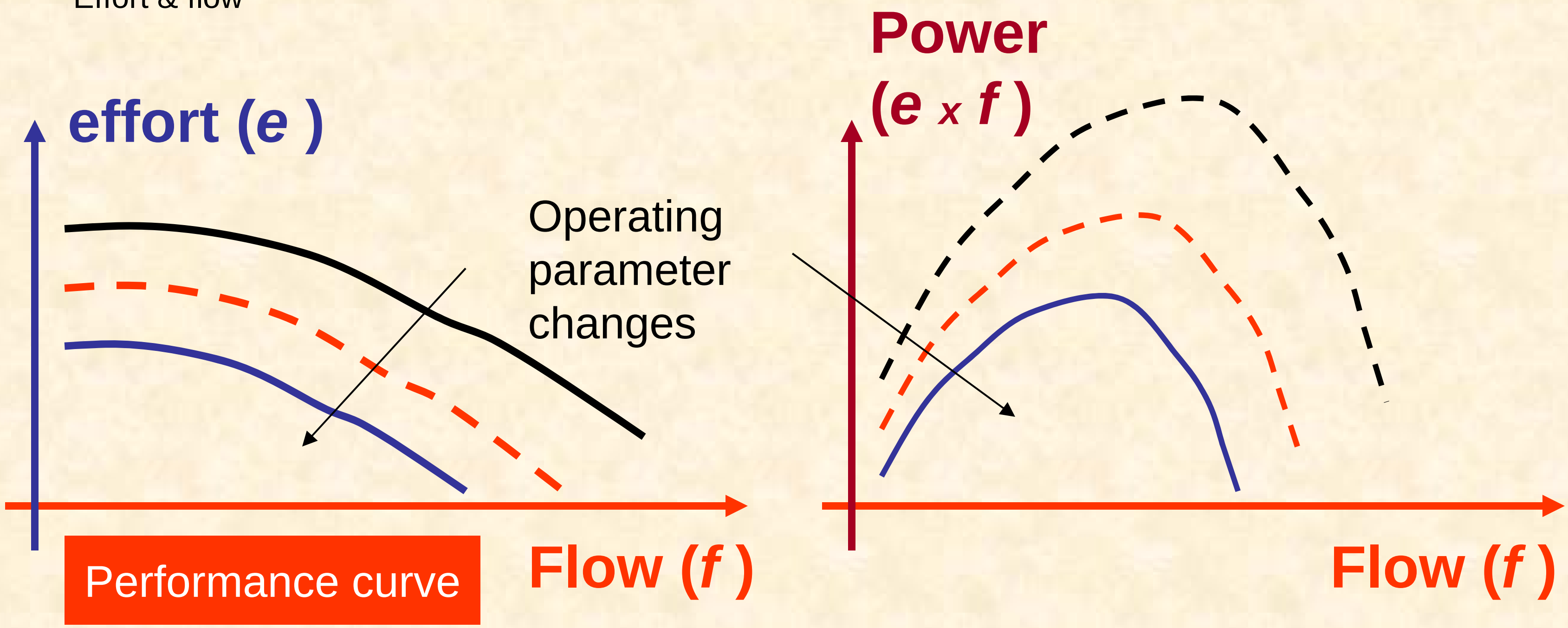


Demands TOO large effort at low flows AND TOO large flows at low efforts

Real driver: effort and flow



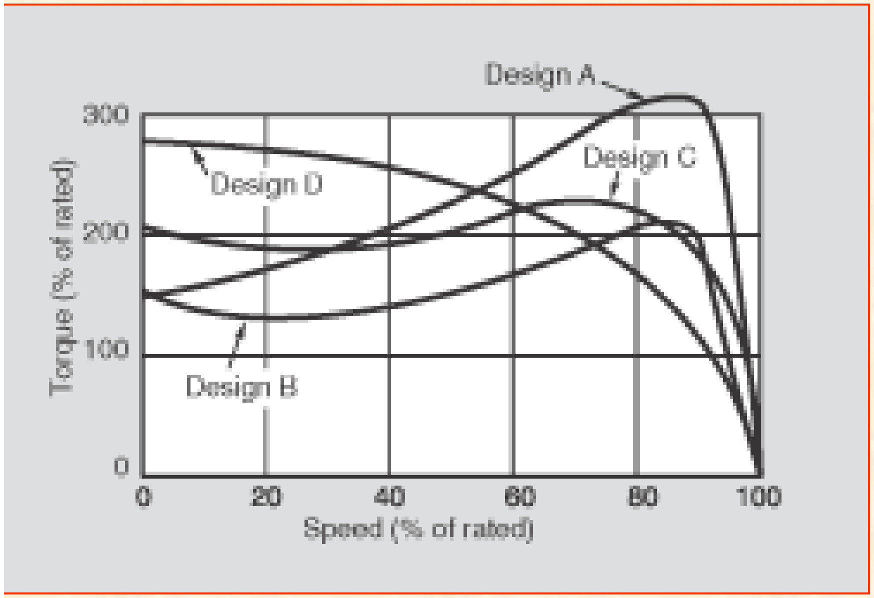
Power $(P) = e \times f$ (W)



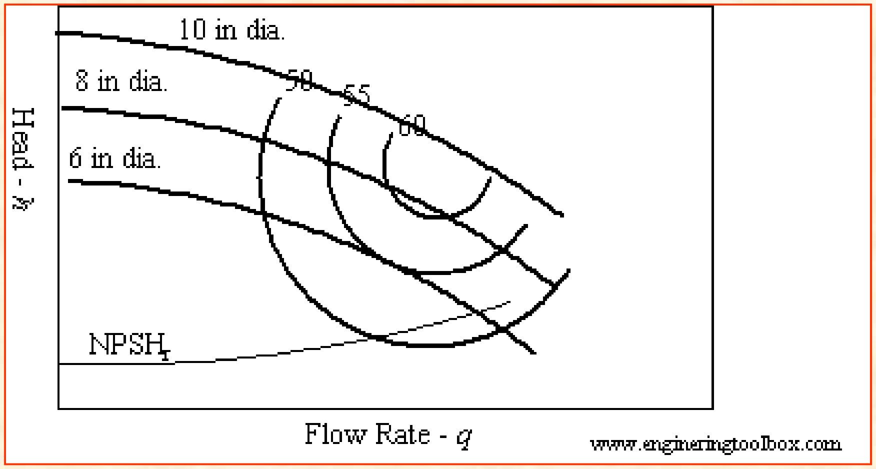
Actual drivers deliver limited power!

Typical performance maps

http://www.electricmotors.machinedesign.com/guiEdits/Content/bdeee11/bdeee11_7.aspx



http://www.engineeringtoolbox.com/pump-system-curves-d_635.html#

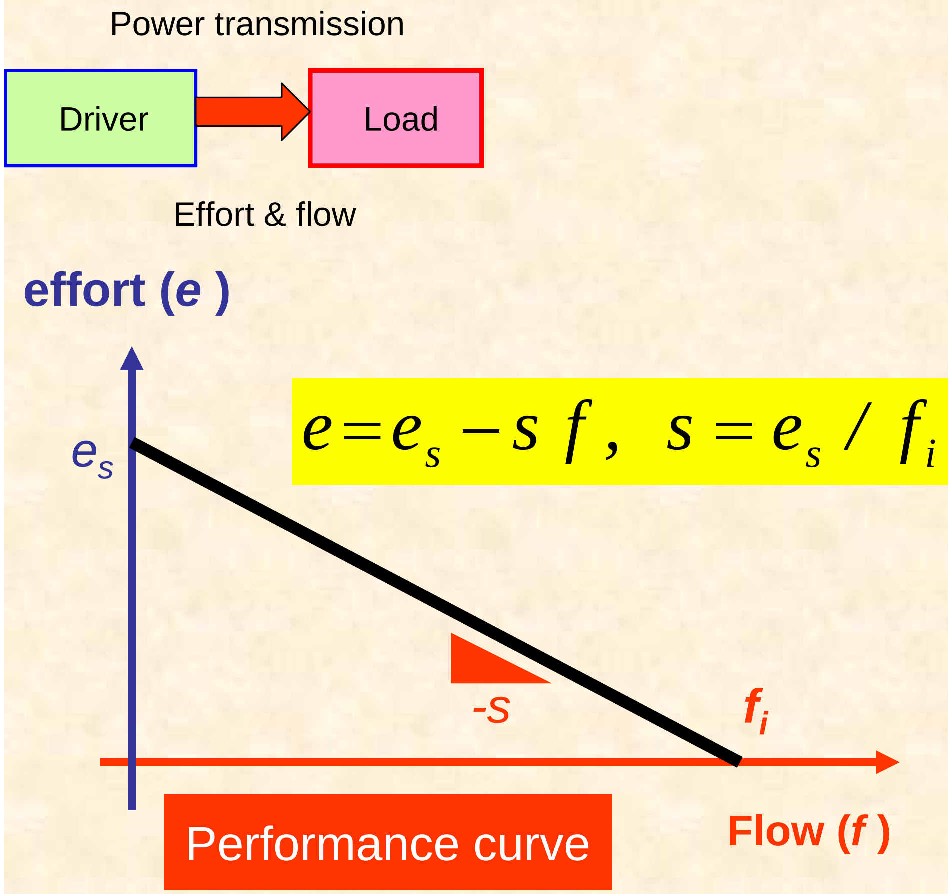


Electrical motor

Pump

All engineered products (drivers) come with a PERFORMANCE CURVE. You must request one if not given by OEM (original equipment manufacturer)

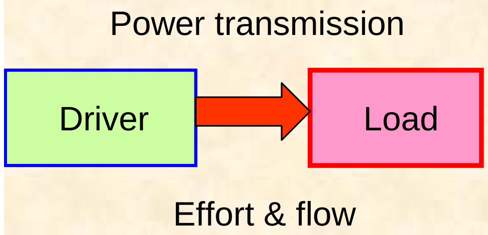
Simplest real driver

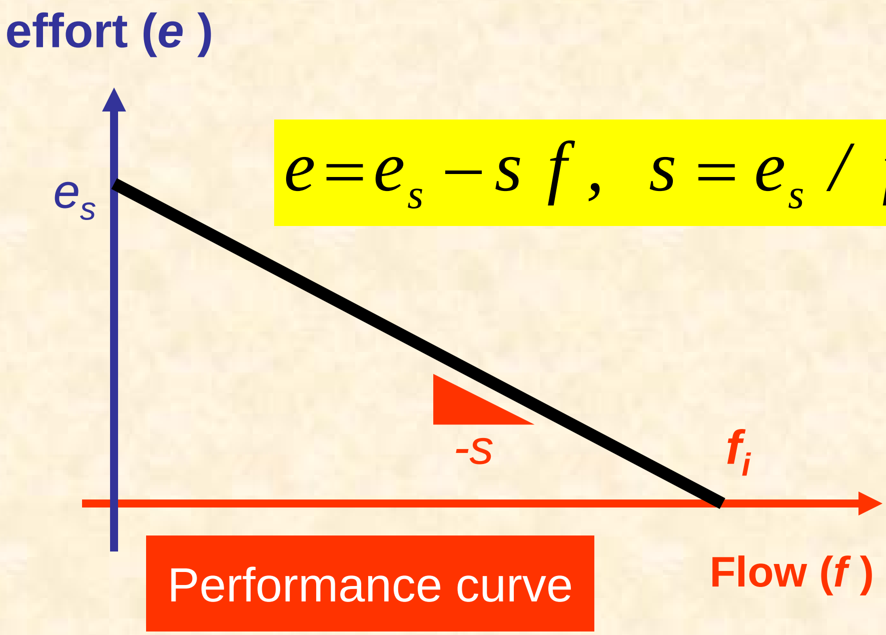


where e_s is the effort at zero flow, i.e. that required to **stall** (stop) the driver; while f_i is the flow at **idle** conditions (maximum flow with no effort).

The slope of the effort vs. flow curve is (-*s*) <0

Simplest real driver



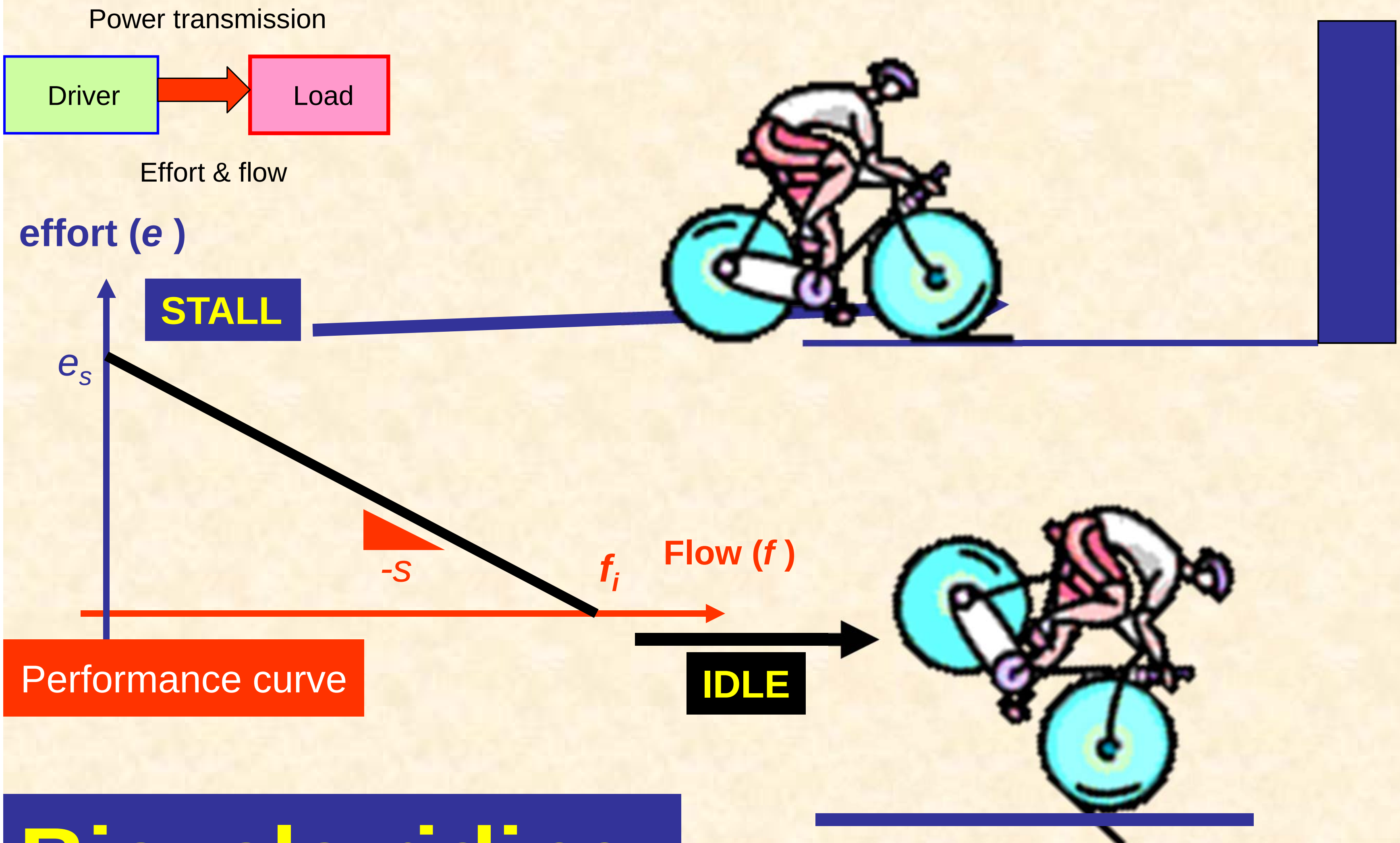


$$e = e_s - s\,f\,,\quad s = e_s / f_i$$

The *s* parameter is known as the
driver impedance
(Units of *e/f*).

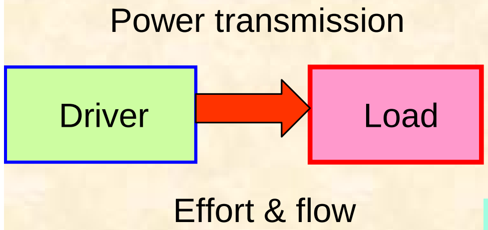
Drivers deliver high effort with little flow OR low effort with high flows. But not both (large *e* & *f*)

Real driver: stall and idle



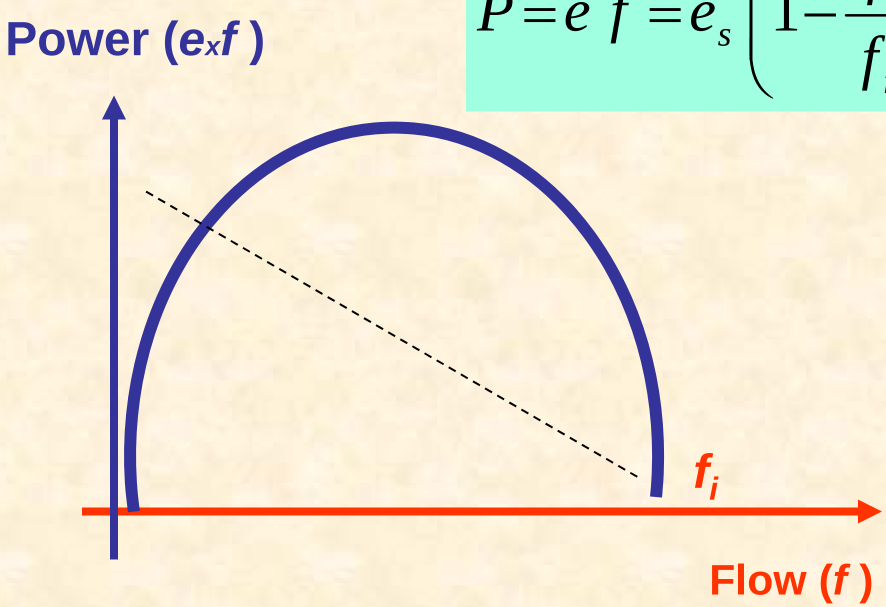
Bicycle riding

Power for simplest real driver



$$e = e_s - s\,f\,,\;\; s = e_s / f_i$$

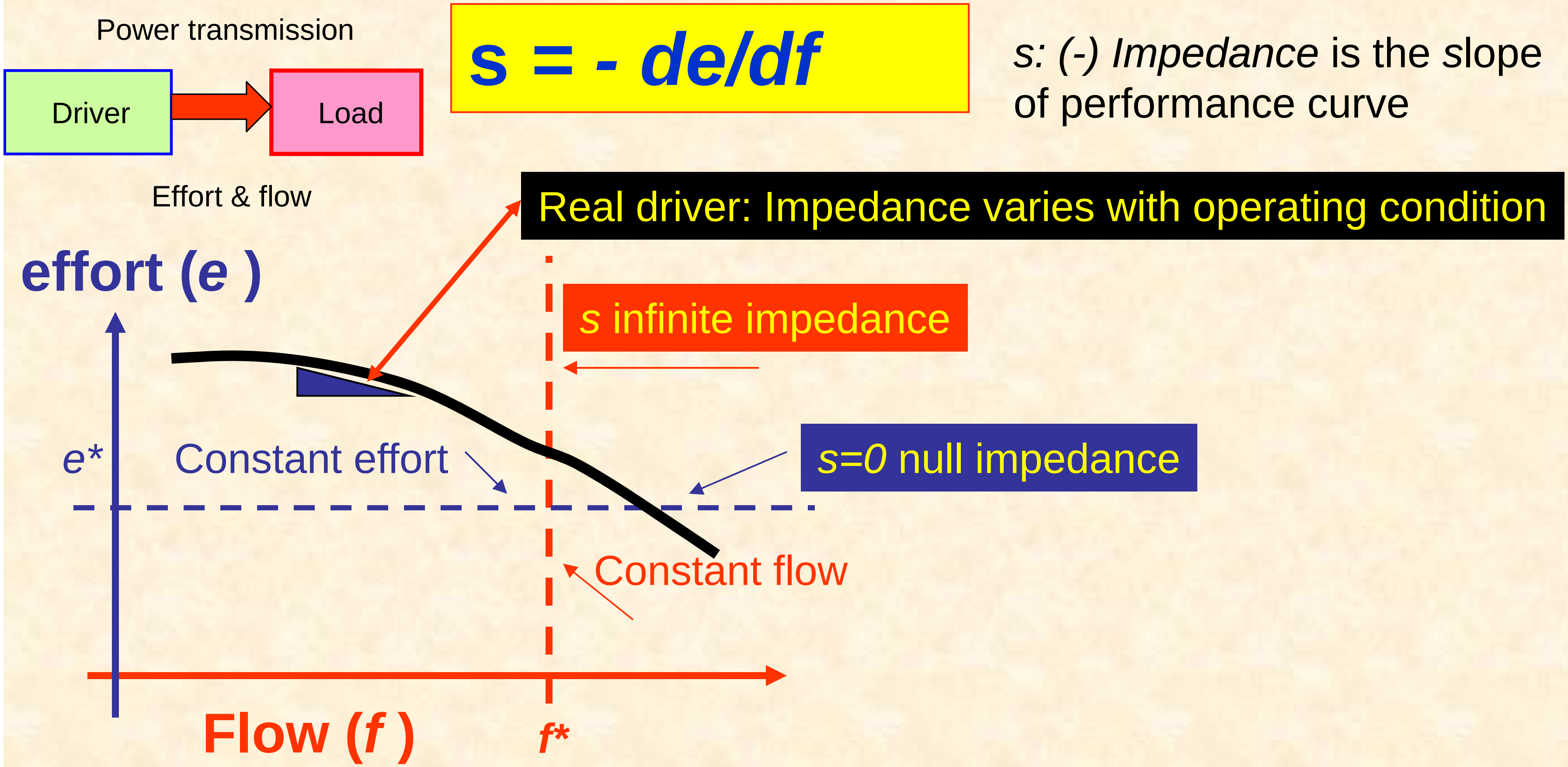
$$P = e\,f = e_s\left(1 - \frac{f}{f_i}\right) f$$



Power ***P*** is a quadratic function of the flow f. Power increases from zero towards a maximum value at a certain flow, and then decreases towards null power at f_i

Drivers deliver limited power! Drivers are not effective to transmit or deliver power at either large flows or low efforts!

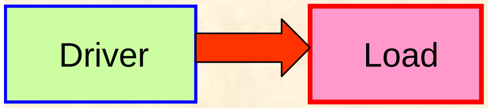
Idealized & real: impedances



s: (-) Impedance is the slope of performance curve

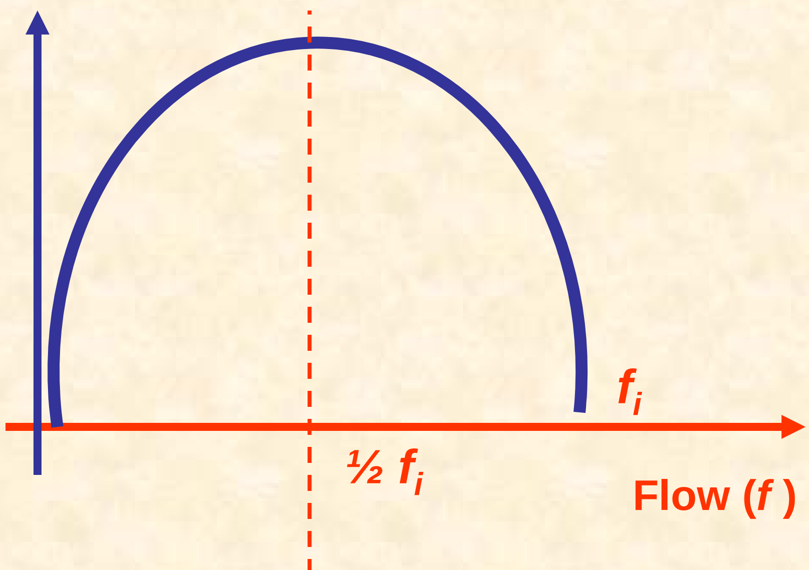
Real sources have impedances that change with operating condition

Peak power for simplest driver



$$e = e_s - s\,f\,,\quad s = e_s / f_i$$

$$P = e\,f = e_s\left(1 - \frac{f}{f_i}\right) f$$



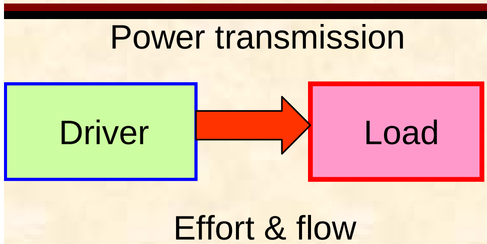
The maximum power available from the driver is obtained from (*dP/df* =0) and equals

$$P_{max} = \frac{e_s\,f_i}{4} = \frac{e_s^2}{4\,s}$$

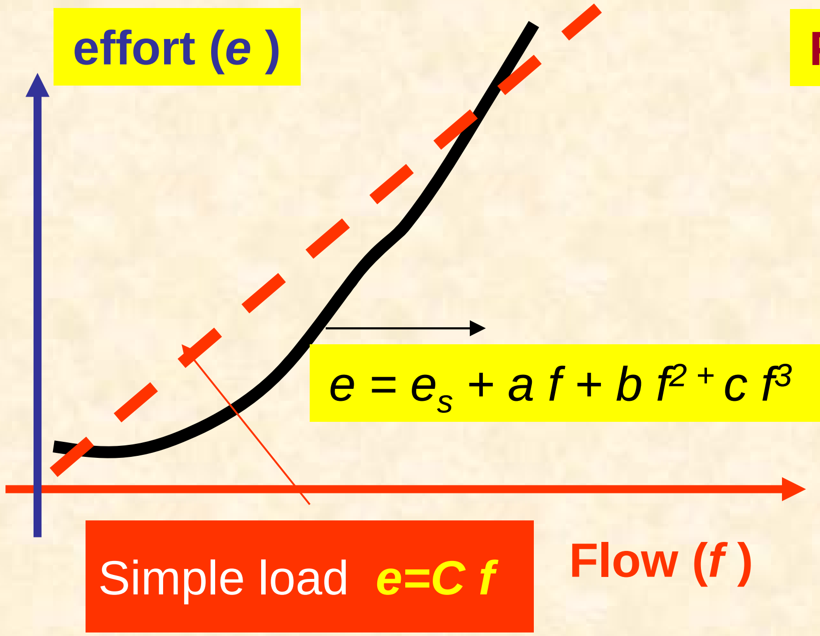
at $f^* = ½\, f_i$

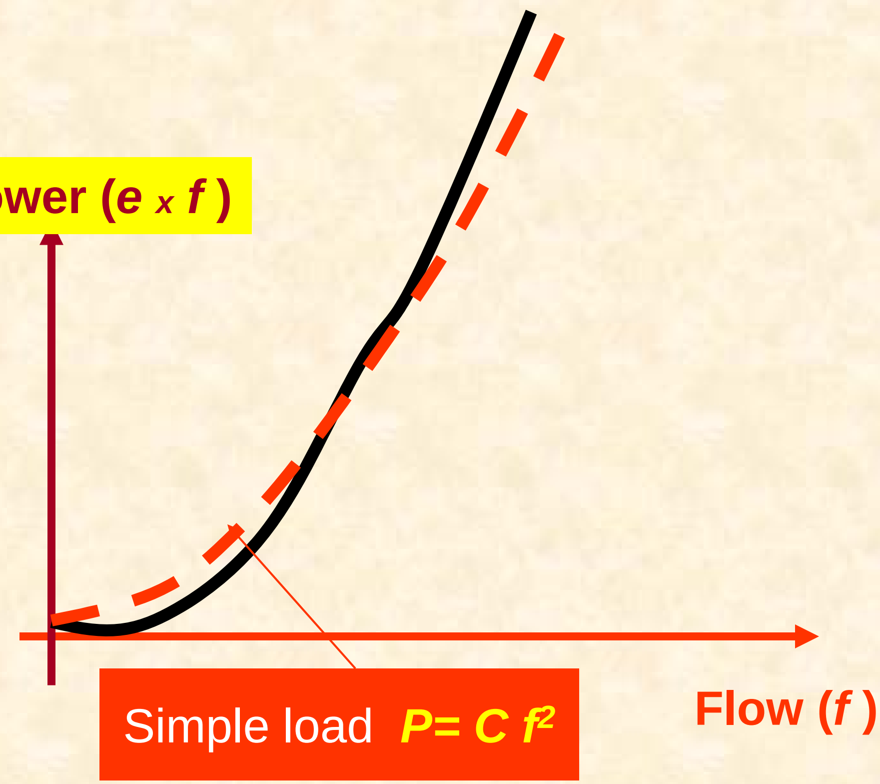
Maximum (peak) power occurs at a flow equal to 50% of the idle or maximum flow condition

Real loads: effort and flow



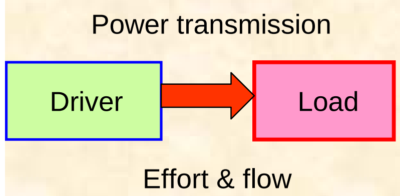
Actual loads show complicated curves: *effort vs flow*

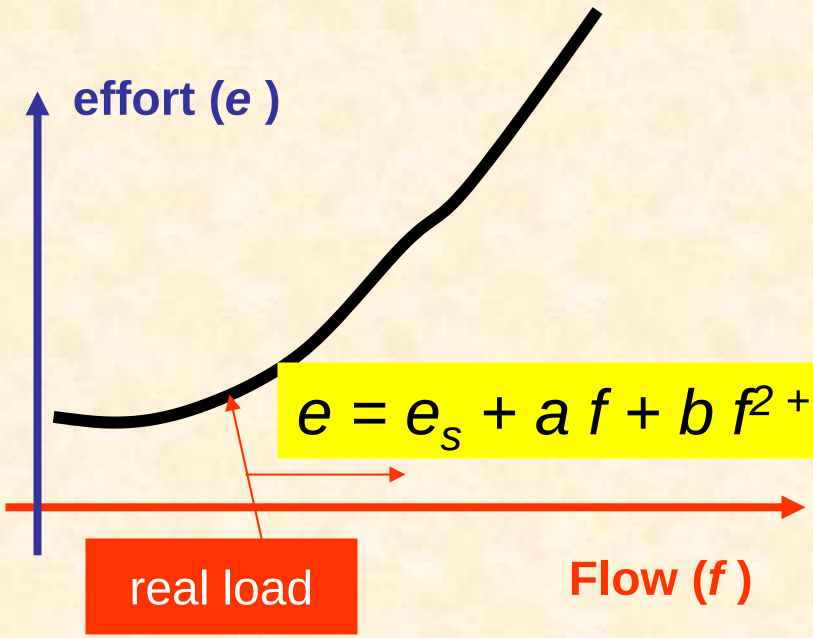




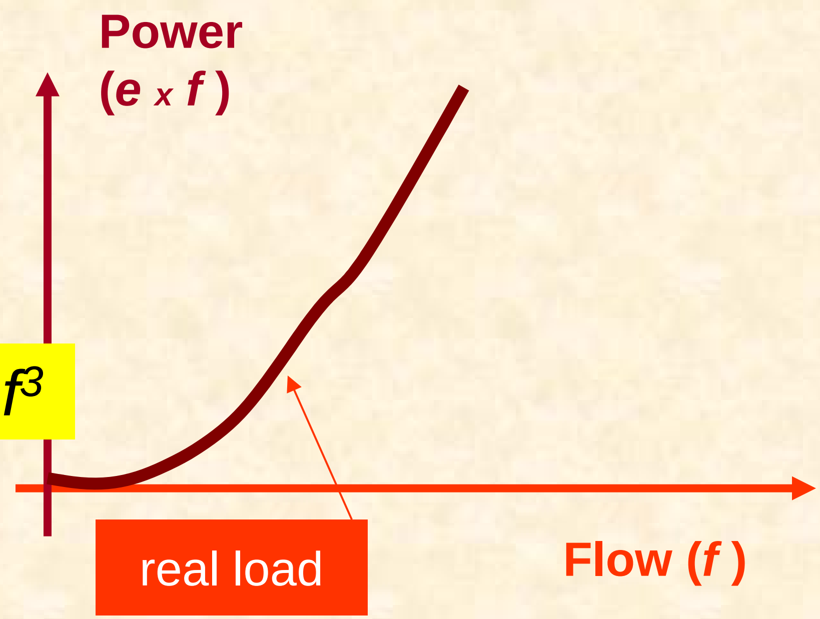
Loads demand (draw) lots of power to perform at high flows

Real loads: effort and flow





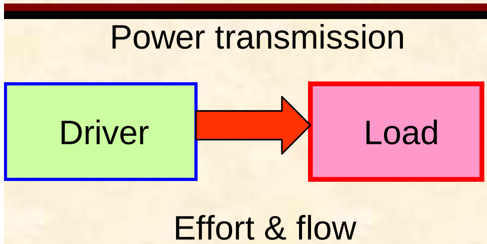
$$e = e_s + a\,f + b\,f^2 + c\,f^3$$

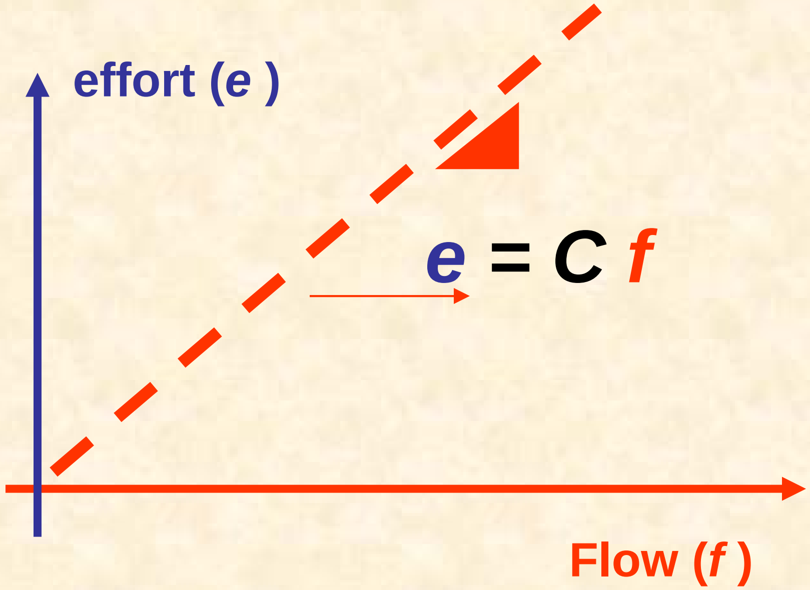


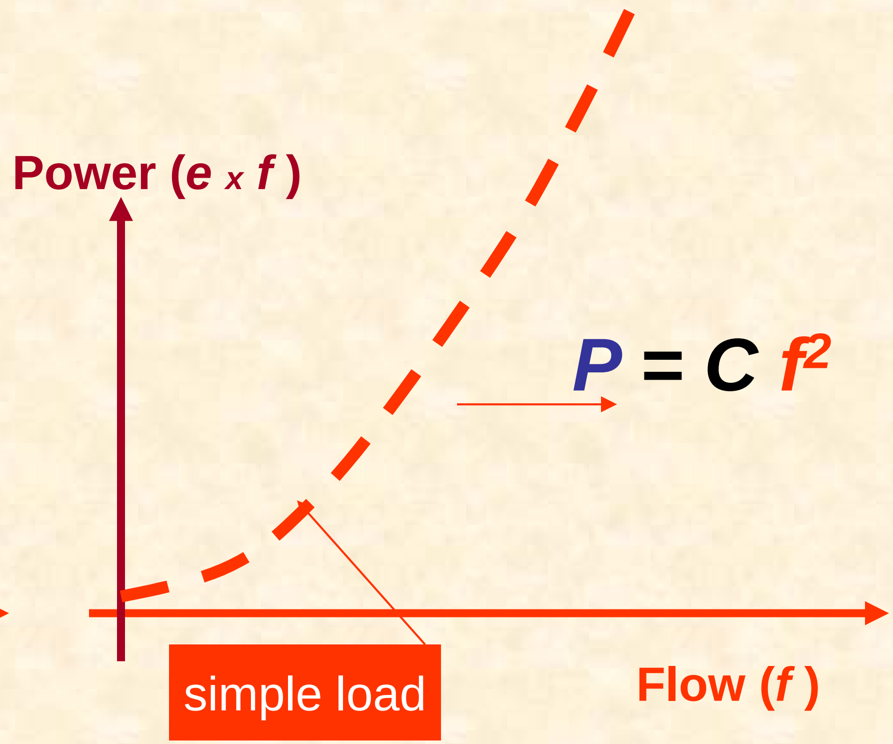
Example: **DRAG forces (or moments)**
= dry friction + viscous drag + aerodynamic drag + ……

A LOAD becomes a DRIVER when used for energy conversion
(Imagine motor-pump-fluid system)

Simple load: effort and power

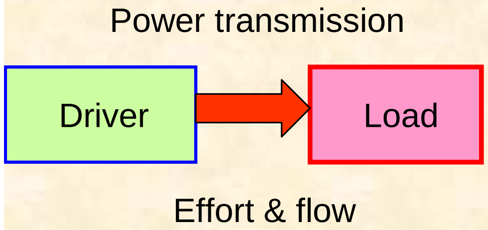




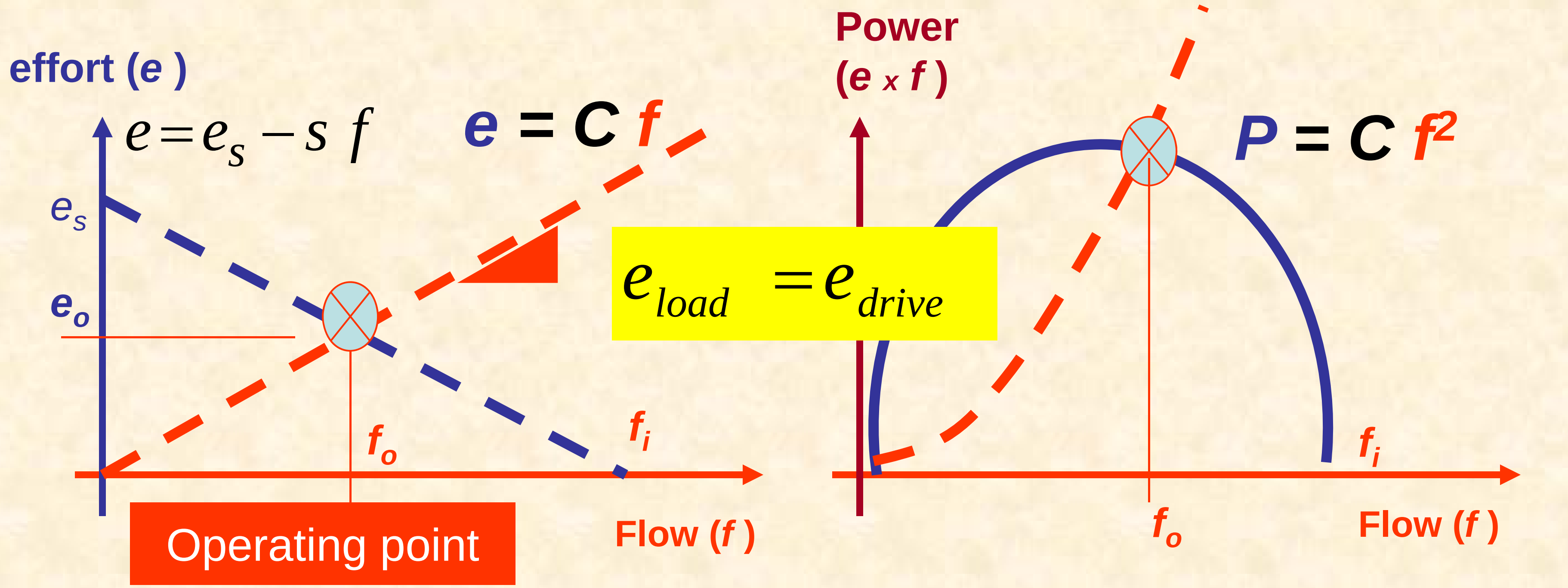


C is known as the <u>load</u> impedance

Connect driver to load



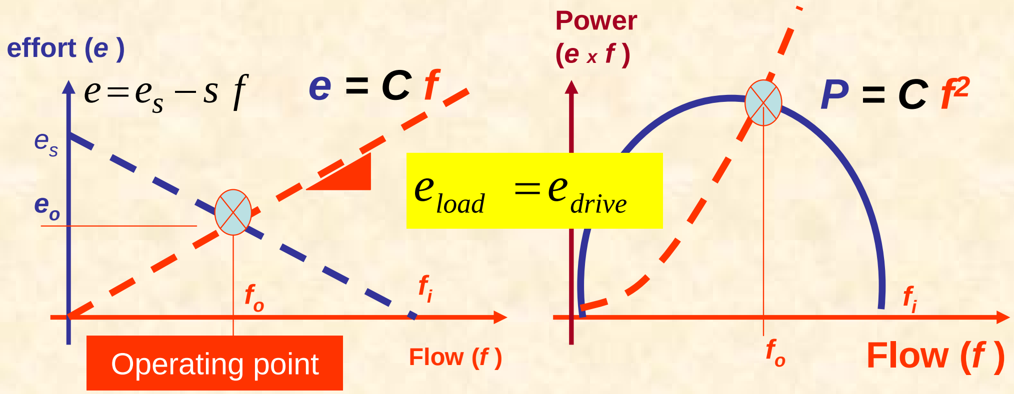
When the load is connected to the driver, an "equilibrium position" or **operating point** is achieved at steady-state operation. The operating point = balance of effort and flow



The "operating point" (flow & effort) & transmitted power from driver to load =

$$f_o = \frac{e_s}{s + C};\ e_o = C\ f_o;\ P_o = \frac{C\,e_s^2}{(s + C)^2}$$

Load impedance for max power



Find the condition at which the power transmission maximizes given a certain load (of impedance *C*).

$$f_o = \frac{e_s}{s+C};\; e_o = C\,f_o;\; P_o = \frac{C\,e_s^2}{(s+C)^2}$$

Determine (*dP*$_o$*/dC=0*):

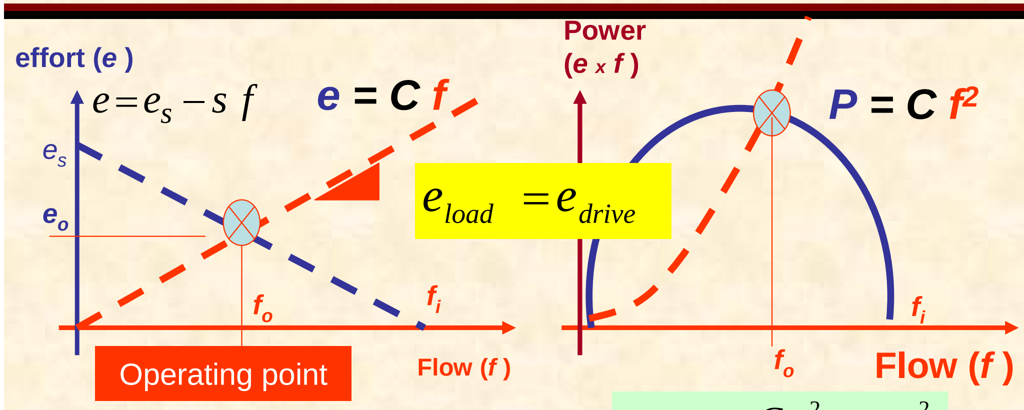
$$\frac{dP_o}{dC} = 0 = \frac{e_s^2\,(s+2C-C)}{(s+C)^2} = 0 \;\rightarrow\; C=s$$

With maximum transmitted power

$$P_{max} = \frac{C\,e_s^2}{(s+C)^2} = \frac{e_s^2}{4s}$$

Thus, maximum power transmission occurs when the load impedance (*C*) = the driver impedance (*s*).

Impedance matching



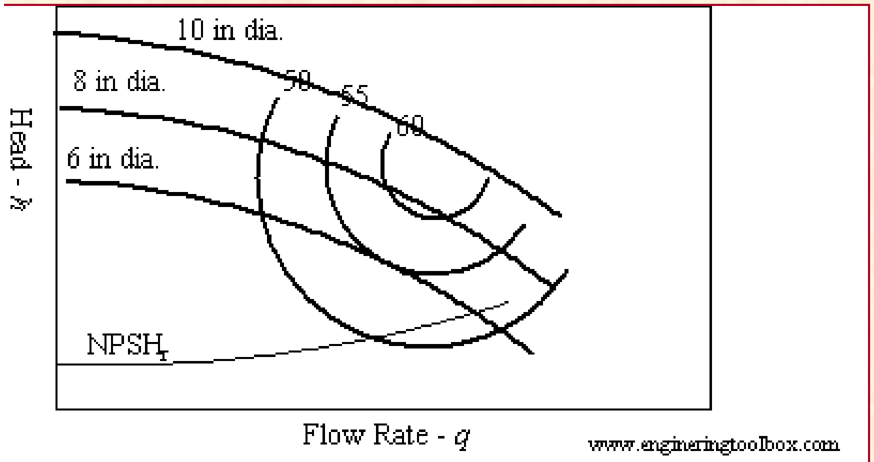
$$P_{max} = \frac{C e_s^2}{(s+C)^2} = \frac{e_s^2}{4s}$$

Maximum power transmission occurs when the load impedance (*C*) = the driver impedance (*s*)

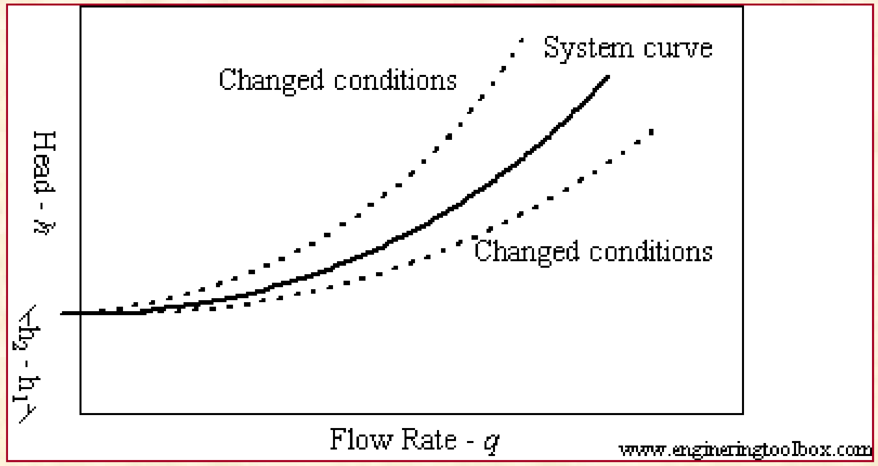
The analysis is known as IMPEDANCE MATCHING. It is useful to ensure maximum power transmission (and efficiency) in the operation of systems. The procedure demonstrates the NEED to appropriately select drivers to accommodate (or satisfy) the desired loads

Pump & system load matching

http://www.engineeringtoolbox.com/pump-system-curves-d_635.html#

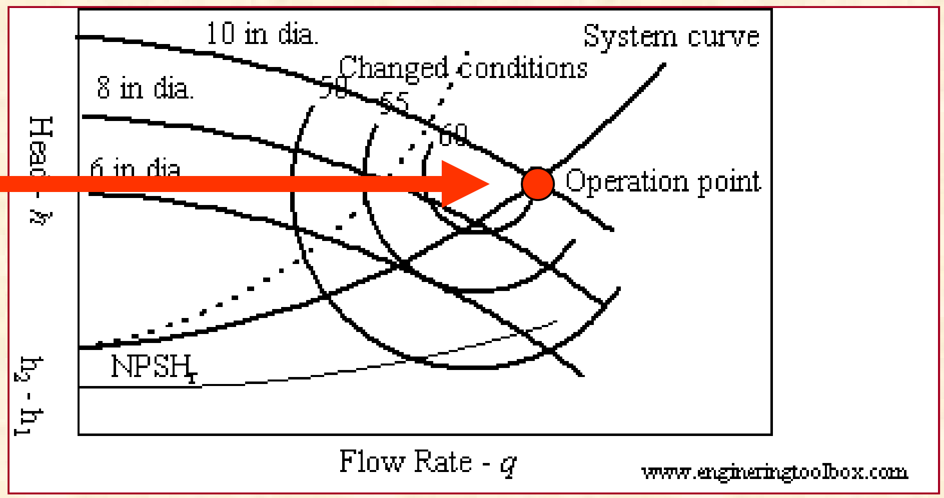


Pump

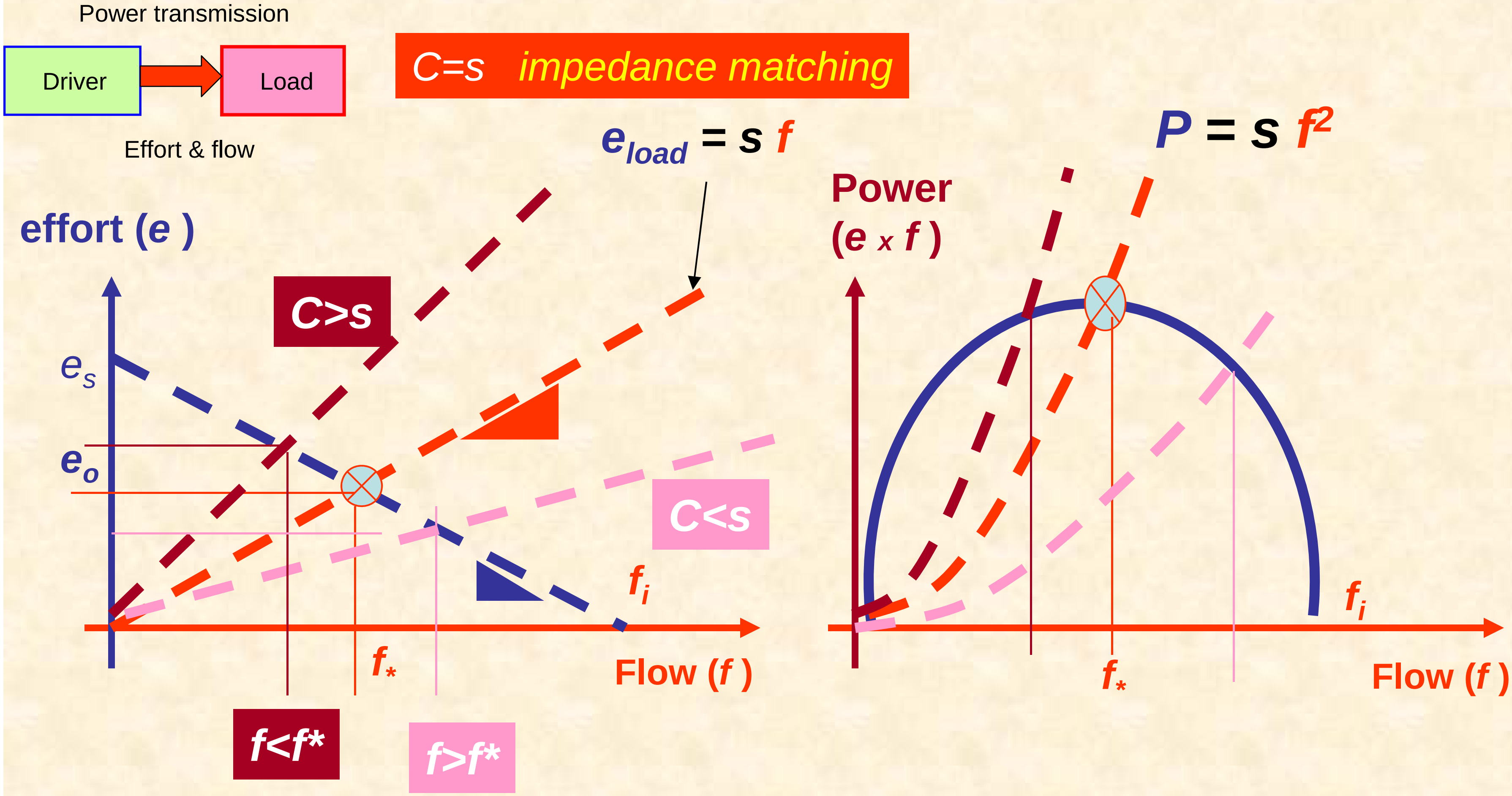


System (load) – pumping demand

FINDING OPERATING POINT (MATCHING of DRIVE to LOAD)



Impedance mismathcing



There is a NEED to appropriately select drivers to accommodate (or satisfy) the desired loads

Example

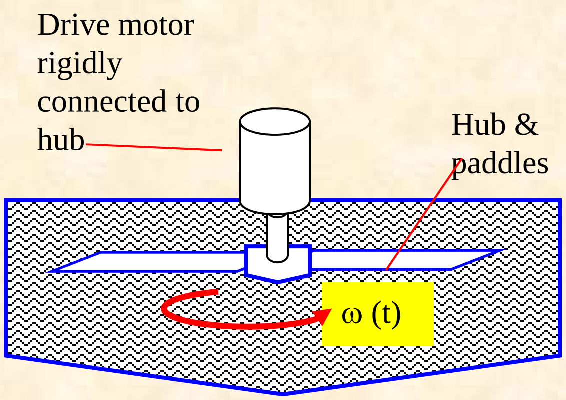
A fluid mixer is composed of the paddles and rigid hub connected directly to a DC drive electric motor. The motor characteristic performance curve as a function of angular speed (ω) is shown. The mass moment of inertia (***I***) of the hub and blades is 2 kg.cm^2**. When mixed,** the painting introduces a viscous drag moment or torque $M = D_\theta \omega$ with $D_\theta = 1\times10^{-2}$ N.cm.sec/rad**.**

a) The mixer is stationary and the motor is turned on. What is the steady state angular speed of the mixer?

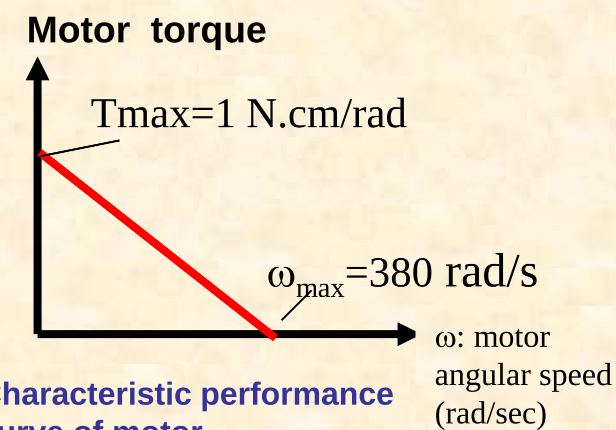
b) What would be this speed if the painting were twice as viscous?

c) How viscous must the painting be to stall the motor?

d) If the mixer is suddenly removed from the paint bucket, how fast will the motor spin? Is this a potentially dangerous event?



Schematic view of mixer



Characteristic performance curve of motor

Example

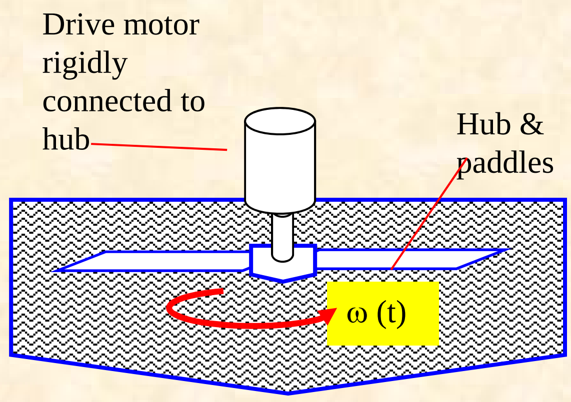
The motor torque equals
$$T_M = T_{max}\left(1 - \frac{\omega}{\omega_{max}}\right)$$
and at the operating point the motor torque must equal the load torque (drag moment). The operating point is defined by the speed ω_o and load=motor torque T_o

$$T_{drag} = D_\theta\, \omega_o = T_{max} - \frac{T_{max}}{\omega_{max}}\,\omega_o$$

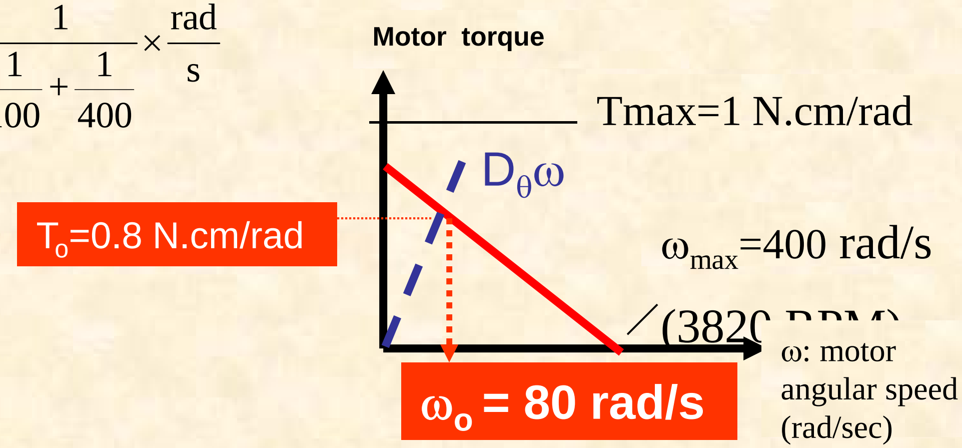
and

$$\omega_o = \frac{T_{max}}{\left(D_\theta + \frac{T_{max}}{\omega_{max}}\right)} = \frac{0.01\,\text{N.m}}{0.0001\,\text{N.m} + \frac{0.01}{400}\,\text{N.m}} \times \frac{\text{rad}}{\text{s}} = \frac{1}{\frac{1}{100} + \frac{1}{400}} \times \frac{\text{rad}}{\text{s}}$$

$$\omega_o = \frac{400}{1 \times 4 + 1} \times \frac{\text{rad}}{\text{s}} = 80 \times \frac{\text{rad}}{\text{s}} \times \frac{60\text{ s}}{1\text{min}} \times \frac{1\text{ rev}}{2\pi\text{ rad}} = 764\,\text{RPM}$$

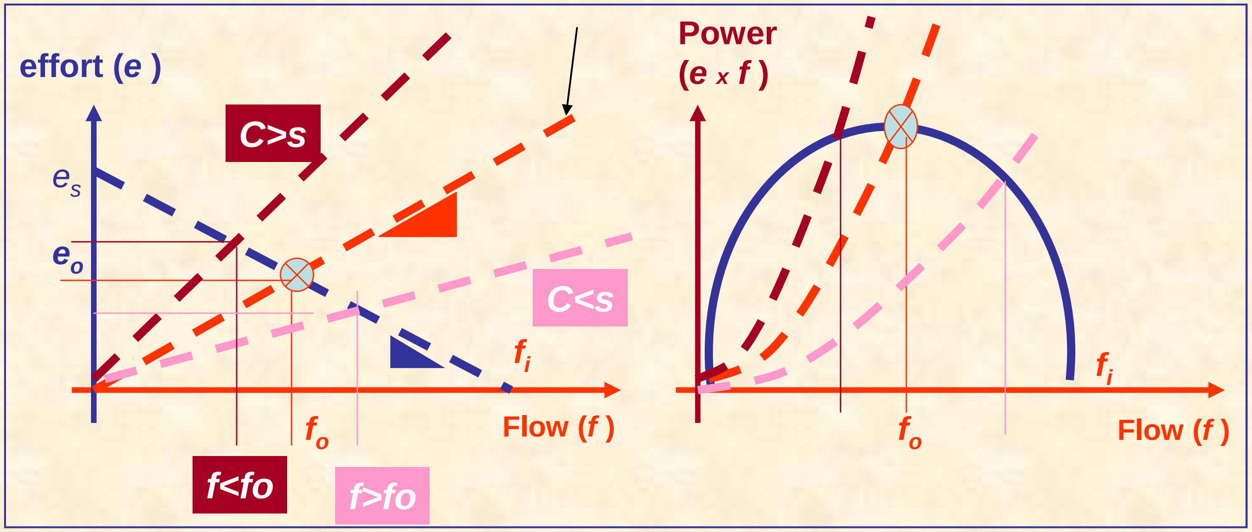


Schematic view of mixer



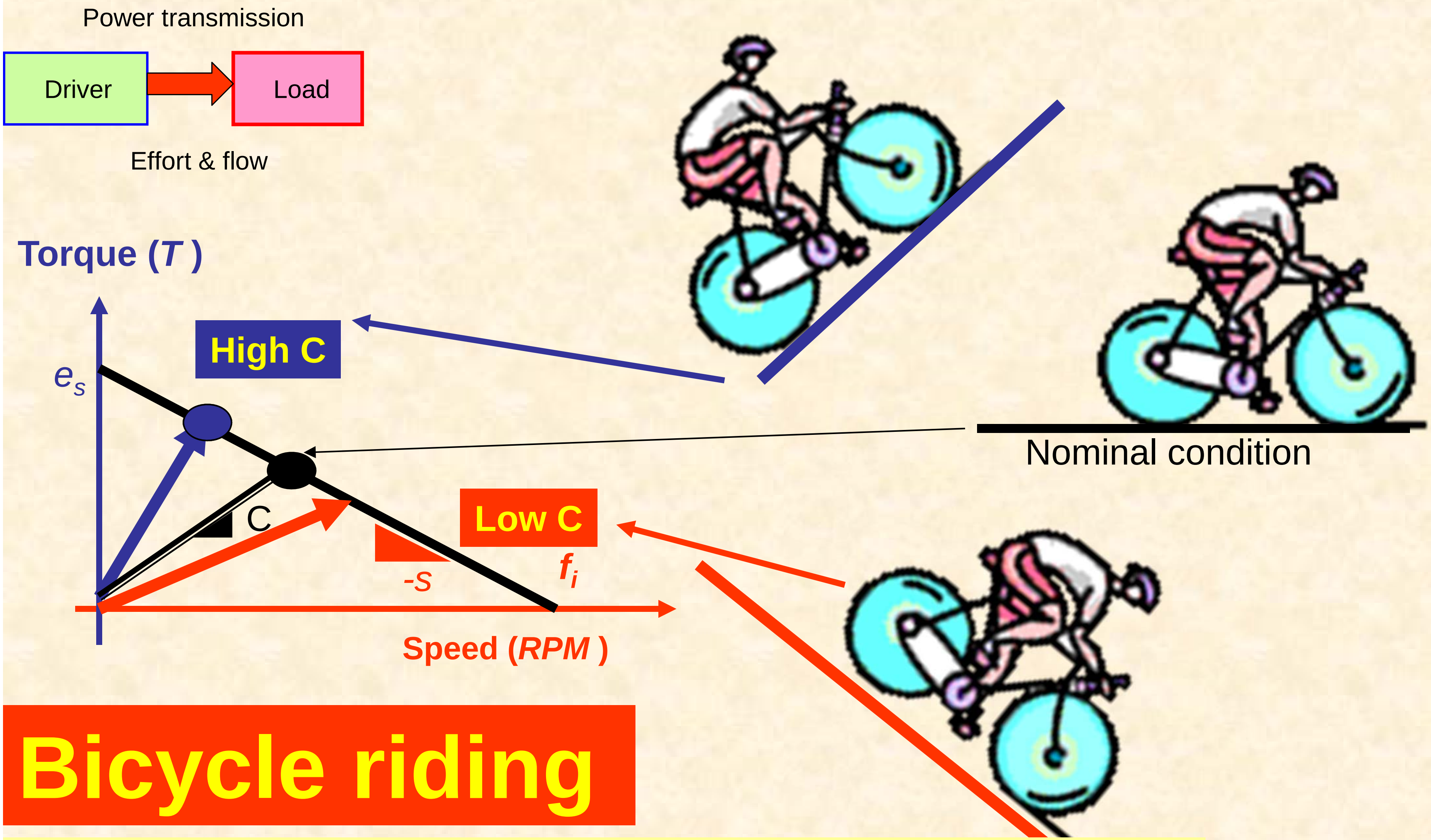
Students continue work…..

Impedance mismatching



The analysis also indicates that if a driver is selected to operate a load with optimum transmission; then, variations in the load (changes such that $C \neq s$) will cause an IMPEDANCE MISMATCHING and inefficient operation; i.e. away from optimum or maximum power transmission.

Varying load impedance (road slope)



Bicycle riding

How **riding a bicycle** works? What are the gears for?

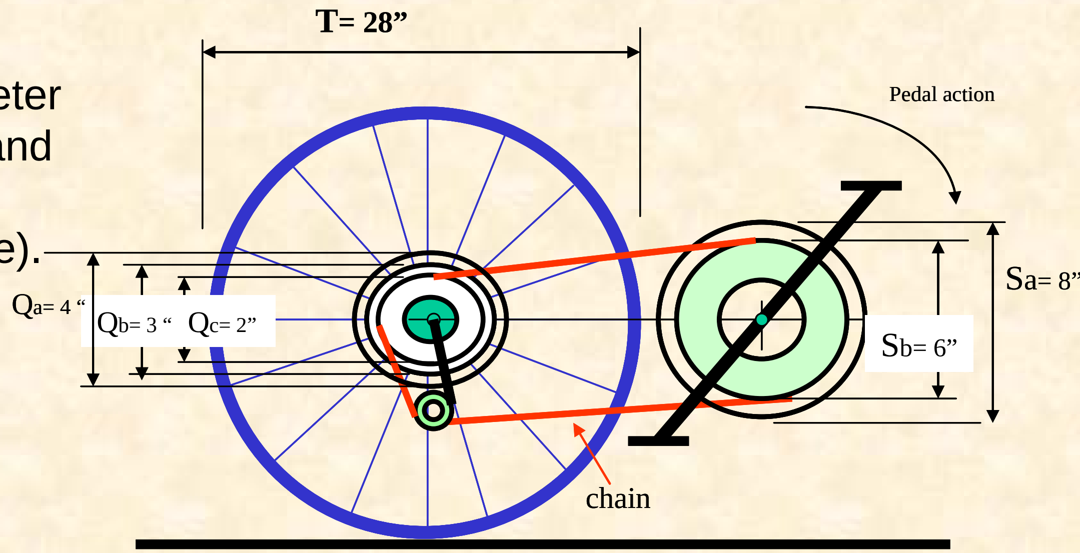
Bicycle riding

Consider a bicycle gear & chain drive mechanism: S and Q denote the diameter of the sprockets (gears) for the pedal and bike wheel, respectively. T denotes the outer diameter of the bicycle wheel (tire). All diameters are in inch.

The rider pedals at a rate Npedal= 75 turns/min.

a) Find a simple formula to calculate the translational speed of the bicycle as a function of pedaling speed (Npedal), sprocket diameters (S, Q) and wheel diameter (T). You must list any important physical assumptions, writing full sentences explaining your work.

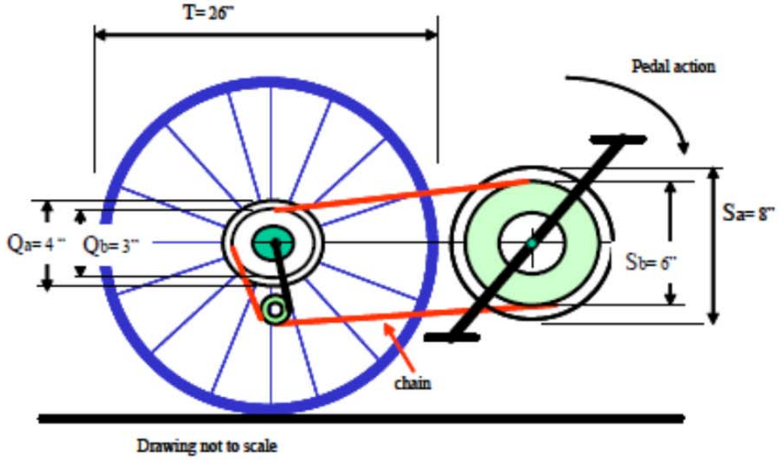
b) **How many speed changes are possible What combination of gears (S &D) will give the highest and lowest bike speeds??**



Drawing not to scale

For the given dimensions and the pedaling rate noted, find the bike highest and lowest translational speeds in miles/hour. (mph=5275 ft/3600 sec)

Bicycle riding



$S_a := 8 \cdot in \quad S_b := 6 \cdot in$ Pedal gear diameters

$Q_a := 4 \cdot in \quad Q_b := 2 \cdot in$ Wheel gear diameters

$T := 26 \cdot in$ bicycle tire diameter

$$RPM := \frac{2 \cdot \pi \cdot rad}{60 \cdot sec}$$

Given pedal rate: turns or revolutions per minute $N_{pedal} := 75$ in radian/sec $\omega_{pedal} := N_{pedal} \cdot \frac{2 \cdot \pi}{sec \cdot 60}$

The chain speed equals to $V_{chain} = \frac{S}{2} \cdot \omega_{pedal}$ = radius of sprocket x angular speed [1]

The chain drives the back wheel sprocket or gear; hence the angular speed of the tire wheel equals

Assume: No slip of chain $\omega_{tire} = \frac{V_{chain} \cdot 2}{Q}$ = chain speed/sprocket radius [2]

The bicycle wheel rolls w/o slipping (contact point = ground); and hence its translational speed equals

Assumed: No slipping of tire $V_{bicycle} = \omega_{tire} \cdot \frac{T}{2}$ [3]

$$\omega_{pedal} = 7.85 \frac{rad}{sec}$$

Bicycle riding

hence, combining equations [1] thru [3]

$$V_{bicycle} = \omega_{tire} \cdot \frac{T}{2} = \frac{V_{chain} \cdot T}{Q} = \frac{S}{2} \cdot \omega_{pedal} \cdot \frac{T}{Q} = N_{pedal} \cdot \frac{\pi}{60 \cdot sec} \cdot T \cdot \frac{S}{Q}$$

$$mph := \frac{5275 \cdot ft}{3600 \cdot sec}$$

$$V_{bicycle} = N_{pedal} \cdot \frac{\pi}{60sec} \cdot T \cdot \frac{S}{Q} \qquad [4]$$

Thus, the **translational speed of the bike depends on the ratio of sprocket diameters** (S/Q).

Max. speed $V_{max} := N_{pedal} \cdot \frac{\pi}{60 \cdot sec} \cdot T \cdot \frac{S_a}{Q_b}$ $\quad \frac{S_a}{Q_b} = 4$ largest S with smallest Q

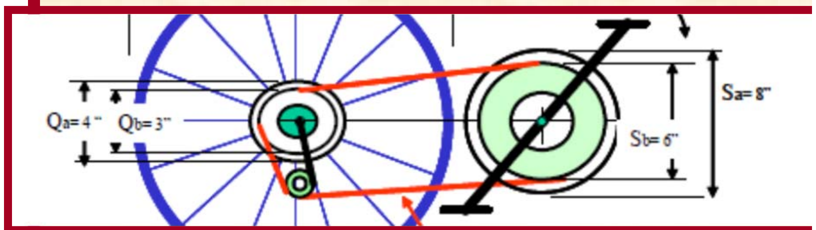
Min speed $V_{min} := N_{pedal} \cdot \frac{\pi}{60 \cdot sec} \cdot T \cdot \frac{S_b}{Q_a}$ $\quad \frac{S_b}{Q_a} = 1.5$ smallest S with largest Q

ALL gear ratios $\frac{S_a}{Q_b} = 4 \quad \frac{S_b}{Q_b} = 3 \quad \frac{S_a}{Q_a} = 2 \quad \frac{S_b}{Q_a} = 1.5$ **FOUR SPEED bicycle**

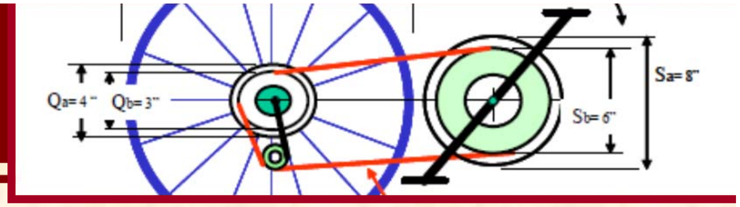
bicycle speed in miles/hour

$V_{max} = 34.03 \frac{ft}{sec}$ $\quad V_{max} = 23.23\ mph$

$V_{min} = 12.76 \frac{ft}{sec}$ $\quad V_{min} = 8.71\ mph$



Match load impedance

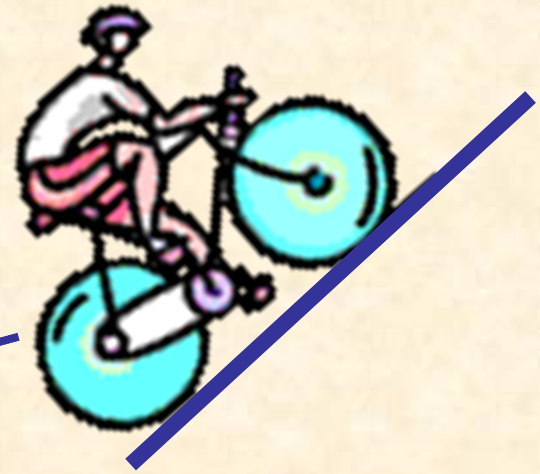


Power transmission

Driver → Load

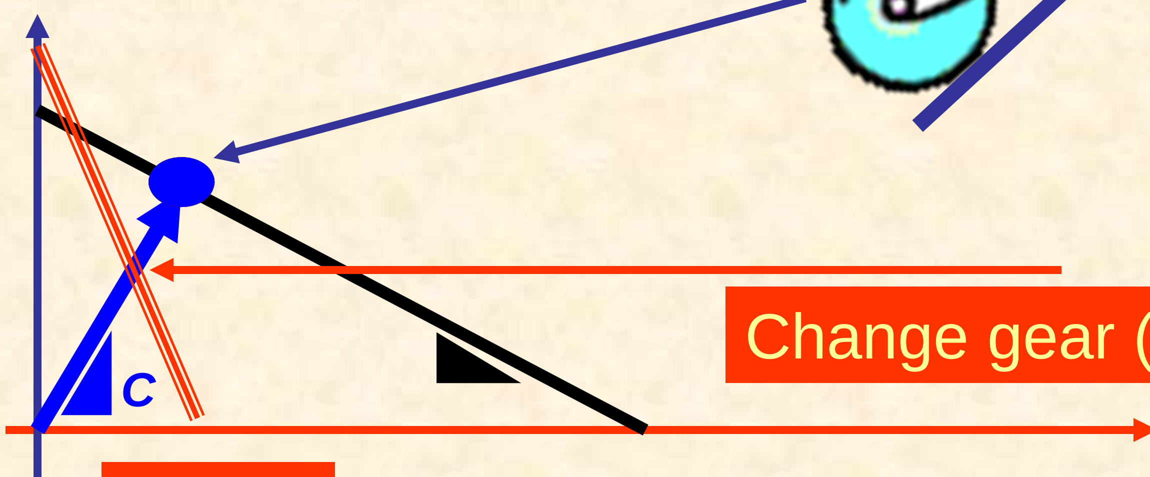
Effort & flow

High C



(-s)=C

Torque (*T*)



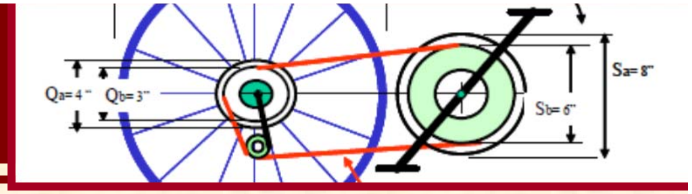
Change gear (S small, large Q)

High (-s)

speed (*RPM*)

Riding uphill

Match load impedance



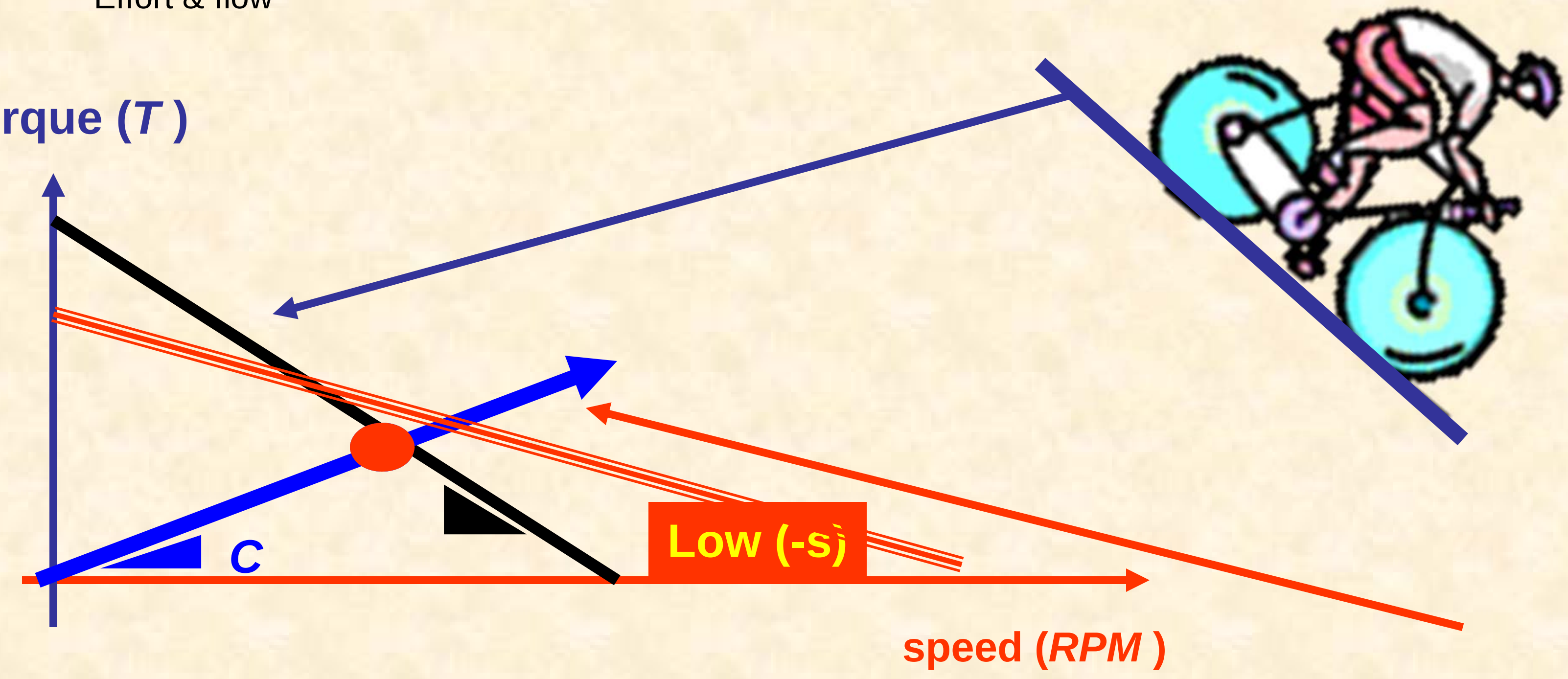
Power transmission

Driver → Load

low C

Effort & flow

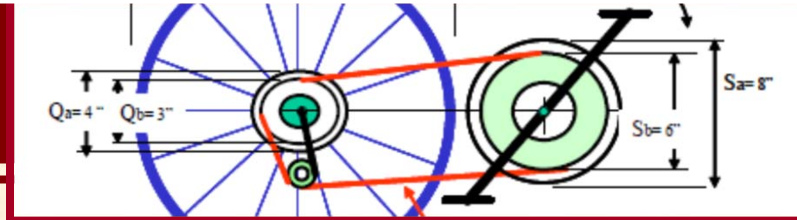
(-s)=C



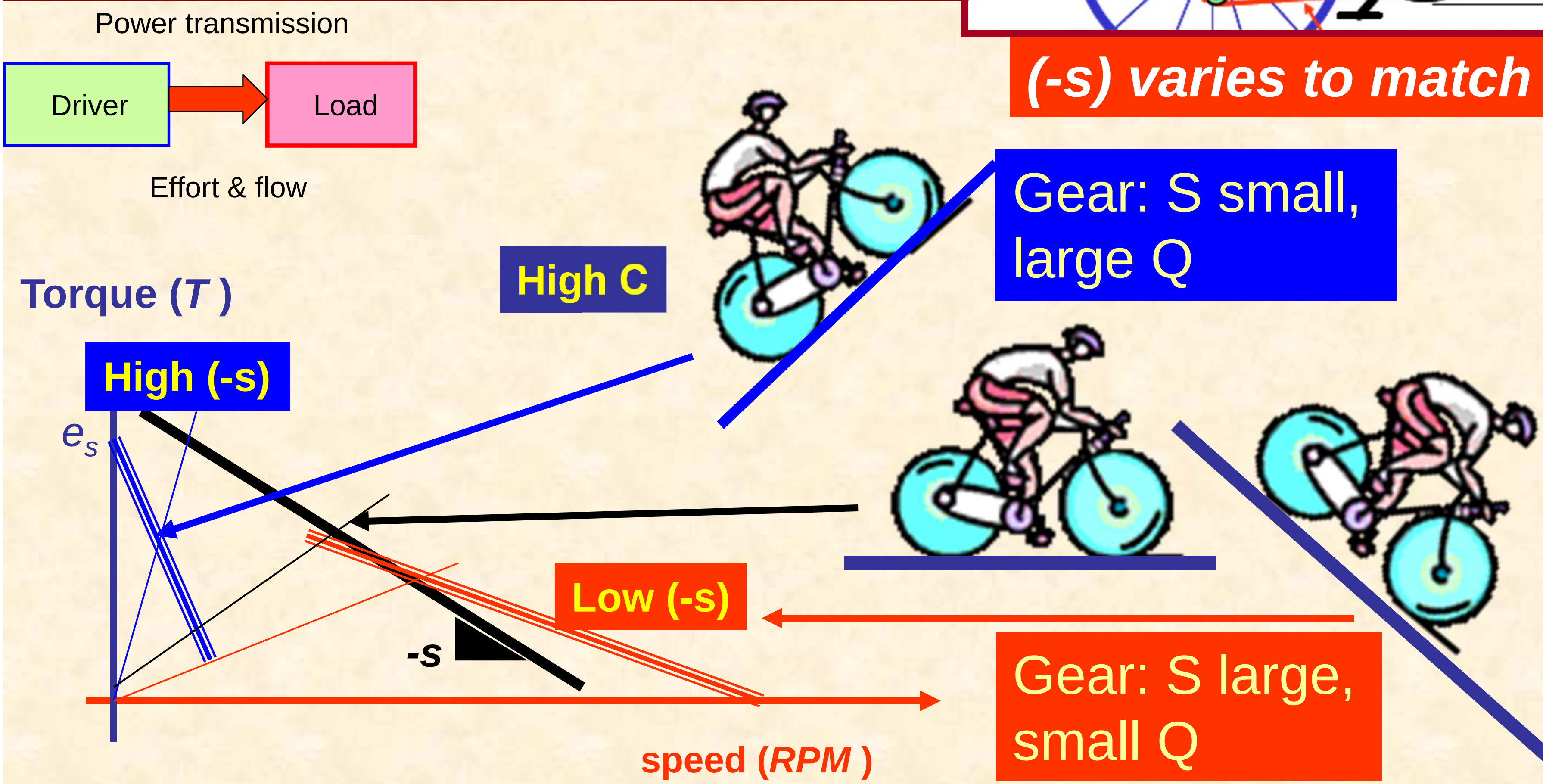
Change gear (S large, small Q)

Riding downhill

Variable speed bike



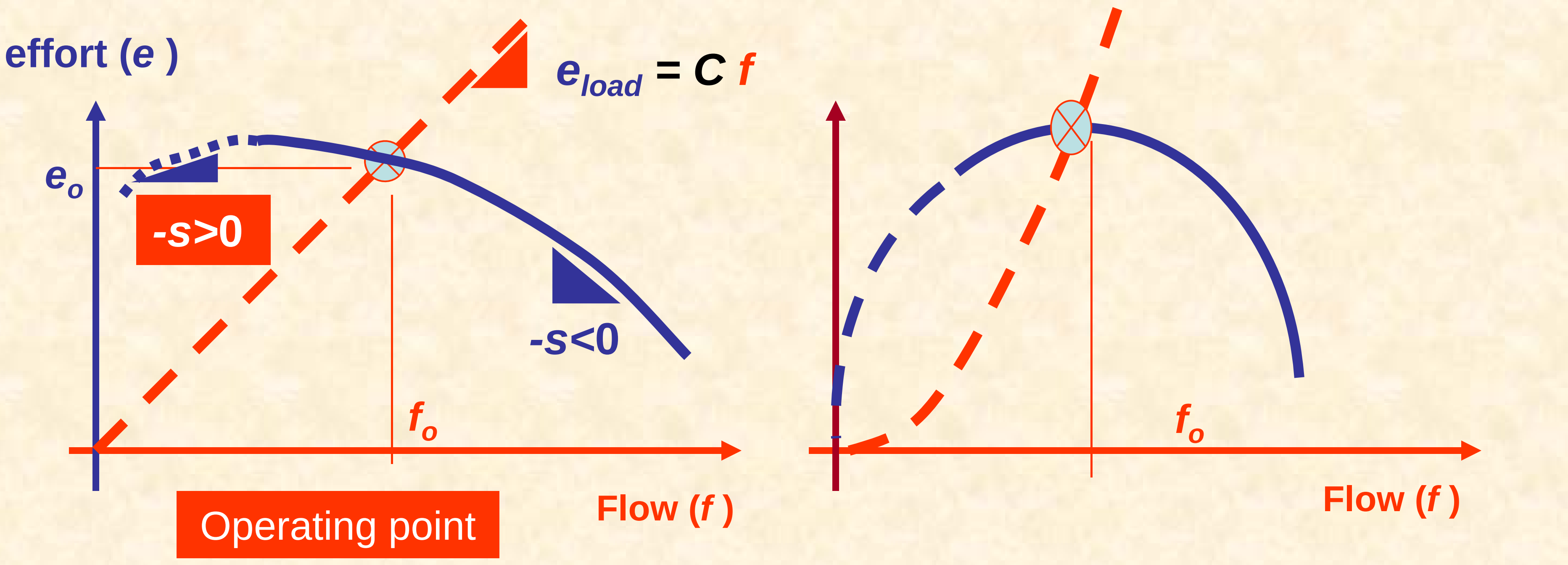
(-s) varies to match load



Match driver to load impedance

Real drive: negative impedance

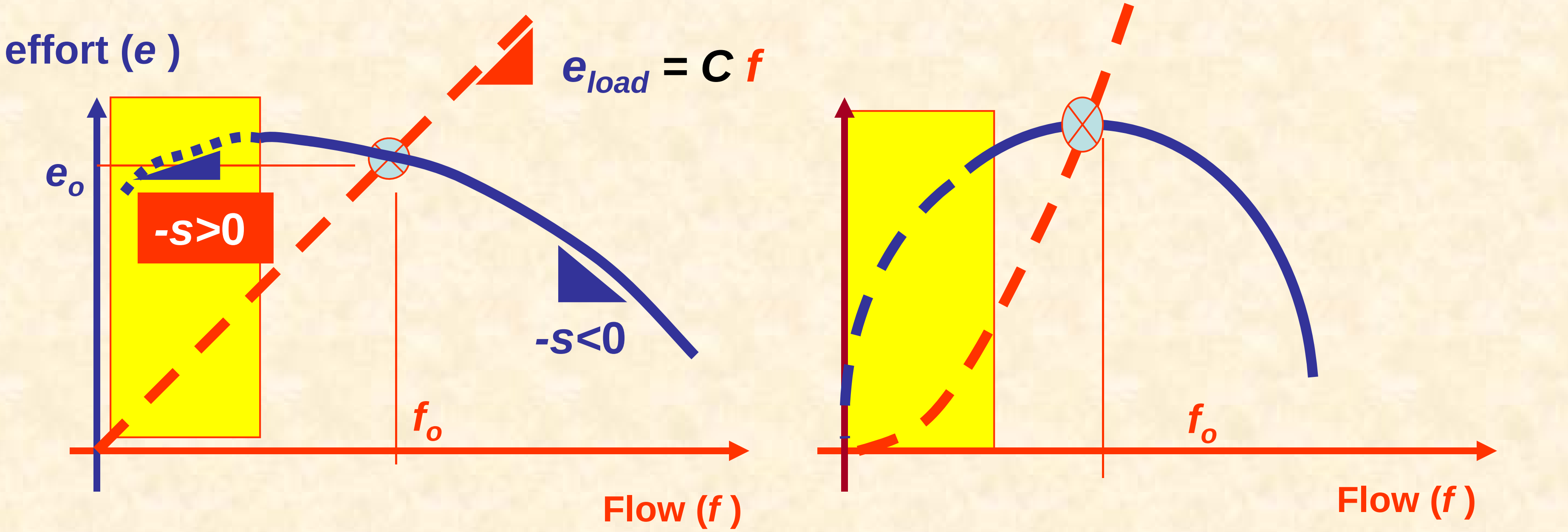
Actual drivers do not show "ideal" performance curves. Most notably compressors show *effort vs. flow* curves as below. Note that in actual hardware, the driver impedance (*s*) varies with the flow *(f)* in a complicated form. One should never allow operation of this type of driver in a flow region where the slope is positive (–*S*>0), i.e., a negative impedance.



C and s correspond to load and driver impedances (slopes)

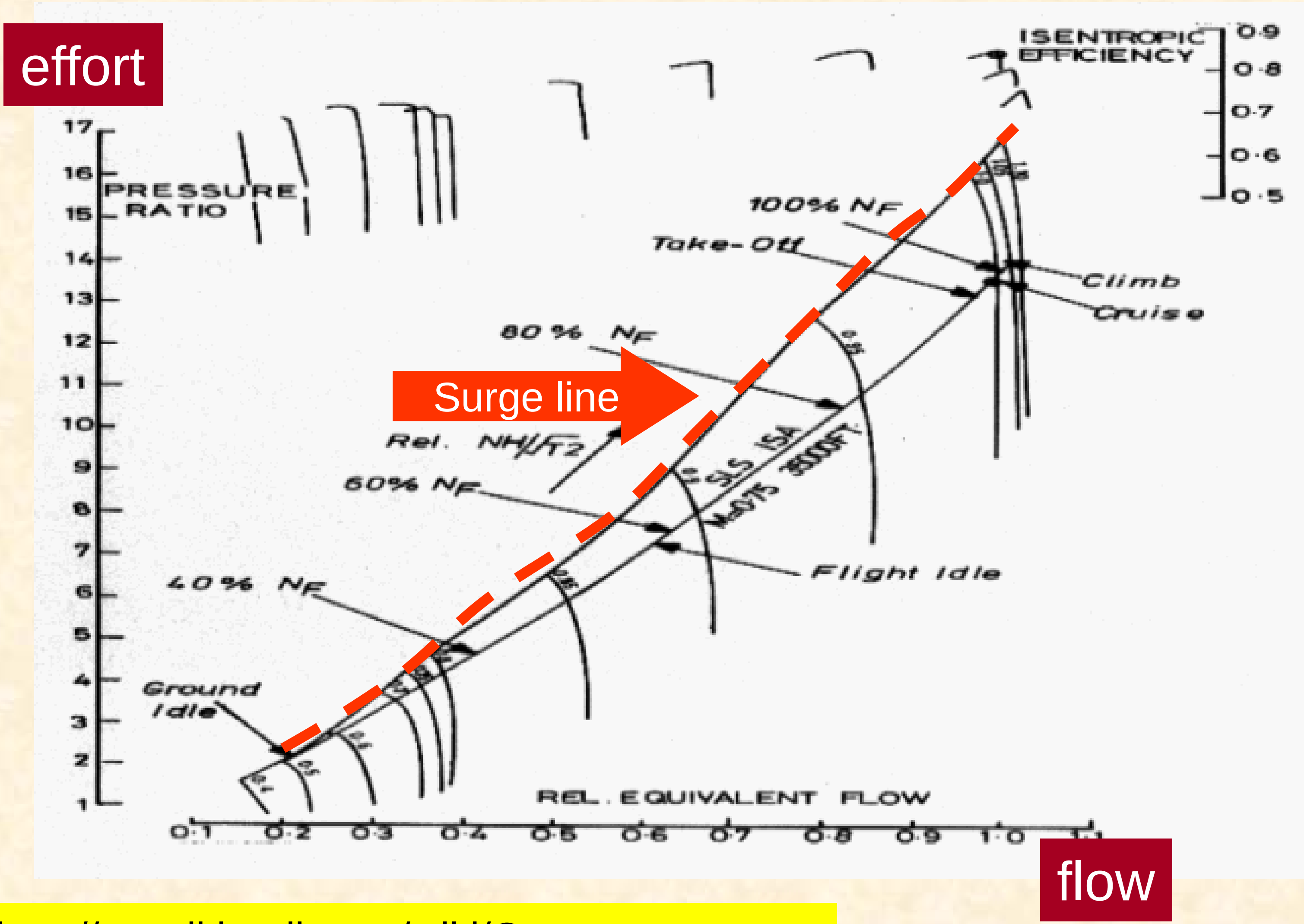
Real drive: instability

Do NOT never operate a driver in a flow region where its impedance is negative, –*s*>0. Attempts to operate at this (typically) low flow condition, will cause damage to the equipment since severe flow instabilities (+ large vibrations, +large forces, +loss in efficiency) will occur. This is the case of compressors undergoing surge and stall, for example.



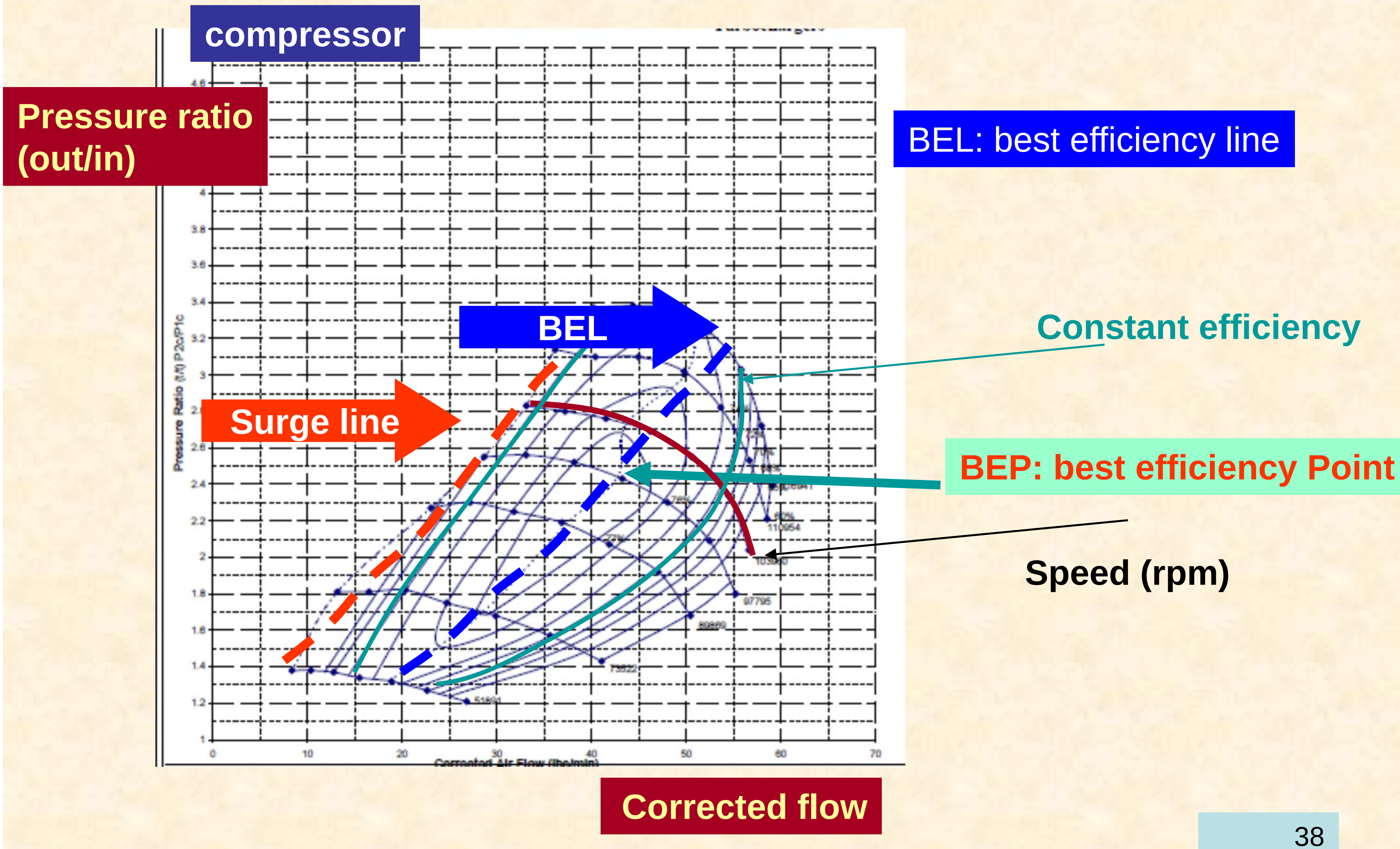
Yellow zone indicates region of instability – forbidden (NO-NO) operation.

Compressor Map

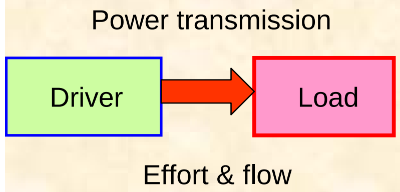


http://en.wikipedia.org/wiki/Compressor_map

Turbocharger: compressor map



Closure: impedance analysis



The knowledge gained will allow you to properly select the best pair of audio speakers that match an audio amplifier, for example.

However, the most enduring concepts for you to ponder are those of driver and load impedances and the importance of matching impedances in an actual engineering application.

Whenever designing or specifying components for a system, do apply these important concepts.

Impedance matching

Why not taught in Eng courses?

• Lecturers lack practical engineering experience. They are good at research and independent topic. Lack knowledge in system integration.

• Materials requires engineering know-how (how things work) & demands of cross-disciplinary learning & practice.

• Material considered too simple for an engineering class. It should be "obvious." Simple use of product catalogs.

Practices of Modern Engineering

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