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Spontaneous Fruit Fly Optimisation for truss weight minimisation: Performance evaluation based on the "no free lunch" theorem

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Abstract
In recent years, researchers have presented various optimisation algorithms for truss design. The "no free lunch" theorem implies that no optimisation algorithm fits all problems; therefore, the interest is not only in their accuracy and convergence rate but also the tuning effort and population size to achieve the optimal result. The latter is particularly crucial for computationally-intensive or high-dimensional problems. Contrast-based Fruit-fly Optimisation Algorithm (c-FOA) proposed by Kanarachos et al. in 2017 is based on the efficiency of fruit flies in food foraging by olfaction and visual contrast. The proposed Spontaneous Fruit-fly Optimisation (s-FOA) enhances c-FOA and addresses the difficulty in solving nonlinear optimisation problems by presenting standard parameters and lean population size for use on optimisation problems. The s-FOA's performance is assessed using six benchmark problems. Comparison of the results obtained from documented literature and other investigated techniques demonstrates the competence and robustness of s-FOA in truss optimisation.

1. Introduction

Trusses have found significant applications in modern engineering. Such applications range from use in transmission towers, offshore wind turbine supports, offshore oil and gas platforms, to microstructural applications such as the lattice structures of additive manufacturing [1, 2]. Truss optimisation aims to improve the performance of trusses while minimising the material resource [3]. The objective of the optimisation can be interpreted as a weight minimisation one, bounded by well-defined constraints. The constraints are the allowable stresses and displacements, as subject to high stress the truss members could fail through buckling or tension. There are many forms of optimisation, each with their unique design variables: this study focuses only on size optimisation. The design variable that is the most commonly investigated is the cross-sectional area of the truss member [4].

Optimisation algorithms are used in searching for the optimum solution to a problem. The application of optimisation algorithms to structures has proliferated in the last decade [5]. Many researchers have published on the applications of improved algorithms to truss weight minimisation problems. Kaveh and Mahdavi proposed a Multi-Objective Colliding Bodies Optimisation (MOCBO) algorithm for the optimisation of trusses bounded by an allowable stress limit [6]. A genetic programming methodology was used by Assimi *et al*. in the optimisation of the size and topology of trusses [7]. Another approach was taken by Cheng *et al*., proposing a Hybrid Harmony Search (HHS) algorithm for the design of truss structures with stress limits [8]. Tejani *et al*. made use of the Improved Passing Vehicle Search (IPVS), Improved Heat Transfer Search (IHTS), Improved Water Wave Optimization (IWWO) and Improved Heat Transfer Search (IHTS) to optimise the topology of truss structures with displacement, stress and kinematic stability constraints [9]. An adaptive Symbiotic Organism Search (SOS) was utilised by Tejani *et al*. in truss structural optimisation with frequency constraints [10]. A development of PSO was presented by Kaveh and Zolghadr: Democratic PSO (Particle Swarm Optimisation) algorithm for the optimisation of truss layout and size with frequency constraints [11]. Multi-Class Teaching-Learning-Based Optimisation algorithm (MC-TLBO) was utilised by Farshchin *et al*. for truss design with frequency constraints [12]. Rajan used a Genetic Algorithm (GA) to optimise the shape, size and topology of truss structures [13].

Through all these research studies, the efficiency of optimisation algorithms in solving structural design problems has been established. However, according to the "no free lunch theorem", there exists no single algorithm to solve all optimisation problems. Hence the need to research lean algorithms [10].

In 2011, Pan proposed the FOA algorithm, a population-based technique that mimics the foraging activities of fruit flies [14]. Fruit-flies, compared to other species, possess a better sense of smell and vision which they use to find food efficiently. The algorithm has a framework which is simple, easy to understand, and is easily implementable in tackling various optimisation problems [15]. However, it is characterised by premature convergence (reduced accuracy) and is easily trapped in local optima.

FOA has been applied successfully to a variety of problems. In 2011, Pan applied the FOA algorithm to optimise the General Regression Neural Network and Multiple Regression utilised in modelling the financial distress problem of Taiwan's enterprise [14]. Lu *et al*. in 2015 proposed an adaptive fruit fly optimisation algorithm based on velocity variable (VFOA). The algorithm utilised the particle velocity concept from PSO on FOA to improve its convergence speed and accuracy. The improved algorithm was used to solve 13 mathematical benchmark problems [16]. As another improvement, Kanarachos *et al*. modified the FOA algorithm by including a visual contrast phase. The modification was based on biological discoveries on the complexity of the fruit foraging activities of fruit flies, thus improving its exploration capabilities. The modified algorithm was applied for the first time to solve truss design problems with stress, displacement or frequency constraints [17]. The algorithm was also used to improve the shock performance of vehicle suspension systems to mitigate damages caused by potholes in the UK [18]. Since then it was applied successfully in a range of problems. Wu *et al*. solved 33 mathematical benchmark functions by modifying the FOA to improve its exploration capabilities. A normal cloud generator was introduced to generate new positions of the swarm based on parameters such as possible food position, search range and search stability. It was inspired by the fact that fruit flies are characterised by fuzziness and randomness as they fly towards the food source. A cloud model is a tool used to synthesise the randomness and fuzziness in the algorithm [19]. Mitic *et al*. in 2015 presented the chaotic fruit fly optimisation algorithm to improve the explorative strategy of the algorithm. It does so by using the theory of chaos to relocate the fruit flies. The improved algorithm was used to solve ten one-dimensional benchmark mathematical problems [20]. With the aim of diversifying the solutions to avoid local optima or premature convergence, Yuan *et al*. introduced a Multi-Swarm Fruit Fly Optimisation Algorithm (MSFOA). The enhanced algorithm was used to identify parameters of a synchronous generator and solve six non-linear mathematical functions. In MSFOA, the swarm is divided into several sub-swarms, and the sub-swarms independently explore the search space to find the global optima [21].

FOA solves optimisation problems in two basic phases: the osphresis phase and the vision phase. The fruit fly makes use of its olfactory organ to detect the odour of the food source and then uses its vision capabilities to fly towards the food direction. Metaheuristic optimisation algorithms are characterised by two vital properties: exploration and exploitation. Exploration makes sure the algorithm visits a broader region of the search space (non-visited) for promising solutions. Exploitation aims to search intensively already visited regions of the search space for better solutions. Ensuring a good balance between exploration and exploitation is imperative to improve the performance of an optimisation algorithm and thus defines its success or failure. An unsuitable balance could lead to premature convergence, local optima entrapment and possibly stagnation [22]. Different problems require a different balance between exploitation and exploration, thus the need to adapt the parameters of the algorithm. In many cases, larger population sizes may compensate this problem. However, the requirement for large population sizes becomes problematic in higher dimensional problems.

The performance of optimisation algorithms significantly relies on certain unique parameters. Although many have tried, tuning of the algorithms to achieve the optimal result is not convenient. Algorithms such as GA, DE and PSO have their performances dependent on a number of parameters not known beforehand. The need for algorithms with standard set of parameters to achieve optimum results has therefore become necessary.

This study aims to present the Spontaneous Fruit fly Optimisation Algorithm (s-FOA) as an optimisation technique with a good exploration-exploitation balance characterised by fewer and standard tuning parameters. To this end, six benchmark truss design problems were solved with the s-FOA. The results were compared to several state of the art optimisation algorithms to establish the robustness of the algorithm.

2. Spontaneous Fruit Fly Optimisation Algorithm

Recently, a study by Van Breugel & Dickinson [23] showed that fruit flies exhibit a more complex search behaviour than the one modelled by Pan. Contrast-based Fruit Fly Optimisation (c-FOA) modelled for the first time the food search process of the Fruit Fly. The proposed Spontaneous Fruit Fly optimisation (s-FOA) enhances c-FOA and is described in detail in the following section. Its flowchart is provided in Figure 1.

2.1 Swarm generation, selection and termination

The algorithm starts by arbitrarily defining the position (X_0, Y_0) of the first fruit fly in a coordinate system. Additional N–1 fruit flies are located, randomly, in the vicinity of (X_0, Y_0) according to Eq. (1).

$$X_{ij}[k] = X_{0j}[k] \cdot \left(1 + M \cdot (2 \cdot rand_{N_{res}} - 1\right), j=1,2,\ldots,m \text{ and } i=1,\ldots,N$$
$$Y_{ij}[k] = Y_{0j}[k] \cdot \left(1 + M \cdot (2 \cdot rand_{N_{res}} - 1\right), j=1,2,\ldots,m \text{ and } i=1,\ldots,N \quad (1)$$

Where $k = 1,2, \ldots, K_{max}$ is the iteration number, m is the number of optimisation variables, N is the size of the swarm and $rand_{N_{res}}$ is a random number, sampled from a uniform discrete distribution defined in the interval [1, N_{res}]. M is a scaling parameter that defines how coarse or fine the search strategy is.

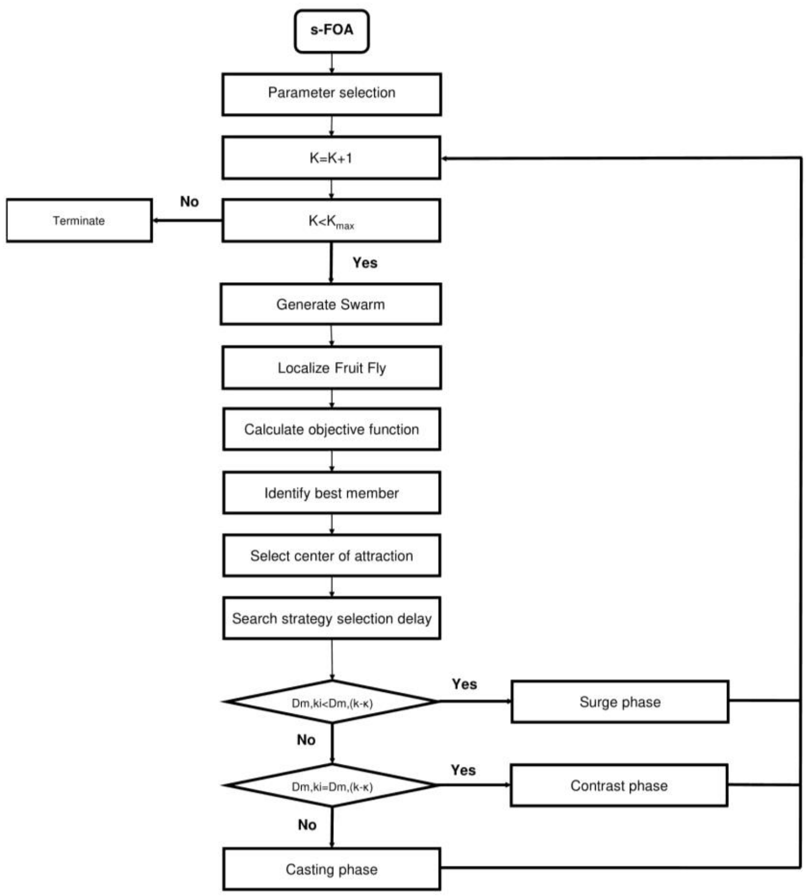

Figure 1. Flowchart of Spontaneous Fruit Fly Optimisation Algorithm (s-FOA).

Each fruit fly is assigned values DI_{ij} based on how close each fruit fly parameter ($X_{ij}[k]$, $Y_{ij}[k]$) is to the origin of the coordinate system:

$$D_{ij}[k] = \sqrt{X_{ij}^2[k] + Y_{ij}^2[k]} \tag{2}$$

$$DI_{ij}[k] = \frac{1}{D_{ij}[k]} \tag{3}$$

For each fruit fly at $\mathbf{d_i}[k]$ an objective function value $Dm_i[k]$ is assigned, $Dm_i[k] = f(\mathbf{d}_\mathrm{i}[k])$.

In the following, the fruit flies are ranked based on their objective function values, and the fruit fly $\mathbf{d}^*[k]$ that achieves the lowest objective function value $Dm_i^*[k]$ at position ($X_i^*[k]$, $Y_i^*[k]$) is identified. In case the objective function value $Dm_i^*[k]$ is lower than the previous centre of attraction $D_0[k]$, then $Dm_i^*[k]$ becomes the new centre of attraction $\mathbf{d}_0[k]$ ($X_0[k]$, $Y_0[k]$).

$$\begin{aligned} &if\ Dm_i^* < D_{m,k0} \\ &then\ X_0[k] = X_i^*[k]\ and\ Y_0[k] = Y_i^*[k] \end{aligned} \tag{4}$$

The algorithm terminates when the maximum number K_{max} of iterations is reached.

In [23], it was shown that fruit flies exhibit three distinct behaviours when searching for food. First, when fruit flies find a better food source, they surge to that location. Second, fruit flies do not change immediately their search strategy when an improvement in food location is not achieved. Instead, they persist for a limited, constant amount of time. Third, fruit flies are attracted by visually contrasting objects. This means that they search also for food by using their eyes, not just smell. The three behaviours were included in s-FOA.

The delay in changing the search behaviour is represented by κ. The fruit flies can change their search strategy only every κ iterations. If the best objective function value $\mathbf{d}^*[k]$ improves over the last κ iterations the swarm enters the "surging" phase, during which the fruit flies surge towards the attraction point $\mathbf{d}_0[k]$:

$$\begin{aligned} &if\,(Dm_i^*[k] < Dm_i^*[k-\kappa]) \\ &M[k+1] = c \cdot M[k] \end{aligned} \tag{5}$$

In case the best objective function value does not change over the last κ iterations the swarm enters the "visual contrast" phase, during which the fruit flies are attracted by an arbitrarily selected point $\mathbf{D}_{i,rand}[k]$:

$$\begin{aligned} &if\,(Dm_i[k] = Dm_0[k-\kappa] \\ &X_0[k] = X_{i,rand}[k]\ and\ Y_0[k] = Y_{i,rand}[k]] \end{aligned} \tag{6}$$

where k is the current iteration.

When a fruit fly does not improve its performance, then the swarm enters the "casting" phase. This behaviour is modelled based on [24]. There it was shown that fruit flies have memory that allows them to make decisions based on how good or bad the memory was.

$$\begin{aligned} &if\ Dm_i[k] > Dm_i[k-1] \\ &then\ X_i[k] = X_i[k-1]\ and\ Y_i[k] = Y_i[k-1] \end{aligned} \tag{7}$$

2.2 Centre of attraction and spontaneous positioning

In most FOA versions, the centre of attraction, the position towards which the fruit fly swarm moves, is usually the fruit fly that achieves the best performance. Although this is a natural step to take, the practice has shown that other algorithms, where the population moves towards random positions, perform much better in some truss optimisation problems. On the downside, this comes at the cost of significant performance degradation (slow convergence) for other problems.

A recent biological study showed that fruit flies have "free will" and can choose where they fly to, in the absence of a stimulus. This finding has triggered the development of the proposed Spontaneous Fruit Fly Optimisation (s-FOA)[1]. In s-FOA the fruit flies are attracted by a centre of attraction, which is defined by the combination of the positions of the fruit fly with the best performance and an arbitrarily selected fruit fly from the swarm.

Specifically, the positions of the fruit fly swarm in the next iterations are calculated according to:

$$\begin{aligned} X_{ij}[k] &= X_i^*[k] + 0.5 \cdot \left(X_0[k] + X_{i,rand}[k]\right) \cdot M \cdot (2 \cdot rand_{N_{res}} - 1) \\ Y_{ij}[k] &= Y_i^*[k] + 0.5 \cdot \left(Y_0[k] + Y_{i,rand}[k]\right) \cdot M \cdot (2 \cdot rand_{N_{res}} - 1) \end{aligned} \tag{8}$$

where j=1,2,…,m and i=1,…,N and $(X_{i,rand}[k], Y_{i,rand}[k])$ is a randomly selected point for each $(X_{ij}[k], Y_{ij}[k])$.

The following pseudocode summarises s-FOA.

begin

Select initial design vector $\mathbf{d_0}$=[DI_{01}, DI_{02}, …, DI_{0m}], m is the number of design variables

Generate initial fruit fly swarm $\mathbf{d_i}$, i=1,2,…, N in the vicinity of $\mathbf{d_0}$ using uniform discrete distribution [1, N_{res}]

Calculate the objective function value $\mathrm{Dm_i}$ at $\mathbf{d_i}$, $\mathrm{Dm_i} = \mathrm{f}(\mathbf{d_i})$

Rank the fruit flies and find the one with the best performance

$\mathrm{Dm}^* = \mathrm{f}(\mathbf{d}^*) = \min(\mathrm{Dm_i})$

If $\mathrm{Dm}^* < \mathrm{Dm_0}$ then $\mathbf{d_0} = \mathbf{d_i^*}$

while (k< K)

Increment k

Reposition the fruit fly swarm $\mathbf{d_i}$[k], near $\mathbf{d_0}$[k] using uniform discrete distribution [1, N_{res}]

Calculate the objective function value $\mathrm{Dm_i}[k] = \mathrm{f}(\mathbf{d_i}[k])$

Rank the fruit flies and find the best:

$\mathrm{Dm}^*[k] = \mathrm{f}(\mathbf{d}^*[k]) = \min(\mathrm{Dm_i}[k])$

If $\mathrm{Dm}^*[k] < \mathrm{Dm_0}[k]$ then $\mathbf{d_0}[k+1] = \mathbf{d}^*[k]$

Increment response time t[k] = t[k−1]+1

If (t[k]>delay time)

If $(\mathrm{Dm}^*[k] < \mathrm{Dm_0}[k - \kappa])$

reduce the search radius $\mathrm{M}[k+1] = c \cdot \mathrm{M}[k]$

(surging phase)

else if $(\mathrm{Dm}^*[k] = \mathrm{Dm_0}[k - \kappa])$

a random candidate, $\mathbf{d}_{rand}[k]$,

[1] https://io9.gizmodo.com/the-crazy-device-that-shows-fruit-flies-have-free-will-1459261376

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            D_rand[k] = f(d_rand[k]), becomes the new attraction point
            d0[k+1]=d_rand[k]

            (contrast-based vision phase)
        end if
    Initialise response time t[k]=0
    end if
End while
Post process results and visualisation
end
```

Figure 2 Pseudo-code of s-FOA.

3. Mathematical formulation of the truss design problem

The goal of truss optimisation is to minimize the weight of the structure such that constraints on its performance are satisfied. In this study, the design variable is chosen as the cross-sectional area of the members of the truss structures. The mathematical formulation of the truss optimisation problem is as shown below:

To minimize the weight of the truss,

$$Minimise\ W = \sum_{i=1}^{m} A_i\, \rho_i L_i \quad i = 1,2,3, \ldots\ldots, m \tag{9}$$

Where W is the weight of the truss structure consisting of m members and A_i, ρ_i and L_i are respectively the cross-sectional area, material density and length of the ith truss member. The design constraints are defined as follows:
Subject to:

$$\begin{aligned}
&\sigma_{min} \leq \sigma_i \leq \sigma_{max}, \qquad i = 1,2,3, \ldots\ldots, m \\
&\delta_{min} \leq \delta_j \leq \delta_{max}, \quad j = 1,2,3, \ldots\ldots, n \\
&\sigma_k^b \leq \sigma_k \leq 0, \qquad k = 1,2,3, \ldots\ldots, mc \\
&A_i \in S = \{A_1, A_2, A_3, \ldots\ldots, A_d\}
\end{aligned} \tag{10}$$

Where σ_i and δ_j are the ith member allowable stress and jth nodal displacement respectively; σ_k is the allowable buckling stress of the ith member under compression; S is a set of discrete cross-sectional area; m, mc and n are the total number of members, members subject to compression and nodes in the truss structure respectively.

In this work, the penalty approach is adopted for the transformation of the constrained optimisation problem to an unconstrained problem. Consequently, the mathematical formulation of the truss optimisation problem becomes:

$$f_{penalty} W = W + \lambda \sum_{r=1}^{nin} [\max(0, g_r)]^2 + \lambda \sum_{s=1}^{neq} [\max(0, |h_s|) - \epsilon]^2 \tag{11}$$

Where nin and neq are the number of equality and inequality constraints; g_r and h_s are the r^{th} equality and s^{th} inequality constraints; λ is the penalty value chosen as 10^5 and is the small positive tolerance for the equality constraints chosen as 10^{-6}.

4. Benchmark truss design problems and discussions

In this section, six benchmark truss problems consisting of 10, 15, 25, 52, 72 and 200 members are studied as a size optimisation problem constrained by nodal displacement and/or allowable stresses in the truss members. For each of the six truss optimisation problems, the performance of the s-FOA algorithm is investigated and compared to other implemented algorithms such as TLBO (Teaching Learning Based Optimisation), GA, DE (Differential Evolution) and PSO. A comparison is also made to the results reported in published literatures. All the benchmark problems were coded and the optimisation algorithms implemented in MatLab R2017a. A statistical stochastic performance analysis of each algorithm is also investigated by conducting 30 independent runs per problem. The DE, PSO, TLBO and GA algorithm used in this study are sourced from [25], [26], [27] and [28] respectively.

The operational parameters of the DE algorithm were selected according to [29]. The population size (N), cross-over probability (Cr), mutation factor (F), parameters chosen, and the maximum number of functional evaluations are as displayed on Table 1. The population members were created by a uniform random distribution. All variables were treated internally as floating variables by DE. The algorithm was terminated when the maximum number of functional evaluations is reached.

Table 1 Selected parameters for DE algorithm

DE						
Truss Problem	10 Bar	15 Bar	25 Bar	52 Bar	72 Bar	200 Bar
Number of Fitness Evaluations	2000	2000	2000	14000	10500	48000
Number of unknowns	10	15	8	12	16	29
Population size	12	28	12	28	37	75
Crossover probability (Cr)	0.2368	0.9426	0.2368	0.9426	0.9455	0.8803
Mutation factor (F)	0.6702	0.6607	0.6702	0.6607	0.6497	0.4717
Number of generations	167	71.429	167	500	283.78	600

For the PSO algorithm, the number of agents in the population (N), inertia weight, maximum velocity chosen and maximum number of functional analysis as shown on Table 2 were obtained from [30]. The initial swarm members were created by a random distribution. Subsequently, new members of the swarm were created using:

$$v_{i+1} = \omega \cdot v_i + c_1 \cdot r_p \cdot (p_i - x_i) + c_2 \cdot r_g \cdot (p_g - x_i) \quad (12)$$

$$x_{i+1} = x_i + v_{i+1}$$

Where parameters c_1 and c_2 were chosen to be 1.5 and 2.0 respectively. The inertia weight was selected within the range [0.8, 1.2] to prevent weak exploration and local optima entrapment. The algorithm was terminated when the number the maximum of functional evaluations is reached.

Table 1 Selected parameters for PSO algorithm

PSO						
Truss Problem	10 Bar	15 Bar	25 Bar	52 Bar	72 Bar	200 Bar
Number of Fitness Evaluations	2000	2000	2000	14000	10500	48000
Number of unknowns	10	15	8	12	16	29
Population size	12	28	37	28	37	75
Number of generations	167	71	54	500	284	600
Inertia weight	0.8					
Vmax	Upper bound Cross-Sectional Area					

To ensure optimal performance of the GA algorithm, tuning parameters were selected from [31]. The size of the population (N), cross-over probability (Cr), mutation factor (F) as well as the maximum number of structural analyses selected are shown in Table 3. Initial population members were generated by a uniform random distribution. The parents in each generation were selected stochastically and then weighted by the crossover operator for the creation of new members. Mutation diversified the population members through random selection.

Table 2 Selected parameters for GA algorithm

GA						
Truss Problem	10 Bar	15 Bar	25 Bar	52 Bar	72 Bar	200 Bar
Number of Fitness Evaluations	2000	2000	2000	14000	10500	48000
Number of unknowns	10	15	8	12	16	29
Population size	37	37	37	28	37	75
Number of generations	54	54	54	500	284	600

The TLBO algorithm utilised is that from [27] with a maximum number of analysis as shown on Table 4. The class members were created randomly using a uniform distribution. The best student is selected as the class teacher in the teacher phase. In the learner's phase, the class members improve the individual and class performance through student-student interaction. Successive implementation of both phases goes on until the number of maximum of functional evaluations is reached.

Table 4 Selected parameters for TLBO algorithm

TLBO						
Truss Problem	10 Bar	15 Bar	25 Bar	52 Bar	72 Bar	200 Bar
Number of Fitness Evaluations	2000	2000	2000	14000	10500	48000
Number of unknowns	10	15	8	12	16	29
Population size	28	28	12	28	37	75
Number of generations	36	36	84	250	142	300

The parameters for the c-FOA algorithm as shown on Table 5 are selected as: Population size=50, κ=320, M=0.95, N_{res}=50 and c=0.92.

Table 5 Selected parameters for cFOA algorithm

cFOA						
Truss Problem	10 Bar	15 Bar	25 Bar	52 Bar	72 Bar	200 Bar
Number of Fitness Evaluations	2000	2000	2000	14000	9600	45000
Number of unknowns	10	15	8	12	16	29
Population size	10	15	8	20	16	60
Number of generations	200	130	250	700	600	800

The parameters for the s-FOA algorithm as shown on Table 5 are selected as: Population size=50, κ=5, M=0.95, N_{res}=10 and c=0.9.

Table 6 Selected parameters for s-FOA algorithm

s-FOA						
Truss Problem	10 Bar	15 Bar	25 Bar	52 Bar	72 Bar	200 Bar
Number of Fitness Evaluations	2000	1950	2000	14000	9600	45000
Number of unknowns	10	15	8	12	16	29
Population size	10	15	8	20	16	60
Number of generations	200	130	250	700	600	800

4.1 : Benchmark 1: 10-Bar planar truss

4.1.1 Benchmark 1: Problem Description

The first truss problem is a 10-bar planar structure as shown in Figure 3. The design was examined by Sadollah *et al*. [33], Li *et al*. [34], Ho-Huu *et al*. [35], Do and Lee [36] and many others as a size optimisation problem with ten design variables. The design variable is defined as the cross-sectional area of the truss members and is chosen from a discrete set of data S= {1.62, 1.80, 1.99, 2.13, 2.38, 2.62, 2.88, 2.93, 3.09, 3.13,3.38, 3.47, 3.55, 3.63, 3.84, 3.87, 3.88, 4.18, 4.22, 4.49, 4.59, 4.80,4.97, 5.12, 5.74, 7.22, 7.97, 11.50, 13.50, 13.90, 14.20, 15.50, 16.00,16.90, 18.80, 19.90, 22.00, 22.90, 26.50, 30.00, 33.50}in^2. Each member of the truss has a material density and a modulus of elasticity as defined in Table 6. Two vertical loads of 10000 lbs each are vertically applied to the truss at nodes 2 and 4 with each member and node constrained as also shown in Table 6.

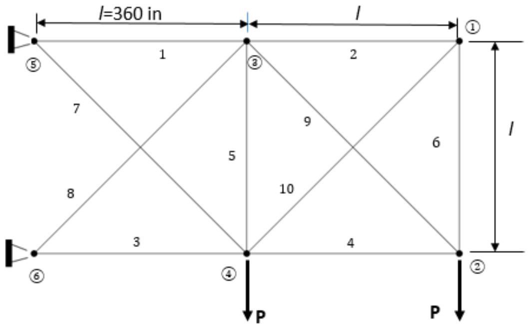

Figure 3 Benchmark Case 1: The 10-bar planar truss structure

Table 7 *Material properties and design constraints of the benchmark truss problems*

Truss Problem	10 Bar	15 Bar	25 Bar	52 Bar	72 Bar	200 Bar
Material Density ?	0.1 lb/in3	7800 kg/m3	0.1 lb/in3	7800 kg/m3	0.1 lb/in3	0.283 lb/in3
Modulus of Elasticity E	104 ksi	200 GPa	104 ksi	2.07 x105 MPa	104 ksi	30000 ksi
Nodal Displacement Constraint δ	± 2 inches the x and y direction	± 10 mm the x and y direction	± 0.35 inches the x and y direction	N/A	± 0.25 inches the x and y direction	N/A
Stress Constraint σ	±25ksi in tension and compression	±120MPa in tension and compression	±40ksi in tension and compression	±180MPa in tension and compression	±25ksi in tension and compression	±10 ksi in tension and compression

4.1.2 Benchmark 1: Results and discussion

The optimal results obtained from the run of all five algorithms and the MBA [33], HPSO [34], aeDE [35], SOS [36] and mSOS [36] are presented in Tables 7 and 8. The results highlight the size variables, best weight, mean weight, standard deviation and number of functional evaluations for all algorithms in this study and in reported literature. From the results, it can be observed that the s-FOA produces the best continuous solution without any constraint violation as 5421.2 lb compared to the other algorithms implemented in the investigation. Relatively, the s-FOA also proves to be the most consistent of the implemented algorithms by recording the lowest standard deviation of 41.62 lb. It further displays the best mean weight of 5506.2 lb.

The best discrete optimum result is achieved by s-FOA,DE, PSO, TLBO, SOS, mSOS, and aeDE with a weight of 5490.738 lb. However, the maximum number of functional analyses used in this research is small compared to that implemented in the reported literatures.

Table 8 Benchmark Case 1: Performance rankings of the algorithms in solving the 10 bar truss problem

	PERFORMANCE RANKINGS					
	DE	PSO	GA	TLBO	cFOA	s-FOA
Best Value	4	3	6	5	2	1
Mean Value	5	2	6	4	3	1
Number of Functional Evaluations per 10,000	0.2	0.2	0.2	0.2	0.2	0.2
Standard deviation	5	3	6	4	2	1
Number of tuning parameters changed from one benchmark problem to another	5	3	3	3	3	3
Total Score	16.2	9.2	18.2	13.2	10.2	6.2

Table 9 Optimal results of the 10-Bar planar truss structure optimisation obtained from different algorithms from previous studies

Area group	PREVIOUS STUDY					
	HPSO [34]	MBA [33]	aeDE [35]	DE [36]	SOS [36]	mSOS [36]
in^2	Discrete optimum	Discrete optimum	Discrete optimum	Discrete optimum	Discrete optimum	Discrete optimum
A1	30	30	33.5	33.5	33.5	33.5
A2	1.62	1.62	1.62	1.62	1.62	1.62
A3	22.9	22.9	22.9	22.9	22.9	22.9
A4	13.5	16.9	14.2	14.2	14.2	14.2
A5	1.62	1.62	1.62	1.62	1.62	1.62
A6	1.62	1.62	1.62	1.62	1.62	1.62
A7	7.97	7.97	7.97	7.97	7.97	7.97
A8	26.5	22.9	22.9	22.9	22.9	22.9
A9	22	22.9	22	22	22	22
A10	1.8	1.62	1.62	1.62	1.62	1.62
Weight (lb)	5531.98	5507.75	5490.738	5490.738	5490.738	5490.738
Mean weight (lb)	-	-	-	-	-	-
Standard Deviation	-	-	-	-	-	-
Number of Functional Evaluations	50000	3600	2380	300,000	300,000	300,000

Table 10 Optimal results of the 10-Bar planar truss structure optimisation obtained from different algorithms in this study

Area group	THIS STUDY											
	TLBO		PSO		DE		GA		cFOA		s-FOA	
in^2	Continuous Optimum	Discrete optimum	Continuous Optimum	Discrete optimum	Continuous Optimum	Discrete optimum	Continuous Optimum	Discrete optimum	Continuous Optimum	Discrete Optimum	continuous	Discrete optimum
A1	32.7009	33.5	31.7749	33.5	33.5	33.5	32.7403	30	32.554386	33.5	31.6141	33.5
A2	1.62	1.62	1.6295	1.62	1.6462	1.62	3.0609	2.93	1.613358	1.62	1.6218	1.62
A3	23.8969	22.9	23.906	22	22.4672	22.9	25.5496	22.9	22.782222	22.9	23.3547	22.9
A4	15.0507	14.2	15.146	14.2	15.4151	14.2	19.8758	18.8	15.63543	14.2	14.8084	14.2
A5	1.636	1.62	1.6201	1.62	1.6209	1.62	1.8382	1.8	1.615788	1.62	1.1627	1.62
A6	1.6396	1.62	1.62	1.62	1.6203	1.62	2.4316	2.38	1.622592	1.62	1.2364	1.62
A7	8.146	7.97	8.6446	7.97	7.9847	7.97	11.7973	7.97	8.411202	7.97	8.269	7.97
A8	21.7906	22.9	22.8267	22.9	23.3522	22.9	19.1187	18.8	22.61358	22.9	22.8569	22.9
A9	22.2357	22	21.1247	22	20.9034	22	17.9177	16.9	21.460788	22	21.534	22
A10	1.6508	1.62	1.62	1.62	1.62	1.62	2.6401	2.38	1.618056	1.62	1.6257	1.62
Weight (lb)	5495.8	5490.738	5485.2	5490.738	5487.8	5490.738	5698.5	5526.962	5484.2	5491.717	5421.2	5490.738
Mean weight (lb)	5570.5		5539.8		5688.9		6223.7		5555		5506.2	
Standard Deviation	130.9274		61.3838		253.0152		359.0314		46.1695		41.62	
Number of Functional Evaluations	2000		2000		2000		2000				2000	

4.2 : Benchmark 2: 15-Bar planar truss

4.2.1 Benchmark 2: Problem description

Another benchmark truss problem considered is the 15-bar planar truss. The problem has been investigated by researchers including Li *et al.* [34]. Each member of this truss problem has its material property and design constraints defined on Table 6. The problem has 15 design variables chosen from a discrete set of data S = {113.2, 143.2, 145.9, 174.9, 185.9, 235.9, 265.9, 297.1, 308.6, 334.3, 338.2, 497.8, 507.6, 736.7, 791.2, 1063.7} mm^2. The load cases acting on the structure are: Case 1: P1 = 35 kN, P2 = 35 kN, P3 = 35 kN; Case 2: P1 = 35 kN, P2 = 0 kN, P3 = 35 kN; Case 3: P1 = 35 kN, P2 = 35 kN, P3 = 0 kN
Figure 4 shows the geometry and loading arrangement of the truss.

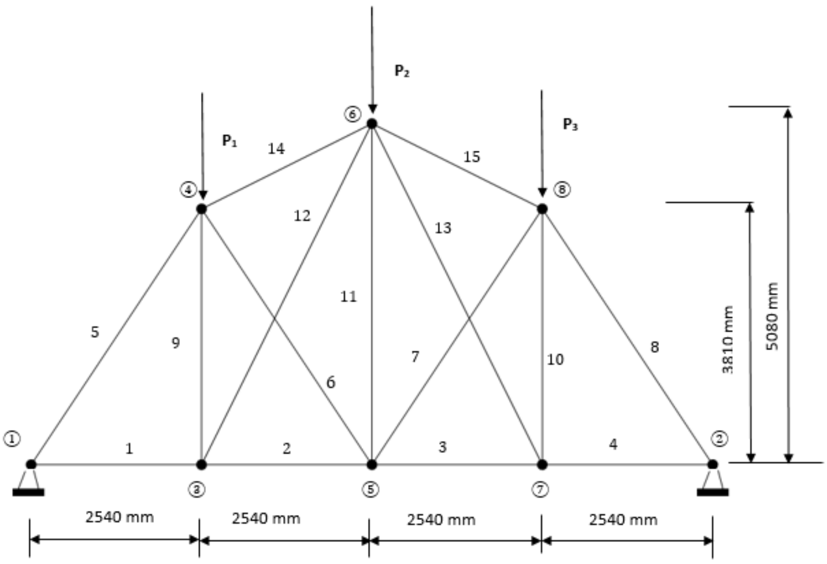

Figure 4 Benchmark Case 2: The 15-Bar planar truss structure

4.2.2 Benchmark 2: Results and discussion

The optimal design of the truss is obtained by taking all three load cases into consideration. A summary of the result is presented in Tables 9 and 10. From the analysis of the continuous results investigated in this study, the s-FOA produces the lightest truss weight of 86.57 kg compared to the other algorithms. s-FOA also records the lowest standard deviation of 1.70 kg and mean weight of 89.75 kg, yet again proving the stability and robustness of the algorithm. From the discrete solution analysis, the s-FOA, TLBO, DE, and HPSO [34] provide the best weight of 105.74 kg each compared to that of cFOA, GA and PSOPC [34] with weights of 107.35kg, 117.96 kg and 108.96 kg.

Table 11 Benchmark Case 2: Performance rankings of the algorithms for the 15 bar truss problem

	PERFORMANCE RANKINGS					
	DE	PSO	GA	TLBO	cFOA	s-FOA
Best Value	5	2	6	3	4	1
Mean Value	5	2	6	3	4	1
Number of Functional Evaluations per 10,000	0.2	0.2	0.2	0.2	0.2	0.195
Standard deviation	5	4	6	3	1	2
Number of tuning parameters changed from one benchmark problem to another	5	3	3	3	3	3
Total Score	17.2	10.2	18.2	11.2	12.2	6.195

Table 12 Optimal results of the 15-bar planar truss structure optimisation obtained from different algorithms

				15 Bar											
Area group	PREVIOUS STUDY			THIS STUDY											
	PSO [34]	PSOPC [34]	HPSO [34]	TLBO		PSO		DE		GA		cFOA		s-FOA	
mm²	Discrete optimum	Discrete optimum	Discrete optimum	Continuous Optimum	Discrete optimum	Continuous Optimum	Discrete optimum	Continuous Optimum	Discrete optimum	Continuous Optimum	Discrete optimum	Continuous Optimum	Discrete Optimum	Continuous Optimum	Discrete optimum
A1	185.9	113.2	113.2	113.2	113.2	113.2	113.2	113.2	113.2	135.8017	113.2	122.58428	113.2	139.0209	113.2
A2	113.2	113.2	113.2	120.7607	113.2	113.2	113.2	113.2	113.2	128.8207	113.2	166.90208	145.9	114.3433	113.2
A3	143.2	113.2	113.2	113.2	113.2	113.2105	113.2	141.3093	113.2	113.2	113.2	123.8974	113.2	129.614	113.2
A4	113.2	113.2	113.2	143.9061	143.2	113.2	113.2	113.2	113.2	176.152	174.9	134.06276	113.2	92.4504	113.2
A5	736.7	736.7	736.7	526.5156	736.7	525.8146	736.7	539.219	736.7	673.1077	736.7	530.1722	736.7	528.5308	736.7
A6	143.2	113.2	113.2	118.266	113.2	113.2	113.2	129.2336	113.2	123.0755	113.2	117.93176	113.2	81.6285	113.2
A7	113.2	113.2	113.2	118.606	113.2	113.2	113.2	141.3216	113.2	113.2	113.2	121.53152	113.2	87.3564	113.2
A8	736.7	736.7	736.7	531.2776	736.7	525.8101	736.7	527.013	736.7	788.847	736.7	526.06304	736.7	530.342	736.7
A9	113.2	113.2	113.2	113.6073	113.2	113.2	113.2	124.7797	113.2	138.0283	113.2	152.6502	145.9	88.3752	113.2
A10	114.2	113.2	113.2	120.7674	113.2	113.2	113.2	123.2264	113.2	169.4542	145.9	126.35384	113.2	110.3134	113.2
A11	113.2	113.2	113.2	113.2	113.2	113.2	113.2	125.7362	113.2	202.6266	185.9	118.10156	113.2	83.0209	113.2
A12	116.2	113.2	113.2	113.2	113.2	113.2114	113.2	113.2	113.2	117.936	113.2	112.9736	113.2	105.2534	113.2
A13	117.2	185.9	114.2	113.2	113.2	113.2088	113.2	120.6281	113.2	196.3723	185.9	119.79956	113.2	103.5667	113.2
A14	334.3	334.3	334.3	322.484	334.3	321.2347	334.3	528.2988	334.3	540.2041	507.6	338.64912	334.3	325.5745	334.3
A15	334.3	334.3	334.3	328.466	334.3	321.2326	334.3	335.0375	334.3	476.7704	338.2	333.3174	334.3	344.8298	334.3
Weight(kg)	108.84	108.96	105.735	91.8692	106.329	90.093	105.735	99.1078	105.735	125.2476	117.956	95.2911	107.335	86.5694	105.735
Mean weight (kg)	-	-	-	94.1729		91.9209		105.5734		152.0246		96.544		89.7546	
Standard Deviation	-	-	-	1.8892		5.2483		6.4822		14.0832		1.2195		1.7082	
Number of Functional Evaluations	25000	25000	25000	2000		2000		2000		2000				1950	

4.3 : Benchmark 3 – 25-Bar planar truss

4.3.1 Benchmark 3: Problem description

The third benchmark truss illustrated in Figure 5 is the 25-bar space truss. The truss design problem previously investigated by Sadollah *et al.*, Li *et al.*, Wu *et al.* and Lee *et al.* [33, 34, 37, 38] has its material properties and design constraints defined in Table 6. The loading configuration of the truss is defined in Table 11 with each member categorised into a group of 8 representing the design variables and chosen from a discrete set of data S= {0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.6, 2.8, 3.0, 3.2, 3.4} (in^2). The 8 design groups are as follows: (1) A1; (2) A2–A5; (3) A6–A9; (4) A10–A11; (5) A12–A13; (6) A14–A17; (7) A18–A21, and (8) A22–A25.

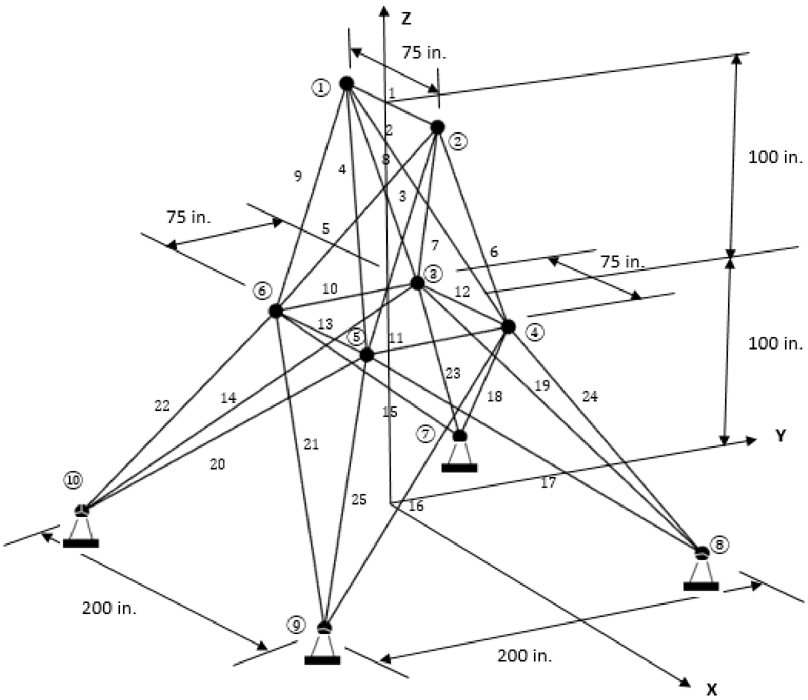

Figure 5 *Benchmark Case 3: The 25-Bar spatial truss structure*

Table 13 Benchmark Case 3: Loading Configuration of the 25-bar truss

Nodes	Loads		
	P_x (kips)	P_y (kips)	P_z (kips)
1	1	−10	−10
2	0	−10	−10
3	0.5	0	0
6	0.6	0	0

4.3.2 Benchmark 3: Results and discussion

A comparison of the optimal and statistical result obtained for the 25-bar truss design is as addressed in Table 12 and 13. The best continuous truss design weighing 483.9 lb is provided by s-FOA and cFOA. The s-FOA also gives the lowest standard deviation and second best mean weight of 9.97 lb and 492.9 lb respectively as compared to the PSO, DE, GA and TLBO algorithms. Therefore, it can be expressed that the s-FOA exhibits more robustness and accuracy over the compared algorithms from the view point of continuous optimum design.

From a discrete optimisation perspective, the design found by the s-FOA and cFOA proves to be the lightest truss weighing 483.87lb compared to the, MBA [33], HPSO [34], SOS [36], mSOS [36] and HS [38] of 484.85lb each, SGA [37] of 486.29 lb, DE of 485.57 lb, TLBO of 487.87 lb, PSO of 488.57 lb and GA of 489.49 lb. The performance of the s-FOA is further evidenced by the attainment of the solution under a significantly lower number of structural analysis of 1950 compared to 300,000 analyses required by the SOS [36] and mSOS [36], 40000 analyses by the SGA [37], 25000 analyses by the HPSO [34], 18734 analyses by the HS [38] and 3750 by the MBA [33].

Table 14 Benchmark Case 3: Performance rankings of the algorithms for the 25-bar truss problem

	PERFORMANCE RANKINGS					
	DE	PSO	GA	TLBO	cFOA	s-FOA
Best Value	5	4	6	3	1	1
Mean Value	5	4	6	1	3	2
Number of Functional Evaluations per 10,000	0.2	0.2	0.2	0.2	0.2	0.2
Standard deviation	2	6	5	4	3	1
Number of tuning parameters changed from one benchmark problem to another	5	3	3	3	3	3
Total Score	15.2	14.2	17.2	9.2	10.2	7.2

Table 15 Optimal results of the 25-bar space truss structure optimisation obtained from different algorithms from previous study

25 Bar							
PREVIOUS STUDY							
Area group	SGA [37]	HS [38]	HPSO [34]	MBA [33]	DE [36]	SOS [36]	mSOS [36]
in^2	Discrete optimum	Discrete optimum	Discrete optimum	Discrete optimum	Discrete optimum	Discrete optimum	Discrete optimum
A1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
A2	0.5	0.3	0.3	0.3	0.3	0.3	0.3
A3	3.4	3.4	3.4	3.4	3.4	3.4	3.4
A4	0.1	0.1	0.1	0.1	0.1	0.1	0.1
A5	1.5	2.1	2.1	2.1	2.1	2.1	2.1
A6	0.9	1	1	1	1	1	1
A7	0.6	0.5	0.5	0.5	0.5	0.5	0.5
A8	3.4	3.4	3.4	3.4	3.4	3.4	3.4
Weight (lb)	486.29	484.85	484.85	484.85	484.85	484.85	484.85
Mean weight	-	-	-	-	-	-	-
Standard Deviation	-	-	-	-	-	-	-
Functional Evaluations	40000	18734	25000	3750	300,000	300,000	300,000

Table 16 Optimal results of the 25-bar space truss structure optimisation obtained from different algorithms in this study

THIS STUDY												
Area group	**TLBO**		**PSO**		**DE**		**GA**		**cFOA**		**s-FOA**	
in^2	Continuous Optimum	Discrete optimum	Continuous Optimum	Discrete optimum	Continuous Optimum	Discrete optimum	Continuous Optimum	Discrete optimum	Continuous Optimum	Discrete Optimum	Continuous Optimum	Discrete optimum
A1	0.1012	0.1	0.1	0.1	0.1	0.1	0.1043	0.1	0.14331	0.1	0.14331	0.1
A2	0.3621	0.4	0.3163	0.4	0.3864	0.4	0.7715	0.7	0.39318	0.4	0.39318	0.4
A3	3.3967	3.4	3.4	3.4	3.4	3.4	3.345	3.4	3.4493	3.4	3.4493	3.4
A4	0.1019	0.1	0.1	0.1	0.1	0.1	0.126	0.1	0.25793	0.2	0.25793	0.2
A5	1.8504	1.9	2.0739	2	1.7151	1.8	1.4307	1.5	1.75214	1.7	1.75214	1.7
A6	0.9904	1	0.9809	1	0.9456	1	0.8461	0.8	0.93785	0.9	0.93785	0.9
A7	0.5107	0.5	0.5055	0.5	0.5812	0.5	0.5675	0.6	0.45127	0.5	0.45127	0.5
A8	3.4	3.4	3.4	3.4	3.3885	3.4	3.3693	3.4	3.4493	3.5	3.4493	3.5
Weight (lb)	484.3278	487.07	484.3331	488.57	484.9171	485.57	489.6039	489.49	483.8996	483.67	483.8986	483.67
Mean weight (lb)	491.4399		494.4975		496.2969		516.2156		493.22		492.8937	
Standard Deviation	13.3153		21.1506		10.2382		16.6668		12.243		9.9664	
Functional Evaluations	2000		2000		2000		2000				2000	

4.4 : Benchmark 4 - 52 Bar planar truss

4.4.1 Benchmark 4: Problem description

The 52-bar planar truss is shown in Figure 6. Researchers such as Li et al, Sadollah et al and Do and Lee [34, 36] have studied this problem. The material properties and design constraints of the truss are highlighted in Table 6. Two vertical loads of 10000 lbs each are vertically applied to the truss at nodes 2 and 4. The design variables are categorised into 12 groups as follows: (1) A_1–A_4; (2) A_5–A_{10}; (3) A_{11}–A_{13},;(4) A_{14}–A_{17}; (5) A_{18}–A_{23}; (6) A_{24}–A_{26}; (7) A_{27}–A_{30}; (8) A_{31}–A_{36}; (9)A_{37}–A_{39}; (10) A_{40}–A_{43}; (11) A_{44}–A_{49}, and (12) A_{50}–A_{52}. The discrete cross-sectional areas are chosen according to the AISC codes presented in Table 14.

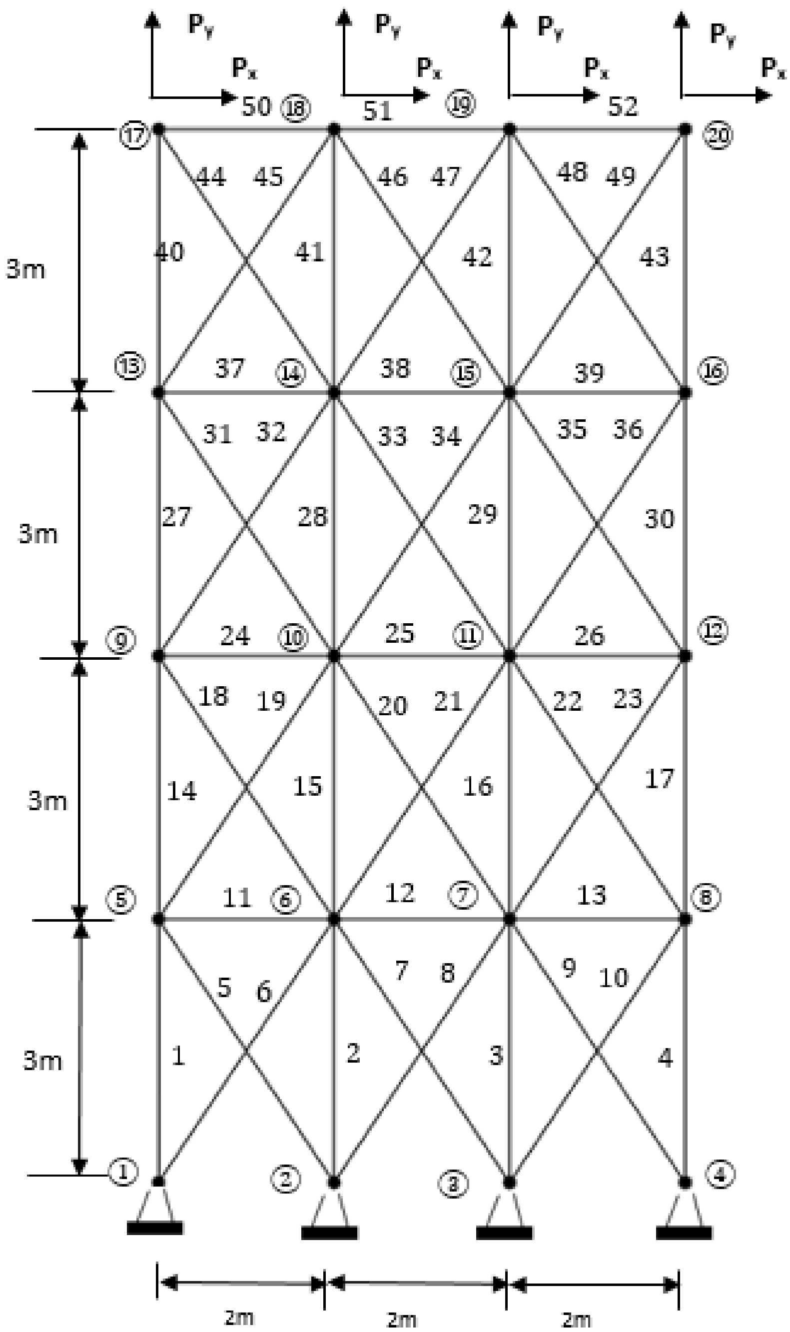

Figure 6 Benchmark Case 4: The 52-Bar planar truss structure

Table 17 The available cross-sectional areas of the AISC code

No	in^2	mm^2	No	in^2	mm^2	No	in^2	mm^2	No	in^2	mm^2
1	0.111	71.613	17	1.563	1008.385	33	3.840	2477.414	49	11.500	7419.340
2	0.141	90.968	18	1.620	1045.159	34	3.870	2496.769	50	13.500	8709.660
3	0.196	126.451	19	1.800	1161.288	35	3.880	2503.221	51	13.900	8967.724
4	0.250	161.290	20	1.990	1283.868	36	4.180	2696.769	52	14.200	9161.272
5	0.307	198.064	21	2.130	1374.191	37	4.220	2722.575	53	15.500	9999.980
6	0.391	252.258	22	2.380	1535.481	38	4.490	2896.768	54	16.000	10322.560
7	0.442	285.161	23	2.620	1690.319	39	4.590	2961.284	55	16.900	10903.204
8	0.563	363.225	24	2.630	1696.771	40	4.800	3096.768	56	18.800	12128.008
9	0.602	388.386	25	2.880	1858.061	41	4.970	3206.445	57	19.900	12838.684
10	0.766	494.193	26	2.930	1890.319	42	5.120	3303.219	58	22.000	14193.520
11	0.785	506.451	27	3.090	1993.544	43	5.740	3703.218	59	22.900	14774.154
12	0.994	641.289	28	3.130	2019.351	44	7.220	4658.055	60	24.500	15806.420
13	1.000	645.160	29	3.380	2180.641	45	7.970	5141.925	61	26.500	17096.740
14	1.228	792.456	30	3.470	2238.705	46	8.530	5503.215	62	28.000	18064.480
15	1.266	816.773	31	3.550	2290.318	47	9.300	5999.988	63	30.000	19354.800
16	1.457	939.998	32	3.630	2341.931	48	10.850	6999.986	64	33.500	21612.860

4.4.2 Benchmark 4: Results and discussion

Tables 15 and 16 present the optimal solutions of the 52-bar truss problem. From observation, it can be seen that the TLBO algorithm produces the best truss design with a corresponding weight of 1816.4 kg compared to the s-FOA which obtains a weight of 1819.2kg, DE algorithm with 1820.9 kg, PSO with 1822.5 kg, cFOA with 1839.7 kg and GA with 4207.3 kg. However, from the statistical analysis, the DE algorithm attains the best mean weight of 1830.5 kg and standard deviation of 7.40 kg compared to a mean weight and standard deviation weight of 1834.3 kg in TLBO and 1840.5 kg and 16.73 kg in s-FOA. Hence DE is the most robust and consistent algorithm for solving the 52-bar truss problem of all algorithms implemented.

Nevertheless, from a discrete perspective, the restricted number of structural analysis of 2000 significantly affects the performance of the algorithms (DE, TLBO, PSO, GA, cFOA and s-FOA) investigated in this study as compared that reported in other literatures; 150,000 analyses in HPSO [34], 300,000 analyses in SOS [36] and mSOS [36]. The s-FOA, PSO and TLBO algorithm record the lightest discrete weight of 1912.6 kg which is 1.01% heavier than that obtained by mSOS [36] of 1899.7 kg. The DE algorithm, cFOA and GA produces a weight of 1914.1 kg, 1927.828 kg and 4063.5 kg respectively, 1.01%, 1.02% and 2.14% worse than mSOS [36].

Table 18 Benchmark Case 4: Performance rankings of the algorithms in solving the 52 bar truss problem

	PERFORMANCE RANKINGS					
	DE	PSO	GA	TLBO	cFOA	s-FOA
Best Value	3	4	6	1	5	2
Mean Value	1	5	6	2	4	3
Number of Functional Evaluations per 10,000	1.4	1.4	1.4	1.4	1.4	1.4
Standard deviation	1	5	6	2	4	3
Number of tuning parameters changed from one benchmark problem to another	5	3	3	3	3	3
Total Score	11.4	16.4	19.4	9.4	17.4	12.4

Table 19 Optimal results of the 52-bar planar truss structure optimisation obtained from different algorithms from previous study

52 Bar					
	PREVIOUS STUDY				
Area group	HPSO [34]	MBA [33]	DE [36]	SOS [36]	mSOS [36]
mm^2	Discrete optimum	Discrete optimum	Discrete optimum	Discrete optimum	Discrete optimum
A1	4658.06	4658.06	4658.06	4658.06	4658.06
A2	1161.29	1161.29	1161.29	1161.29	1161.29
A3	363.225	494.193	494.193	494.193	494.193
A4	3303.22	3303.22	3303.22	3303.22	3303.22
A5	940	940	940	940	940
A6	494.193	494.193	506.451	494.193	506.451
A7	2238.71	2238.71	2238.71	2238.71	2238.71
A8	1008.39	1008.39	1008.39	1008.39	1008.39
A9	388.386	494.193	388.386	494.193	388.386
A10	1283.87	1283.87	1283.87	1283.87	1283.87
A11	1161.29	1161.29	1161.29	1161.29	1161.29
A12	792.256	494.193	506.451	494.193	506.451
Weight (kg)	1905.49	1902.605	1899.654	1902.605	1899.654
Mean weight	-	-	-	-	-
Standard Deviation	-	-	-	-	-
Functional Evaluations	150000	5450	300,000	300,000	300,000

Table 20 Optimal results of the 52-bar planar truss structure optimisation obtained from different algorithms in this study

52 Bar												
	THIS STUDY											
Area group	TLBO		PSO		DE		GA		cFOA		s-FOA	
mm^2	Continuous Optimum	Discrete optimum	Continuous Optimum	Discrete optimum	Continuous Optimum	Discrete optimum	Continuous Optimum	Discrete optimum	Continuous Optimum	Discrete Optimum	Continuous Optimum	Discrete optimum
A1	4386.8	4658.055	4390.6	4658.055	4396.1	4658.055	7880.6	7419.34	4363.48751	4658.055	4417.6	4658.055
A2	1129.2	1161.288	1124.8	1161.288	1126.7	1161.288	2008.6	1993.544	1163.195636	1161.288	1121.7	1161.288
A3	318.4	252.258	282.5	252.258	293.2	285.161	837.8	792.256	369.2652732	363.225	266.6	252.258
A4	3376.2	3703.218	3372.4	3703.218	3375.6	3703.218	5822	5503.215	3445.265624	3703.218	3393.2	3703.218
A5	861.9	939.998	870.3	939.998	885.5	939.998	1810.5	1696.771	878.6127357	939.998	877.8	939.998
A6	236	252.258	271.3	252.258	252.9	252.258	1091.6	1045.159	316.0639755	285.161	241	252.258
A7	2295.7	2290.318	2281.3	2290.318	2304.2	2290.318	6212	5999.998	2292.890711	2290.318	2312.4	2290.318
A8	968.3	1008.385	973.6	1008.385	964.3	1008.385	2522.9	2503.221	981.384552	1008.385	960.3	1008.385
A9	282.7	285.161	294.9	285.161	268.9	285.161	3246.8	3206.445	260.1915129	285.161	257.3	285.161
A10	1305	1283.868	1281.7	1283.868	1312.5	1283.868	3926.7	3703.218	1314.09855	1374.191	1345.9	1283.868
A11	1062	1161.288	1092.8	1161.288	1068.9	1161.288	1337.1	1696.771	1081.592623	1045.159	1050	1161.288
A12	451.9	494.193	500.1	494.193	433.3	494.193	8666.8	7419.34	432.4494231	494.193	414.4	494.193
Weight (kg)	1816.4	1912.524	1822.5	1912.524	1820.9	1914.076	4207.3	4063.478	1839.7	1927.828	1819.2	1912.524
Mean weight (kg)	1834.3		2270.8		1830.5		5295.4		1882.6		1840.5	
Standard Deviation	15.6369		621.5253		7.4022		875.5519		38.5744		16.7269	
Functional Evaluations	14000		14000		14000		14000				14000	

4.5 : Benchmark 5 – 72-Bar planar truss

4.5.1 Benchmark 5: Problem description

The 72-bar planar truss is selected as the fourth problem as shown in Figure 7. The material properties and design constraints of the truss are indicated in Table 6. Table 17 defines the load cases acting on the truss structure. All structural members are categorised into 16 design variables as follows; (1) A1–A4, (2) A5–A12, (3) A13–A16, (4) A17–A18, (5) A19–A22, (6) A23–A30 (7) A31–A34, (8) A35–A36, (9) A37–A40, (10) A41–A48, (11) A49–A52, (12) A53–A54, (13) A55–A58, (14) A59–A66 (15) A67–A70, (16) A71–A72. A discrete set of data as displayed in Table 14 is given for the design.
The truss problem has been previously treated by Li *et al*. and Kaveh and Mahdavi [34, 39].

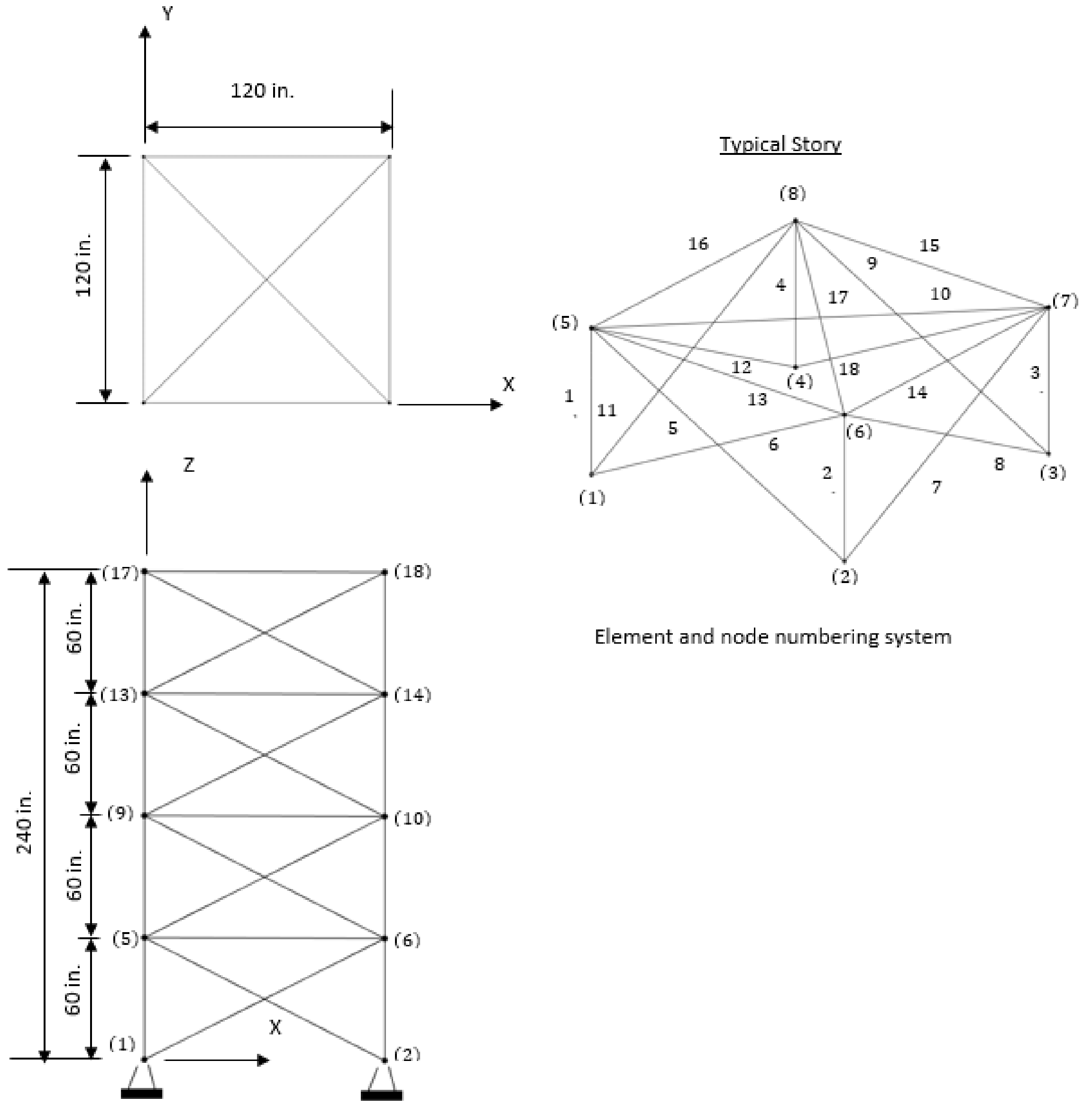

Figure 7 Benchmark Case 5: The 72-Bar Spatial truss structure

Table 21 Benchmark Case 5: Loading Configuration of the 72-bar truss

Nodes	Load case 1			Load case 2		
	P_x (kips)	P_y (kips)	P_z (kips)	P_x (kips)	P_y (kips)	P_z (kips)
17	5	5	-5	0	0	-5
18	0	0	0	0	0	-5
19	0	0	0	0	0	-5
20	0	0	0	0	0	-5

4.5.2 Benchmark 5: Results and discussion

The optimum 72-bar truss design found by the s-FOA is compared in Tables 18 and 19 to those obtained by other algorithms in this study and other reported literatures. As can be seen, the s-FOA produces yet again the best continuous optimal weight of 403.55lb compared to other algorithms examined. However, the TLBO algorithm attains the best mean weight and standard deviation of 404.12 lb and 0.45 lb respectively compared to the s-FOA algorithm with a mean weight of 406.17 lb and standard deviation of 1.64 lb. The ECBO [39] presents the best discrete truss design with a weight of 389.33 lb. Nevertheless, the number of structural analysis utilised by the ECBO algorithm to achieve the results is 7320 less than that utilised in this study. With a maximum number of functional analysis of 2000, the s-FOA, TLBO, DE, PSO algorithms obtained a discrete weight of 403.22 lb each while the cFOA and GA algorithm produced a weight of 408.51 lb and 594.42 lb respectively.

Table 22 Benchmark Case 5: Performance rankings of the algorithms in solving the 72-bar truss problem

	PERFORMANCE RANKINGS					
	DE	PSO	GA	TLBO	cFOA	s-FOA
Best Value	4	2	6	3	5	1
Mean Value	5	4	6	1	3	2
Number of Functional Evaluations per 10,000	1.05	1.05	1.05	1.05	1.05	0.96
Standard deviation	4	5	6	1	3	2
Number of tuning parameters changed from one benchmark problem to another	5	3	3	3	3	3
Total Score	17.05	13.05	19.05	9.05	15.05	8.96

Table 23 Optimal results of the 72-bar space truss structure optimisation obtained from different algorithms

				72 Bar												
Area group	PREVIOUS STUDY								THIS STUDY							
	SGA [26]	HPSO [22]	CBO [28]	ECBO [28]	**TLBO**		**PSO**		**DE**		**GA**		**cFOA**		**s-FOA**	
in²	Discrete optimum	Discrete optimum	Discrete optimum	Discrete optimum	Continuous Optimum	Discrete optimum	Continuous Optimum	Discrete optimum	Continuous Optimum	Discrete optimum	Continuous Optimum	Discrete optimum	Continuous Optimum	Discrete optimum	Continuous Optimum	Discrete optimum
A1	0.196	4.97	1.62	1.99	1.9691	1.8	1.9119	1.8	2.1191	1.8	2.5972	2.38	1.9495041	1.99	1.9877	1.8
A2	0.602	1.228	0.563	0.563	0.5344	0.563	0.5354	0.563	0.4829	0.563	0.3776	0.391	0.5275275	0.563	0.5367	0.563
A3	0.307	0.111	0.111	0.111	0.1111	0.111	0.111	0.111	0.111	0.111	0.3366	0.307	0.1427016	0.141	0.1086	0.111
A4	0.766	0.111	0.111	0.111	0.1118	0.111	0.111	0.111	0.1282	0.111	0.1524	0.141	0.1601286	0.141	0.111	0.111
A5	0.391	2.88	1.457	1.288	1.3882	1.457	1.3905	1.457	1.5096	1.457	1.3759	1.266	1.3779207	1.457	1.3584	1.457
A6	0.391	1.457	0.442	0.442	0.5921	0.602	0.5936	0.602	0.5753	0.602	1.116	1	0.5971467	0.563	0.6117	0.602
A7	0.141	0.141	0.111	0.111	0.1112	0.111	0.111	0.111	0.111	0.111	0.3481	0.307	0.1151736	0.111	0.0882	0.111
A8	0.111	0.111	0.111	0.111	0.1129	0.111	0.111	0.111	0.1116	0.111	0.7811	0.766	0.1137861	0.111	0.079	0.111
A9	0.8	1.563	0.602	0.563	0.4895	0.442	0.5408	0.442	0.4232	0.442	2.6031	2.38	0.5016645	0.442	0.4876	0.442
A10	0.602	1.228	0.563	0.563	0.5577	0.563	0.5564	0.563	0.6054	0.563	0.4919	0.563	0.5509707	0.563	0.5575	0.563
A11	0.141	0.111	0.111	0.111	0.111	0.111	0.111	0.111	0.1145	0.111	0.1121	0.111	0.1581861	0.141	0.1106	0.111
A12	0.307	0.196	0.111	0.111	0.111	0.111	0.111	0.111	0.128	0.111	0.1184	0.111	0.1618047	0.141	0.1103	0.111
A13	1.563	0.391	0.196	0.196	0.1552	0.141	0.1543	0.141	0.1674	0.141	3.2152	3.13	0.1502829	0.141	0.1563	0.141
A14	0.766	1.457	0.602	0.563	0.5713	0.563	0.5699	0.563	0.5973	0.563	0.5636	0.563	0.5735703	0.563	0.5803	0.563
A15	0.141	0.766	0.391	0.391	0.408	0.391	0.4152	0.391	0.3945	0.391	0.2581	0.307	0.3995112	0.391	0.4351	0.391
A16	0.111	1.563	0.563	0.563	0.5604	0.563	0.5545	0.563	0.5077	0.563	0.6043	0.602	0.5398152	0.563	0.4881	0.563
Weight (lb)	427.203	393.09	391.07	389.33	404.1224	403.22	404.0382	403.22	409.1085	403.22	615.4444	594.42	409.1748	408.51	403.5532	403.22
Mean weight (lb)	-	-	-	-	404.6884		416.7609		420.1844		1268.7		410.6061		406.1689	
Standard Deviation	-	-	-	-	0.4552		24.263		8.9041		461.4097		1.6644		1.6442	
Functional Evaluations	60000	50000	4500	3180	10500		10500		10500		10500				9600	

4.6 : Benchmark 6 – 200-Bar planar truss

4.6.1 Benchmark 6: Problem description

Figure 8 illustrates the 200-bar planar truss which has been examined by Lee and Geem [40]. The material properties and design constraints on this example are displayed in Table 6. The members are lined into 29 design categories as shown in Table 20. The minimum possible cross-sectional area is 0.1 in^2. The truss structure is subjected to three loading conditions as follows: (1) 1 kip in positive x-axis at nodes of 1, 6, 15, 20, 29, 43,48 57, 62 and 71; (2) 10 kips in negative y-axis at nodes of 1, 2, 3, 4,5, 6, 8, 10, 12, 14, 15, 16, 17, 18, 19, 20, 22, 24, 26, 28, 29, 30, 31, 32,33, 34, 36, 38, 40, 42, 43, 44, 45, 46, 47, 48, 50, 52, 54, 56, 58, 59, 60,61, 62, 64, 66, 68, 70, 71, 72, 73, 74 and 75, and (3) both case 1 and case 2.

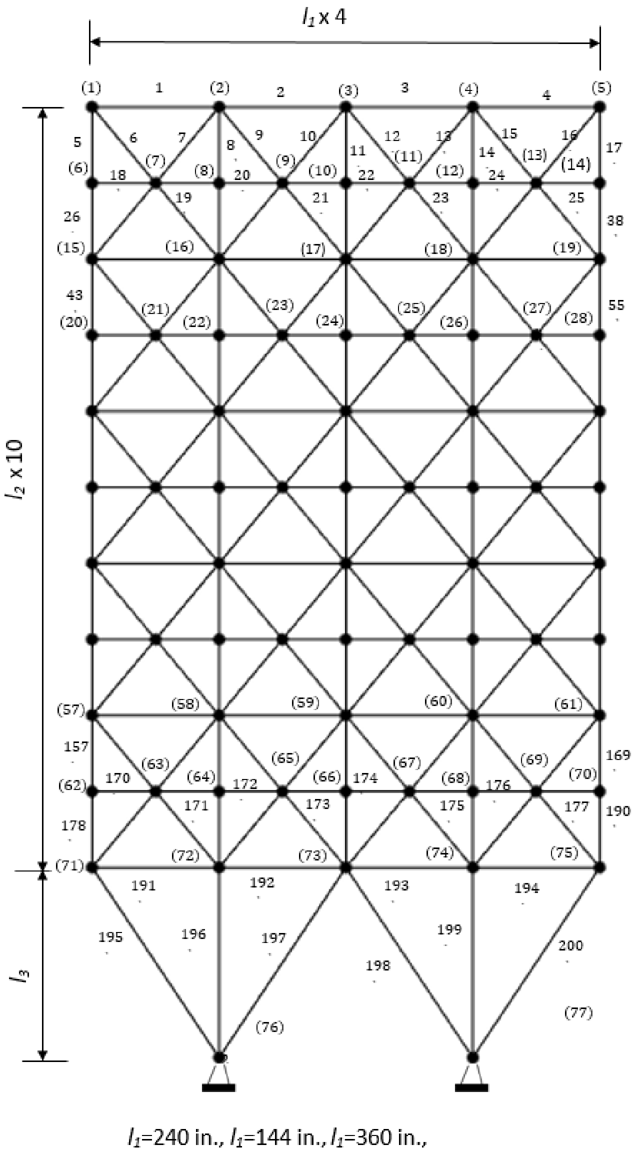

Figure 8 Benchmark Case 6: The 200-Bar planar truss structure

Table 24 Benchmark Case 6: Design variable classification of the 200-bar truss

Element Group Number	Members in the group
1	1,2,3,4
2	5,8,11,14,17
3	19,20,21,22,23,24
4	18,25, 56, 63, 94, 101, 132, 139, 170, 177
5	26, 29, 32, 35, 38
6	6,7,9,10, 12, 13, 15, 16, 27, 28, 30, 31, 33, 34, 36, 37
7	39, 40, 41, 42
8	43, 46, 49, 52, 55
9	57, 58, 59, 60, 61, 62
10	64, 67, 70, 73, 76
11	44, 45, 47, 48, 50, 51, 53, 54, 65, 66, 68, 69, 71, 72, 74, 75
12	77, 78, 79, 80
13	81, 84, 87, 90, 93
14	95, 96, 97, 98, 99, 100
15	102, 105, 108, 111, 114
16	82, 83, 85, 86, 88, 89, 91, 92, 103, 104, 106, 107, 109, 110, 112, 113
17	115, 116, 117, 118
18	119, 122, 125, 128, 131
19	133, 134, 135, 136, 137, 138
20	140, 143, 146, 149, 152
21	120, 121, 123, 124, 126, 127, 129, 130, 141, 142, 144, 145, 147, 148, 150, 151
22	153, 154, 155, 156
23	157, 160, 163, 166, 169
24	171, 172, 173, 174, 175, 176
25	178, 181, 184, 187 190
26	158, 159, 161, 162, 164, 165, 167, 168, 179, 180, 182, 183, 185, 186, 188, 189
27	191, 192, 193, 194
28	195, 197, 198, 200
29	196, 199

4.6.2 Benchmark 6: Results and discussion

Tables 21 and 22 display the results obtained from the considered algorithms with other optimisation techniques. The results show that the s-FOA algorithm proposes a truss design with a weight of 25,505lb. The HS based algorithm [40] ranks first amongst the algorithms considered in the study with a weight of 25,447.1lb. However, the number of functional evaluations utilised by the HS based algorithm [40] is 3000 more than the other algorithms considered. Nonetheless, the s-FOA algorithm is placed third in the comparison. The truss weights obtained from the TLBO, DE, cFOA GA and PSO algorithms are 25497 lb, 26189 lb, 26746 lb, 31264 lb, and 25962 lb respectively. The result also indicates that the s-FOA compared to other algorithms is placed third in terms of the best mean weights and standard deviation with values of 26891lb and 537lb respectively. The HS based algorithm [40] is exempted from this analysis as there is no information reported on the mean weight and standard deviation to establish a comparison.

Table 25 Benchmark Case 6: Performance rankings of the algorithms in solving the 200-bar truss problem

	PERFORMANCE RANKINGS					
	DE	PSO	GA	TLBO	cFOA	s-FOA
Best Value	4	3	6	1	5	2
Mean Value	2	5	6	1	4	3
Number of Functional Evaluations per 10,000	4.5	4.5	4.5	4.5	4.5	4.5

Standard deviation	1	5	6	2	4	3
Number of tuning parameters changed from one benchmark problem to another	5	3	3	3		3
Total Score	16.5	17.5	21.5	11.5	17.5	12.5

Table 26 Optimal results of the 200-bar planar truss structure optimisation obtained from different algorithms

	200 Bar						
	PREVIOUS STUDY	THIS STUDY					
	Lee and Geem [40]	**TLBO**	**PSO**	**DE**	**GA**	**cFOA**	**s-FOA**
Area group in^2	Continuous Optimum	Continuous Optimum	Continuous Optimum	Continuous Optimum	Continuous Optimum	Continuous Optimum	Continuous Optimum
A1	0.1253	0.1002	0.1	0.1464	0.1976	0.84276	0.1008
A2	1.0157	0.9446	0.9763	1.1397	1.0795	1.42356	0.9946
A3	0.1069	0.35	0.1	0.2703	0.1196	0.1783	0.1035
A4	0.1096	0.1124	0.1	0.1481	0.1992	0.97888	0.1184
A5	1.9369	1.9457	1.9781	2.2242	4.5599	2.00472	1.9909
A6	0.2686	0.2879	0.145	0.2667	0.1386	0.30422	0.1651
A7	0.1042	0.1449	0.4787	0.187	0.1442	0.44632	0.3790
A8	2.9731	3.1724	3.0254	3.2557	5.8761	3.20697	3.0480
A9	0.1309	0.1011	0.114	0.1	0.2381	0.20619	0.1100
A10	4.1831	4.1576	4.049	4.2936	4.5466	4.13978	4.0586
A11	0.3967	0.3097	0.3991	0.3697	0.2686	0.41515	0.5880
A12	0.4416	0.1824	0.4129	0.3315	0.2601	0.17381	0.2053
A13	5.1873	5.3714	5.2903	5.5847	4.9692	5.29643	5.3936
A14	0.1912	0.1417	0.4163	0.1	1.1107	0.83119	0.2111
A15	6.241	6.4221	6.2992	6.481	5.7642	6.25224	6.3832
A16	0.6994	0.4274	0.7178	0.6262	0.9245	0.79046	0.4990
A17	0.1158	0.548	0.3752	0.4016	0.5035	0.13696	0.3609
A18	7.7643	7.7648	8.0483	8.0576	7.1131	8.64241	7.8628
A19	0.1	0.1099	0.6505	0.1297	0.9186	0.12575	0.1003
A20	8.8279	8.7661	9.0707	8.9544	7.9984	8.98813	8.8603
A21	0.6986	0.7582	1.1272	0.7092	0.6227	0.6403	0.6673
A22	1.5563	0.502	0.2648	1.2072	5.5998	0.20019	0.4757
A23	10.9806	10.6533	11.7034	11.3708	8.6013	10.60984	10.6563

A24	0.1317	0.6135	0.9308	0.3314	1.8583	0.33525	0.2405
A25	12.1492	11.6602	12.7045	12.4438	9.6234	11.65307	11.6516
A26	1.6373	1.3	1.6645	1.6162	2.3529	0.86093	1.0313
A27	5.0032	6.4437	4.1803	5.0998	10.7055	6.94793	6.9146
A28	9.3545	10.5826	9.0751	10.0299	14.4036	11.96894	10.8883
A29	15.091	13.9279	15.4446	14.6571	12.9477	13.27711	13.5963
Weight (lb)	25447.1	25497	25962	26189	31264	26746	25505
Mean weight (lb)	-	26173	28655	26735	41393	27727	26891
Standard Deviation	-	516.6024	18526	267.126	52944	766	537
Functional Evaluations	48000	45000	45000	45000	45000		45000

5. Conclusion

In this study, the Spontaneous Fruit-fly Optimisation Algorithm (s-FOA) is presented to design optimal trusses, subject to stress and displacement bounds. An improved vision phase is proposed as an improvement to the FOA, in order to improve the exploitative capabilities of the algorithm in the search space. The improved vision phase aims to improve control of exploration and exploitation. Standard tuning parameters for use in any structural design are also presented. This eliminates the problems associated with the selection of the right parameters for the s-FOA algorithm. The effectiveness of the s-FOA algorithm is tested on six benchmark truss problems of size optimisation. The design variable is selected as the cross-sectional area of the truss members.

This study compares the performance of the s-FOA algorithm with the cFOA, TLBO, DE, PSO and the GA algorithms as well as other algorithms reported in several literatures such as mSOS, SOS, MBA, HSPO, HS, PSOPC, aeDE and SGA. It is observed that in most of the truss problems the s-FOA obtains the best continuous results in terms of the lowest weight, mean weight and standard deviation compared to other algorithms, despite a restricted number of structural analysis. The s-FOA is also seen to be competitive in discovering discrete optimal designs to the truss problems notwithstanding restricted computing resources. Therefore, s-FOA can be employed by industries with limited computation power in designing optimal truss structures.

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