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Investigating the impacts of price-taking and price-making energy storage in electricity markets through an equilibrium programming model

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Abstract: The envisaged decarbonisation of electricity systems has attracted significant interest around the role and value of energy storage systems (ESSs). In the deregulated electricity market, there is a need to investigate the complex impacts of ESSs, considering the potential exercise of market power by strategic players. This study aims at comprehensively analysing the impacts of both price-taking and price-making storage behaviours on energy market efficiency, corresponding to potential settings with small and large storage players, respectively. In order to achieve this and in contrast to previous papers, this work develops a multi-period equilibrium programming market model to determine market equilibrium stemming from the interactions of independent strategic producers and ESSs, while capturing the time-coupling operational constraints of ESSs as well as network constraints. The results of case studies on a test market capturing the general conditions of the GB electricity system demonstrate that the introduction of ESSs mitigates market power exercise and improves market efficiency, with this beneficial impact being higher when ESSs act as price takers. When the electricity network is congested, the location of ESSs also affects the market outcome, with their beneficial impact on market efficiency being higher when they are located in higher-priced areas.

Nomenclature

Indices and sets

$t \in T$	time periods
$n, m \in M$	network buses
M_n	buses connected to bus n through a line
$i \in I$	producers
$i-$	producers other than producer i
$j \in J$	demands
$k \in K$	energy storage systems (ESSs)
I_n, J_n, K_n	producers, demands and ESSs connected to bus n
$b \in B$	generation blocks
$c \in C$	demand blocks

Parameters

τ, N_T	temporal resolution and length of the market horizon
$\bar{F}_{n,m}$	capacity of the line connecting buses n and m (MW)
$x_{n,m}$	reactance of line connecting buses n and m (p.u.)
$\lambda^G_{i,b}$	marginal cost of block b of producer i (£/MWh)
$\bar{g}_{i,b}$	maximum power output limit of block b of producer i (MW)
$\lambda^D_{j,t,c}$	marginal benefit of block c of demand j at period t (£/MWh)
$\bar{d}_{j,t,c}$	maximum power input limit of block c of demand j at period t (MW)
$\bar{s}_k$	power capacity of ESS k (MW)
E^{cap}_k	energy capacity of ESS k (MWh)
$\underline{E}_k, \bar{E}_k$	minimum and maximum energy limits of ESS k (MWh)
E^0_k	initial energy level in ESS k (MWh)
η^c_k, η^d_k	charging and discharging efficiency of ESS k

Variables

$\theta_{n,t}$	voltage angle at bus n and period t (rad)
$v_{i,t}$	economic withholding strategy of producer i at period t
$g_{i,t,b}$	power output of block b of producer i at period t (MW)
$d_{j,t,c}$	power input of block c of demand j at period t (MW)
$w_{k,t}$	capacity withholding strategy of ESS k at period t
$s^c_{k,t}, s^d_{k,t}$	charging and discharging power of ESS k at period t (MW)
$E_{k,t}$	energy level in ESS k at the end of period t (MWh)
$\lambda_{n,t}$	dual variables of constraints (5) or equivalently locational marginal prices at bus n and period t (£/MWh)
$\mu^-_{i,t,b}, \mu^+_{i,t,b}$	dual variables of constraints (6) (£/MW)
$\nu^-_{j,t,c}, \nu^+_{j,t,c}$	dual variables of constraints (7) (£/MW)
$\xi_{k,t}$	dual variables of constraints (8) (£/MW)
$\pi^-_{k,t}, \pi^+_{k,t}$	dual variables of constraints (9) (£/MW)
$\rho^-_{k,t}, \rho^+_{k,t}$	dual variables of constraints (10) (£/MW)
$\sigma^-_{k,t}, \sigma^+_{k,t}$	dual variables of constraints (11) (£/MW)
φ_k	dual variables of constraints (12) (£/MW)
$\chi^-_{n,m,t}, \chi^+_{n,m,t}$	dual variables of constraints (13) (£/MW)
$\psi^-_{n,t}, \psi^+_{n,t}$	dual variables of constraints (14) (£/rad)
δ_t	dual variables of constraints (15) (£/rad)

1 Introduction

1.1 Background and motivation

Electricity systems are facing the important challenges of deregulation and decarbonisation. In the deregulated electricity market, participants do not necessarily behave as *price-takers*. Participants of large size and/or strategically located in the transmission network are able to manipulate the electricity prices and increase their economic surpluses beyond competitive levels, through strategic bids and offers. This effect is known as *market power* and results in higher price levels and loss of social welfare [1, 2].

In parallel, the envisaged decarbonisation of electricity systems introduces fundamental techno-economic challenges associated with the high variability and limited controllability of renewable generation as well as the increasing demand peaks driven by the electrification of transport and heat sectors. In this setting, *energy storage systems* (ESSs) have attracted great interest as their flexibility can support system balancing and limit peak demand levels, improving the cost efficiency of low-carbon electricity systems [3].

As a result of the deregulation and decarbonisation trends, there is an emerging need to investigate the impacts of ESSs in electricity markets through suitable analytical models, considering the potential exercise of market power. This task involves two equally significant perspectives: the perspective of price-taking ESSs and the perspective of price-making ESSs. Under the first one, ESS owners are assumed to behave competitively and reveal their actual techno-economic characteristics to the market. The validity of this assumption is likely for independent, small-scale, distributed ESSs [3], which cannot unilaterally affect the market outcome. A wide literature has presented optimal coordination models and explored the impacts of price-taking ESSs on different aspects of the power system short-term operation and long-term planning, including [4–12].

However, their impact on the extent of market power exercised by price-making electricity producers has not been comprehensively investigated yet. To the best of the authors' knowledge, their previous paper [13] constitutes the only work exploring this aspect, by employing a multi-period *bi-level optimisation* model, which is solved after converting it to a *mathematical program with equilibrium constraints* (MPEC). However, the adopted modelling framework exhibits two fundamental shortcomings: (a) all the price-making electricity producers are assumed to collectively determine their optimal offering strategies in the market through a single bi-level/MPEC problem, implying that a monopoly setting rather than a realistic oligopoly setting is modelled and (b) the network constraints are neglected, implying that locational market power effects are not examined.

Under the second perspective, ESS owners are assumed to behave strategically and misreport their techno-economic characteristics to the market, i.e. they exercise market power. The validity of this assumption is likely for single large-scale, bulk ESS [3] or a number of smaller ESSs operated by the same market entity (e.g. an aggregator), which can affect the market outcome through their individual actions.

A large number of previous papers [14–30] have modelled price-making ESSs and quantitatively analysed their ability to exercise market power. However, the modelling approaches adopted in these works exhibit certain limitations. In [14, 15], the author analytically calculates market equilibrium in a highly simplified two-period market model without network constraints; this complex analytical calculation cannot be easily extended to realistic market models involving a much larger number of clearing periods and network constraints. In [16–20], the impact of strategic ESSs on market prices is modelled through an inverse demand function. However, the parameters of this function are determined through exogenous data and therefore cannot accurately capture the impacts of other market participants' characteristics on price formation. To address this limitation, the authors of [21–30] model the decision making of a single price-maker ESS through bi-level optimisation approaches, which endogenously represent the market clearing and price formation process. Nevertheless, all these papers assume that only the single examined ESS acts strategically and electricity producers are price-takers (i.e. the market outcome is determined through a single bi-level/MPEC model), implying that the realistic interactions between strategic producers and strategic ESSs are neglected.

1.2 Scope and contributions

In summary, previous works investigating the impacts of both price-taking and price-making ESSs neglect realistic aspects of deregulated electricity markets, since they all assume that a single player determines the market outcome by relying on single bi-level/MPEC models. This study aims at addressing this challenge by developing a *multi-period equilibrium programming market model*, which determines *oligopolistic market equilibrium* stemming from the interactions of independent strategic producers and ESSs. In this model, the decision making of each strategic producer or ESS is modelled through a bi-level optimisation problem, capturing the time-coupling operational constraints of ESSs as well as network constraints. This problem is converted to an MPEC and linearised through suitable techniques in order to solve it efficiently. Oligopolistic market equilibrium is determined by employing the *iterative diagonalisation* method.

Through the application of this model on a test market capturing the general conditions of the GB electricity system, this study aims at comprehensively analysing the impacts of both price-taking and price-making ESSs on energy market efficiency. The results demonstrate that the introduction of the ESS mitigates market power exercise and improves the market efficiency, with this beneficial impact being lower – yet persisting – when ESSs act strategically. When the electricity network is congested, the location of the ESS also affects the market outcome, with their beneficial impact on market efficiency being higher when they are located in higher-priced areas.

More specifically, the novel contributions of this work are as follows:

- A multi-period equilibrium programming electricity market model is developed, capturing the time-coupling operational characteristics of ESSs as well as network constraints. In contrast to single bi-level/MPEC models employed in [13, 21–30] and representing an unrealistic monopoly setting, the proposed model captures more accurately the reality, where the market outcome is determined by the interactions of multiple independent and strategic electricity producers and ESSs.
- Quantitative analysis with the developed model demonstrates that the price-taking ESSs mitigate market power exercise by strategic producers during peak periods and enhance it during off-peak periods, with the former mitigation significantly dominating the latter enhancement and resulting in an overall positive impact in terms of market efficiency. When the ESSs behave strategically, they exercise capacity withholding in order to maintain the price differential between peak and off-peak periods at higher levels and increase their arbitrage revenues. As a result, their flattening effect on system demand is limited and the market outcome is less efficient with respect to the price-taking case, although it is still more efficient than the case without storage in the system. This result persists irrespectively of network congestion conditions and the location of storage, implying that the envisaged penetration of storage capacity is likely to reduce the extent of market power and improve market efficiency, with this benefit being higher if this storage capacity is shared by a large number of independent small players who cannot unilaterally affect the market outcome.
- In cases where the electricity network is congested, apart from the market behaviour of storage, its location also affects considerably the market outcome. Specifically, the presence of storage at a particular location mitigates market power exercise by collocated strategic producers and improves the market position of collocated consumers. Its overall impact on market efficiency is positive irrespective of its location, but this benefit is higher when it is located in areas with more costly generation and higher demand, which are more prone to market power exercise. These locational effects persist but are less pronounced when storage behaves strategically.

1.3 Paper structure

The remaining of this paper is organised as follows. Section 2 formulates the developed equilibrium programming electricity market model, considering both price-taking and price-making ESS behaviours. Section 3 presents the examined case studies and analyses quantitative results. Finally, Section 4 outlines the conclusions of this work.

2 Equilibrium programming electricity market model with energy storage participation

2.1 Modelling assumptions

The main assumptions of this work are listed below:

i. The examined market is a pool-based energy-only market and the objective function of the clearing algorithm is the maximisation of the short-term social welfare.
ii. The market clearing algorithm incorporates a DC power flow model of the transmission network and produces *locational marginal prices* (LMPs) $\lambda_{n,t}$ for every bus n and time period t.
iii. Each producer i operates a single generation unit and submits an increasing stepwise offer curve to the market at every period t, consisting of a number of blocks b. This assumption is made for presentation clarity reasons and does not sacrifice generality.
iv. Adopting the approach of [31–33], producer i can exercise market power through *economic withholding* (i.e. offering higher prices than its actual marginal costs). Specifically, its market strategy at period t is expressed through a decision variable $v_{i,t} \geq 1$. If $v_{i,t} = 1$, producer i behaves as a price-taker and offers its actual marginal costs ($\lambda^G_{i,b}, \forall b$) to the market at t. If $v_{i,t} > 1$, producer i behaves as a price-maker and submits an offer price higher than its actual marginal costs ($v_{i,t}\lambda^G_{i,b}, \forall b$) to the market at t.
v. Each demand participant j submits a decreasing stepwise bid curve (capturing the effect of demand's price elasticity [17, 19, 20, 27, 34]) to the market at every period t, consisting of a number of blocks c. The price/quantity bids are time-specific and location-specific parameters, capturing the differentiated preferences of consumers across different time periods and the differentiated geographical conditions. Demand participants are assumed price-takers, bidding based on their actual techno-economic parameters.
vi. Following the approach adopted in [4–7, 9, 11, 13–15, 17, 22–30], a generic, technology-agnostic model is employed for the representation of the technical characteristics of the ESS, which includes charging and discharging efficiencies, energy balance constraints as well as minimum and maximum energy and power limits.
vii. Regarding the economic properties of the ESS, following the model employed in [14, 15, 21, 25, 27, 30], the ESSs do not submit a price offer or bid to the market, since they do not exhibit inherent cost or benefit components (as electricity producers and demands, respectively) apart from their negligible operation and maintenance costs. However, ESS k can exercise market power through *capacity withholding* (i.e. submitting a power capacity lower than its actual value). Specifically, its market strategy at period t is expressed through a decision variable $0 \leq w_{k,t} \leq 1$. If $w_{k,t} = 0$, storage behaves competitively and submits its actual capacity ($\bar{s}_k$) to the market at t. If $w_{k,t} > 0$, storage behaves strategically and submits lower than its actual capacity ($(1 - w_{k,t})\bar{s}_k$) to the market at t.
viii. ESS k can be modelled as a price-taker by forcing $w_{k,t} = 0, \forall t$ in the model; in this case, the underlying physical assumption is that this ESS is formed by an independent, small-scale, distributed ESS, which cannot unilaterally affect the market outcome. When ESS k is modelled as a price-maker (allowing $0 \leq w_{k,t} \leq 1, \forall t$), the underlying physical assumption is that this ESS corresponds to a single large-scale, bulk ESS or is formed by a number of smaller ESSs operated by the same market entity (e.g. an aggregator), which can affect the market outcome through their individual actions.

2.2 Bi-level optimisation model of strategic player

Adopting the approach of [21–37], the decision-making process of each strategic producer or ESS is modelled through a bi-level optimisation problem, the structure of which is illustrated in Fig. 1.

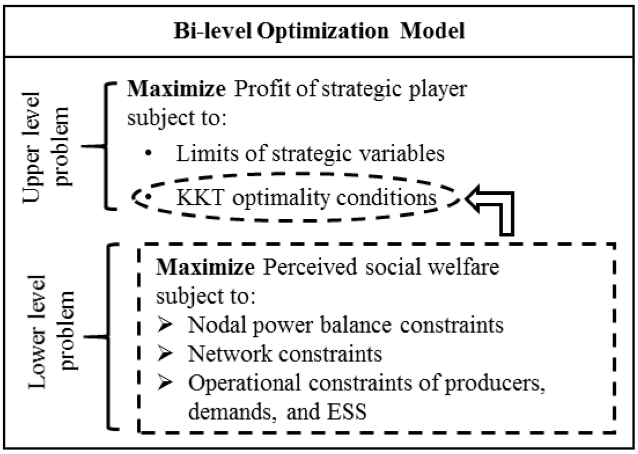

Fig. 1 *Structure of bi-level optimisation model of strategic player's decision making*

The upper level (UL) problem determines the optimal economic or capacity withholding strategies maximising the profit of the producer or ESS, respectively, and is subject to the lower level (LL) problem representing the market clearing process.

In the case of a strategic producer i, this bi-level optimisation model is formulated as follows:

(Upper level)

$$\max_{\{v_{i,t}\}} \sum_{t,b} \left[\left(\lambda_{(n:i \in I_n),t} - \lambda^G_{i,b} \right) g_{i,t,b} \right] \tag{1}$$

subject to

$$v_{i,t} \geq 1, \quad \forall t. \tag{2}$$

(Lower level)

$$\min_{V^{\text{LL}}} \sum_{i,t,b} v_{i,t}\lambda^G_{i,b} g_{i,t,b} - \sum_{j,t,c} \lambda^D_{j,t,c} d_{j,t,c}, \tag{3}$$

where

$$V^{\text{LL}} = \left\{ g_{i,t,b},\ d_{j,t,c},\ s^c_{k,t},\ s^d_{k,t},\ E_{k,t},\ \theta_{n,t} \right\}, \tag{4}$$

subject to

$$\sum_{(j \in J_n),c} d_{j,t,c} + \sum_{k \in K_n} \left(s^c_{k,t} - s^d_{k,t} \right) - \sum_{(i \in I_n),b} g_{i,t,b} + \sum_{m \in M_n} \frac{\theta_{n,t} - \theta_{m,t}}{x_{n,m}} = 0 : \lambda_{n,t}, \quad \forall n, \forall t, \tag{5}$$

$$0 \leq g_{i,t,b} \leq \bar{g}_{i,b} : \mu^-_{i,t,b}, \mu^+_{i,t,b}, \quad \forall i,\ \forall t,\ \forall b, \tag{6}$$

$$0 \leq d_{j,t,c} \leq \bar{d}_{j,t,c} : \nu^-_{j,t,c}, \nu^+_{j,t,c}, \quad \forall j,\ \forall t,\ \forall c, \tag{7}$$

$$E_{k,t} = E_{k,(t-1)} + \tau s^c_{k,t}\eta^c_k - \frac{\tau s^d_{k,t}}{\eta^d_k} : \ \xi_{k,t}, \quad \forall k,\ \forall t, \tag{8}$$

$$\underline{E}_k \leq E_{k,t} \leq \bar{E}_k : \ \pi^-_{k,t}, \pi^+_{k,t}, \quad \forall k,\ \forall t, \tag{9}$$

$$0 \leq s^c_{k,t} \leq (1 - w_{k,t})\bar{s}_k : \rho^-_{k,t}, \rho^+_{k,t}, \quad \forall k,\ \forall t, \tag{10}$$

$$0 \leq s^d_{k,t} \leq (1 - w_{k,t})\bar{s}_k : \sigma^-_{k,t}, \sigma^+_{k,t}, \ \forall k,\ \forall t \tag{11}$$

$$E^0_k = E_{k,N_T} : \ \varphi_k, \quad \forall k, \tag{12}$$

$$-\bar{F}_{n,m} \le \frac{\theta_{n,t}-\theta_{m,t}}{x_{n,m}} \le \bar{F}_{n,m}: \chi^-_{n,m,t}, \chi^+_{n,m,t}, \quad \forall n, \ \forall m \in M_n, \ \forall t, \tag{13}$$

$$-\pi \le \theta_{n,t} \le \pi: \psi^-_{n,t}, \psi^+_{n,t}, \quad \forall n, \forall t, \tag{14}$$

$$\theta_{1,t} = 0: \delta_t, \quad \forall t. \tag{15}$$

The UL problem maximises the short-term economic surplus of producer i (1). This maximisation is subject to the limits of the strategic variables (2) and the LL problem (3)–(15). The latter corresponds to the market clearing algorithm, maximising the *perceived short-term social welfare* (3), subject to nodal power balance constraints (5), the operational constraints of the producers (6), the demands (7), and the ESS (including energy balance constraints (8), minimum and maximum energy and power limits (9)–(11) and the energy neutrality assumption (12)), as well as network constraints (13)–(15).

In the case of a strategic ESS k, the UL problem is formulated through (16) and (17), while the LL problem formulation does not change

$$\max_{\{w_{k,t}\}} \sum_t \lambda_{(n:k \in K_n),t}\left(s^d_{k,t} - s^c_{k,t}\right) \tag{16}$$

subject to

$$0 \le w_{k,t} \le 1, \quad \forall t. \tag{17}$$

2.3 MPEC model of strategic player

Each of the above bi-level optimisation problems is converted to a single-level MPEC, after replacing its LL problem by its *Karush–Kuhn–Tucker* (KKT) optimality conditions (Fig. 1). This conversion is enabled by the continuity and convexity of the LL problem.

In the case of a strategic producer i, this MPEC is formulated as follows:

$$\max_V \sum_{t,b} \left[\left(\lambda_{(n:i \in I_n),t} - \lambda^G_{i,b}\right) g_{i,t,b}\right], \tag{18}$$

where

$$V = \{v_{i,t}, V^{LL}, \lambda_{n,t}, \mu^-_{i,t,b}, \mu^+_{i,t,b}, \nu^-_{j,t,c}, \nu^+_{j,t,c}, \xi_{k,t}, \pi^-_{k,t}, \pi^+_{k,t}, \rho^-_{k,t}, \rho^+_{k,t}, \sigma^-_{k,t}, \sigma^+_{k,t}, \chi^-_{n,m,t}, \chi^+_{n,m,t}, \psi^-_{n,t}, \psi^+_{n,t}, \delta_t\}, \tag{19}$$

subject to

(2), (5), (8), (12), (15)

$$v_{i,t}\lambda^G_{i,b} - \lambda_{(n:i \in I_n),t} - \mu^-_{i,t,b} + \mu^+_{i,t,b} = 0, \quad \forall t, \forall b, \tag{20}$$

$$v_{i-,t}\lambda^G_{i-,b} - \lambda_{(n:i- \in I_n),t} - \mu^-_{i-,t,b} + \mu^+_{i-,t,b} = 0, \quad \forall i-, \forall t, \forall b, \tag{21}$$

$$-\lambda^D_{j,t,c} + \lambda_{(n:j \in J_n),t} - \nu^-_{j,t,c} + \nu^+_{j,t,c} = 0, \quad \forall j, \forall t, \ \forall c, \tag{22}$$

$$\lambda_{(n:k \in K_n),t} - \tau\xi_{k,t}\eta^c_k - \rho^-_{k,t} + \rho^+_{k,t} = 0, \quad \forall k, \forall t, \tag{23}$$

$$-\lambda_{(n:k \in K_n),t} + \frac{\tau\xi_{k,t}}{\eta^d_k} - \sigma^-_{k,t} + \sigma^+_{k,t} = 0, \quad \forall k, \forall t, \tag{24}$$

$$\xi_{k,t} - \xi_{k,(t+1)} - \pi^-_{k,t} + \pi^+_{k,t} = 0, \quad \forall k, \forall t < N_T, \tag{25}$$

$$\xi_{k,N_T} - \pi^-_{k,N_T} + \pi^+_{k,N_T} - \varphi_k = 0, \quad \forall k, \tag{26}$$

$$\sum_{m \in M_n} \frac{\lambda_{n,t} - \lambda_{m,t}}{x_{n,m}} + \sum_{m \in M_n} \frac{\chi^+_{n,m,t} - \chi^+_{m,n,t}}{x_{n,m}} - \sum_{m \in M_n} \frac{\chi^-_{n,m,t} - \chi^-_{m,n,t}}{x_{n,m}} + \psi^+_{n,t} - \psi^-_{n,t} + (\delta_t)_{n=1} = 0, \quad \forall n, \forall t, \tag{27}$$

$$0 \le \mu^-_{i,t,b} \perp g_{i,t,b} \ge 0, \quad \forall i, \forall t, \forall b, \tag{28}$$

$$0 \le \mu^+_{i,t,b} \perp \left(\bar{g}_{i,b} - g_{i,t,b}\right) \ge 0, \quad \forall i, \forall t, \forall b, \tag{29}$$

$$0 \le \nu^-_{j,t,c} \perp d_{j,t,c} \ge 0, \quad \forall j, \forall t, \forall c, \tag{30}$$

$$0 \le \nu^+_{j,t,c} \perp \left(\bar{d}_{j,t,c} - d_{j,t,c}\right) \ge 0, \quad \forall j, \forall t, \forall c, \tag{31}$$

$$0 \le \pi^-_{k,t} \perp (E_{k,t} - \underline{E}_k) \ge 0, \quad \forall k, \forall t, \tag{32}$$

$$0 \le \pi^+_{k,t} \perp \left(\bar{E}_k - E_{k,t}\right) \ge 0, \quad \forall k, \forall t, \tag{33}$$

$$0 \le \rho^-_{k,t} \perp s^c_{k,t} \ge 0, \quad \forall k, \forall t, \tag{34}$$

$$0 \le \rho^+_{k,t} \perp \left((1 - w_{k,t})\bar{s}_k - s^c_{k,t}\right) \ge 0, \quad \forall k, \forall t, \tag{35}$$

$$0 \le \sigma^-_{k,t} \perp s^d_{k,t} \ge 0, \quad \forall k, \forall t, \tag{36}$$

$$0 \le \sigma^+_{k,t} \perp \left((1 - w_{k,t})\bar{s}_k - s^d_{k,t}\right) \ge 0, \quad \forall k, \forall t, \tag{37}$$

$$0 \le \chi^-_{n,m,t} \perp \left(\bar{F}_{n,m} + \frac{\theta_{n,t} - \theta_{m,t}}{x_{n,m}}\right) \ge 0, \quad \forall n, \forall m \in M_n, \forall t, \tag{38}$$

$$0 \le \chi^+_{n,m,t} \perp \left(\bar{F}_{n,m} - \frac{\theta_{n,t} - \theta_{m,t}}{x_{n,m}}\right) \ge 0, \quad \forall n, \forall m \in M_n, \forall t, \tag{39}$$

$$0 \le \psi^-_{n,t} \perp (\pi + \theta_{n,t}) \ge 0, \quad \forall n, \forall t, \tag{40}$$

$$0 \le \psi^+_{n,t} \perp (\pi - \theta_{n,t}) \ge 0, \quad \forall n, \forall t. \tag{41}$$

The set of decision variables (19) of the MPEC includes the primal decision variables of both UL and LL problems as well as the dual variables corresponding to the constraints of the LL problem. Equations (20)–(41) express the KKT optimality conditions of the LL problem.

In the case of a strategic ESS k, the objective function of the MPEC problem is formulated through (16), and is subject to constraints (17), (5), (8), (12), (15) as well as the KKT optimality conditions (20)–(41).

2.4 Mixed-integer linear problem (MILP) model of strategic player

The above MPEC is nonlinear implying that any solution produced by commercial optimisation solvers is not guaranteed to be globally optimal. Therefore this MPEC is converted to a MILP, which commercial branch-and-cut solvers can efficiently solve to global optimality [36].

More specifically, both the MPEC corresponding to producer i and the MPEC corresponding to ESS k include two types of nonlinearities. The first one involves the bilinear terms $\sum_{t,b} \lambda_{(n:i \in I_n),t} g_{i,t,b}$ and $\sum_t \lambda_{(n:k \in K_n),t}\left(s^d_{k,t} - s^c_{k,t}\right)$ in the objective functions (18) and (16), respectively. The linearisation approach proposed in [34] is employed in order to replace these bilinear terms with linear expressions. The resulting linear reformulations of (18) and (16) are given by (42) and (43), respectively, with the detailed derivations presented in the Appendix (see (42))

(see (43)) .

The second non-linearity involves the bilinear terms in inequalities (28)–(41), which can be written in the general form $0 \le \mu \perp p \ge 0$, where μ and p represent general dual and primal terms, respectively. The linearisation approach proposed in [38] replaces each of these inequalities with the set of mixed-integer

$$\begin{aligned}\max \sum_{j,t,c}\left(\lambda^{D}_{j,t,c}d_{j,t,c}-\nu^{+}_{j,t,c}\bar{d}_{j,t,c}\right)-\sum_{i-,t,b}\left(\lambda_{(n:i-\in I_n),t}g_{i-,t,b}+\mu^{+}_{i-,t,b}\bar{g}_{i-,b}\right)\\+\sum_{k,t}\left[\pi^{-}_{k,t}\underline{E}_k-\pi^{+}_{k,t}\bar{E}_k-(\rho^{+}_{k,t}+\sigma^{+}_{k,t})(1-w_{k,t})\bar{s}_k\right]\\+\sum_{k}(\varphi_k-\xi_{k,1})E^{0}_k-\sum_{n,(m\in M_n),t}(\chi^{-}_{n,m,t}+\chi^{+}_{n,m,t})\bar{F}_{n,m}\\-\sum_{n,t}(\psi^{-}_{n,t}+\psi^{+}_{n,t})\pi-\sum_{i,t,b}\lambda^{G}_{i,b}g_{i,t,b},\end{aligned} \tag{42}$$

$$\begin{aligned}\max \sum_{j,t,c}\left(\lambda^{D}_{j,t,c}d_{j,t,c}-\nu^{+}_{j,t,c}\bar{d}_{j,t,c}\right)-\sum_{i,t,b}\left(v_{i,t}\lambda^{G}_{i,b}g_{i,t,b}+\mu^{+}_{i,t,b}\bar{g}_{i,b}\right)\\-\sum_{n,(m\in M_n),t}(\chi^{-}_{n,m,t}+\chi^{+}_{n,m,t})\bar{F}_{n,m}-\sum_{n,t}(\psi^{-}_{n,t}+\psi^{+}_{n,t})\pi\,.\end{aligned} \tag{43}$$

linear conditions $\mu \geq 0$, $p \geq 0$, $\mu \leq \omega M^{D}$, $p \leq (1-\omega)M^{P}$, where ω is an auxiliary binary variable, and M^{D} and M^{P} are large positive constants.

2.5 Determining the oligopolistic market equilibrium

The above bi-level/MPEC/MILP models express the decision making of a single strategic player participating in the market. To determine the oligopolistic market equilibrium stemming from the interactions of multiple strategic players, the *iterative diagonalisation* method, which was introduced in the mathematical paper [39] and employed in [31, 32, 35] was adopted.

This iterative procedure is illustrated in Fig. 2 and involves three steps:

i. The players' strategic variables are initialised, the iteration counter is set to 1 and the convergence tolerance is determined. In this work, the initial values of the players' strategic variables are set as $v^{[0]}_{i,t}=1, \forall i,\ \forall t$ and $w^{[0]}_{k,t}=0,\ \forall k,\ \forall t$.
ii. At every iteration r, each strategic player solves its respective MILP, considering the strategies of the other players as fixed parameters, equal to their values at iteration $r-1$.
iii. The vector of all players' strategic variables at iteration r is compared to the one at iteration $r-1$. If their distance is lower than ε, the iterative procedure terminates. As discussed in [31, 32, 35], the resulting outcome after convergence corresponds by definition to a *pure strategy Nash equilibrium* of the market, since none of the players can increase their surpluses by unilaterally modifying their strategic variables.

As discussed in the relevant literature, Nash equilibrium is not generally guaranteed to exist or to be unique [31, 32, 35]. Moreover, the diagonalisation method is not generally guaranteed to converge, even if equilibrium exists [31, 32, 35]. In the context of this work, however, this method has converged after a relatively small number of iterations in every examined case study (Section 3.4). This outcome, combined with the focus of this work on investigating the impact of different ESS behaviours on the market outcome, sets a detailed analysis of existence, uniqueness, and convergence to equilibrium out of the scope of this study.

3 Case studies

3.1 Input data and implementation

The impacts of price-taking and price-making ESSs are quantitatively analysed by employing a test market with the day-ahead horizon and hourly resolution, capturing the general conditions of the GB electricity system, and examining a single representative day. A 16-bus model of the GB transmission network is employed (Fig. 3), where the area of Scotland corresponds to buses 1–6 while the area of England corresponds to buses 7–16 [40].

Seven electricity producers participate in the market, with their location, the linear l_i and quadratic q_i coefficients of their quadratic

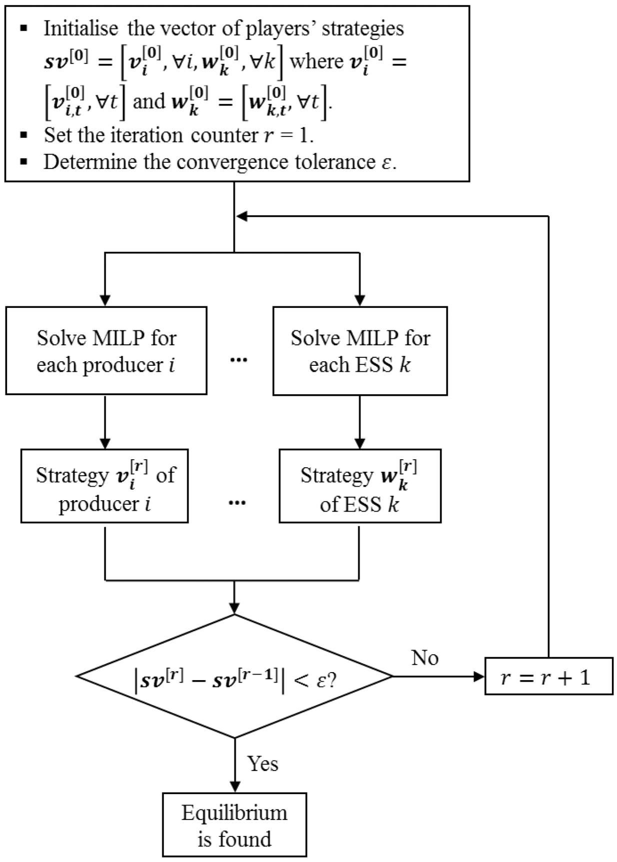

Fig. 2 *Flowchart of iterative diagonalisation method to determine oligopolistic market equilibrium*

operating cost curve, and their maximum output $g^{\max}_{i}$ presented in Table 1 [41, 42]. This data reflects the reality in the GB system, where England is characterised by more expensive generation. The quadratic operating cost curves have been transformed to piecewise linear curves of five blocks, which implies stepwise marginal cost curves, as prescribed by the formulation of this study.

Thirteen demands also participate in the market, with their location presented in Table 2. The relative size of each demand with respect to the system demand is assumed identical for every time period (i.e. $\frac{\bar{d}_{j,t=1,c}}{\sum_j \bar{d}_{j,t=1,c}}=\frac{\bar{d}_{j,t=2,c}}{\sum_j \bar{d}_{j,t=2,c}}=\cdots=\frac{\bar{d}_{j,t=24,c}}{\sum_j \bar{d}_{j,t=24,c}},\forall j,\forall c$) and is presented in % terms in Table 2. This data reflects the reality in the GB system, where the largest demand centres are located in England. The benefit and maximum input parameters of these demands are collected from [41, 42].

The total power and energy capacities of ESSs in the system are assumed equal to $\bar{s}=9.4\,\mathrm{GW}$ and $E^{\mathrm{cap}}=18.8\,\mathrm{GWh}$, respectively, while the assumed values of the rest of the ESS operational parameters are presented in Table 3. Two different market behaviours of ESSs are examined: (i) price-taking behaviour, which involves the underlying assumption that the above storage capacity is owned by a large number of independent, small-scale ESSs, which cannot unilaterally affect the market outcome and (ii) price-making behaviour, which involves the underlying assumption that the above storage capacity is owned by a single market entity, which can thus affect the market outcome through its individual actions.

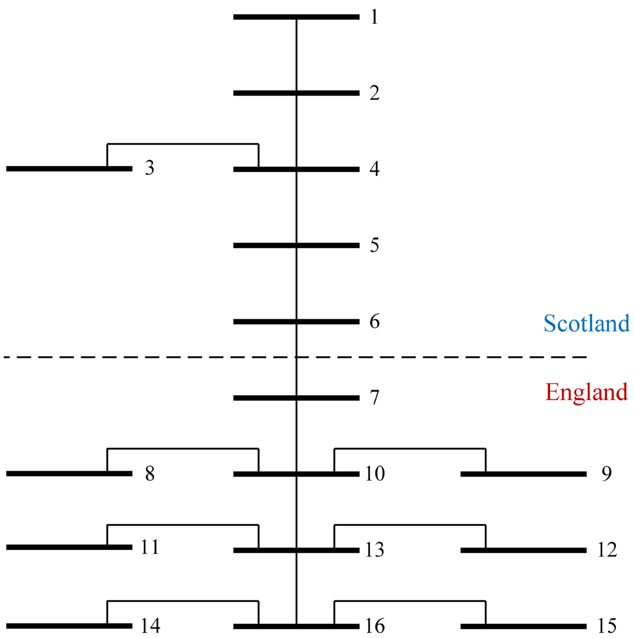

Fig. 3 *Model of GB transmission network*

Table 1 Characteristics of electricity producers

Producer *i*	1	2	3	4
bus	3	5	6	9
l_i, £/MW	10	23	50	15
q_i, £/MW^2	0.0001	0.0014	0.0042	0.0006
$g_i^{\max}$, MW	13,170	7,560	6,500	11,520

Producer *i*	5	6	7
bus	11	15	16
l_i, £/MW	35	70	100
q_i, £/MW^2	0.0026	0.0065	0.001
$g_i^{\max}$, MW	6,670	5,760	5,500

The optimisation software FICO$^{\text{TM}}$ Xpress [43] has been used to implement and solve the developed equilibrium programming market model, on a computer with an 8-core 3.47 GHz processor and 192 GB RAM. Two alternative computation approaches have been implemented and compared. Under the first one (*sequential computation*), the MILP corresponding to the different strategic players of the market is solved sequentially at every iteration. Under the second one (*parallel computation*), these MILPs are solved in parallel at every iteration; specifically, each MILP is solved at a different core of the employed computer. This parallel computation approach is enabled by the structure of the iterative diagonalisation algorithm since the MILP of each strategic player at iteration r requires information only from the solution of the rest of the MILP at iteration $r-1$ (Section 2.5). Irrespective of the computation approach, the diagonalisation algorithm has converged and thus a market equilibrium has been reached after a relatively small number of iterations in every examined case study (Section 3.4).

3.2 Impact of ESS: uncongested network

This section analyses the results corresponding to a case where the network capacity limits are neglected and therefore the network is not congested. As a result, the LMPs are identical to every bus of the network [1], i.e. $\lambda_{1,t} = \lambda_{2,t} = \cdots = \lambda_{16,t}, \forall t$; therefore, for simplicity reasons, the subscript n is omitted from the expression of prices in this section. Three test scenarios are examined and compared:

NO-ESS: The system does not include any ESS.
C-ESS: The system includes competitive (price-taking) ESS with the parameters given in Section 3.1.
S-ESS: The system includes strategic (price-making) ESS with the parameters given in Section 3.1.

In all the above test scenarios, the producers behave strategically. To characterise the extent of market power exercised, each of these test scenarios (indicated by the superscript ts) are compared against a benchmark scenario (indicated by the superscript bn) involving an ideal, perfectly competitive market, where all market players behave competitively.

Fig. 4 presents the hourly net demand of the system (considering the charging and discharging power of the ESS) and Fig. 5 presents the increase of market prices driven by market power exercise (i.e. the difference $\lambda_t^{\text{ts}} - \lambda_t^{\text{bn}}$), for each of the considered scenarios. In the No-ESS scenario, as demonstrated in [1, 13], the increase of market prices driven by the producers' strategic actions is significantly higher during peak periods, due to the increasing slope of the producers' offer curves at higher demand levels and the higher need to utilise available generation capacity in the system. In other words, peak periods are the most critical ones concerning the exercise of market power by strategic producers.

Table 2 Characteristics of electricity demands

Demand *j*	1	2	3	4	5	6	7
bus	1	2	4	5	6	7	8
size, %	1.8	2.0	3.6	5.6	0.8	19.0	14.1

Demand *j*	8	9	10	11	12	13
bus	9	11	12	14	15	16
size, %	5.6	5.9	6.3	10.4	2.6	22.3

Table 3 Characteristics of energy storage

Parameter	$\underline{E}$	$\bar{E}$	E^0	η^c	η^d
value	$0.2E^{\text{cap}}$	E^{cap}	$0.25E^{\text{cap}}$	0.9	0.9

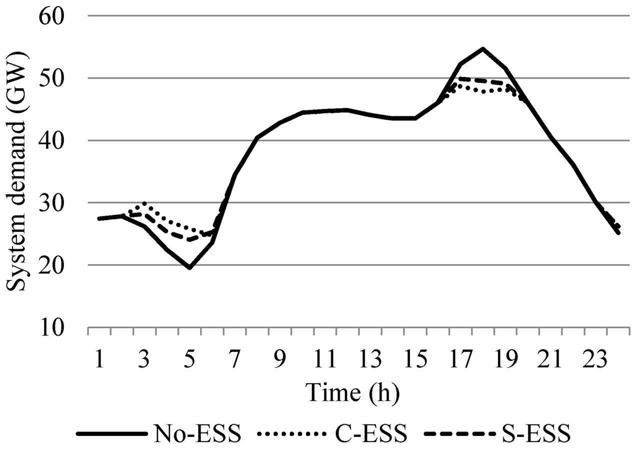

Fig. 4 *Hourly net demand of the system for different ESS scenarios*

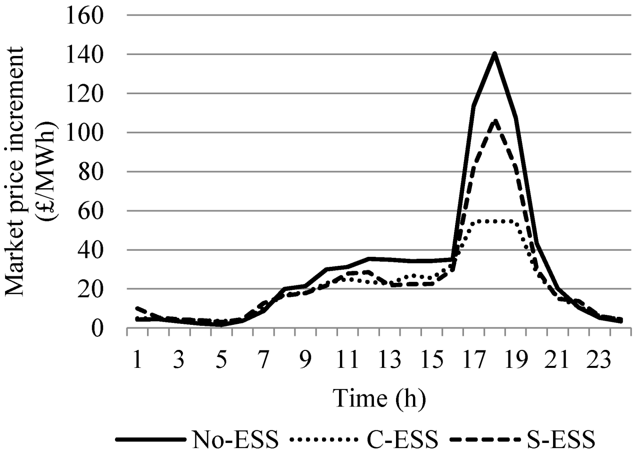

Fig. 5 *Hourly market price increase driven by market power exercise for different ESS scenarios*

The operation of the price-taking ESS flattens the system demand profile by (i) charging and thus increasing demand during off-peak periods and (ii) discharging and thus reducing demand during peak periods. This drives a similar flattening effect on the price increments: the price increments increase at off-peak periods due to the increase in demand and reduce at peak periods due to the reduction in demand. However, the latter reduction is prominently greater than the former increase due to the criticality of the peak periods discussed above. As a result, the operation of the price-taking ESS results in an overall reduction of the market power exercised by the generation side.

When the ESS is assumed to behave strategically, it can also exercise market power through capacity withholding (Section 2.1). In other words, the ESS submits a power capacity lower than its actual value, leading to a reduction of its flattening effect on the system demand (Fig. 4); as a result, the market price differential between peak and off-peak periods is maintained at higher levels, increasing the arbitrage revenues of the ESS. Therefore, it can be observed that under a price-making ESS behaviour, the strategic price increments at peak periods are higher than under a price-taking ESS behaviour (Fig. 5). In other words, price-making ESS behaviour results in a less efficient market outcome than price-taking ESS behaviour. However, the market outcome is still more efficient than the No-ESS scenario, since the existence of the ESS still flattens the system demand profile (although to a smaller extent than under a price-taking behaviour).

The above effects of the price-taking and price-making ESS on the market outcome are further justified through the quantification of the average offering strategies of the producers over the critical peak periods $T^{\text{peak}} = \{17, 18, 19\}$, i.e. $\bar{v} = \text{average}_{i,t \in T^{\text{peak}}} v_{i,t}$, presented in Table 4 and the market indexes presented in Table 5. The extent of market power exercised by strategic producers over the peak periods, expressed by the size of $\bar{v}$ in Table 4, is higher in

Table 4 Average offering strategies of electricity producers over peak periods for different ESS scenarios

No-ESS	C-ESS	S-ESS
5.30	3.99	4.17

Table 5 Market indices for different ESS scenarios

	Generation surplus, mil.£	Demand surplus, mil.£	ESS surplus, mil.£	Social welfare, mil.£
No-ESS	86.57	301.32	\	387.90
C-ESS	62.44	332.17	1.23	395.84
S-ESS	76.16	313.56	1.71	391.42

the No-ESS scenario, since the higher peak demand levels create a higher need to utilise available generation capacity, a condition which is exploited by the producers through the submission of more strategic (higher) offering prices. On the other hand, this extent of market power is lower in the C-ESS scenario due to the lower peak demand levels, a condition which results in more competition among the producers. Finally, an intermediate extent of market power is observed in the S-ESS scenario, since the ESS reduces the peak demand levels with respect to the No-ESS scenario, although to a smaller extent than under the C-ESS scenario. In other words, the adoption of price-making behaviour by the ESS facilitates the adoption of the same behaviour by electricity producers.

As expected, the ESS makes a higher profit by behaving strategically, since it increases the market price differential between peak and off-peak periods (Table 5). Furthermore, following the trend observed in market prices (Fig. 5), the total surplus of the strategic producers is the highest in the No-ESS scenario, the lowest in the C-ESS scenario where the ESS flattens the demand and price profiles and an intermediate value in the S-ESS scenario where the ESS flattens to a smaller extent the demand and price profiles. A similar trend is observed with respect to the total surplus of the demand side, implying that the ESS enables consumers to shield their economic surplus from market power exercise by large players, with the price-taking ESS behaviour enhancing this benefit with respect to the price-making ESS behaviour. Finally, a similar trend is observed with respect to social welfare which constitutes a global metric of the market efficiency. This implies that the presence of the ESS improves the efficiency of the market and this benefit is more significant when the ESS behaves competitively.

3.3 Impact of ESS: congested network

This section analyses the results corresponding to a case where the network capacity limits are accounted for. In this case, the transmission line (6, 7) gets congested during some peak hours, reflecting the reality in the GB system where network corridors connecting Scotland with England are congested due to the transmission of Scotland's cheaper generation to England's large demand centres. This congestion results in a locational price differential between Scotland (buses 1–6) and England (buses 7–16), as illustrated in Fig. 6 (the prices of all buses in Scotland are identical, i.e. $\lambda_{1,t} = \lambda_{2,t} = \cdots = \lambda_{6,t}, \forall t$, and the prices of all buses in England are identical, i.e. $\lambda_{7,t} = \lambda_{8,t} = \cdots = \lambda_{16,t}, \forall t$, since none of the other lines is congested). Specifically, during periods of congestion, England – exhibiting more costly generation and higher demand – has a higher price than the one observed in the uncongested case, while Scotland – exhibiting less costly generation and lower demand – has a lower price than the one observed in the uncongested case. Five test scenarios are examined and compared, in all of which the producers behave strategically:

No-ESS: The system does not include any ESS.

C-ESS-SC: The system includes competitive (price-taking) ESS with the parameters given in Section 3.1, and located in Scotland (specifically at bus 3).

C-ESS-EN: The system includes competitive (price-taking) ESS with the parameters given in Section 3.1, and located in England (specifically at bus 16).

S-ESS-SC: The system includes strategic (price-making) ESS with the parameters given in Section 3.1, and located in Scotland (specifically at bus 3).

S-ESS-EN: The system includes strategic (price-making) ESS with the parameters given in Section 3.1, and located in England (specifically at bus 16).

Table 6 presents the average offering strategies of the producers over the critical peak periods $T^{\text{peak}} = \{17, 18, 19\}$ and Table 7 presents market indexes for each of the above scenarios. Before analysing the impact of the ESS behaviour and location, let us analyse the implications of network congestion which are crucial for understanding the subsequent analysis; in order to do this, we compare the results of the No-ESS scenario against the respective scenario of Section 3.2 which neglects network capacity limits and therefore congestion effects. As previously discussed, congestion on line (6, 7) increases the prices in England and reduces the prices in Scotland with respect to the uncongested case. Since higher price levels favour the exercise of market power (Section 3.2), congestion results in more strategic (higher) offering prices by producers in England and less strategic (lower) offering prices by producers in Scotland, with respect to the uncongested case (as observed by comparing the first two rows of Table 6). As a result, congestion creates a more favourable economic setting (i.e. higher surplus) for producers in England and demands in Scotland and a less favourable setting (i.e. lower surplus) for producers in Scotland and demands in England (as observed by comparing the first two rows of Table 7). Finally, the overall impact of network congestion on the efficiency of the market is negative (as justified by the observed increase in the overall average offering strategy of the producers in Table 6 and the reduction of the social welfare in Table 7); in other words, congestion aggravates the extent of market power exercise, as also demonstrated in [31, 32].

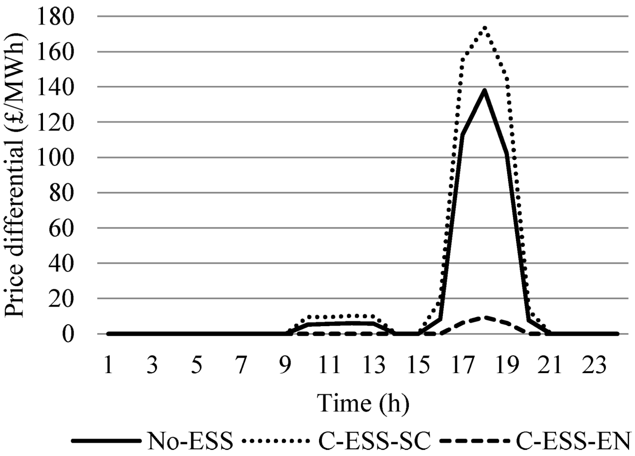

Fig. 6 *Price differential between Scotland and England for scenarios No-ESS, C-ESS-SC and C-ESS-EN*

Let us now examine the impacts of introducing the price-taking ESS in different locations. When such an ESS is located in Scotland, it flattens the demand profile in Scotland and thus aggravates congestion and increases the price differential between the two areas, with respect to the No-ESS scenario (Fig. 6). Following the above discussion regarding the impacts of congestion, the offering strategies and surplus of producers in Scotland/England are reduced/increased, while the surplus of consumers in Scotland/England is increased/reduced with respect to the No-ESS scenario (Tables 6 and 7). In other words, the presence of the price-taking ESS in Scotland creates a less favourable setting for producers in Scotland and demands in England and a more favourable setting for producers in England and demands in Scotland.

On the other hand, when the price-taking ESS is located in England, it flattens the demand profile in England and thus relieves congestion and reduces the price differential between the two areas, with respect to the No-ESS scenario (Fig. 6). As a result, the offering strategies and surplus of producers in Scotland/England are increased/reduced, while the surplus of consumers in Scotland/England is reduced/increased with respect to the No-ESS scenario (Tables 6 and 7). In other words, the presence of the price-taking ESS in England creates a less favourable setting for producers in England and demands in Scotland and a more favourable setting for producers in Scotland and demands in England.

As expected, ESS makes a higher surplus when it is located in the higher-priced area (England). The overall impact of the price-taking ESS on market efficiency is positive irrespective of its location, as in both C-ESS-SC and C-ESS-EN scenarios the overall average offering strategy of the producers is reduced (Table 6) and the social welfare is increased (Table 7), with respect to the No-ESS scenario. However, this positive impact is higher when it is located in the higher-priced area (England) which is more prone to market power exercise by strategic producers.

Let us now examine the impacts of strategic behaviour by the ESS, depending on its location. When the ESS is located in Scotland, its capacity withholding actions limit its flattening effect on Scotland's demand profile and thus relieve congestion with respect to the C-ESS-SC scenario (Fig. 7). Following the above discussion regarding the impacts of congestion, the offering strategies and surplus of producers in Scotland/England are increased/reduced, while the surplus of consumers in Scotland/England is reduced/increased with respect to the C-ESS-SC

Table 6 Average offering strategies of electricity producers over peak periods for different ESS scenarios

	Scotland	England	Overall
No-ESS (uncongested)	6.93	4.09	5.30
No-ESS	6.86	4.51	5.52
C-ESS-SC	6.39	4.56	5.34
C-ESS-EN	6.91	4.05	5.32
S-ESS-SC	6.78	4.53	5.49
S-ESS-EN	6.89	4.07	5.41

Table 7 Market indexes for different ESS scenarios

	Generation surplus, mil.£		Demand surplus, mil.£		ESS surplus, mil.£	Social welfare, mil.£
	Scotland	England	Scotland	England		
No-ESS (uncongested)	45.87	40.71	41.48	259.84	—	387.90
No-ESS	37.52	56.09	51.72	226.48	—	377.99
C-ESS-SC	35.73	59.89	56.18	222.36	1.17	383.90
C-ESS-EN	44.76	32.83	42.86	263.87	1.35	386.45
S-ESS-SC	36.86	57.67	53.55	224.16	1.50	380.22
S-ESS-EN	43.13	39.98	49.06	244.97	1.81	383.64

scenario (Tables 6 and 7). As expected, the ESS makes a higher profit by behaving strategically (Table 7). Finally, by comparing the overall average offering strategies of the producers (Table 6) and the social welfare (Table 7), it can be concluded that, in a similar line with the results observed in the uncongested case (Section 3.2), the price-making ESS behaviour results in a less efficient market outcome overall than the price-taking ESS behaviour, which is, however, still more efficient than the scenario without ESS. In other words, the presence of the ESS in Scotland improves the efficiency of the market and this benefit is more significant when the ESS behaves competitively.

When the ESS is located in England, its capacity withholding actions limit flattening effect on England's demand profile and thus aggravate congestion with respect to the C-ESS-EN scenario (Fig. 8). As a result, the offering strategies and surplus of producers in Scotland/England are reduced/increased, while the surplus of consumers in Scotland/England is increased/reduced with respect to the C-ESS-EN scenario (Tables 6 and 7). Once again, the ESS makes a higher profit by behaving strategically (Table 7). Finally, in a similar line with previous results, the price-making ESS behaviour results in a less efficient market outcome overall than the price-taking ESS behaviour, which is, however, still more efficient than the scenario without ESS. In other words, as in the case it is connected to Scotland, the presence of the ESS in England improves the efficiency of the market and this benefit is more significant when the ESS behaves competitively.

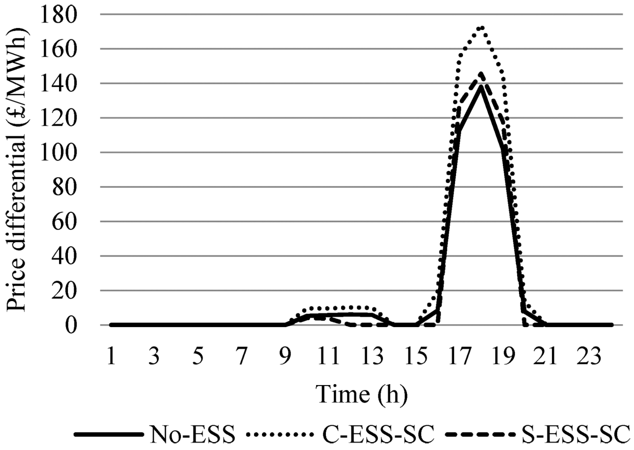

Fig. 7 *Price differential between Scotland and England for scenarios No-ESS, C-ESS-SC and S-ESS-SC*

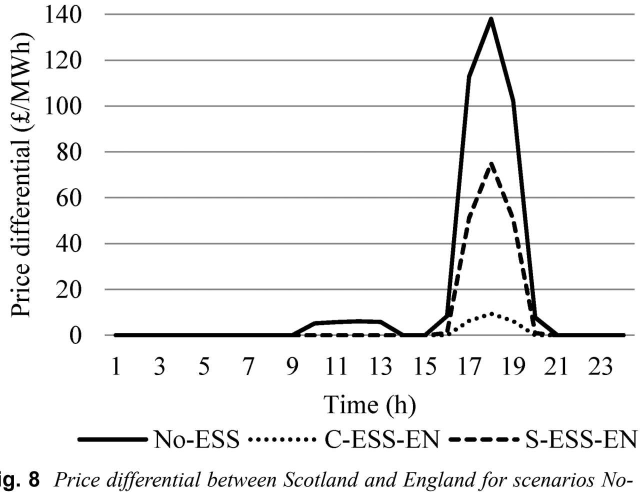

Fig. 8 *Price differential between Scotland and England for scenarios No-ESS, C-ESS-EN and S-ESS-EN*

3.4 Computational requirements

Table 8 summarises the computational performance of the developed equilibrium programming market model, by presenting the number of iterations and the total CPU time required by the diagonalisation algorithm for each of the examined scenarios. The total CPU time is reported for both sequential and parallel computation approaches discussed in Section 3.1. The sequential approach leads to long computation times which may constitute an obstacle for the practical application of the model. The parallel computation approach reduces significantly the required CPU time by solving the MILP corresponding to all strategic players simultaneously at every iteration.

In both uncongested and congested network cases, the computational requirements are the lowest when the ESS and its time-coupling constraints are not present (No-ESS scenario), and the highest when the ESS behaves strategically (S-ESS scenarios) since an additional strategic player is included in the model with respect to the C-ESS scenarios. Furthermore, the computational burden is higher when the network is congested.

4 Conclusions

This study has comprehensively analysed the impacts of energy storage in electricity markets, considering both price-taking and price-making storage behaviours, corresponding to potential settings with independent, small-scale, distributed ESSs and large storage capacities owned by the same market entity, respectively. In order to achieve this and in contrast to previous works, this paper has developed a multi-period equilibrium programming market model, determining market equilibrium stemming from the interactions of independent strategic producers and ESSs, while capturing the time-coupling operational constraints of the ESS as well as network constraints.

Case studies on a test market capturing the general conditions of the GB system have demonstrated that the presence of the price-taking ESS in the system mitigates market power exercise by strategic producers during peak periods and enhances it during off-peak periods, with the former mitigation significantly dominating the latter enhancement and resulting in an overall positive impact in terms of market efficiency. When the ESSs behave strategically, they exercise capacity withholding in order to maintain the price differential between peak and off-peak periods at higher levels and increase their arbitrage revenues. As a result, their flattening effect on system demand is limited and the market outcome is less efficient with respect to the price-taking case, although it is still more efficient than the case without storage in the system. This result persists irrespectively of network congestion conditions and the location of storage, implying that the envisaged penetration of storage capacity is likely to reduce the extent of market power and

Table 8 Computational performance of equilibrium programming market model

Scenario		Number of iterations	Total CPU time with sequential computation, (s)	Total CPU time with parallel computation, (s)
uncongested network	No-ESS	14	3,111	437
	C-ESS	22	3,746	549
	S-ESS	34	5,540	702
congested network	No-ESS	30	7,722	1,088
	C-ESS-SC	33	7,856	1,146
	C-ESS-EN	36	7,809	1,134
	S-ESS-SC	48	14,445	1,849
	S-ESS-EN	45	12,422	1,555

improve the efficiency of the market, with this benefit being higher if this storage capacity is shared by a large number of independent small players who cannot unilaterally affect the market outcome.

In cases with network congestion, apart from the market behaviour of storage, its location also affects considerably the market outcome. Specifically, the presence of storage at a particular location mitigates market power exercise by collocated strategic producers and improves the market position of collocated consumers. Its overall impact on market efficiency is positive irrespective of its location, but this benefit is higher when it is located in areas with more costly generation and higher demand, which are more prone to market power exercise. These locational effects persist but are less pronounced when storage behaves strategically.

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6 Appendix: Derivation of MILP objective functions

The linearisation approach proposed in [27] exploits the strong duality theorem, according to which the optimal values of the primal and dual objective functions of a convex problem are equal. In the case of the convex LL problem (3)–(15), this theorem implies that (see (44)) . By multiplying both sides of (20) by $g_{i,t,b}$, summing for every t and b and rearranging some terms we get

$$\sum_{t,b} v_{i,t}\lambda^{G}_{i,b}g_{i,t,b} = \sum_{t,b}\left(\lambda_{(n:i\in I_n),t}g_{i,t,b} + \mu^{-}_{i,t,b}g_{i,t,b} - \mu^{+}_{i,t,b}g_{i,t,b}\right). \quad (45)$$

By making use of (28), (45) becomes

$$\sum_{t,b} v_{i,t}\lambda^{G}_{i,b}g_{i,t,b} = \sum_{t,b}\left(\lambda_{(n:i\in I_n),t}g_{i,t,b} - \mu^{+}_{i,t,b}g_{i,t,b}\right). \quad (46)$$

By making use of (29), (46) becomes

$$\sum_{t,b} v_{i,t}\lambda^{G}_{i,b}g_{i,t,b} = \sum_{t,b}\left(\lambda_{(n:i\in I_n),t}g_{i,t,b} - \mu^{+}_{i,t,b}\bar{g}_{i,b}\right). \quad (47)$$

Analogously, by multiplying both sides of (21) by $g_{i-,t,b}$, summing for every $i-$, t and b, making use of (28) and (29) and rearranging some terms we get

$$\sum_{i-,t,b} v_{i-,t}\lambda^{G}_{i-,b}g_{i-,t,b} = \sum_{i-,t,b}\left(\lambda_{(n:i-\in I_n),t}g_{i-,t,b} - \mu^{+}_{i-,t,b}\bar{g}_{i-,b}\right). \quad (48)$$

By substituting (47) and (48) into (44) and rearranging some terms we get (see (49)) .

Therefore, the bilinear terms $\sum_{t,b} \lambda_{(n:i \in I_n),t} g_{i,t,b}$ in the objective function (18) of the MPEC problem corresponding to producer i can be replaced with the expression in the right side of (49), which is linear. The resulting objective function of the MILP formulation is (see (50)) . Following the same rationale, the bilinear terms $\sum_t \lambda_{(n:k \in K_n),t}\left(s_{k,t}^d - s_{k,t}^c\right)$ in the objective function (16) of the MPEC corresponding to ESS k can be reformulated, and its resulting objective function of the MILP formulation is

$$\max \sum_{j,t,c}\left(\lambda_{j,t,c}^D d_{j,t,c} - \nu_{j,t,c}^+ \bar{d}_{j,t,c}\right) - \sum_{i,t,b}\left(v_{i,t}\lambda_{i,b}^G g_{i,t,b} + \mu_{i,t,b}^+ \bar{g}_{i,b}\right) - \sum_{n,(m \in M_n),t}(\chi_{n,m,t}^- + \chi_{n,m,t}^+)\bar{F}_{n,m} - \sum_{n,t}(\psi_{n,t}^- + \psi_{n,t}^+)\pi . \quad (51)$$

$$\sum_{i,t,b} v_{i,t}\lambda_{i,b}^G g_{i,t,b} - \sum_{j,t,c}\lambda_{j,t,c}^D d_{j,t,c} = -\sum_{i,t,b}\mu_{i,t,b}^+ \bar{g}_{i,b} - \sum_{j,t,c}\nu_{j,t,c}^+ \bar{d}_{j,t,c} + \sum_{k,t}\left[\pi_{k,t}^- \underline{E}_k - \pi_{k,t}^+ \bar{E}_k - (\rho_{k,t}^+ + \sigma_{k,t}^+)(1 - w_{k,t})\bar{s}_k\right] + \sum_k(\varphi_k - \xi_{k,1})E_k^0 - \sum_{n,(m \in M_n),t}(\chi_{n,m,t}^- + \chi_{n,m,t}^+)\bar{F}_{n,m} - \sum_{n,t}(\psi_{n,t}^- + \psi_{n,t}^+)\pi . \quad (44)$$

$$\sum_{t,b}\lambda_{(n:i \in I_n),t} g_{i,t,b} = \sum_{j,t,c}\left(\lambda_{j,t,c}^D d_{j,t,c} - \nu_{j,t,c}^+ \bar{d}_{j,t,c}\right) - \sum_{i-,t,b}\left(\lambda_{(n:i- \in I_n),t} g_{i-,t,b} + \mu_{i-,t,b}^+ \bar{g}_{i-,b}\right) + \sum_{k,t}\left[\pi_{k,t}^- \underline{E}_k - \pi_{k,t}^+ \bar{E}_k - (\rho_{k,t}^+ + \sigma_{k,t}^+)(1 - w_{k,t})\bar{s}_k\right] + \sum_k(\varphi_k - \xi_{k,1})E_k^0 - \sum_{n,(m \in M_n),t}(\chi_{n,m,t}^- + \chi_{n,m,t}^+)\bar{F}_{n,m} - \sum_{n,t}(\psi_{n,t}^- + \psi_{n,t}^+)\pi . \quad (49)$$

$$\max \sum_{j,t,c}\left(\lambda_{j,t,c}^D d_{j,t,c} - \nu_{j,t,c}^+ \bar{d}_{j,t,c}\right) - \sum_{i-,t,b}\left(\lambda_{(n:i- \in I_n),t} g_{i-,t,b} + \mu_{i-,t,b}^+ \bar{g}_{i-,b}\right) + \sum_{k,t}\left[\pi_{k,t}^- \underline{E}_k - \pi_{k,t}^+ \bar{E}_k - (\rho_{k,t}^+ + \sigma_{k,t}^+)(1 - w_{k,t})\bar{s}_k\right] + \sum_k (\varphi_k - \xi_{k,1})E_k^0 - \sum_{n,(m \in M_n),t}(\chi_{n,m,t}^- + \chi_{n,m,t}^+)\bar{F}_{n,m} - \sum_{n,t}(\psi_{n,t}^- + \psi_{n,t}^+)\pi - \sum_{i,t,b}\lambda_{i,b}^G g_{i,t,b} . \quad (50)$$