

# Liquefaction resistance of fibre-reinforced silty sands under cyclic loading

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## Abstract

Whether the so-called double porosity in soils with a loose and natural packing state is a concept with real-world implications is a fundamental yet controversial question in the study of cyclic undrained shear behaviour of fibre-reinforced silty sands. An attempt is made here to clarify the question by means of particle-level modelling combined with 41 undrained cyclic triaxial shear tests. The study shows that the initial Random Loose Packing changes to Random Close Packing and then Close Packing with silt content increments. The transition from random to close packing occurs at a threshold silt content which is relatively lower in coarser sands. For sands with <40% silt content, the rate of pore pressure growth with loading-unloading cycles increase with silt content increment. Reverse trend applies to silty sands with >40% silt content. Irrespective of fine content, fibres tend to sit deep into the silt pellets and encrust the macro-pore spaces. Generally, increasing fibre content leads to an increase in the average number of contacts per particle, dilation and easier dissipation of excess pore water pressure, a decrease in contact forces and improved liquefaction resistance. For sands with >40% silt content, effectiveness of fibre reinforcement diminishes with increasing sand median size.

**Keywords:** Liquefaction; silty sand; fibre; cyclic; shear; packing

## 27 **1. Introduction**

28 For a traffic infrastructure (e.g. roads, highways, railway tracks) constructed on low embankments or built  
29 directly on sandy grounds, the compressibility of the subsoil has substantial effects on the stability,  
30 operational speed (i.e. serviceability) and the maintenance costs of the system. Deformation of the subsoil  
31 is dependent on their strength and stiffness, as well as applied loading conditions. Subsoils under static  
32 loading fundamentally behave differently than subsoils under repeated traffic loading of the same order of  
33 magnitude. Depending on the magnitude of the individual wheel load, and the number and frequency of  
34 repetitions, subsoils may experience excessive deformations and in some cases liquefaction and flow.  
35 Liquefaction is the substantial loss of strength in soil in response to a sudden change in stress state mostly  
36 caused by earthquake and seismic loads (for example from the inertia loads in onshore wind turbines during  
37 emergency stop), but also traffic loads (Wichtmann et al. 2004) and loads associated with high-speed  
38 moving vehicles (Naeini and Gholampoor 2014). Liquefaction is mostly expected in loose saturated clean  
39 sands although it may also occur in clayey and silty sands. In principle, soil liquefies as the excess pore  
40 water pressure reduces the effective stress to dangerously low levels, leading to catastrophic floating  
41 (Sawicki and Mierczyński 2015), sinking of structures (Ardeshiri-Lajimi et al. 2016) and ground subsidence  
42 (Romeo et al. 2015). For traffic infrastructure, elevated levels of excess pore water pressure have adverse  
43 impacts on the operational speed and stability of the system, regardless of liquefaction. In the short term,  
44 liquefaction is unlikely to occur in dense granular soils. In the long term, at shallow depths and under certain  
45 combinations of temperature, soil suction, soil permeability, and availability of water, weathering and  
46 subsequent particle-level events can lead to the development of new open structures in soil (Li and Selig  
47 1995), thereby a potential of liquefaction. This ‘progressive loosening’ problem is more common in finer  
48 soils and upon the progressive flow of fines and softening (Ghadr and Assadi-Langroudi 2019), or dilation  
49 associated with strain softening (Indraratna et al. 2011). The effect of fine fraction and reinforcement  
50 element (e.g. fibre) content on liquefaction resistance of soils have received much interest but the studies  
51 are constrained to granular mixtures compacted to a constant and high relative density. Little is known on

52 the cyclic behaviour of relatively loose silty sands in reinforced and natural forms and compressed to their  
53 natural random packing state. The aim of this paper is to show to what extent the cyclic response of loose  
54 sands compares with that of the loose sand-silt and sand-silt-fibre mixtures and to provide evidence to the  
55 influence of some packing parameters of the materials on fibre-reinforced soils' mechanical response.

### 56 *1.1 Progressive Loosening: Rationale*

57 In the long-term, sand-silt mixtures progressively develop a quality of open porosity that can cause  
58 uncertainties in serviceability of the built settings they underpin. Such unwelcomed implications are  
59 generally overlooked in the literature and have motivated this study. Khosravani (2014) and Assadi-  
60 Langroudi (2014) modelled loosely packed cemented silty soils in the form of three-phase discontinuous  
61 mediums composed of sub-rounded. Mono-dispersed, silt particles of  $R$  diameter bridged with water  
62 menisci and bonding minerals. They adopted a homogenization framework, taking the soil system as a  
63 representative elementary volume (REV) composed of particles that interact with one another via a suite of  
64 traction forces ( $t_i(x)$ ), and expressed the effective stress as a function of local micro-scale variables.  
65 Khosravani (2014) derived a the tensorial form of the effective stress equation as a function of  $\chi_{ij}$  and  $B_{ij}$   
66 effective stress parameters, and expanded the formulation for the benchmark REV. She assumed that the  
67 continuity of the pore network is a valid simplification regardless of soil's structure and its dependence on  
68 the matric suction. For cemented Body-centred Cubic (BCC) regular packings, Assadi-Langroudi (2014)  
69 used the double-porosity theorem and proposed an improved form of the tensorial effective stress equation  
70 and suggested that the  $B_{ij}$  the parameter is inversely proportional to  $\chi_{ij}$  (the Bishop property) and is a direct  
71 function of  $\sigma_{ij}^d$ , a periodic hydro-dynamic boundary level stress acting on buttress units during the flow of  
72 liquid between micro- and macro-pore phases.

73 In this, Khosravani (2014) and Assadi-Langroudi (2014) formulated the inter-particle forces acting at  
74 contact points which at macro-scale represents the effective stress,  $\sigma'_{ij}$ , that applies to the solid skeleton in  
75 a soil. The water influx into loose lightly bonded granular assemblies initially impacts the buttress units.

76 Upon full saturation of buttress units, pore water advances into the macro-pore spaces. As the matric suction  
77 drops below the air entry value, air pockets relocate from macro-pores into micro-pores within the buttress  
78 units. In fact, macro-pore air begins to dissolve in micro-pore water before the influx of water into the  
79 macro-pore voids. This leads to the collapse of buttress units into macro-pore spaces and their coagulation.  
80 Upon drying, the macro-pore phase lose water while the  $\sigma_{ij}^d$  remains at its nominal minimum. Under  
81 constant net stress and matric suction at micro-pore level,  $(u_a - u_w)_m$ , and in the absence of an immediate  
82 head gradient between micro- and macro-pore phases, effective stress decreases, resulting in a rebound  
83 volume change and expansion. Assadi-Langroudi (2014) showed that this progressive volumetric expansion  
84 under constant net pressure continues over subsequent wetting-drying cycles. In the long-term, this induced  
85 open structure in engineered silty sands can cause uncertainties in serviceability of the built settings they  
86 underpin, a matter which has been broadly overlooked in the literature.

## 87 *1.2 Question of Packing*

88 In the terrestrial system, sand is fundamentally a binary particulate matter (Yamamuro and Lade, 1999) and  
89 almost always comprises an inherently crushable quartz silt fraction (Assadi-Langroudi et al. 2014). The  
90 physical nature and mechanical response of sand are dependent on its silt content (Lade and Yamamuro,  
91 1997; Thevanayagam et al. 1997; Thevanayagam,1998 ). Yamamuro and Covert (2001) discussed the effect  
92 of silt fraction variation on soil packing state. In their pivotal work, they proposed three transitional zones  
93 and three extreme packing states that can theoretically appear in any sand-silt composite (identified as  
94 points labelled as A, B and C in Fig. 1).

95 **Figure 1.** The schematic explanation of packing for sand-silt composites (Yamamuro and Corvet, 2001)

96 In a binary mixture of particles with two sizes, and where the fines content is low, fine matters tend to sit  
97 in macro-void spaces between larger particles. An increase in fines content leads to an increase in the  
98 packing density, causing particles to be more closely 'knitted' together (Fonseca et al. 2013). Yamamuro  
99 and Covert (2001) suggested that the packing density reaches the maximum at a critical threshold fines

100 content of around 20% (by weight). For the fine matter to fit into macro-voids, the diameter of the fine  
101 particles should be at least 6.5 times smaller of the large particles. Further increment of fines content (from  
102 20 to 50%) leads to a decrease in packing density as the fine fraction begins to push the larger particles  
103 apart, separate the large particles and cause the increment of global void ratio until the volume of fine  
104 particles become large enough to accommodate larger particles as isolated inclusions (Xenaki and  
105 Athanasopoulos, 2003; Chang et al. 2015).

106 Sladen et al. (1985) showed a decrease in strength of sand for the addition of small contents of non-plastic  
107 silt. Pitman et al. (1994) argued these findings based on findings from a programme of undrained triaxial  
108 compression tests on the standard Ottawa Sand. For soils with up to 40% silt content, they showed an  
109 increase in the strength of Ottawa Sand that mitigates the rapid shearing-induced collapsibility typically  
110 appearing in clean sands. Verdugo and Ishihara (1996) presented experimental evidence for the contractive  
111 response of silty sands, and consequently the high risk of flow failure and liquefaction. The dispute over  
112 the role of silt in mechanical behaviour of sand continued into the 21<sup>st</sup> century, and emphasis was put on  
113 the interplay between silt content and packing quality, and also the complex mechanics of binary mixtures  
114 (Bouckovalas et al. 2003; Thevanayagam and Martin, 2002; Xenaki and Athanasopoulos, 2003; Yang et al.  
115 2006; Yin et al. 2014). Lade et al. (2009) studied the role of fine fragments in the formation of highly  
116 compressible, liquefiable microstructures in sands. Monkul (2013) downplayed the importance of fines  
117 content, and sand and silt gradation quality on strength parameters. Studying the cyclic behaviour of silt  
118 and sandy silt soils, El Takch et al. (2016) presented evidence in support of susceptibility of non-plastic  
119 silts to liquefaction and similarities between the strain and excess pore water generation patterns in sand  
120 and silt under dynamic excitation. They also suggested that the cyclic stress ratio (*CSR*, or the ratio of the  
121 cyclic deviator stress to the initial confining stress) increases with increasing silt content at constant global  
122 void ratio (also see Noorzad and Amini 2014; Karim and Alam 2014). The majority of published research  
123 is based on experimental works on soils compacted to a constant low void ratio. Little is known on the  
124 cyclic response of loose saturated silty sands with a naturally open structure. This, together with the often-

125 contradictory perception of fines influence on the cyclic behaviour of granular soils have led to broad  
126 uncertainties in cyclic properties of composite granular geomaterials and their dependence on packing  
127 quality of constituting particles.

128 In this, the macro-scale cyclic response of testing materials here are discussed in terms of stress evolutions  
129 at fractal pore phases and through relating sand particle shape, sorting, and mean size to soil packing state  
130 parameters. Global void ratio is idealized into micro- and macro-pore level void ratios and the impact of  
131 silt and fibre content variation on packing quality, likelihood of particle contact destruction and  
132 reorientation, and inter-particle stress evolution are discussed in length. As such, this work employs a  
133 micro-to-macro approach in studying the cyclic response of geocomposite systems.

### 134 *1.3 Fibre-reinforced systems*

135 The use of fibres for ground improvement origins from the principles of naturally reinforced soils by tree  
136 root systems. It is now well established that randomly distributed fibres in sand generally improves the  
137 drained and undrained mechanical properties of soils, including shear strength and bearing resistance  
138 (Sadek et al. 2010; Kutanaei and Choobbasti 2016; Mirzababaei et al. 2017). An important early  
139 contribution to the dynamic response of fibre-reinforced sand systems is the work of Noorany and  
140 Uzdavines (1989). They reported findings from a programme of cyclic triaxial tests and discussed the  
141 liquefaction resistance of saturated sands reinforced with a suite of reinforcing elements including fine  
142 steel-wire mesh, polypropylene fabric, nylon netting, and fine polypropylene fibres. For all fibre types, they  
143 reported an improvement in liquefaction resistance. The closing of the 20<sup>th</sup> century saw a hike in  
144 experimental dynamic studies on fibre-reinforced soils. Maher and Woods (1990) studied the anisotropy in  
145 fibre reinforced sands through a programme of torsional shear and resonant-column tests. They showed the  
146 advantages of fibres in improving the dynamic shear modulus of sandy soils. A particularly distinctive  
147 feature of fibre-reinforced soils is the anisotropic behaviour they may demonstrate under various loading  
148 conditions (Diambra et al. 2010; Gao and Zhao 2013). Most recently, Ghadr et al., (2019) and Ghadr (2020)

149 showed that upon fibre reinforcement, the contractive behaviour of saturated loose sands - that is  
150 predominant as the inclination angle of the principal stress increases beyond  $60^\circ$  - changes to dilative. They  
151 also showed that the undrained performance for fibres are poor when soil carries a combination of torsional  
152 and compressive stresses. The undrained performance of fibres improves when soil carries a combination  
153 of extension and compressive stresses. The undrained performance of fibres is maximum when soil carries  
154 a combination of extension and torsional stresses, and further enhances with principal stress direction  
155 increments. Maher and Ho (1993) studied the behaviour of fibre-reinforced cemented sands under cyclic  
156 loading and reported enhanced cyclic strength upon fibre addition. Ibraim et al. 2010 and Liu et al. 2011  
157 attributed that improvement to the role of fibres in restricting lateral movement of particles. This brings the  
158 question of fibre shape, dimensions, and texture, and how and if these play a role in soils dynamic  
159 behaviour. Li and Ding (2002) used discrete crimped polypropylene fibres and suggested that fibre content  
160 (in sand) and small-strain stiffness are directly correlated. They suggested that fibre reinforcement leads to  
161 a decrease in static flow and liquefaction potential. Their observations were later repeated and confirmed  
162 for both compression and extension loading environments (Ibraim et al. 2010; Ghadr and Bahadori 2019).  
163 Michalowski and Cermák (2002) casted doubts on the effectiveness of fibre reinforcement in sands with  
164 relatively finer particle size and angular shape. At the extreme end of angularity and silt size and for  
165 liquefiable fly ash, Boominathan and Hari (2002) showed an increase in liquefaction resistance as fibres  
166 provide enhanced interlocking, which then leads to a decrease in the excess pore water pressure. More  
167 recently, Noorzad and Amini (2014) reported similar findings but argued that advantages are more  
168 pronounced in soils of medium density as compared to loosely compacted soils. This can be a serious  
169 drawback to the stability of fibre-reinforced sands and is a principal objective of the present work.

## 170 **2. Materials and Methods**

### 171 *2.1 Materials*

172 Two angular sands of varied mean diameter are used in this study. Sand A is moderately-well-sorted  
173 standard F161 (Firoozkuh 161) sand with particle sizes ranging 0.80 to 1.18 mm. A UB200i Lacet

174 transmitting light microscope (integrated with DCM-900 digital camera) was used to capture the sand  
175 particles' shape through two benchmark sphericity ( $r_s$ ) and roundness ( $r_r$ ) parameters in compliance with  
176 the Wadell (1932) method. Sphericity ( $r_s$ ) is a measure of convergence of particle's dimensions in the three-  
177 dimensional coordinate system. Particles with the highest sphericity contain minimum eccentricity and  
178 flatness. Roundness ( $r_r$ ) is a measure of surface features, relative to the radius of the particle. Angular  
179 particles typically gain small roundness index values. The size, shape and packing properties of Sand A  
180 ( $r_s = 0.6, r_r = 0.38, D_{50} = 0.27 \text{ mm}, C_u = 1.78, e_{min} = 0.548, e_{max} = 0.874$ ) is nearly identical to those  
181 of standard Ottawa C109 sand ( $r_s = 0.69, r_r = 0.40, D_{50} = 0.37 \text{ mm}, C_u = 1.80, e_{min} = 0.503, e_{max} =$   
182  $0.811$ ) and also has some resemblance with standard Fraser River and Hostun RF sands (Gue and Su 2007;  
183 Cho et al. 2006). The second sand material investigated (Sand B) was well-sorted standard Firoozkuh 141  
184 (F141) Sand, with particle sizes ranging 0.80 to 1.18 mm. Sand B is generally coarser than Sand A with  
185 almost similar shape properties. The size and packing properties of Sand B are brought in Table 1 ( $D_{50} =$   
186  $0.48 \text{ mm}, C_u = 1.16, e_{min} = 0.587, e_{max} = 0.892$ ). X-Ray Fluorescence (XRF) analysis on both sands  
187 confirmed their predominantly siliceous composition ( $\text{SiO}_2 > 96\%$ ,  $\text{Fe}_2\text{O}_3 = 0.2\text{-}0.7\%$ ,  $\text{Al}_2\text{O}_3 = 0.5\text{-}1.6\%$ ,  
188  $\text{CaO} = 0.2\text{-}0.5\%$ ,  $\text{Na}_2\text{O} = 0.03\text{-}0.08\%$ ,  $\text{K}_2\text{O} = 0.03\text{-}0.10\%$ ).

189 The sands were mixed with a sharp non-plastic silt soil produced in-house by controlled grinding of Sand  
190 A. Fig. 2 illustrates the particle size distribution, shape, and texture of silt and sand materials. Table 1  
191 summarizes the geometrical and physical properties of sand, silt and fibre. A common and commercially  
192 available thermoplastic polymeric micro-synthetic fibre with a ribbed linear texture (to improve adhesion  
193 with surrounding soil) and wave-shaped cross-section was adopted as the reinforcement components  
194 (MEX200, also Fig. 3). This fibre is commonly used in the construction industry, primarily as tension-  
195 resistant elements in concrete. Fibres used were 15 mm in length ( $l_f$ ) and 0.20 mm in equivalent diameter  
196 ( $D_f$ ), with a tensile strength of 450 MPa, ignition point of 450°C and melting point of 160°C. The adopted  
197 dimensions bring the fibre aspect ratio for reinforced systems ( $\text{AR}_F = l_f/D_{50}$ ) within the range reported in  
198 previously published research (e.g. Ghadr et al. 2019). The upper-bound and lower-bound value for fibre



199 aspect ratio are recommended as 100 (Shukla 2017) and 10 (Diambra and Ibraim 2015); This is to ensure  
200 reasonable levels of interaction between soil and reinforcement elements. The  $AR_F$  for fibres in the present  
201 work is in between the two extreme boundaries. The specimen aspect ratio (i.e. ratio of test specimen  
202 diameter to the length of fibres,  $AR_S=D/l_f$ ) in fibre reinforced systems conventionally varies between 0.17  
203 and 10.2. For example and for compacted silty clay loams, Ang and Loehr (2003) suggested an optimum  
204 1.25 specimen aspect ratio based on strength and stress-strain behaviour and following a series of  
205 unconfined compression tests. The  $AR_F$  and  $AR_S$  for test specimens are reported in Table 1.

206 **Table 1** Geometrical and physical properties of testing specimen constituents

207 **Figure 2.** Particle size distribution, shape, and texture of silt and sand particles

208 **Figure 3.** Illustration of fibres used in this study

## 209 **2.2 Equipment**

210 A pneumatic controlled cyclic triaxial testing apparatus manufactured by the British manufacturer VJ Tech  
211 was employed to conduct the experimental investigation and to evaluate the large-strain dynamic properties  
212 of clean sands and sands in mixtures with silt and fibre. The equipment generates dynamic frequencies in  
213 the range of 0.01 Hz to 2.00 Hz using a pneumatic pressure system. The system is controlled by digital  
214 Servo Controller which provides 16-bit resolution with an analogue and digital filter, for high and low  
215 frequencies respectively. The test settings were assigned using a Computer-Aided Testing System (CATS)  
216 software (VJTech Dynamic Suite 1.7). A solenoid valve and Swagelok controller control the cell and back  
217 pressures. A 5 kN external load cell is attached to a ram and allows measurement of applied axial dynamic  
218 loads. A linear variable differential transformer (LVDT) transducer is integrated into the actuator and allows  
219 the precise monitoring of axial displacements at 0.001 mm precision and within a +/- 40 mm range. The  
220 equipment includes three standard pressure transducers to measure the variation in cell pressure, pore water  
221 pressure and backpressure. The pressure transducers have the capacity of 1000 kPa and a sensitivity of 2  
222 kPa (+/- 0.2% accuracy). Figure 4 shows a front and rear view of the triaxial machine used in this study.

223 **Figure 4.** The pneumatic cyclic triaxial machine [a] front view, [b] rear view

### 224 **2.3 Specimen preparation**

225 For the purpose of triaxial tests, the soil is conventionally placed in split moulds in moist, dry or saturated  
226 state; it can be placed by dry deposition, water sedimentation, pouring or spooning; or can be compacted  
227 by tapping, tamping, or vibration (Consoli et al. 2005; Soriano et al. 2017; Mirzababaei et al. 2018). Dry  
228 sands were first mixed with sharp silt measuring 0%, 10%, 20%, 30%, 40% and 50% by dry mass in a  
229 container and a small amount of water; subsequently, the required amount of polypropylene fibres was  
230 added to the wet mix and they were thoroughly mixed. Moistening the silty sand (Hyde et al. 1993; Sadeghi  
231 and Beigi 2014; Ye et al. 2017) prior to addition of fibres is commonly used by many previous workers to  
232 avoid segregation and floating of fibres, also to facilitate the even distribution of fibres in soil. In the present  
233 study, water content was brought up to 1.5 to 2.5%, which almost equals the hygroscopic moisture content.  
234 Fibre content was varied from 0 to 1.5% in increments of 0.5%. The concentration of fibres included in a  
235 composite is defined as a proportion of the dry weight of soil,  $\chi = W_f/W_s$ , where  $W_f$  is the weight of fibres  
236 and  $W_s$  is the weight of the dry soil. The adopted range of fibre contents here accommodates the optimum  
237 fibre content of 0.3% to 0.5% reported in previous studies (Hoare 1979; Mercer et al. 1984; Al-Refeai and  
238 Al-Suhaibani 1998; Ghadr et al. 2019; Ghadr 2020). For Sand B, a 1.0% maximum fibre content was  
239 adopted for testing due to some operational constraints. Fibres were added ‘randomly’ to the mix in small,  
240 equal, and controlled amounts. The split mould (i.e., sample former) sized 50 mm in diameter and 100 mm  
241 in height. The adopted diameter to height ratio complies with the ASTM D3999-91 recommendations. A  
242 small amount of body powder (hydrated magnesium silicate) was applied to the edges of the sample former;  
243 The two halves of the sample former were then secured against one another by placing and tightening a  
244 clamp. A membrane was placed over the base pedestal and fixed in place with an O-ring. The excess  
245 membrane was then folded up over the O-ring before a second O-ring was placed over the base pedestal  
246 and rolled just above the first O-ring. The free top of the membrane was folded over the sample former. A  
247 small vacuum pressure of 15 kPa was applied to two points around the perimeter of the split mould to

248 stretch the membrane against the inner wall of the former. Sand-fibre mixtures were poured into the split  
249 mould resting on a flat working surface and total mass of mixture contained in mould was measured. The  
250 mixture was decanted, divided into five equivalent parts, and carefully poured into the split mould fixed on  
251 the lower porous disc on the platen (i.e., base pedestal) of the triaxial cell in five layers. Dry mixtures were  
252 poured through a funnel with a nozzle opening of 25 mm in diameter into the former in sequential layers.  
253 The initial void ratio was kept at the natural high, but low enough to minimize the entanglement and  
254 distortions in slender fibres potentially caused by the large compaction effort needed in the case of high  
255 fibre content (Gray and Ohashi, 1983).

256 A loading cap was placed on the top of the specimen and the split mould was the removed whilst a small  
257 14 to 15 kPa vacuum pressure was applied through the back pressure and upper outlets to allow the sample  
258 to maintain its cylindrical shape after the moulds were removed. The pressure chamber was assembled.  
259 Afterwards, a small cell pressure of 50 kPa was applied through the hydraulic devices, and the vacuum  
260 pressure was simultaneously released.

261 The initial void ratio of metastable samples is reported in Table 2 and marked as  $e_0$ . Gaseous  $\text{CO}_2$  and de-  
262 aired water were then gently introduced through the bottom drainage to the soil specimens and were driven  
263 upwards through the specimens. Pore water pressure and cell pressure were then increased simultaneously  
264 and in equal increments to raise the B-Skempton value to 0.96 at which state soil was deemed fully  
265 saturated. Depending on the silt content, the time required to accomplish saturation ranged from 30 min to  
266 5 hours. Specimens were then isotopically consolidated to a range of initial void ratios ( $e_c$  in Table 2) under  
267 a constant 200 kPa initial confining pressure (here reported as initial effective mean principal stress,  $P'_c$ ).  
268 The  $e_c$  fell in the range of 0.44 to 0.81 for unreinforced Sand A and 0.40 to 0.88 for unreinforced Sand B.  
269 A harmonically varying cyclic load was applied to the saturated specimens during the cyclic axial test, and  
270 the variation of excess pore water pressure and axial stress and strain of the specimen were continuously  
271 recorded during cyclic loading.

272 Table 2 summarises the 41 triaxial specimens tested in this study, their initial mean effective stress, initial  
273 and post-consolidation global void ratio, idealised void ratios, fine content (*FC*) and fibre content. Each  
274 specimen is numbered and given a test ID. The first letter indicates the sand type, silt content is indicated  
275 in brackets, followed by the fibre content in per cent; for example, A(20):1.5 stands for Sand A mixed with  
276 20% by mass silt and 1.5% fibre. The *CSR* is the cyclic stress ratio; that is the ratio of the cyclic deviator  
277 stress ( $\sigma_d$ ) to the initial confining stress,  $P'_c$ ,  $CSR = \sigma_d / 2P'_c$ . In this study, the *CSR* was set at 0.15, which  
278 corresponded to deviator stress of 60 kPa. The relatively small *CSR* makes the influence of fibre content  
279 more remarkable (Ye et al. 2017), avoids additional compaction induced by cyclic loading-unloading  
280 (Sadeghi and Beigi 2014), and allows the relative loose testing specimens here to reach the state of  
281 liquefaction (Xu et al. 2015). A sinusoidal waveform loading with the frequency ( $f$ ) of 0.05 Hz was applied  
282 on the specimens until liquefaction occurred. Typical traffic load frequencies fall within the 0 to 10 Hz  
283 range (Hyde et al. 1993) whilst a range of 0.1 to 1.5 Hz is well established for the cyclic loading frequency  
284 in most geotechnical problems and broadly applied by many experimental workers (Al-Refeai and Al-  
285 Suhaibani, 1998; Ye et al. 2017). The choice of low frequency is consistent with the relatively open packed  
286 porous state of test specimens. The liquefaction onset was considered as the mean effective stress when  
287 first reaching the zero-stress state, that is when the generated excess pore water pressure ratio was equal to  
288 or higher than 0.96. Here, we employ a parameter  $N_l$ , which is defined as the cycle number at which the  
289 excess pore water pressure ratio first reached 1.0 to indicate the loading cycle number for triggering  
290 liquefaction. In this, larger values of  $N_l$  corresponds to greater resistance to liquefaction in the sand. Table  
291 2 summarises the achieved values of  $N_l$ ; these fell in the range of 3 to 19 for unreinforced Sand A and 7 to  
292 52 for unreinforced Sand B. The excess pore water pressure ratio is presented in Table 2 with  $r_u$  and is  
293 defined as  $r_u = \Delta u / \sigma_3$ , where  $\Delta u$  is the excess pore water pressure.

294 **Table 2** Testing itinerary and specimens ( $f=0.05$  Hz,  $CSR= 0.15$ ,  $\sigma_d= 60$  kPa)

295

296

297

### 298 3. Scale-dependent Porosity

299 Packing density is a macro-scale parameter measured via the specimen mass and volume (Fonseca et al.  
300 2013) and is the fraction of the total packing volume occupied by solid particles in a domain of soil. For  
301 example, for an assembly of particles with a void ratio of 1.0, the packing density is about 0.5. Packing  
302 density is commonly quantified using the de Larrard (1999) equation as a function of packing density of  
303 soil components (here sand and silt), solid volumetric fraction of less dominant fraction (here silt) and  
304 interaction functions that are directly correlated with particle size ratio (i.e., the ratio of mean particle size  
305 of dominant, to less dominant soil component). Metastable granular materials can generally adopt two  
306 random structures: The Random Loose Packing (*RLP*) and the Random Close Packing (*RCP*). For granular  
307 materials containing monodisperse spheres, the *RLP* quality occurs at void ratios above 0.6. The *RCP*  
308 quality occurs at global void ratios between 0.5 and 0.6 (Xu et al. 2015). Void ratios lower than 0.5 represent  
309 Close Packing (*CP*) quality that occurs in most natural soils. In Table 2, for Sand A, irrespective of fibre  
310 content, consolidated soils adopted an *RLP* state (with a packing density of around 0.539) for 20% fine  
311 fraction (*FC*). As the fine fraction increases to 40%, the packing state changes to *RCP* (with a packing  
312 density of around 0.690). With further increase in fine fraction to values above 40%, soil adopts a close  
313 packing state (i.e. dense state). In this and for Sand A, the 40% fine content marks a critical transition point  
314 from a metastable random packing to a stable close packing (*CP*). For Sand B, the metastable *RLP* state  
315 changes to stable *CP* at the slightly lower 30% fine fraction. The threshold silt content appeared in the form  
316 of a dip in the minimum global void ratio ( $e_{min}$ ) plot against silt content in Figure 5 for Sand A. The two  
317 extreme void ratio values ( $e_{max}$  and  $e_{min}$ ) are effectively density related indices that limit them physically  
318 attainable void ratios in binary granular materials.

319 **Figure 5.** Minimum and maximum void ratios for sand-silt composite mixes

320 To better understand the links between the packing state and fines content, the pore phase is idealised as  
321 micro- and macro-pore spaces. The testing material here ranges from pure sand to sandy silt; As such, only

322 trans-assemblage (macro) and inter-assemblage (micro) void spaces are likely to form the soils' void  
 323 structure. Equations 1 to 4 formulate the void ratio at the macro ( $e_s$ ) and micro ( $e_f$ ) scale using formulations  
 324 proposed in Thevanavagain and Mohan (2000), and Thevanayagam (1998). The method is also discussed  
 325 in Bouckovalas et al. (2003), Thevanayagam and Martin (2002), Xenaki and Athanasopoulos (2003), and  
 326 Yang et al. (2006).

$$327 \quad e_s = \frac{e_c + (1 - b_i)FC}{1 - (1 - b_i)FC} \quad (1)$$

$$328 \quad e_f = \frac{e_c}{FC + \frac{(1 - FC)}{R_d^m}} \quad (2)$$

329 where  $e_c$  is the global void ratio at the end of the triaxial consolidation stage,  $FC$  is the fines content,  $m$  is  
 330 the improvement factor that ranges from 0 to 1, and the parameters  $R_d$  (the size disparity) and  $b_i$  are defined  
 331 as follows.

$$332 \quad R_d = \frac{D_{50(sand)}}{D_{50(silt)}} \quad (3)$$

$$333 \quad b_i = \left(1 - e_c^{-\left[\frac{f^n}{1-r^{0.25}}\right]}\right) \left(r \frac{FC}{FC_t}\right)^r \quad (4)$$

$$334 \quad m = \log\left(\frac{111 - 91b_i}{11 + 199b_i}\right) \quad (5)$$

335 where  $D_{50(sand)}$  and  $D_{50(silt)}$  are the mean particle sizes of the sand and silt fractions, respectively;  $FC_t$  is  
 336 the threshold fines content,  $f$  is the loading frequency, parameter  $n$  governs the initial rate of increase in  $b_i$   
 337 with increasing  $FC$ , thereby  $n = \partial b_i / \partial FC$ , and  $r = D_{50(silt)} / D_{10(sand)}$ , where  $D_{50(silt)}$  and  $D_{10(sand)}$  are  
 338 the mean and effective particle sizes for the silt and sand fractions, respectively. Note parameter  $b_i$   
 339 represents the relative effectiveness of the soil's fine fraction in carrying interparticle stresses (i.e., the  
 340 contribution of separating fines to active contacts,  $(0 < b_i < 1)$ ). The  $b_i$  magnitude directly proportional to  
 341 the fines content.

342 Figure 6 illustrates the variation with silt content of trans-assembly and inter-assembly void ratios for  
343 Sand A. Immediate observations suggest that silt content is inversely proportional with the inter-assembly  
344 pore void ratio ( $e_f$ ) and directly proportional to the trans-assembly void ratio ( $e_s$ ). In other words, for a  
345 silty sand soil and for silt contents lower than the threshold, an increase in silt content leads to a substantial  
346 decrease in micro-scale void ratio. This indicates the tendency of silt to coagulate into silt globules, a  
347 consequent decrease in micro-scale pore size and increase in matric suction. Fibre reinforcement generally  
348 decreases the global void ratio (Fig. 6a) but has little effect on idealised void ratios.

349 The idealisation of void spaces was recently explored for binary sand-clay mixtures in Ghadr and Assadi-  
350 Langroudi (2018). They discussed the fractal pore phases in the context two conceptual *Small Clay* and  
351 *Large Clay* models and through relating sand particle shape, sorting, and mean size to soil packing state  
352 and hydromechanical properties. Building on those findings, two conceptual models are proposed for the  
353 sand-silt mixtures: *Small Silt* and *Large Silt*. At low silt contents (fine fraction below 40% - here termed as  
354 *Small Silt*), the variation of silt content has little effect on trans-assembly and substantial effect on inter-  
355 assembly pore spaces (Fig. 6b). On addition of silt, trans-assembly void spaces marginally expand in  
356 volume. Silt initially sits at sand-sand contact trapdoors and form silt aggregates, pushing sand grains apart  
357 and expanding the trans-assembly void space: Initially, silt assemblies are loose and honeycomb in  
358 shape. As silt content increase, additional silt particles rest in the intra-lattice space within the honeycomb  
359 aggregates. This leads to the gradual transformation of the loose silt pellets to densely packed, tightly  
360 interlocked silt coagulates. This explains the inverse relationship between silt content and inter-assembly  
361 void ratio (for *Small Silt*). For *Large Silt* systems (silt fraction above 40%), the variation of silt fraction has  
362 little effect on inter-assembly pore spaces and substantial effect on trans-assembly pore spaces. As silt  
363 content increase, densely packed silt coagulates expand into and occupy the trans-assembly pore phase.  
364 With an exception of soils with  $FC < 10\%$ , fibres appear to have almost no influence on post-consolidation  
365 inter- and the trans-assembly void ratio at high silt contents. In determination of void ratios at the end of  
366 consolidation (that account for densification during saturation and consolidation), fibres were treated as

367 being part of the solid in calculations. This is a consistent approach with the recent similar experimental  
368 attempts (e.g. Zhang and Russell 2020).

369

370 **Figure 6.** (a) Global void ratios for sand-silt-fibre mixtures [Sand A] (b) intergranular and inter-fine void  
371 ratios for sand-silt-fibre mixtures [Sand A]

#### 372 **4. Results and Observations**

373 A total of 41 cyclic triaxial experiments were conducted in this research in compliance with ASTM D5311  
374 standard. The excess pore water pressure ratio ( $r_u$ ), axial strain ( $\epsilon_c$ ) and axial stress ( $\sigma_d$ ) were measured at  
375 regular periodic intervals during the course of cyclic sinusoidal waveform load application. The load was  
376 retained on specimens until the initial liquefaction when the  $r_u$  became equal to 1.0. Findings from a typical  
377 cyclic triaxial experiment conducted on A(0):0 (see Table 2) are presented in Fig. 7a-e. The development  
378 of strains with growing cycles is not uniform. This is clear in Fig. 7a, where the greatest strain increments  
379 appear to have taken place with a distance from the core of stress-strain loops. This suggests that  
380 liquefaction failure in unreinforced granular soils is a sudden phenomenon with not much early warning in  
381 the form of ground movements. In Fig. 7a and for the initial cycles, the hysteresis loops adopt an ellipse  
382 shape with narrow width and sharp corners that mark sharp load reversal events. This implies a viscoelastic  
383 behaviour. The hysteresis loops become wider and adopt a parallelogram shape with further  
384 loading/unloading cycles. The variation of  $\epsilon_c$  and cyclic deviator stress is minimal during the early stages  
385 of cyclic loading (Fig. 7d). The excess pore water pressure however gradually and constantly grows with  
386 an increasing number of loading cycles. The  $r_u$  sharply increases from 78% to 93% and the axial strain  
387 begins to rise at the 16<sup>th</sup> cycle of loading. On the 18<sup>th</sup> cycle, the  $r_u$  tends to nearly 1.0 (Fig. 7b) and the axial  
388 strain gains a double amplitude axial strain of about 5.5%, indicating a complete state of liquefaction (Fig.  
389 7c). At this stage, the stress state is likely to have touched the failure envelope.

390



391 **Figure 7.** (a) stress-strain relationship for A(0):0, Time histories of (b) excess pore water pressure ratio, (c)  
392 Axial strain, (d) deviator stress, (e) Effective stress path for test specimen 1 (pure sand)

393

#### 394 ***4.1 Effect of fine fraction and underlying micro-mechanisms***

395 The core objective of this study is to gain a better understanding of the impact of non-plastic fines and fibres  
396 on dynamic properties of loose saturated sands. The excess pore water pressure ratio ( $r_u$ ) and the number  
397 of cycles required to cause initial liquefaction ( $N_l$ ) are adopted as benchmark parameters that quantify the  
398 liquefaction potential. The formation and growth of excess pore water pressure under undrained loading  
399 conditions is well-established as the fundamental mechanism related to liquefaction. Initial liquefaction  
400 happens when excess pore water pressure ratio ( $r_u$ ) reaches 1. The general  $r_u - N$  trends for two sands were  
401 found to be similar. For Sand A, the excess pore water pressure ratio is plotted against the number of loading  
402 cycles in Fig. 8 for  $CSR = 0.15$  and unreinforced silty sand specimens.

403 **Figure 8.** Pore pressure response against loading cycles for unreinforced silty sands [Sand A] and  $CSR$   
404  $= 0.15$

405 Initial inspection of Fig. 8 suggests that the rate of pore pressure generation in the sand–silt mixtures are  
406 generally higher than that of clean sand. It is clear that the development of  $r_u$  with loading, cycles are  
407 dependent on the fines content, with two explicitly different patterns seen for fines content lower and higher  
408 than a critical threshold ( $FC_t$ ). The fundamentals of this critical fines content threshold were earlier  
409 discussed in Fig. 1 and Fig. 6.

410 A combination of contact destruction and contact reorientation events lead to the progressive weakening of  
411 the contact force network until it becomes insufficient to sustain any further shearing at which point soil  
412 experiences large deformations and liquefies. Xu et al. (2015) recently highlighted the dominant role of  
413 inter-particle friction as the governing micro-parameter for liquefaction resistance. These fundamental

414 concepts are discussed here in the context of the conceptual *Small-Silt Large-Silt* model, a graphical  
415 illustration of which is presented in Fig. 9.

416 **Figure 9.** Conceptual idealised void spaces model for sand-silt mixtures: transition of packing quality with  
417 increasing fines content

418 For fines content lower than  $FC_t$  (*Small Silt*), loose honeycomb silt pellets appear at sand-sand contact  
419 points. In Table 2, the average coordination number for Small Silt soils is generally low, leading to a greater  
420 likelihood of carrying the higher interparticle stresses (McDowell and Bolton 2000). Here, the coordination  
421 number ( $CN$ ) is a scalar fabric quantity and a measure of the number of contacts per particle (Thornton  
422 2000; Fonseca et al. 2013) used to link the macroscopic behaviour to the changes in the microstructure. The  
423 parameter was first formulated by Smith et al. (1929) as a function of void ratio. Ode 1977 and Nolan and  
424 Kavanagh 1992 presented extensive experimental data to correlate the coordination number with the void  
425 ratio in two- and multi-mixed particulate assemblies. Assadi-Langroudi et al. 2018 showed that the Smith  
426 and colleagues' equation is in excellent agreement with the experimental equations of Ode 1977, once the  
427 void ratio is set to the idealised values. Increasing silt content up to  $FC_t$  leads to the densification of silt  
428 pellets with little impact on trans-assemblage void spaces. This has a number of implications. Silt pellets  
429 interfere with the interlocking of sand particles at sharp contact points, leading to a decrease in inter-particle  
430 friction. Silt pellets act as lubricating agents and facilitate displacement of sand grains with increasing  
431 loading cycles. In other words, stronger pellets function as hinge elements with trapdoor effect, allowing  
432 sand particles to roll over one another. Initially, contact destruction is limited mainly due to the isotropic  
433 consolidation. Contact destruction then rapidly gains momentum with increasing cycles. Increasing fines  
434 content forms more competent silt pellets which further facilitates rotation of sand particles within the trans-  
435 assemblage void phase, their clashing and destruction. This has reflected in the faster growth of  $r_u$  at higher  
436 fines content in Fig. 8 for *Small Silt* soils. For silty sands with fines content greater than  $FC_t$  (*Large Silt*),  
437 the excess fines expand into the trans-assemblage void space, increase the coordination number (see Table  
438 2) and decrease the average skeletal force magnitude. The excess fines at macro-voids act as cushions,

439 damp the energy and provide a degree of additional support to sand particles. Increasing silt content beyond  
440  $FC_t$  leads to a decrease in contact destruction. This has reflected in the slower growth of  $r_u$  at higher fines  
441 content in Fig. 8 for *Large Silt* soils. Soil develops a greater potential of liquefaction with increasing fines  
442 content to a limiting  $FC_t = 40\%$  (common in both sands as seen from the  
443  $N_l - FC$  pattern in Table 2). Any further increase in fines content would decrease the liquefaction potential.

#### 444 **4.2 Effect of fibre inclusion and underlying micro-mechanisms**

445 For both sands, the number of cycles to liquefaction ( $N_l$ ) increase with increasing fibre content. For clean  
446 Sand A, the  $N_l$  increased approximately 2.4 times from 19 in A(0):0 to 46 in A(0):1. The increase in  $N_l$  for  
447 Sand, A is remarkably greater than in clean Sand B, where the  $N_l$  increased approximately 1.2 times from  
448 62 in B(0):0 to 80 in B(0):1. Whilst this may imply the greater efficiency of fibre reinforcement in sands of  
449 relatively lower median size, it is important to take note of the substantially lower [potential of clean coarse  
450 Sand B as compared to Sand A.

451 Figure 10 plots the evolution of  $r_u$  with the number of loading cycles towards initial liquefaction for  
452 isotopically consolidated fibre-reinforced sand mixtures (Sand A) at a confining pressure of 200 kPa.  
453 Important observations are made here. First, the addition of fibres seems to have decreased the  $r_u$ . This is  
454 a welcomed phenomenon and in agreement with earlier findings (e.g. Ye et al. 2017), where the rate of  
455 increase of the excess pore water pressure was reported to become lower with the increment of fibre  
456 contents. Whilst  $r_u$  gains value with increasing loading cycles, rapid sawtooth fluctuations in the  $r_u - N$   
457 sinusoidal curves reflect rapid contact reorientation events which theoretically last until particles attain a  
458 more stable state. Relatively stronger fluctuations can be seen in soils with lower fibre content at any  $N$   
459 cycle. This lends evidence to the encrusting role of fibres as they brace the trans-assemblage pore spaces  
460 and arrest contact reorientation. It is also evident that with the application of constant cyclic stress,  $r_u$  only  
461 increases after a certain number of cycles (let this be  $N_1$ ) before increasing dramatically towards the first

462 liquefaction event. The  $N_1$  is directly correlated with fibre content. This infers that unlike unreinforced  
463 sands, liquefaction in reinforced sands is not sudden and can easier be detected at early stages.

464 **Figure 10.** Effect of fibre content on the progress of excess pore water pressures (a) Clean sand (b) F161  
465 Sand A + 10% silt (c) Sand A + 20% silt (d) Sand A + 30% silt (e) Sand A + 40% silt (f) Sand A + 50%  
466 silt (for a constant CSR= 0.15).

#### 467 *Small Silt*

468 In Fig. 11 and for mixtures with  $FC = 20\%$  (*Small Silt* – Sand A), the global, micro- and macro-scale void  
469 ratio only marginally vary with increasing fibre content. For 0% to 1.5% fibre content,  $e_f = 1.48 \pm 0.01$ ,  
470  $e_s = 0.86 \pm 0.01$  and the packing adopts a global void ratio of  $e = 0.58 \pm 0.01$ . For fines content lower  
471 than the threshold  $FC_t$ , the coordination number and the liquefaction resistance increase with increasing  
472 fibre content. This is evident in Table 2 and from the inverse relationship between  $N_l$  and fibre content in  
473 specimen N° 3, 9, 15 and 21. Here,  $N_l$  sees an increase from 11 to 34 with increasing fibre content (to 1.5%).  
474 Similar trends for Sand B (specimen N° 27, 33, 38) followed, although as in the case of clean sands, the  
475 increase in  $N_l$  was found to be moderate. One should also note that unlike in Sand A, the global and idealised  
476 void ratios exhibit a more marked decrease on fibre addition to Sand B mixed with 20% silt content  
477 (representing *Small Silt*). In Fig. 11 and for mixtures with  $FC = FC_t = 40\%$  (limiting or threshold fines  
478 content) and for 0% to 1.5% fibre content, the global void ratio varies only marginally and adopts a mean  
479  $e = 0.45 \pm 0.01$  value. At that constant global void ratio, increasing fibre content leads to a modest 2.8%  
480 decrease in macro-scale void ratio from  $e_s = 1.086$  to  $e_s = 1.055$  and a more substantial 4.0% decrease in  
481 micro-scale void ratio from  $e_f = 0.846$  to  $e_f = 0.812$ . This suggests that fibres tend to sit deep into silt  
482 pellets and encrust the trans-assembly void spaces. Increasing fibre content leads to an increase in the  
483 average coordination number (see Table 2), a decrease in contact forces, a decrease in contact destruction  
484 and consequently enhanced liquefaction resistance. The encrusting fibres also generate some degree of  
485 dilation which then leads to a decrease in excess pore water pressure ratio (see Fig. 10). Lower liquefaction

486 potential is evident for Sand A in Table 2 and from the inverse relationship between  $N_l$  and fibre content in  
487 specimen N° 5, 11, 17 and 23. Here, the  $N_l$  sees an increase from 3 to 13 with increasing fibre content.  
488 Similar to Sand A, the  $N_l$  increased with increasing fibre content in Sand B specimens mixed with the  
489 threshold  $FC_t = 40\%$  silt content.

490 Figure 12 illustrates the first and second stress-strain hysteresis loops for A(0):0, A(20):0 and A(20):1.5  
491 (Fig. 12a) and B(0):0, B(20):0 and B(20):1.0 (Fig. 12b) for *Small Silt* specimens. In all classes, silt content  
492 is 20% which brings the soils into the *Small Silt* category specimens. Larger strain increments in Sand B  
493 specimens is consistent with generally greater  $N_l$  (i.e. number of cycles to liquefaction) and hence flow  
494 resistance in sands with coarser grain size. This is mainly attributed to the higher gravitational forces at the  
495 particle level and a greater degree of direct sand-to-sand interlocking. Larger strains also imply a generally  
496 less sudden and more gradual increase in axial strains in sands with coarser grain size. Reinforcement of  
497 Sand B with fibre continue to increase the axial strain for each loading cycle as in Sand A, but to a more  
498 moderate level. This is in conjunction with the trend of  $N_l$  with fibre content in Table 2 suggest a lesser  
499 efficiency of fibres for sands of larger median diameter.

#### 500 ***The rigid wall boundary effect***

501 An interesting observation here is the peculiar trends of global and idealised void ratio variation with fibre  
502 content in fibre reinforced Sand B specimens. At threshold fine content, both types of void ratio increased  
503 with increasing fibre content from 0.5 to 1.0% (see Specimen N° 35, 40). Among the idealised void ratios,  
504 the increase is more pronounced at the micro-pore level (i.e.  $e_f$ ). For fibre-reinforced coarse sand at  
505 threshold silt content, this implies disturbance of dense silt pellets upon further addition of fibres, which  
506 then probably has led to the migration of liberated fines to new accumulation sites at fibre-to-sand and fibre-  
507 to-fibre contact points. This is quite speculative, and confirmation of such events does need advanced  
508 imaging, which in the case of non-cohesive sands is technically very hard. Similar trends were observed  
509 for Sand B at  $FC = 50\%$  (*Large Silt* system). It appears that the presence of fibres in the coarser Sand B  
510 has a substantial impact on the porosity in their vicinity. Events here may relate to the ‘rigid wall boundary

511 effect', in which particulate matters (here large sand particles) near a rigid wall boundary (here fibres)  
512 develop higher local porosity. The radius of this 'perturbed zone' where fresh pellets of silt form is  
513 estimated between 10% (De Larrad 2014) to 100% (Soriano et al. 2017) of average sand size in fibre-  
514 reinforced sand systems, whilst this radius theoretically can be as large as 4 to 5 times average particle size  
515 diameter (Suzuki et al. 2008). Observations here demand further investigation. Given the small size of the  
516 dataset here, findings cautiously suggest that the efficiency of fibre reinforcement in *Large Silt* system may  
517 be compromised when the median size of sand increase beyond a certain critical value. Identification of  
518 that critical sand size is significantly important but remains out of scopes of this study.

519

## 520 *Large Silt*

521 In Fig. 11 and for mixtures with  $FC = 50\%$  (*Large Silt*), and for 0% to 1.5% fibre content, the global void  
522 ratio varies only marginally with a mean  $e = 0.43 \pm 0.01$  value. At that constant global void ratio,  
523 increasing fibre content leads to a modest 3.2% decrease in macro-scale void ratio from  $e_s = 1.307$  to  $e_s =$   
524  $1.275$  and a more substantial 4.2% decrease in micro-scale void ratio from  $e_f = 0.713$  to  $e_f = 0.683$ .  
525 Similar underlying mechanisms to the  $FC = FC_t$  the case applies here. The direct correlation between the  
526 liquefaction resistance and fibre content ties in with the inverse relationship between  $N_l$  and fibre content  
527 in Table 2.

528 **Figure 11.** Variation of void ratio with the number of loading cycles to liquefaction (a) global void ratio  
529 (b) skeletal void ratio (c) Fine void ratio

530 **Figure 12.** The stress-strain relationship for Sand A and B at FC=0% and FC=20% in unreinforced and  
531 reinforced forms (a) Sand A, (b) Sand B

## 532 *Shear modulus*

533 The secant shear modulus ( $G$ ) is determined in compliance with methods proposed by Kokusho 1980 and  
534 through analysing the second hysteresis loop curves. In reflection of the cycle number, the shear modulus  
535 is presented here with  $G_2$  and plotted against fines content in Fig. 13.

536 **Figure 13.** Effect of fines and fibre content on shear modulus

537 Similar to the  $r_u$ , the shear modulus ( $G_2$ ) decreases with increasing fines content for *Small Silt* soils, which  
538 can be attributed to the contractive behaviour of silty sands (Fig. 9). Similar findings were reported for  
539 clayey sands in Sadeghi and Beigi 2014 and  $\chi$ (= 0, 0.5, 1%). To bring this observation in numbers, for  
540 Sand A, the percentage increase in the 2<sup>nd</sup> cycle shear modulus corresponding to 0.5%, 1% and 1.5% fibre  
541 content amounted to 11%, 28%, and 46% respectively. For Sand A mixed with 30% fines content (*Small*  
542 *Silt*), the increase in  $G_2$  corresponding to 0.5%, 1% and 1.5% fibre content amounted to 537%, 881%, and  
543 1405% respectively. Findings here are consistent with earlier findings in Maher and Woods (1990) and  
544 Noorzad and Amini (2014) for fibre-reinforced cohesionless soils.

545

## 546 **5. Conclusions**

547 Key findings are summarised below.

548 1. For both reinforced and base soils, the initial Random Loose Packing (RLP) state change to a  
549 Random Close Packing (RCP) as silt content is increased beyond 20%. The metastable random  
550 packing changes to a stable Close Packing (CP) state as silt content in further increased beyond  
551 40%. The latter transition accounts for the transition from *Small Silt* to *Large Silt* system and takes  
552 place at a lower threshold silt content in coarser sands.

553 2. In a silty sand system and for silt contents lower than the threshold (*Small Silt* system), an increase  
554 in silt content leads to a substantial decrease in micro-scale void ratio. Loose silt pellets gradually  
555 transform into densely packed, tightly interlocked silt coagulates. Formation of silt coagulates leads

556 to a gradual expansion of sand structure and the macro-pore space volume. The consequent  
557 decrease in micro-scale pore size leads to a gradual increase in the matric suction.

558 With an exception of soils with  $FC < 10\%$ , fibre reinforcement generally decreases the global void  
559 ratio but has little effect on idealised void ratios and hence water retention capacity of the soil  
560 system.

561 3. Variation of global and idealised void ratios with fibre content in fibre reinforced sandy specimens  
562 and when the sand of a larger median diameter was used follows peculiar trends that signify the  
563 importance of sand size. Findings cautiously suggest that the efficiency of fibre reinforcement in  
564 *Large Silt* system may be compromised when the median size of sand increases beyond a certain  
565 critical value. Identification of that critical sand size is important but remains out of scopes of this  
566 study.

567 4. Liquefaction failure takes place following a combination of contact destruction and contact  
568 reorientation events that lead to the progressive weakening of the contact force network, until  
569 interlocking at particle-to-particle contact points become insufficient to sustain any further shearing  
570 at which point soil experiences large deformations, flow and liquefaction. In unreinforced silty  
571 sands, failure is sudden with not much early warning in the form of ground movements.

572 5. The impact of fines content on the  $r_u - N$  interplay is explicitly different between *Small Silt* and  
573 *Large Silt* systems. A micro-to-macro approach is adopted here.

574 As silt pellets gain strength, the interlocking between sand particles and hence inter-particle friction  
575 decreases. Silt pellets facilitate displacement of sand grains and hence contact destruction with  
576 increasing loading cycles. At macro-scale, this leads to faster growth of  $r_u$  with  $N$  at rates greater  
577 than those in clean sand systems. On further addition of silt to silty sand and as silt content increase  
578 beyond the threshold, silt pellets expand to macro-void phase to increase the coordination number,  
579 damping, and overall resistance of sand grains against contact destruction. At macro-scale, this  
580 leads to slower growth of  $r_u$  with  $N$  with increasing silt content.



581 The mechanisms discussed here can be extended to any natural hazard that involves in a sudden  
582 flow of muddy waters into sandy grounds, riverbanks and hillslopes, and explains how the interplay  
583 between independent events such as earthquake, debris flow and liquefaction can cause interacting  
584 problems.

585 6. Irrespective of fine content, fibres tend to sit deep into the silt pellets and encrust the trans-  
586 assemblage void spaces. At the micro-scale, footprints of the hypothesised encrusting (bracing)  
587 function of fibres at trans-assemblage pores were found on the  $r_u - N$  envelope and in the form of  
588 rapid sawtooth fluctuations that mark rapid contact reorientation events. Increasing fibre content  
589 leads to an increase in the average coordination number, dilation and easier dissipation of excess  
590 pore water pressure, a decrease in contact forces, a decrease in contact destruction and consequently  
591 enhanced liquefaction resistance.

592

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