ANALYSIS OF INDETERMINATE STRUCTURES
BY COMBINING REDUNDANTS

by

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SYNOPSIS

In the analysis of highly indeterminate structures, the task of setting up and solving the elastic equations becomes time-consuming. Several methods have been developed to reduce the work considerably. Analysis by combining redundants is one of them. Each method has its own advantages. Two illustrative examples are solved to compare the combined redundant method with the method of consistent deformations.

INTRODUCTION

In the study of indeterminate structural analysis the usual procedure is to solve simultaneous equations obtained by the method of consistent deformations. The number of equations involved is equal to the degree of indeterminancy of the structure. For high degrees of indeterminancy, the solution of the simultaneous equations becomes a tedious task. The combined redundant method eliminates the time-consuming procedure of solving these simultaneous equations. The purpose of this study was to become familiar with this method of analysis and its applications to structural analysis problems. To illustrate this method, three numerical examples are solved. First, a truss with three degrees

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of indeterminancy (7), second, a flexural bent with three degrees of indeterminancy, and third, a truss, indeterminate to the eighth degree. To compare this method with another method, problems 2 and 3 are also solved by the method of consistent deformations.

The usual procedure for the analysis of indeterminate structures is to select a statically determinate structure by removal of redundant forces. This is done by cutting through the members, inserting hinges and removing reaction components. The number of these redundant forces is always n for an n-times indeterminate case. A set of simultaneous equations results:

$$\sum \delta_{ij} X_j + \delta_{io} = 0$$

where X_j are the redundants. The subscript i and j are integers from 1 to n. The terms δ_{ij} are coefficients depending only on the size and shape of the original structure and the terms δ_{io} are the loading terms which depend on geometry and on the load (4).

The primary purpose of the method of combining redundants is to get a system of linear equations which can very easily be solved. This involves the determination of the coefficients of the "combined redundants". After these coefficients have been found, member stresses corresponding to the given condition of loading or distortion may be evaluated. These coefficients are independent of the type of loading a structure has to bear (10).

OUTLINE OF THE METHOD

The 'm' times statically indeterminate structure is first reduced to a statically determinate form by removing a number of redundants equal to the degree of indeterminancy. The system of simultaneous equations thus obtained is:

$$
\begin{aligned}
&x_1 \delta_{11} + x_2 \delta_{12} + \cdots\cdots + x_m \delta_{1m} - \Delta_1 = 0 \\
&x_1 \delta_{21} + x_2 \delta_{22} + \cdots\cdots + x_m \delta_{2m} - \Delta_2 = 0 \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \qquad \text{Eq.1.} \\
&x_1 \delta_{m1} + x_2 \delta_{m2} + \cdots\cdots + x_m \delta_{mm} - \Delta_m = 0
\end{aligned}
$$

The proposed method, the orthogonalization of the above equations, gives the following form.

$$
\begin{aligned}
&x_1' \delta_{11}' && - \Delta_1' = 0 \\
&\quad x_2' \delta_{22}' && - \Delta_2' = 0 \quad \cdots\text{Eq. 2.} \\
&\qquad \ddots \; x_m' \delta_{mm}' && - \Delta_m' = 0
\end{aligned}
$$

To establish this pattern of equations, the principle of virtual work will be used. It states that during any virtual displacement of an elastic body the net work done by all the forces is zero. The internal andexternal forces must be in equilibrium and the virtual displacement must be small and compatible with the condition of the constraints. (12).

For example, assume a truss which is three times statically indeterminate Fig.1(a). If the final stress in a member is denoted by S, then by the principle of superposition

$$S = x_1 s_1 + x_2 s_2 + x_3 s_3 + S_0 \qquad \cdots\cdots (3)$$

where s_1, s_2, s_3 are the bar stresses due to unit value of redundants x_1, x_2, x_3 and S_o is the bar stress due to external loading on the statically determinate structure. If the constant term L/AE for each bar is represented by α , then the corresponding elongation is given by

$$\Delta_L = \alpha S = \alpha(x_1 s_1 + x_2 s_2 + x_3 s_3 + S_0) \qquad \cdots\cdots (4)$$

Considering the case of Fig. 1(c) where a unit load is applied at point H, the bar stress s_1 results. The system will be in equilibrium due to this force. If a small displacement is given to the truss, it causes a virtual displacement in all the members of the truss. By the principle of virtual work, the net work done by all the forces due to this virtual displacement is zero. Internal and external forces must be in equilibrium and the virtual displacement must be small and compatible with the condition of the constraints (12). The following equation may be written:

$$\sum \alpha S s_1 = 0 = x_1 \sum \alpha s_1^2 + x_2 \sum \alpha s_1 s_2 + x_3 \sum \alpha s_1 s_3 + \sum \alpha s_1 S_0 \quad \text{.....(A)}$$

This equation contains three unknown redundant forces, x_1, x_2, x_3. Applying the redundant forces as shown in Fig. 1(d) and Fig. 1(e) the bar stresses s_2 and s_3 will result. Two similar equations could be obtained as given below.

$$\sum \alpha S s_2 = 0 = x_1 \sum \alpha s_1 s_2 + x_2 \sum \alpha s_2^2 + x_3 \sum \alpha s_2 s_3 + \sum \alpha s_2 S_0 \quad \text{.....(B)}$$

and

$$\sum \alpha S s_3 = 0 = x_1 \sum \alpha s_3 s_1 + x_2 \sum \alpha s_3 s_2 + x_3 \sum \alpha s_3^2 + \sum \alpha s_3 S_0 \quad \text{......(C)}$$

This summation extends over all the members of the truss. The above three equations are set up in the form given in equation (1), and can be solved simultaneously for the unknown redundant forces x_1, x_2, x_3. In the combined redundant method, the redundants chosen from Fig. 1(c) to Fig. 1(e) are combined with each other in such a manner that the quantity $\sum \alpha s_i s_j$ becomes zero, where $i \neq j$. Thus reducing the system of equations to the form given in equation (2). This method is called the combined unit redundants method as the unit redundants are combined with each other to obtain the form of equation (2). It is to be noted that only one of the combined redundants will have a unit value. The bar stresses due to 'n' combined redundants will be denoted by S_n. The first combined redundant will be equal to one; Fig. 1(c) and corresponding bar stresses are given by

$$S_1 = s_1 \qquad \text{. . . 5(a)}$$

The second combination of redundants from Fig. 1(c) and Fig. 1(d) is made in such a way that bar stresses due to these forces are

$$S_2 = C_{21} S_1 + s_2 \qquad \text{. . . 5(b)}$$

The third combination of Fig. 1(c), (d), (e) such that bar stresses are

$$S_3 = C_{31} S_1 + C_{32} S_2 + s_3 \qquad \text{. . . 5(c)}$$

The coefficients C_{21}, C_{31}, and C_{32} are constants, yet to be determined (12).

With these new bar stresses S_1, S_2, and S_3, the values of the redundant forces X_1, X_2 and X_3 can be obtained and by the principle of superposition the final stresses are given by

$$S = S_1 X_1 + S_2 X_2 + S_3 X_3 + S_0 \qquad \text{. . . (6)}$$

It is worth noting that the redundant forces X_1, X_2 and X_3 are quite different from x_1, x_2 and x_3. The corresponding elongation is given by

$$\Delta_L = \alpha S = \alpha (X_1 S_1 + X_2 S_2 + X_3 S_3 + S_0)$$

Applying the virtual work principle, the set of equations similar to equations (A), (B), and (C) can be written as

$$\Sigma \alpha S S_1 = 0 = X_1 \Sigma \alpha S_1^2 + X_2 \Sigma \alpha S_1 S_2 + X_3 \Sigma \alpha S_1 S_3 + \Sigma \alpha S_1 S_0 \quad \text{------} (A_1)$$

$$\Sigma \alpha S S_2 = 0 = X_1 \Sigma \alpha S_2 S_1 + X_2 \Sigma \alpha S_2^2 + X_3 \Sigma \alpha S_2 S_3 + \Sigma \alpha S_2 S_0 \quad \text{..} (B_1)$$

$$\Sigma \alpha S S_3 = 0 = X_1 \Sigma \alpha S_3 S_1 + X_2 \Sigma \alpha S_3 S_2 + X_3 \Sigma \alpha S_3^2 + \Sigma \alpha S_3 S_0 \quad \text{--} (C_1)$$

This summation extends over all the members of the truss. Actually, as per the scheme, one needs to establish a condition such as that

$$\Sigma \alpha S_1 S_2 = 0 \quad \text{.............} \; 7(a)$$

$$\Sigma \alpha S_1 S_3 = 0 \quad \text{.............} \; 7(b)$$

$$\Sigma \alpha S_2 S_3 = 0 \quad \text{.............} \; 7(c)$$

This can be done by suitably choosing the constant terms of equations (5). We have from equation 7 (a) $\Sigma \alpha S_1 S_2 = 0$
Substituting the value of S_2 from equation 5 (b)

$$\Sigma \alpha S_1 (C_{21} S_1 + S_2) = 0$$

or $C_{21} \Sigma \alpha S_1^2 + \Sigma \alpha S_1 S_2 = 0$

$$\therefore C_{21} = - \frac{\Sigma \alpha S_1 S_2}{\Sigma \alpha S_1^2} \quad \text{.............} \; 8(a)$$

Similarly, substituting the value of S_3 from equation 5 (c) in equation 7 (b)

$$\Sigma \alpha S_1 (C_{31} S_1 + C_{32} S_2 + S_3) = 0$$

or $C_{31} \Sigma \alpha S_1^2 + C_{32} \Sigma \alpha S_1 S_2 + \Sigma \alpha S_1 S_3 = 0$

but $\Sigma \alpha S_1 S_2 = 0$ from equation 7(a)

$$\therefore C_{31} = - \frac{\Sigma \alpha S_1 S_3}{\Sigma \alpha S_1^2} \quad \text{........} \; 8(b)$$

and finally, substituting the value of S_3 in equation 7(c)

$$\Sigma \alpha S_2 (C_{31} S_1 + C_{32} S_2 + s_3) = 0$$

or $C_{31} \Sigma \alpha S_2 S_1 + C_{32} \Sigma \alpha S_2^2 + \Sigma \alpha S_2 s_3 = 0$

but $\Sigma \alpha S_2 S_1 = 0$ from equation 7(a)

$$\therefore C_{32} = - \frac{\Sigma \alpha S_2 s_3}{\Sigma \alpha S_2^2} \quad . . . 8(c)$$

In order to compute the value of constant C_{ij}, first one needs to know the bar stresses s_1, s_2 and s_3 due to a unit value of redundants applied as shown in Fig. 1(c), (d), and (e). The first combined redundant is chosen equal to one and thus the bar stress $S_1 = s_1$. With this value of S_1 known, the coefficient C_{21} can very easily be obtained from the equation 8(a). Once the value of coefficient C_{21} is known, the magnitude of S_2 can very easily be found from the equation 5(b) and thus the coefficients C_{31} and C_{32} can be found from equation 8(b) and 8(c) and therefore the magnitude of S_3 may be computed. Once the coefficients C_{ij} are known, the bar stresses (due to combined redundants) S_1, S_2 and S_3 can be found from equation (5) with thecombined redundants as shown in Fig. 2. The bar stresses produced are S_1, S_2 and S_3. With these values of bar stresses, the equations (A_1), (B_1) and (C_1) reduce to

$$X_1 \Sigma \alpha S_1^2 + \Sigma \alpha S_1 S_0 = 0 \quad . . . . . 9(a)$$

$$X_2 \Sigma \alpha S_2^2 + \Sigma \alpha S_2 S_0 = 0 \quad . . . . . 9(b)$$

$$X_3 \Sigma \alpha S_3^2 + \Sigma \alpha S_3 S_0 = 0 \quad . . . . . 9(c)$$

The unknown X_1, X_2 and X_3, thus obtained, will give final bar stresses by substitution into equation (6). It is worth noting that the constants C_{ij} and the bar stresses S_n are independent of the type of loading, that is, with a different type of loading, one needs to calculate only equation (6) and (9). The remaining values are unaltered. (12)

GENERAL THEORY

To generalize the above methods, let there be a "m" times statically indeterminate structure. Let the internal stresses produced due to redundants $x_1, x_2 \ldots x_m$ be $s_1, s_2 \ldots s_m$. The structure is made statically determinate by removing the redundant forces. In order that no external work will be done by interior redundants, they are cut or hinged so that equal and opposite forces or moments can be applied at points infinitely close together. Therefore, the only external work that can be done by the combined unit redundant system during its virtual displacement is due to yielding of supports. (12)

As shown in equation (5), the internal stresses $S_1, S_2 \ldots S_m$ produced by combining unit redundants can be put in the following pattern:

$$
\begin{aligned}
S_1 &= s_1 \\
S_2 &= C_{21} S_1 + s_2 \\
S_3 &= C_{31} S_1 + C_{32} S_2 + s_3 \\
&\vdots \\
S_m &= C_{m1} S_1 + C_{m2} S_2 + \cdots + C_{m(m-2)} S_{m-2} + C_{m(m-1)} S_{m-1} + s_m
\end{aligned}
\qquad \ldots\ldots (10)
$$

The internal stresses $s_1, s_2 \ldots s_m$ are produced due to external forces like axial forces, shear forces, couples or any combination of such functions. (12)

The coefficients $$C_{ij} = -\frac{\int \alpha S_j S_i \, d_v}{\int \alpha S_j^2 \, d_v} \quad \text{for } i > j$$

here α is the constant term $= \frac{L}{AE}$

where L - is the length of the member

A - is the Cross Section area of the member

E - is modulus of elasticity for a given member

With the arrangement shown in equation (10)

$\int \alpha S_i S_j \, d_v = 0$ for $i \neq j$. This results in an orthogonal form of the linear equations. By the principle of superposition, the internal stresses in the actual structure is given by

$$S = X_1 S_1 + X_2 S_2 + X_3 S_3 + \cdots + X_m S_m + S_o \quad \text{(11)}$$

where S_0 is the stress due to external loading on the statically determinate structure.

Corresponding elongation is given by

$$\delta = \alpha S + \delta_t = \alpha (X_1 S_1 + X_2 S_2 + X_3 S_3 + \cdots X_m S_m + S_o) + \delta_t$$

where δ_t represents internal strain due to temperature changes, due to loosening of connection, etc. For flexural members, that is, members subjected to pure bending, the term δ_t is neglected as its magnitude is negligible in comparison with stresses due to bending.

Applying the principle of virtual work, the unknowns $X_1, X_2 \ldots X_m$ can be determined from the following equations.

$$X_1\int\alpha S_1^2 dv + \int\alpha S_0 S_1 dv + \int\delta_t S_1 dv + \Sigma R_{1p}\Delta_p = 0$$

$$X_2\int\alpha S_2^2 dv + \int\alpha S_0 S_2 dv + \int\delta_t S_2 dv + \Sigma R_{2p}\Delta_p = 0 \quad \ldots\ldots (12)$$

$$- -$$

$$X_m\int\alpha S_m^2 dv + \int\alpha S_0 S_m dv + \int\delta_t S_m dv + \Sigma R_{mp}\Delta_p = 0$$

where $\Sigma R_{ip}\Delta_p$ denotes the external work done due to settlement of the p^{th} support, which is equal to the external force R_{ip} at the p^{th} support due to combined redundant at point i multiplied by the settlement Δ_p of the p^{th} support. Let the external force or moment at p, for the system s_n, be r_{np} and for the system S_n, be R_{np}. It can be seen that these are the same for the system S_1 as for the system s_1 that is $R_{1p} = r_{1p}$.
Writing these equations in a form similar to equation (10)

$$R_{1p} = \gamma_{1p}$$

$$R_{2p} = C_{21}R_{1p} + \gamma_{2p} \quad \ldots\ldots (13)$$

$$- - - - - - - - - - -$$

$$R_{np} = C_{n1}R_{1p} + C_{n2}R_{2p} + \qquad + C_{n(n-1)}R_{(n-1)p} + \gamma_{np}$$

from equation (13) values of R_{ip} ; $i = 1, 2 \ldots n$, can be determined and if substituted in equation (12), unknown redundants X_1, X_2, . . X_m can be determined. (12)

Actually the terms representing strain due to temperature changes and support settlements are neglected since these cause a stress which is of opposite nature to the stress due to live-loads. Moreover, its magnitude is negligible as compared to that due to live loads.

COMBINED REDUNDANTS FOR FLEXURAL STRUCTURES

The method of combined redundants applied to structures in pure bending, compares favourably with result obtained by the elastic center method. For such structures the work done by the axial force andby shear or torsional force is neglected as it is of very small magnitude in comparison with the work done by bending. For the sake of illustration, let us assume a bent with fixed end support as shown in Fig. 3(a).

The given structure is made statically determinate as shown in Fig. 3(b). The three unknown redundants are moment, horizontal force and vertical force. These forces and the moment are applied at the free end 'A'. These forces at the free end will cause moments at each section of the given bent.

Let the moments at any point be $m_1 = 1$; $m_2 = x$; $m_3 = y$. Now as per equation (10), the combined unit redundants will be

$$M_1 = m_1 = 1$$

$$M_2 = C_{21} M_1 + m_2 = C_{21} + m_2 = C_{21} + x$$

$$M_3 = C_{31} M_1 + C_{32} M_2 + m_3 = C_{31} + C_{32}(C_{21} + x) + y \qquad \ldots\ldots (14)$$

Here the coefficient C_{ij}, $(i \neq j)$ has a meaning similar to that for truss members.

$$C_{21} = -\frac{\int M_1 m_2 \frac{ds}{EI}}{\int M_1^2 \frac{ds}{E.I.}}$$

where 1/EI is constant for flexural member.

From equation (14)

$$C_{21} = -\frac{\int m_1 m_2 \frac{ds}{EI}}{\int m_1^2 \frac{ds}{EI}} = -\frac{\int x \frac{ds}{EI}}{\int \frac{ds}{EI}} \quad \ldots (15)a$$

Coefficient $$C_{31} = -\frac{\int m_1 m_2 \frac{ds}{EI}}{\int m_1^2 \frac{ds}{EI}} = -\frac{\int y \frac{ds}{EI}}{\int \frac{ds}{EI}} \quad \ldots (15)b$$

$$C_{32} = -\frac{\int M_2 m_3 \frac{ds}{EI}}{\int M_2^2 \frac{ds}{EI}}.$$

For C_{32}, the numerator is $= \int (C_{21} + x)\, y \frac{ds}{EI}.$

$$= \int \left\{ -\frac{\int x \frac{ds}{EI}}{\int \frac{ds}{EI}} + x \right\} y \frac{ds}{E}$$

$$= \left\{ \frac{-\int x \frac{ds}{EI} + x \int \frac{ds}{EI}}{\int \frac{ds}{EI}} \right\} \int y \frac{ds}{EI}.$$

$$= \left\{ -\frac{\int xy \frac{ds}{EI} + \left(x \int \frac{ds}{EI}\right)\left(\int y \frac{ds}{EI}\right)}{\int \frac{ds}{EI}} \right\}$$

and the denominator is

$$= \int (C_{21} + x)^2 \frac{ds}{EI}$$

$$= \int \left\{ (C_{21})^2 + 2 . C_{21} . x + x^2 \right\} \frac{ds}{EI}.$$

$$= \int \left\{ \left(-\frac{x \int \frac{ds}{EI}}{\int \frac{ds}{EI}} \right)^2 - 2x \frac{\int \frac{ds}{EI}}{\int \frac{ds}{EI}} . x + x^2 \right\} \frac{ds}{EI}.$$

$$= \frac{\int \left\{ \left(\int x \frac{ds}{EI} \right)^2 - 2 \int x \frac{ds}{EI} \cdot \int \frac{ds}{EI} \cdot x \cdot + x^2 \int \left(\frac{ds}{EI} \right)^2 \right\} \frac{ds}{EI}}{\int \left(\frac{ds}{EI} \right)^2}$$

$$= \frac{\left\{ \int \left(x \frac{ds}{EI} \right)^2 - \int x^2 \frac{ds}{EI} \right.}{\int \frac{ds}{EI}}$$

$$C_{32} = - \frac{\left\{ - \int x y \frac{ds}{EI} + \left(\int x \frac{ds}{EI} \right) \left(\int y \cdot \frac{ds}{EI} \right) \right\} \Big/ \int \frac{ds}{EI}}{\left\{ \int \left(x \frac{ds}{EI} \right)^2 - \int \left(x^2 \frac{ds}{EI} \right) \right\} \Big/ \int \frac{ds}{EI}}$$

Multiplying the denominator and numerator by (-1)

$$C_{32} = - \frac{\int x y \frac{ds}{EI} - \left(\int x \frac{ds}{EI} \right) \left(\int y \frac{ds}{EI} \right) \Big/ \left(\int \frac{ds}{EI} \right.}{\int x^2 \frac{ds}{EI} - \left(\int x \frac{ds}{EI} \right)^2 \Big/ \left(\int \frac{ds}{EI} \right)} \quad \ldots (15)c$$

With these known values of coefficients C_{21}, C_{31} and C_{32} the moments M_1, M_2 and M_3 can be found from the equation (14) and the multiplier X_1, X_2 and X_3 can very easily be found from equation (12) where $S_1 = M_1$; $S_2 = M_2$ etc. M_0 is the bending moment due to the external load on the statically determinate structure. The integration of the bending moment is over the whole structure.

Actually the bending moment diagram is split up into various parts, rectangles, triangles, trapezium etc. for which ready-made integration formulas are available (6).

PHYSICAL MEANING OF THE METHOD

To demonstrate the physical meaning of the given method, assume a continuous beam with four spans as shown in Fig. 4(a). This structure is statically indeterminate to the third degree.

For the sake of illustration let the three redundants be the reactions applied at points 1, 2, and 3. By removing the intermediate supports, a simply supported beam will result with its deflection curve as shown in Fig. 4(b). Thisrepresents the state of S_0, that is the deflection due to applied loading. Applying a unit redundant at point (1), the deflection curve will be as shown in Fig. 4(c). Fig. 4(d) represents the deflection curve which is due to the combination of redundants at the point (1) and (2). It may be observed that the selection of the coefficient C_{21} is such as to cause the deflection at point (1) to be equal to zero. The deflection curve shown in Fig. 4(e) is the superposition of the deflection curve shown in Fig. 4(c) and 4(d) plus that due to the unit redundant at point (3). Thus in Fig. 4(e) the coefficients C_{31} and C_{32} nullify the deflection at points (1) and (2).

Now, from state S_1, S_2 and S_3, only state S_1 causes the deflection at point (1) but the multiplier X_1 is so chosen that the deflection at point (1) becomes equal to zero (due to state S_0). The state S_2 causes the deflection at point (2) but multiplier X_2 nullifies the deflection at point (2) (due to states $S_0 + X_1 S_1$). Similarly the scheme is extended for other redundants. The superposition of all the different states is shown in Fig. 4(f) (Ref. 12).

RELATION TO OTHER METHODS

The application of this method to flexural structures has been established. It can be shown that this method is very closely related to the well-known elastic center and column analogy methods.

To illustrate this, consider the same bent as shown in Fig. 3(a). The structure is made statically determinate by making end 'A' free. Applying three redundants for moment, horizontal force and vertical force such that $m_1 = 1$; $m_2 = x$ and $m_3 = y$. Rewriting the equation (14).

$$M_1 = m_1 = 1$$

$$M_2 = C_{21}M_1 + m_2 = C_{21} + x$$

$$M_3 = C_{31}M_1 + C_{32}M_2 + m_3 = C_{31} + C_{32}(C_{21} + x) + y$$

The coefficients: $$C_{21} = -\frac{\int x \frac{ds}{EI}}{\int \frac{ds}{EI}}$$

$$C_{31} = -\frac{\int y \cdot \frac{ds}{EI}}{\int \frac{ds}{EI}}$$

$$C_{32} = -\frac{\int xy \frac{ds}{EI} - \left(\int x \cdot \frac{ds}{EI}\right)\left(\int y \cdot \frac{ds}{EI}\right) / \left(\int \frac{ds}{EI}\right)}{\int x^2 \frac{ds}{EI} - \left(\int x \frac{ds}{EI}\right)^2 / \left(\int \frac{ds}{EI}\right)}$$

In the elastic center method, the point of application of redundants is so chosen that

$$\int x_1 \frac{ds}{EI} = 0$$

$$\int y_1 \frac{ds}{EI} = 0$$

$$\int x_1 y_1 \frac{ds}{EI} = 0$$

where x_1 and y_1 are the distances to the center of gravity of the closed structure.

$\int x_1 \frac{ds}{EI}$ and $\int y_1 \frac{ds}{EI}$ represent the statical moments of the area about Y-axis and X-axis respectively from the fixed point 'A' Fig. (3); (1).

Therefore, coefficient $-C_{21} = \dfrac{\int x \frac{ds}{EI}}{\int \frac{ds}{EI}} = \bar{x}$ . . . . . . 15(a_1)

where ds represents the length of thesmall element and 1/EI the width of that small element.

Therefore $\int \frac{ds}{EI}$ = Area of the whole structure.

Similarly $-C_{31} = \dfrac{\int y \frac{ds}{EI}}{\int \frac{ds}{EI}} = \bar{y}$ . . . . . . 15(b_1)

From equation 15)c) it can be seen that the numerator

$\int xy \frac{ds}{EI} - \left(\int x \frac{ds}{EI}\right)\left(\int y \frac{ds}{EI}\right) / \left(\int \frac{ds}{EI}\right)$ represents the product of inertia about the X_2-Y_2 axis and the denominator

$\int \frac{x^2 ds}{EI} - \left(\int x \frac{ds}{EI}\right)^2 / \left(\int \frac{ds}{EI}\right)$ represents the moment of inertia about the Y_2-axis.

thus $-C_{32} = \dfrac{\int x_2 y_2 \frac{ds}{EI}}{\int x_2^2 \frac{ds}{EI}} = \dfrac{I_{x_2 y_2}}{I_{y_2}}$ . . . . 15(c_1)

If the redundant forces and moment are applied at the origin of principal axis then,

$$C_{21} = - \frac{\int x_1 \frac{ds}{EI}}{\int \frac{ds}{EI}} = 0$$

$$C_{31} = - \frac{\int y_1 \frac{ds}{EI}}{\int \frac{ds}{EI}} = 0$$

$$C_{32} = - \frac{\int x_1 y_1 \frac{ds}{EI}}{\int x_1^2 \frac{ds}{EI}} = 0$$

The coefficient C_{32} becomes zero on the centroidal axis if the given structure is symmetrical as shown in Fig. 3 (d).

Thus the elastic center method becomes a special case of the proposed method.

Rewriting equation (14) we have,

$$M_1 = m_1 = 1$$

$$M_2 = C_{21}M_1 + m_2 = C_{21} + m_2 = C_{21} + x \qquad \ldots\ldots (14)$$

$$M_3 = C_{31}M_1 + C_{32}M_2 + m_3 = C_{31} + C_{32}(C_{21} + x) + y$$

With these known values of M_1, M_2 and M_3, multipliers X_1, X_2 and X_3 can very easily be found from equation (9) that is,

$$X_1 = - \frac{\int M_o \frac{ds}{EI}}{\int \frac{ds}{EI}} \qquad \ldots\ldots 16(a)$$

$$X_2 = - \frac{\int M_2 M_o \frac{ds}{EI}}{\int M_2^2 \frac{ds}{EI}} \qquad \ldots\ldots 16(b)$$

$$X_3 = - \frac{\int M_3 M_o \frac{ds}{EI}}{\int M_3^2 \frac{ds}{EI}} \qquad \ldots\ldots 16(c)$$

The final moment at any point is given by

$$M = X_1M_1 + X_2M_2 + X_3M_3 + M_0 \qquad \ldots\ldots\ldots\ldots (17)$$

To show the relation of the method of combining redundants with the well-known column analogy method:

From the fundamentals of column analogy (9)

$$\int M_0 \frac{ds}{EI} = P \;:\; \int \frac{ds}{EI} = A$$

$$\int x_2 y_2 \frac{ds}{EI} = I_{x_2y_2} \;:\; \int x_2^2 \frac{ds}{EI} = I_{y_2} \;:\; \int y_2^2 \frac{ds}{EI} = I_{x_2} \qquad \ldots\ldots\ldots\ldots (18)$$

$$\int M_0 x_2 \frac{ds}{EI} = M_{y_2} \;:\; \int M_0 y_2 \frac{ds}{EI} = M_{x_2}$$

Simplifying equation (16)

$$X_1 = -\frac{\int M_0 \frac{ds}{EI}}{\int \frac{ds}{EI}} = -\frac{P}{A} \qquad \ldots\ldots\ldots\ldots 19(a)$$

$$X_2 = -\frac{\int (C_{21}+x) M_0 \frac{ds}{EI}}{\int (C_{21}+x)^2 \frac{ds}{EI}} = -\frac{\int (x-\bar{x}) M_0 \frac{ds}{EI}}{\int (x-\bar{x})^2 \frac{ds}{EI}}$$

From the equation $15(a_1)$

$$X_2 = -\frac{\int M_0 x_2 \frac{ds}{EI}}{\int x_2^2 \frac{ds}{EI}} = -\frac{M_{y_2}}{I_{y_2}} \qquad \ldots\ldots\ldots\ldots 19(b)$$

$$X_3 = -\frac{\int (C_{31} + C_{32} C_{21} + C_{32} x + y) M_0 \frac{ds}{EI}}{\int (C_{31} + C_{32} C_{21} + C_{32} x + y)^2 \frac{ds}{EI}}$$

$$X_3 = -\frac{\int \left\{ -\bar{y} + \left(-\frac{I_{x_2y_2}}{I_{y_2}}\right)(-\bar{x}) + \left(-\frac{I_{x_2y_2}}{I_{y_2}}\right) x + y \right\} M_0 \frac{ds}{EI}}{\int \left\{ -\bar{y} + \left(-\frac{I_{x_2y_2}}{I_{y_2}}\right)(-\bar{x}) + \left(-\frac{I_{x_2y_2}}{I_{y_2}}\right) x + y \right\}^2 \frac{ds}{EI}}$$

$$X_3 = - \frac{\left\{ M_{x_2} - \frac{I_{x_2y_2}}{I_{y_2}} \cdot M_{y_2} \right\}}{\left\{ I_{x_2} - 2\frac{I_{x_2y_2}{}^2}{I_{y_2}} + \frac{I_{x_2y_2}}{I_{y_2}} \right\}} = - \left\{ \frac{M_{x_2} - \frac{I_{x_2y_2}}{I_{y_2}} \cdot M_{y_2}}{I_{x_2} - \frac{I_{x_2y_2}{}^2}{I_{y_2}}} \right\} \quad 19(c)$$

Substituting into equation (17) from equations (14) and (19) we obtain the following equation.

$$M = M_0 - \frac{P}{A} - \frac{M_{y_2}}{I_{y_2}} \cdot x_2 - \left\{ \frac{M_{x_2} - \frac{I_{x_2y_2}}{I_{y_2}} \cdot M_{y_2}}{I_{x_2} - \frac{I_{x_2y_2}{}^2}{I_{y_2}}} \right\} \left(y_2 - \frac{I_{x_2y_2}}{I_{y_2}} \cdot x_2 \right)$$

$$= M_0 - \frac{P}{A} - \left\{ \frac{M_{x_2} - \frac{I_{x_2y_2}}{I_{y_2}} \cdot M_{y_2}}{I_{x_2} - \frac{I_{x_2y_2}}{I_{y_2}}} \right\} - \frac{M_{y_2} x_2}{I_{y_2}} + \left\{ \frac{M_{x_2} - \frac{I_{x_2y_2}}{I_{y_2}} M_{y_2}}{I_{x_2} - \frac{I_{x_2y_2}^2}{I_{y_2}}} \right\} \left(\frac{I_{x_2y_2}}{I_{y_2}} \cdot x_2 \right)$$

$$= M_0 - \frac{P}{A} - \left\{ \frac{M_{x_2} - \frac{I_{x_2y_2}}{I_{y_2}} M_{y_2}}{I_{x_2} - \frac{I_{x_2y_2}{}^2}{I_{y_2}}} \right\} y_2 - \frac{M_{y_2} \cdot x_2 \left(I_{x_2} - \frac{I_{x_2}{}^2 y_2}{I_{y_2}} \right) + M_{x_2} \cdot I_{x_2y_2} \cdot x_2}{I_{x_2} I_{y_2} - I_{x_2 \cdot y_2}{}^2} \cdots - \frac{\frac{I_{x_2y_2}{}^2}{I_{y_2}} \cdot M_{y_2} x_2}{I_{x_2} \cdot I_{y_2} - I_{x_2}{}^2 y_2}$$

$$= M_0 - \frac{P}{A} - \left\{ \frac{M_{x_2} - \frac{I_{x_2y_2}}{I_{y_2}} \cdot M_{y_2}}{I_{x_2} - \frac{I_{x_2y_2}2}{I_{y_2}}} \right\} y_2 - \left[\frac{M_{y_2} x_2 I_{x_2} + M_{y_2} \cdot x_2 \cdot \frac{I_{x_2}{}^2 y_2}{I_{y_2}} + M_{x_2} I_{x_2y_2} \cdot x_2}{I_{x_2} I_{y_2} - I_{x_2 \cdot y_2}{}^2} \right.$$

$$\left. - \frac{I_{x_2y_2}^2 \cdot \frac{M_{y_2}}{I_{y_2}} \cdot x_2}{I_{x_2} \cdot I_{y_2} - I_{x_2 \cdot y_2}^2} \right]$$

$$= M_0 - \frac{P}{A} - \left\{ \frac{M_{x_2} - \frac{I_{x_2y_2}}{I_{y_2}} \cdot M_{y_2}}{I_{x_2} - \frac{I_{x_2y_2}^2}{I_{y_2}}} \right\} y_2 - \left\{ \frac{M_{y_2} I_{x_2} - I_{x_2y_2} \cdot M_{x_2}}{I_{x_2} I_{y_2} - I_{x_2 y_2}^2} \right\} x_2$$

$$= M_0 - \frac{P}{A} - \left\{ \frac{M_{x_2} - \frac{I_{x_2y_2}}{I_{y_2}} M_{y_2}}{I_{x_2} - \frac{I_{x_2}{}^2 y_2}{I_{y_2}}} \right\} y_2 - \left\{ \frac{M_{y_2} - \frac{I_{x_2y_2}}{I_{x_2}} \cdot M_{x_2}}{I_{y_2} - \frac{I_{x_2}{}^2 y_2}{I_{x_2}}} \right\} x_2$$

This is the general form of the column analogy method for unsymmetrical bent. Substituting $I_{x_2 y_2} = 0$ that is, considering the symmetrical frame of Fig. 3(d)

$$M = M_0 - \frac{P}{A} - \frac{M_{x_2}}{I_{y_2}} \cdot y_2 - \frac{M_{y_2}}{I_{y_2}} \cdot x_2$$

Thus the method of combined redundants reduces to the column analogy method.

ILLUSTRATIVE PROBLEMS

To illustrate the mothod of combining redundants, several problems will be solved, involving the application of this method:

PROBLEM: 1

A truss shown in Fig. 5(a) with 12^k load applied at point H, acting downward (7).

Required: Bar stresses due to the given load:

Solution: The given frame is statically indeterminate to the third degree. The truss is made statically determinate by removing the intermediate support and cutting bars CH and CF.

The combined redundants are shown in Fig. 5 the required computations are framed in Table 1.

$$S_1 = s_1$$

$$C_{21} = - \frac{\sum \alpha S_1 s_2}{\sum \alpha S_1^2} = - \frac{\sum \alpha s_1 s_2}{\sum \alpha s_1^2} = \frac{0.85}{3.25} = 0.262$$

$$S_2 = C_{21} S_1 + s_2 = 0.262 s_1 + s_2$$

$$C_{31} = - \frac{\sum \alpha S_1 s_3}{\sum \alpha S_1^2} = - \frac{\sum \alpha s_1 s_3}{\sum \alpha s_1^2} = 0.262$$

$$C_{32} = - \frac{\sum \alpha S_2 s_3}{\sum \alpha S_2^2} = - \frac{\sum \alpha (0.262 s_1 + s_2) s_3}{\sum \alpha (0.262 s_1 + s_2)^2} = -0.1105$$

$$S_3 = C_{31} S_1 + C_{32} S_2 + s_3 = C_{31} S_1 + C_{32}(C_{21} + s_2) + s_3$$

$$= 0.262\ s_1 - 0.1105\ (0.262\ s_1 + s_2) + s_3$$

With these known values of C_{21}, C_{31}, C_{32} and hence, S_1, S_2 and S_3, unknown redundants could be computed as follows.

$$X_1 = - \frac{\sum \alpha S_1 S_0}{\sum \alpha S_1^2} = - \frac{\sum \alpha s_1 S_0}{\sum \alpha s_1^2} = \frac{22.88}{3.254} = 7.0650$$

$$X_2 = - \frac{\sum \alpha S_2 S_0}{\sum \alpha S_2^2} = \frac{20.71}{3.775} = 5.48$$

$$X_3 = - \frac{\sum \alpha S_3 S_0}{\sum \alpha S_3^2} = - \frac{1.32}{3.736} = - 0.353$$

The final bar stresses due to the given loading are shown in column 22 of Table 1.

For the sake of convenience, $\frac{L}{A}$ has been chosen equal to unity.
($\frac{L}{A}$ same for all members)

PROBLEM: 2

A flexural bent is shown in Fig. 6(a) with both ends fixed. The loading is as shown.

Required: Horizontal reaction due to given load and the bending moment diagram.

Solution: The given bent is statically indeterminate to the third degree. The end A is freed and the following three redundant forces are applied; counter-clockwise moment X_1, horizontal force X_2 and vertical force X_3 at A. The moment diagrams for these forces are shown in Fig. 6(c), (d) and (e). Fig. 6(b) is the moment diagram due to the applied load on the statically determinate structure.

With the use of formulas given in Tafel der Werte (6), the necessary coefficients can be obtained.

(Detail calculations are not shown)

$EI\ \delta_{20} = 2,295,000$ kip ft.

$EI\ \delta_{30} = -4,360,000$ kip ft.

$EI\ \delta_{10} = 78,000$ kip ft.

$EI\ \delta_{22} = 182,250$ kip ft.

$EI\ \delta_{32} = -141,750$ kip ft.

$EI\ \delta_{33} = 234,000$ kip ft.

$EI\ \delta_{21} = 4,725$ kip ft.

$EI\ \delta_{31} = -4,500$ kip ft.

$EI\ \delta_{11} = 150$ kip ft.

$$c_{21} = -\frac{\int M_1 m_2 \frac{ds}{EI}}{\int M_1^2 \frac{ds}{EI}} = -\frac{4,725}{150} = -31.5$$

$$c_{31} = -\frac{\int M_1 m_3 \frac{ds}{EI}}{\int m_1^2 \frac{ds}{EI}} = -\frac{4,500}{150} = 30$$

$$c_{32} = -\frac{\int M_2 m_3 \frac{ds}{EI}}{\int M_2^2 \frac{ds}{EI}} = -\frac{\int (c_{21} M_1 + m_2) m_3 \frac{ds}{EI}}{\int (c_{21} M_1 + m_2)^2 \frac{ds}{EI}} = \frac{4500 \times 31.5 - 141750}{\int (c_{21} M_1 + m_2)^2 \frac{ds}{EI}} = 0$$

$$X_1 = -\frac{\int M_1 M_0 \frac{ds}{EI}}{\int M_1^2 \frac{ds}{EI}} = -\frac{7800}{150} = -520$$

$$X_2 = -\frac{\int M_2 M_0 \frac{ds}{EI}}{\int M_2^2 \frac{ds}{EI}} = -\frac{\int (C_{21} m_1 M_0 + m_2 M_0) \frac{ds}{EI}}{\int (C_{21}^2 m_1^2 + 2 C_{21} m_1 m_2 + m_2^2) \frac{ds}{EI}}$$

$$= \frac{-31.5 \times 78000 + 2295000}{(31.5)^2 \times 150 + 2(-31.5)(4725) + 182250}$$

$$= 4.85$$

$$X_3 = -\frac{\int (C_{31} M_1 + m_3) M_0 \frac{ds}{EI}}{\int (C_{31} M_1 + m_3)^2 \frac{ds}{EI}}$$

$$= \frac{30 \times 78000 + (-4360000)}{900 \times 150 + 60(-4500) + 234000}$$

$$= 20.4$$

Final Moments:

$M_A = X_1M_1 + X_2M_2 + X_3M_3 + M_0$

$= -520(1) + 4.85\ (31.5 \times 1 + 0) + 20.4\ (30 \times 1 + 0) + 0$

$= -60$ kip ft.

$M_D = X_1M_1 + X_2M_2 + X_3M_3 + M_0$

$= -520 + 4.85\ (-31.5) + 20.4\ (30-60) + 1200$

$= -84.5$ kip ft.

$M_B = -520 + 4.85\ (-31.5 + 45) + 20.4\ (30 + 0)$

$= 156.5$ kip ft.

$$M_C = -520 + 4.85\,(-31.5 + 45) + 20.4\ \ (30 - 60) + 1200$$
$$= 131.5 \text{ kip ft.}$$

Horizontal Reaction at A $= \dfrac{156.5 + 60}{45} = 4.82$

Horizontal Reaction at D $= \dfrac{131.5 + 84.5}{45} = 4.82$

The final moment diagram is shown in Fig. 6(f)

To compare the solution by the method of combined redundants to the solution by the method of consistent deformations the same problem can be worked solving the resulting equations by the triangular method.

The pattern of the equations is:

$$150\ x_1 + 4725 x_2 + (-4500) x_3 = -78{,}000 \quad \ldots\ldots (I)$$

$$4725\ x_1 \quad 182{,}250\ x_2 + (-141{,}750) x_3 = -2{,}295{,}000 \ \ldots (II)$$

$$-4500\ x_1 + (-141{,}750) x_2 + (234{,}000) x_3 = 4{,}360{,}000 \ . (III)$$

Solving by the triangular method: (3),

$$150\ x_1 + 4725\ x_2 \quad -4500\ x_3 = -78{,}000$$

$$\left\{ \frac{4725}{150}\ (I) + (II) \right\};\ 33250\ x_2 \quad 250\ x_3 = 165{,}000$$

$$\left\{ \frac{4500}{150}\ (I) + (III) \right\};\ 250 x_2 \quad 99{,}000\ x_3 = 2{,}020{,}000$$

Solving the last two equations

$$\left(-\frac{250}{33250}\right);\quad 33250\ x_2 + 250\ x_3 = 165{,}000$$

$$250\ x_2 + 999000\ x_3 = 2{,}020{,}000$$

$$x_3 = 20.4$$

and $x_2 = 4.8$

Substituting these values of x_2 and x_3 in (I)

$$x_1 = -59.5$$

Moments at A $= x_1\ (1) = -9.5$ kip ft.

Moments at D $= -1200 + 59.5 + 60x20.4 = 84.0$ kip ft.

Moments at B $= 59.5 + (045)x4.8 = -156.5$ kip ft.

Moments at C $= -1200 + (-1)\ (-59.5)\ -45\ (4.8) + 60\ (20.4)$

$= -132$ kip ft.

which gives the same bending moment diagram as shown in Fig. 6(f).

PROBLEM: 3

A truss shown in Fig. 7(a) with 8k load applied at point (14), acting downward.

Required: Bar stresses due to given load. (5) page 401.

Solution: The method of combined redundants for higher degree of indeterminancy can be demonstrated by this problem. The given truss is statically indeterminate eight degrees internally. The truss is made statically determinate by cutting one of the diagonal members in each panel Fig. 7(b). The forces in the members due to the applied loads and the redundant forces are shown in Fig. 7(b) and (c) respectively. The systematic calculations for various combined redundant coefficients and the final bar stresses are tabulated in Table 2.

It is to be noted that calculations up to column (31) remain unaltered for different loadings.

In order to compare the combined redundant method with the method of consistent deformations the same problem is solved using Guass's Elimination Method to solve the resulting equations. (Table 4)

The equations are:

$$79.96\, x_1 + 20.45\, x_2 + 140.08 = 0$$
$$20.45\, x_1 + 93.86\, x_2 + 20.45\, x_3 + 169.60 = 0$$
$$20.45\, x_2 + 112.86\, x_3 + 20.45\, x_4 + 193.39 = 0$$
$$20.45\, x_3 + 123.68\, x_4 + 20.45\, x_5 + 206.9 = 0$$
$$20.45\, x_4 + 123.68\, x_5 + 20.45\, x_6 + 206.9 = 0$$
$$20.45\, x_5 + 112.86\, x_6 + 20.45\, x_7 - 11.30 = 0$$
$$20.45\, x_6 + 93.86\, x_7 + 20.45\, x_8 - 508.60 = 0$$
$$20.45\, x_7 + 79.96\, x_8 - 339.40 = 0$$

The solution of simultaneous equation gives:

$$x_8 = \frac{225.7}{75.32} = 2.93$$

$$x_7 = \frac{500.875 - 20.45 x 2.93}{90.04} = 4.86$$

$$x_6 = \frac{41.30 - 20.45 x 4.86}{109.38} = -0.53$$

$$x_5 = \frac{-176.7 + 20.45 x 0.53}{120.17} = -1.37$$

$$x_4 = \frac{-176.20 + 20.45 x 1.37}{119.81} = -1.23$$

$$x_3 = \frac{-162.59 + 20.45 x 1.23}{108.135} = -1.25$$

$$x_2 = \frac{-133.60 + 20.45 x 1.25}{88.635} = -1.21$$

$$x_1 = \frac{-140.08 + 20.45 x 1.21}{79.96} = -1.44$$

The detailed calculations are given in Table 3. The bar stresses by both methods check fairly well.

CONCLUSION

From the illustrative examples it can be observed that the combined redundant method eliminates the task of solving the simultaneous equations, which result from application of the method of consistent deformations. This is replaced by calculation of the coefficients of the combined redundants.

In Table 1, for different loading systems, the calculations remain unaltered except for columns 15 to 22. Therefore, by the principle of superposition the maximum bar stresses can be obtained by suitable combinations of loading systems. If the same problem were to be solved for maximum bar stresses by some other method, for example, consistent deformations, then, for each system of loading, the magnitude of the redundant forces must be calculated by solving the simultaneous equations each time. This is a very tedious and time-consuming approach. Thus it can be seen that this method has an advantage if the maximum bar stresses are to be calculated.

For a flexural structure, the calculations involved by the combined redundant method are the calculations for coefficients of combined redundants. Thus, the problem of solving simultaneous equations is eliminated. The redundant forces in this method of combined redundants, are quite different from those obtained by the consistent deformation method.

It was pointed out by Steven J. Fenves[2] A.M. ASCE (8), that in common types of trusses, if the number of members greatly exceeds the number of redundants, the combined redundant method does not have any computational advantage. Moreover, for higher degree of indeterminancy the accuracy is affected by this method, since more operation of multiplications are involved. However, the same author, Steven J. Fenves, discovered that, for computer programming, this method involves fewer operations than by ordinary methods, that is, matrix inversion by the computer.

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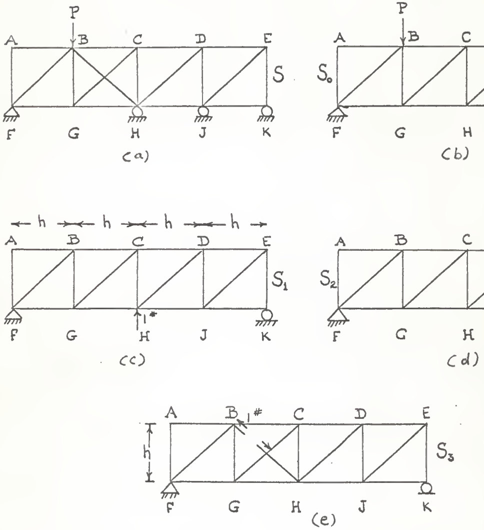

Fig. 1 System of Loads

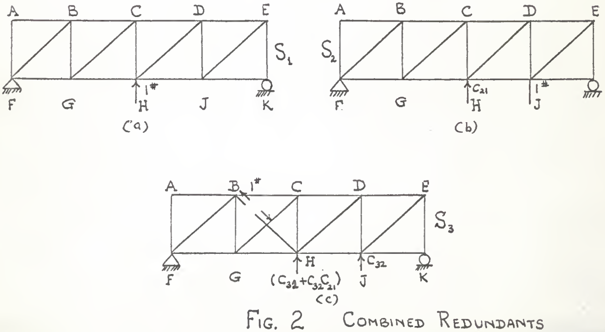

Fig. 2 Combined Redundants

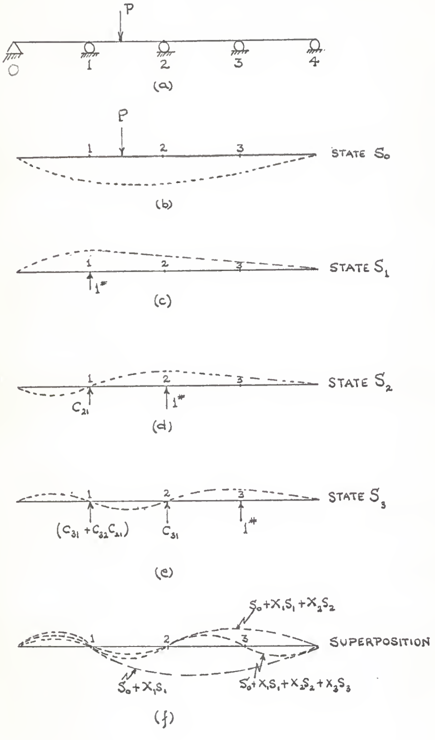
P
0
1
2
3
4
(a)
P
1
2
3
STATE S_0
(b)
1
2
3
STATE S_1
1^*
(c)
1
2
3
STATE S_2
C_{21}
1^*
(d)
1
2
3
STATE S_3
$(C_{31} + C_{32}C_{21})$
C_{31}
1^*
(e)
$S_0 + X_1S_1 + X_2S_2$
1
2
3
SUPERPOSITION
$S_0 + X_1S_1$
$S_0 + X_1S_1 + X_2S_2 + X_3S_3$
(f)

FIG. 4

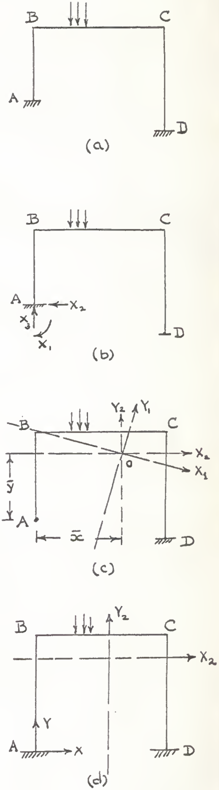
B
C
A
D
(a)
B
C
A
X_2
X_3
X_1
D
(b)
Y_2
Y_1
B
C
X_2
0
X_1
$\bar{y}$
A
$\bar{x}$
D
(c)
Y_2
B
C
X_2
Y
A
X
D
(d)

FIG. 3

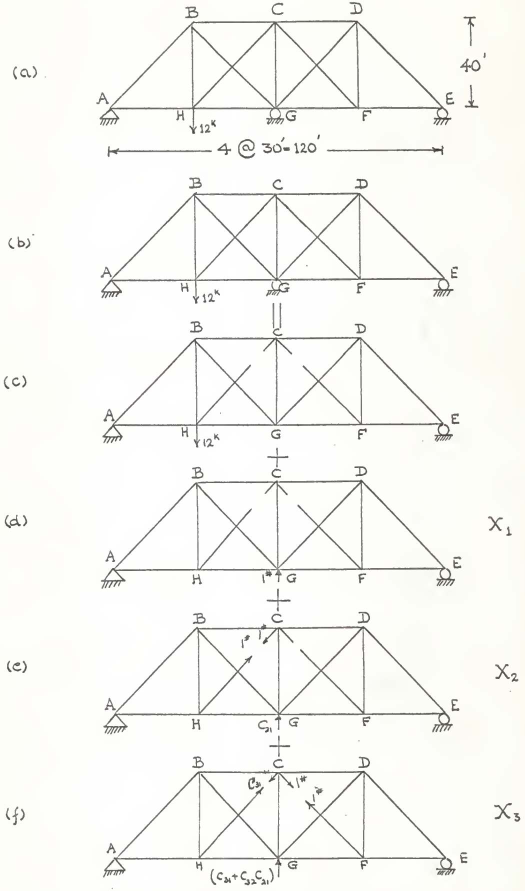

FIG. 5: PROBLEM No. 1 METHOD OF COMBINED REDUNDANTS

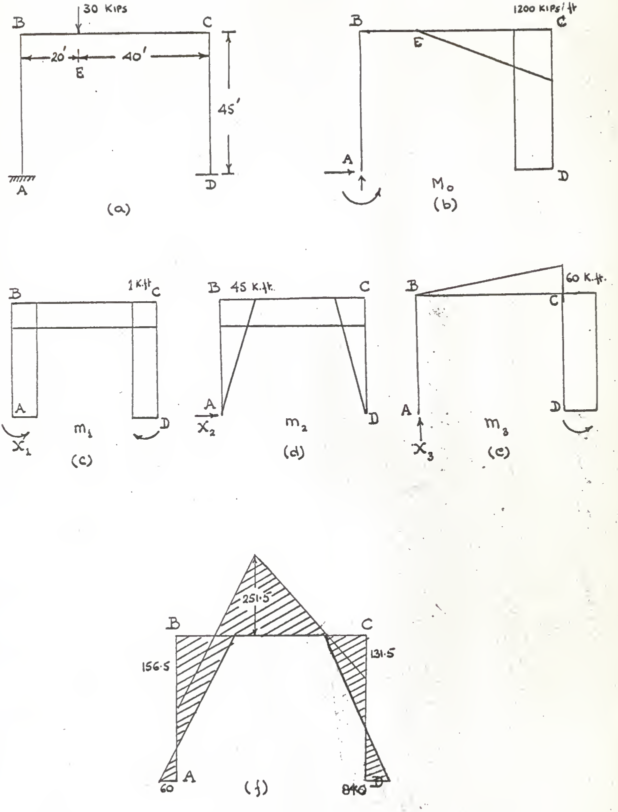

Fig. 6: Problem No. 2: Flexural members by combining Redundants

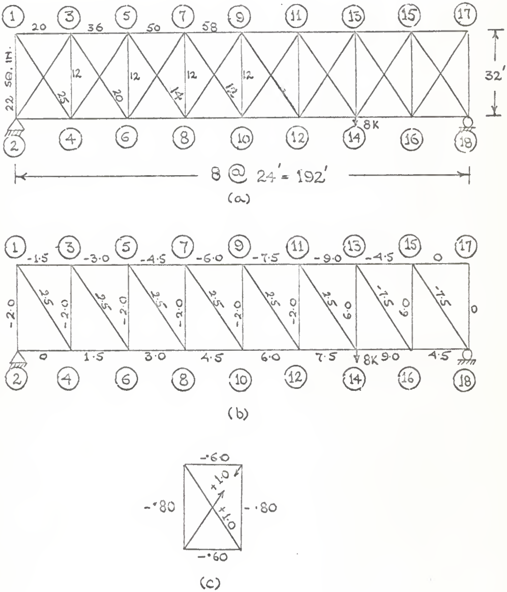

Fig. 7. Problem No. 3: 8th Degree redundant truss by Combining Redundants

Table 1. Third Degree Redundant Frame
by the Method of Combining Redundants

	1	2	3	4	5	6	7	8	9	10
BAR	$s_1 \mp S_1$	s_2	s_3	S_1^2	$S_1 s_2$	$c_{21} S_1$	S_2	$S_1 s_3$	$S_2 s_3$	S_2^2
AB	+0.625	0	0	+0.391	0	+0.164	+0.164	0	0	+0.027
BC	+0.75	-0.60	0	+0.563	-0.45	+0.197	-0.403	0	0	+0.16
CD	+0.75	0	-0.60	+0.563	0	+0.197	+0.197	-0.45	-0.118	+0.039
DE	+0.625	0	0	+0.391	0	+0.164	+0.164	0	0	+0.027
EF	-0.375	0	0	+0.141	0	-0.098	-0.098	0	0	+0.01
FG	-0.375	0	-0.60	+0.141	0	-0.098	-0.098	+0.225	+0.059	+0.01
GH	-0.375	-0.60	0	+0.141	+0.225	-0.098	-0.698	0	0	-0.487
HA	-0.375	0	0	+0.141	0	-0.098	-0.098	0	0	+0.01
BG	-0.625	+1.0	0	+0.391	-0.625	-0.164	+0.836	0	0	+0.70
DG	-0.625	0	+1.0	+0.391	0	-0.164	-0.164	-0.625	-0.164	+0.027
BH	0	-0.80	0	0	0	0	-0.80	0	0	+0.64
CG	0	-0.80	-0.80	0	0	0	-0.80	0	+0.64	+0.64
DF	0	0	-0.80	0	0	0	0	0	0	0
CH	0	+1.0	0	0	0	0	+1.0	0	0	+1.0
CF	0	0	+1.0	0	0	0	0	0	0	0
TOTAL	-	-	-	+3.254	-0.85	-	-	-0.85	+0.417	+3.775

Table 1 (concl.)

	11	12	13	14	15	16	17	18	19	20	21	22
BAR	$c_{31}S_1$	$c_{32}S_2$	S_3	S_3^2	S_0	S_0S_1	S_0S_2	S_0S_3	X_1S_1	X_2S_2	X_3S_3	S
AB	+0.164	±0.0181	+0.146	+0.021	-11.25	-7.03	-1.85	-1.65	+4.42	+0.90	-0.06	-5.99
BC	+0.197	+0.0446	+0.242	+0.059	-4.50	-3.37	+1.82	-1.09	+5.31	-2.22	-0.08	-1.49
CD	+0.197	-0.0218	-0.425	+0.181	-4.50	-3.37	-0.89	+1.92	+5.31	+1.08	+0.16	+2.05
DE	+0.164	-0.0181	+0.146	+0.021	-3.75	-2.35	-0.62	-0.55	4.42	+0.90	-0.06	+1.51
EF	-0.098	+0.0109	-0.087	+0.008	+2.25	-0.84	-0.22	-0.20	-2.66	-0.54	+0.03	-0.92
FG	-0.098	+0.0109	-0.687	+0.472	+2.25	-0.84	-0.22	-1.54	-2.66	-0.54	+0.26	-0.69
GH	-0.098	+0.0771	-0.021	+0.004	+6.75	-2.54	-4.72	-0.14	-2.66	-3.84	+0.01	+0.26
HA	-0.098	+0.0109	-0.087	+0.008	+6.75	-2.54	-0.67	-0.59	-2.66	-0.54	+0.03	+3.58
BG	-0.164	-0.0924	-0.256	+0.065	-3.75	+2.35	-3.13	+0.97	-4.42	+4.61	+0.10	-3.16
DG	-0.164	+0.0181	+0.854	+0.73	+3.75	-2.35	-0.62	+3.21	-4.42	-0.90	-0.32	-1.89
BH	0	++0.0885	+0.089	+0.008	+12.0	0	-9.6	+0.98	0	-4.41	-0.03	+7.56
CG	0	+0.0885	-0.712	+0.507	0	0	0	0	0	-4.41	+0.27	-4.14
DF	0	0	-0.80	+0.64	0	0	0	0	0	0	+0.30	+0.30
CH	0	-.1105	0.1105	+0.012	0	0	0	0	0	+5.48	+0.04	+5.52
CF	0	0	+1.0	+1.0	0	0	0	0	0	0	-0.35	-0.35
TOTAL	-	-	-	+3.736	-	-22.88	-20.71	+1.32	-	-	-	-

EIGHTH DEGREE REDUNDANT FRAME BY COMBINING REDUNDANTS : TABLE 2

BARS	1	2	3	4	5	6	7	8	9	10	11	12	13	14
BARS	L/A	S_1	αS_1^2	$\alpha S_1 S_2$	$C_{21}S_1$	S_2	$\alpha S_2 S_3$	αS_2^2	$C_{32}S_2$	S_3	$\alpha S_3 S_4$	αS_3^2	$C_{43}S_3$	S_4
1-3	14.4	-0.6	5.18		0.153	0.153		0.337	-0.085	-0.035		0.0177	0.0066	0.0066
2-4	14.4	-0.6	5.18		0.153	0.153		0.337	-0.085	-0.035		0.0177	0.0066	0.0066
1-2	11.45	-0.8	11.16		0.204	0.204		0.727	-0.087	-0.047		0.0386	0.0088	0.0088
3-4	32.0	-0.8	20.48	20.48	0.204									
1-4	19.2	1.0	19.20		-0.255	-0.255		1.25	0.069	0.069		0.067	-0.011	-0.011
3-2	19.2	1.0	19.20		-0.255	-0.255		1.25	0.069	0.069		0.067	-0.011	-0.011
		S_2												
3-5	8.0	-0.6				-0.6		2.88	0.137	0.137		0.15	-0.026	-0.026
4-6	8.0	-0.6				-0.6		2.88	0.137	0.137		0.15	-0.026	-0.026
3-4	32.0	-0.8				-0.896		11.33	0.136	0.136		0.884	-0.026	-0.026
5-6	32.0	-0.8				-0.8	20.48	20.48	0.183					
3-6	24.0	1.0				1.0		24.0	-0.229	-0.229		1.26	0.043	0.043
5-4	24.0	1.0				1.0		24.0	-0.229	-0.229		1.26	0.043	0.043
		S_3												
5-7	5.76	-0.6								-0.6		2.075	0.117	0.117
6-8	5.76	-0.6								-0.6		2.075	0.117	0.117
5-6	32.0	-0.8								-0.617		12.28	0.116	0.116
7-8	32.0	-0.8								-0.8	20.48	20.48	0.156	
5-8	34.3	1.0								1.0		34.30	-0.195	-0.195
7-6	34.3	1.0								1.0		34.30	-0.195	-0.195
		S_4												
7-9	4.97	-0.6												-0.6
8-10	4.97	-0.6												-0.6
7-8	32.0	-0.8												-0.644
9-10	32.0	-0.8												-0.8
7-10	40.0	1.0												1.0
9-8	40.0	1.0												1.0
		S_5												
9-11	4.97	-0.6												
10-12	4.97	-0.6												
11-12	32.0	-0.8												
9-10	32.0	-0.8												
11-10	40.0	1.0												
9-12	40.0	1.0												
		S_6												
11-13	5.76	-0.6												
12-14	5.76	-0.6												
13-14	32.0	-0.8												
11-12	32.0	-0.8												
13-12	34.3	1.0												
11-14	34.3	1.0												
		S_7												
13-15	8.0	-0.6												
14-16	8.0	-0.6												
15-16	32.0	-0.8												
13-14	32.0	-0.8												
15-14	24.0	1.0												
13-16	24.0	1.0												
		S_8												
15-17	14.4	-0.6												
16-18	14.4	-0.6												
17-18	11.45	-0.8												
15-16	32.0	-0.8												
17-16	19.2	1.0												
15-18	19.2	1.0												
TOTAL			80.39	20.48			20.48	89.471			20.48	109.05		

BARS	15	16	17	18	19	20	21	22	23	24	25	26	27	28
BARS	$\alpha S_4 S_5$	αS_4^2	$C_{54}S_4$	S_5	$\alpha S_5 S_6$	αS_5^2	$C_{65}S_5$	S_6	$\alpha S_6 S_7$	αS_6^2	$C_{76}S_6$	S_7	$\alpha S_7 S_8$	αS_7^2
1-3														
2-4														
1-2														
3-4														
1-4		0.002		0.002										
3-2		0.002		0.002										
3-5		0.005	0.0044	0.0044										
4-6		0.005	0.0044	0.0044										
3-4		0.022	0.0044	0.0044										
5-6														
3-6		0.045	-0.007	-0.007										
5-4		0.045	-0.007	-0.007										
5-7		0.078	-0.02	-0.02		0.0023	0.0034	0.0034			-0.0006	-0.0006		
6-8		0.078	-0.02	-0.02		0.0023	0.0034	0.0034			-0.0006	-0.0006		
5-6		0.44	-0.02	-0.02		0.013	0.0034	0.0034		0.0004	-0.0006	-0.0006		
7-8														
5-8		1.31	0.0334	0.034		0.040	-0.006	-0.006		0.0012	0.0011	0.0011		
7-6		1.31	0.0334	0.034		0.040	-0.006	-0.006		0.0012	0.0011	0.0011		
7-9		1.785	0.103	0.103		0.063	-0.017	-0.017		0.0014	0.0032	0.0032		
8-10		1.785	0.103	0.103		0.063	-0.017	-0.017		0.0014	0.0032	0.0032		
7-8		13.35	0.11	0.11		0.388	-0.0186	-0.0186		0.011	0.0034	0.0034		0.0004
9-10	20.48	20.48	0.137											
7-10		40.0	-0.171	-0.171		1.17	0.03	0.03		0.036	-0.0066	-0.0066		0.0015
9-8		40.0	-0.171	-0.171		1.17	0.03	0.03		0.036	-0.0066	-0.0066		0.0015
9-11				-0.6		1.785	0.102	0.102		0.062	-0.019	-0.019		0.0018
10-12				-0.6		1.785	0.102	0.102		0.062	-0.019	-0.019		0.0018
11-12				-0.8	20.48	20.48	0.135							
9-10				-0.665		14.10	0.113	0.113		0.41	-0.021	-0.021		0.0141
11-10				1.0		40.0	-0.17	-0.17		1.16	0.0316	0.0316		0.04
9-12				1.0		40.0	-0.17	-0.17		1.16	0.0316	0.0316		0.04
11-13								-0.6		2.075	0.112	0.112		0.072
12-14								-0.6		2.075	0.112	0.112		0.072
13-14								-0.8	20.48	20.48	0.140			
11-12								-0.665		14.20	0.124	0.124		0.485
13-12								1.0		34.30	-0.186	-0.186		1.19
11-14								1.0		34.30	-0.186	-0.186		1.19
13-15												-0.6		2.88
14-16												-0.6		2.88
15-16												-0.8	20.48	20.48
13-14												-0.651		14.0
15-14												1.0		24.0
13-16												1.0		24.0
15-17														
16-18														
17-18														
15-16														
17-16														
15-18														
TOTAL	20.48	120.72			20.48	121.12			20.48	110.35			20.48	91.35

BARS	29	30	31	32	33	34	35	36	37	38	39	40	41	42	
BARS	$C_{87}S_7$	S_8	αS_8^2	S_0	$\alpha S_0 S_1$	$\alpha S_0 S_2$	$\alpha S_0 S_3$	$\alpha S_0 S_4$	$\alpha S_0 S_5$	$\alpha S_0 S_6$	$\alpha S_0 S_7$	$\alpha S_0 S_8$	S	S'	BARS
1-3				-1.5	12.96	-3.3	0.755	-0.143					-0.597	-0.636	1-3
2-4				0	0	0	0	0	0				0.913	0.864	2-4
1-2				-2.0	27.92	-7.125	1.64	0.308					-0.79	-0.848	1-2
3-4				-2.0	51.25	-38.1		0.423					0.135	0.12	3-4
1-4				2.5	40.0	-12.25	2.895	-0.528	0.096				0.960	1.06	1-4
3-2				0	0	0	0	0	0				-1.50	-1.44	3-2
3-5				-3.0		14.4	-8.3	0.625	-0.105				-2.28	-2.272	3-5
4-6				1.5		-7.2	1.66	-0.312	0.05				2.00	2.23	4-6
3-4				-2.0		30.1	-8.7	1.665	-0.28				0.135	0.12	3-4
5-6				-2.0		51.25							-0.0315	-0.03	5-6
3-6				2.5		60.0	-13.75	2.58	-0.42				1.218	1.286	3-6
5-4				0		0	0	0	0				-0.782	-1.214	5-4
5-7				-4.5			16.55	-8.03	0.52	-0.09			-3.75	-3.73	5-7
6-8				3.0			-10.38	2.02	-0.36	0.06			3.75	3.76	6-8
5-6				-2.0			39.0	-7.42	1.28	-0.22			-0.0365	-0.03	5-6
7-8				-2.0			51.25						-0.021	-0.02	7-8
5-8	-0.0002	-0.0002		2.5			85.80	-16.7	-2.02	0.51	-0.08		1.26	1.28	5-8
7-6	-0.0002	-0.0002		0			0	0	0	0	0		-1.25	-1.25	7-6
7-9	-0.0007	-0.0007		-6.0				17.86	-3.07	0.5	-0.09		-5.26	-5.262	7-9
8-10	-0.0007	-0.0007		4.5				-13.40	2.30	-0.37	0.07		5.239	5.238	8-10
7-8	-0.0007	-0.0007		-2.0				41.4	-7.06	1.2	-0.2		-0.021	-0.02	7-8
9-10				-2.0				51.25					0.076	0.08	9-10
7-10	0.0012	0.0012		2.5				100.0	-17.1	3.0	-0.5		1.265	1.27	7-10
9-8	0.0012	0.0012		0				0	0	0	0		-1.235	-1.23	9-8
9-11	0.0042	0.0042		-7.5					22.3	-3.8	0.70	-0.1	-6.70	-6.678	9-11
10-12	0.0042	0.0042		6.0					17.85	3.0	-0.60	0.07	6.79	6.822	10-12
11-12				-2.0					51.25				-0.49	-0.48	11-12
9-10	0.0047	0.0047	0.0007	-2.0					42.25	-7.2	1.34	-0.3	0.076	0.08	9-10
11-10	0.0071	0.0071	0.002	0					0	0	0	0	-1.27	-1.37	11-10
9-12	0.0071	0.0071	0.002	2.5					100.0	-17.0	3.16	0.71	1.122	1.13	9-12
11-13	-0.026	-0.025	0.0036	-9.0						31.1	-5.8	0.13	-8.68	-8.682	11-13
12-14	-0.025	-0.025	0.0036	7.5						-25.9	4.8	-0.10	7.82	7.818	12-14
13-14				6.0						-158.6			2.563	2.636	13-14
11-12	-0.028	-0.028	0.025	-2.0						42.3	-7.34	1.8	-0.49	-0.48	11-12
13-12	0.0417	0.0417	0.060	0						0	0	0	-0.528	-0.53	13-12
11-14	0.0417	0.0417	0.06	2.5						85.75	-16.0	3.5	1.972	1.97	11-14
13-15	0.184	0.184	0.144	-4.5							21.6	-4.8	-7.407	-7.446	13-15
14-16	0.184	0.184	0.144	9.0							-43.2	9.6	6.09	6.084	14-16
15-16	0.18			6.0							-158.5		-0.22	-0.232	15-16
13-14	0.146	0.146	0.68	6.0							-124.5	28.0	2.553	2.596	13-14
15-14	-0.224	-0.224	1.21	0							0	0	4.844	4.86	15-14
13-16	-0.224	-0.224	1.21	-7.5							-180.0	40.4	2.65	-2.64	13-16
15-17		-0.6	5.18	0								0	-1.758	-1.758	15-17
16-18		-0.6	5.18	4.5								-38.9	2.742	2.742	16-18
17-18		-0.8	11.20	0								0	-2.344	-2.344	17-18
15-16		-0.82	12.02	6.0								-117.5	-0.22	-0.232	15-16
17-16		1.0	19.20	0								0	2.93	2.93	17-16
15-18		1.0	19.20	-7.5								-144.0	-4.57	-4.57	15-18
TOTAL			75.54		140.12	85.775	162.35	175.972	110.9	-40.77	-502.56	-221.49			

S = STRESSES BY COMBINING REDUNDANTS

S' = STRESSES BY CONSISTANT DEFORMATIONS

$C_{21} = -\dfrac{\Sigma \alpha S_1 S_2}{\Sigma \alpha S_1^2} = -\dfrac{20.48}{80.39} = -0.255$ $\quad C_{32} = -\dfrac{20.48}{89.471} = -0.229$ $\quad C_{43} = -\dfrac{20.48}{109.05} = -0.188$ $\quad C_{54} = -\dfrac{20.48}{120.72} = -0.171$ $\quad C_{65} = -\dfrac{20.48}{121.12} = -0.1692$ $\quad C_{76} = -\dfrac{20.48}{110.35} = -0.186$ $\quad C_{87} = -\dfrac{20.48}{91.35} = -0.224$

$X_1 = -\dfrac{\Sigma \alpha S_0 S_1}{\Sigma \alpha S_1^2} = -\dfrac{140.12}{80.39} = -1.742$ $\quad X_2 = -\dfrac{85.775}{89.471} = -1.07$ $\quad X_3 = -\dfrac{162.35}{109.05} = -1.49$ $\quad X_4 = -\dfrac{175.972}{120.72} = -1.46$ $\quad X_5 = -\dfrac{110.9}{121.12} = -1.41$ $\quad X_6 = \dfrac{40.77}{110.35} = +0.37$ $\quad X_7 = \dfrac{502.55}{91.35} = +5.5$ $\quad X_8 = \dfrac{221.49}{75.54} = +2.93$

Table 3. Eighth degree redundant frame by consistent deformation method.

BAR	L/A	s	U_n	$U_n^2 \frac{L}{A}$	$U_1U_2 \frac{L}{A}$	$sU_n \frac{L}{A}$
1-3	14.40	-1.5	- .6	5.18		12.98
2-4	14.40	0	- .6	5.18		0
1-2	17.45	-2.0	- .8	11.15		27.90
3-4	32.0	-2.0	- .8	20.45	20.45	51.20
1-4	19.20	2.5	1.0	19.20		48.0
3-2	19.20	0	1.0	19.20		0
				79.96	$U_2U_3 \frac{L}{A}$	140.08
3-5	8	-3.0	- .6	2.88		14.40
4-6	8	1.5	- .6	2.88		-7.20
3-4	32	-2.0	- .8	20.45		51.20
5-6	32	-2.0	- .8	20.45	20.45	51.20
3-6	24	2.5	1.0	24.0		60.0
5-4	24	0	1.0	24.0		0
				93.86	$U_3U_4 \frac{L}{A}$	169.60
5-7	5.76	-4.5	- .6	2.08		15.57
6-8	5.76	3.0	- .6	2.08		-10.38
5-6	32.0	-2.0	- .8	20.45		51.20
7-8	32.0	-2.0	- .8	20.45	20.45	51.20
5-8	34.30	2.5	1.0	34.30		85.80
7-6	34.30	0	1.0	34.30		0
				112.86	$U_4U_5 \frac{L}{A}$	193.39
7-9	4.97	-6.0	- .6	1.79		17.90
8-10	4.97	4.5	- .6	1.79		-13.40
7-8	32.0	-2.0	- .8	20.45		51.20
9-10	32.0	-2.0	- .8	20.45	20.45	51.20
7-10	40.0	2.5	1.0	40.0		100.0
9-8	40.0	0	1.0	40.0		0
				123.68	$U_5U_6 \frac{L}{A}$	206.90
9-11	4.97	-7.5	- .6	1.79		20.40
10-12	4.97	6.0	- .6	1.79		-17.90
11-12	32.0	-2.0	- .8	20.45		51.20
9-10	32.0	-2.0	- .8	20.45	20.45	51.20
11-10	40.0	0	1.0	40.0		0
9-12	40.0	2.5	1.0	40.0		100.0
				123.68	$U_6U_7 \frac{L}{A}$	206.90
11-13	5.76	-9.0	- .6	2.08		31.10
12-14	5.76	7.5	- .6	2.08		-25.90
13-14	32.0	6.0	- .8	20.45	20.45	-153.50
11-12	32.0	-2.0	- .8	20.45		51.20
13-12	34.30	0	1.0	34.30		0
11-14	34.30	2.5	1.0	34.30	$U_7U_8 \frac{L}{A}$	85.80
				112.86		-11.30
13-15	8	-4.5	- .6	2.88		21.6
14-16	8	9.0	- .6	2.88		-43.2
15-16	32	6.0	- .8	20.45		-153.60
13-14	32	6.0	- .8	20.45	20.45	-153.60
15-14	24	0	1.0	24.0		0
13-16	24	-7.5	1.0	24.0		-180.0
				93.86		-508.60

TABLE 3 (concl.).

BAR	L/A	s	U_n	$U_n^2 \frac{L}{A}$	$U_1U_2 \frac{L}{A}$	$sU_n \frac{L}{A}$
15-17	14.40	0	-.6	5.18		0
16-18	14.40	4.5	-.6	5.18		-38.90
17-18	17.45	0	-.8	11.15		0
15-16	32.0	6.0	-.8	20.65		-153.60
17-16	19.20	0	1.0	19.20		0
15-18	19.20	-7.5	1.0	19.20		-144.0
				79.96		339.40

Table 4. Gauss's Elimination Method

Gauss's Elimination Method	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	$= K_1$	K_2
(1)	79.96	20.45							-140.08	
(2)	20.45	93.86	20.45						-169.60	
(3)		20.45	112.86	20.45					-193.39	
(4)			20.45	123.68	20.45				-206.90	
(5)				20.45	123.68	20.45			-206.90	
(6)					20.45	112.86	20.45		11.30	
(7)						20.45	93.86	20.45	508.60	
(8)							20.45	79.96	339.40	

EQUATION	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	K	Check
Equation (1)	79.96	20.45							-140.08	
Sum I = Eq. (1)	79.96	20.45							-140.08	
Equation (2)	20.45	93.86	20.45						-169.60	-34.84
Multi Sum I by -.2555	-20.45	-5.225							+36.0	+10.325
Sum II	0	88.635	20.45						-133.60	-24.515
Equation (3)		20.45	112.860	20.45					-193.39	-39.63
Multi Sum I by 0	0	0							0	0
Multi Sum II by -.231		-20.45	-4.725	0					30.80	5.625
Sum III	0	0	108.135	20.45					-162.59	-36.005
Equation (4)			20.45	123.68	20.45				-206.90	-42.32
Multi Sum I by 0	0	0							0	0
Multi Sum II by 0		0	0						0	0
Multi Sum III by -.189			-20.45	-3.87					+30.7	6.38
Sum IV			0	119.81	20.45				-176.20	-35.94
Equation (5)				20.45	123.68	20.45			-206.90	-42.32
Multi Sum I by 0	0	0							0	0
Multi Sum II by 0		0	0						0	0
Multi Sum III by 0			0	0					0	0
Multi Sum IV by -.1715				-20.45	-3.51				30.2	6.24
Sum V				0	120.17	20.45			-176.7	-36.08
Equation (6)					20.45	112.86	20.45		11.30	165.06
Multi Sum I by 0										
Multi Sum II by 0										
Multi Sum III by 0										

Table 4 (concl.)

EQUATION	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	K	Check
Multi Sum IV by 0										
Multi Sum V by -0.17					-20.45	-3.48			30.0	6.07
Sum VI					0	109.38	20.45		41.30	171.13
Equation (7)						20.45	93.86	20.45	508.60	643.36
Multi Sum I by 0										
Multi Sum II by 0										
Multi Sum III by 0										
Multi Sum IV by 0										
Multi Sum V by 0										
Multi Sum VI by -.187						-20.45	-3.82		-7.725	-31.995
Sum VII						0	90.04	20.45	500.875	611.363
Equation (8)							20.45	79.96	339.40	439.81
Multi Sum I by 0										
Multi Sum II by 0										
Multi Sum III by 0										
Multi Sum IV by 0										
Multi Sum V by 0										
Multi Sum VI by 0										
Multi Sum VII by -0.227							-20.45	-4.64	-113.70	-138.79
Sum VIII							0	75.32	225.7	301.02

ANALYSIS OF INDETERMINATE STRUCTURES
BY COMBINING REDUNDANTS

by

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ABSTRACT

In the analysis of highly indeterminate structures, the task of setting up and solving the elastic equations becomes time-consuming. Several methods have been developed to reduce the work considerably. This paper illustrates the method of combining redundants. By suitably combining the redundant forces, an orthogonalized form of simultaneous equations is achieved. These can easily be solved for unknown redundant forces. Three illustrative examples are solved. To compare this method with another method, two of the problems are also solved by the method of consistent deformations.

This method of combining redundants has great advantage when the bar stresses are to be calculated for different loading conditions. For flexural structures, it is well suited. This method does not show any computational advantage, when the structure is highly indeterminate. In such cases, accuracy is also affected since more operation of multiplications are involved.