

SUGGESTED GOALS FOR CULTIVATING INDIVIDUAL
CREATIVENESS IN GEOMETRY AT THE
HIGH SCHOOL LEVEL

by

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TABLE OF CONTENTS

	PAGE
THE PROBLEM	1
Introduction	1
Statement of the Problem	3
Importance of the Study	3
Limitations and Procedures	4
Definitions	5
REVIEW OF THE LITERATURE	7
GOALS AND SUGGESTED PROBLEMS	12
Goals	12
Problems	17
CONCLUSIONS	27
ACKNOWLEDGEMENTS	30
BIBLIOGRAPHY	31

THE PROBLEM

Introduction

Over the last half century there has been a continually changing amount of emphasis placed on the creative individual and the development of his potentialities. Only in the past ten to fifteen years has there been a surge of interest in the creative student and the failure of our schools to develop his full potential.¹

Prior to the last decade or so, the creative student, in general, was considered to be a trouble-maker in the classroom. The creative student was not usually recognized for his true abilities and potentialities. Instead he was noted as the individual with the unorthodox views, unwilling to conform to the teacher-accepted standard of how a student should act and think.²

Before the World War II period, the bulk of our scientific knowledge dealt with the discovery of basic concepts. Since that time we have realized that a knowledge of the basic concepts is not enough to maintain our rate of scientific growth. It has become apparent that what are needed are more men who can integrate these basic concepts into new and useful ideas.

Since the beginning of time itself we have had discoverers and inventors. The true creators of knowledge have been few and, in general,

¹Regan Carpenter, "Creativity: Its Nature and Nurture," Education, 82:391, March, 1962.

²Jacob W. Getzels and Phillip W. Jackson, Creativity and Intelligence (New York: John Wiley & Sons, Inc., 1962), pp. 30-37.

have not been recognized and appreciated in their own time.

The advent of the Space Age has probably done more to awaken us to the need for creative individuals than any other event in history. What we really need in this day and age are individuals who can work on "true intuition." They must be able to develop ideas for which there may be no immediate need, but which do establish relationships between existing concepts. They must be able to imagine what does not even exist. As Donald N. Frey stated in an address delivered at the forty-fifth annual convention of the National Association of Secondary-School Principals, "True intuition is like a leap forward in the dark, across an empty space, which ends with one finding himself on solid ground again."³

How do we find these individuals? We must develop them. To some, creativity is an innate gift and is expressed naturally throughout life. For others, the potentiality is ever present but needs proper care and development to be realized.⁴

What better place is there to help the student to realize this potentiality than in the high school geometry classroom? The study of geometry in the high school has most generally been defended as a course in reasoning. In no field other than pure logic does an individual learn to present facts with such strict order and precision.⁵

³Donald N. Frey, "Developing Creativity," The Bulletin of the National Association of Secondary-School Principals, 45:421, April, 1961.

⁴Hughes Mearns, Creative Power: The Education of Youth in the Creative Arts (second edition; New York: Dover Publications, Inc., 1958), pp. 263-272.

⁵Jack D. Wilson, "Trends in Geometry," The Mathematics Teacher, 46:67, February, 1953.

The study of geometry is one of the student's first contacts with a fundamental method of mathematics, or of any science. It is at this point that it should be possible to cultivate in the student the nature of inquiry to ask such questions as "Why - - -," "What if - - -," "How - - -." What can we as educators do to help cultivate creativeness in our students? Is it possible to accomplish this through the study of geometry?

Statement of the Problem

It was the purpose of this study to (1) make a general review of the available literature on the meaning, goals, and methods of creativity and the literature on the goals and methods of teaching geometry; (2) make a comparison of the goals of creativity and the goals of geometry and attempt an integration of these findings into their common factors; and (3) make basically personal suggestions of the types of lesson planning and classroom procedures which will enable the classroom teacher to present material so as to realize the goals of teaching geometry and teaching for creativity, thus cultivating individual creativeness in the classroom.

Importance of the Study

It is recognized that our schools need to help individuals cultivate their creative potential. To date the main area of concern has been the study of the nature of creativity and its identification.⁶ Few

⁶Jacob W. Getzels and Phillip W. Jackson, Creativity and Intelligence (New York: John Wiley & Sons, Inc., 1962).

studies have been made on techniques which would be applicable to the classroom situation in mathematics for cultivating creativeness in each individual student.⁷

It is generally agreed that every individual, especially the young child, possesses some degree of creative ability.⁸ Creativity is not something which can be taught, but it can be developed to a certain degree in all individuals.⁹ But, as many times happens, this creative ability is discouraged quite early in the child's school experiences.¹⁰ What are needed at the present time are methods of reactivating these latent abilities and encouraging their natural development and expression.

It has been the attempt of this study to suggest such methods which will enable the high school teacher of geometry to cultivate in the students a realization, understanding, and appreciation of their creative abilities.

Limitations and Procedures

This study was limited to high school mathematics, particularly the study of geometry. The literature dealing with the study of geometry was mainly limited to articles in The Mathematics Teacher and similar professional journals. Actual textbook material was not used in this

⁷Morris I. Stein and Shirley J. Heinze, Creativity and Intelligence (Glencoe, Ill.: The Free Press, 1960), pp. 54-58.

⁸Carpenter, op. cit., p. 392.

⁹Ibid.

¹⁰E. Paul Torrance, Guiding Creative Talent, (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1962), p. 125.

study since the main objective was not to review the material, but the methods of presentation and types of problems adaptable for specific consideration.

The procedures of this study were those of library research. A review was made of the more important literature pertaining to the theories of creativity and the methods of identifying the creative student. A review was made of the literature pertaining to the goals of teaching geometry and its implications for the student. A great deal of literature was reviewed concerning the types of problems that individual classroom teachers have found to be successful in aiding students to realize and appreciate the hidden relationships and implications of geometry.

An attempt was then made to seek out the common factors of the goals of creativity and geometry. On the basis of these findings, certain types of problems were developed which would enable the classroom teacher to present material with the aims of both creativity and geometry in mind.

There was no actual classroom experimentation with the suggestions offered. Suggestions were made mainly on the basis of the findings of the common factors. It should be noted that personal opinions of the writer greatly influenced many of the suggestions.

Definitions

In general, the technical terms pertaining specifically to the study of geometry will be defined in the context. Due to the prime importance of the terms "geometry" and "creativity," they are defined

here as they are used in this study.

Geometry - Due to the nature of this study it is necessary to define geometry as an integration of plane, solid, analytic, descriptive, demonstrative, and intuitive geometry. It is by means of this notion of geometry that we are able to help the student "discover" the relationships and appreciate the complexity of geometry.

Creativity - Creativity will be regarded as the establishment of a relationship between accumulated information and the current situation, such that the individual adds something new to what already exists for himself. It is not necessary that the experience of the individual be novel and of significant value to society. It is necessary only that the experience of the individual be a genuine insight to the relationships mentioned, and not details that have been related to the student by the teacher for memorization.

REVIEW OF THE LITERATURE

Just what is creativity? Carl R. Rogers offers this explanation. "(It) is the emergence in action of a novel relational product, growing out of the uniqueness of the individual on the one hand, and the materials, events, people, or circumstances of his life on the other."¹ Creativity is the process of "bringing something new into birth."² Creativity can also be said to be the process of perceiving missing and disturbing elements, forming and testing hypotheses concerning these elements, and communicating the results.³

During the last ten to fifteen years there has been a marked increase in the number of studies and writings on creativity, a great change over the first half of this century.⁴ Stoddard comments, "Creativity came close to being a lost cause in American education."⁵ He goes on to say that "progressive education . . . helped to revive its spirit."⁶ Other causes for the recent increased interest in creativity

¹Carl R. Rogers, "Toward a Theory of Creativity," Creativity and Its Cultivation, Harold H. Anderson, editor (New York: Harper & Brothers, 1959), p. 71.

²Rollo May, "The Nature of Creativity," Creativity and Its Cultivation, Harold H. Anderson, editor (New York: Harper & Brothers, 1959), p. 57.

³E. Paul Torrance, Guiding Creative Talent (Englewood Cliffs, N.J., Prentice-Hall, Inc., 1962), p. 16.

⁴May, op. cit., p. 55.

⁵George D. Stoddard, "Creativity in Education," Creativity and Its Cultivation, Harold H. Anderson, editor (New York: Harper & Brothers, 1959), p. 181.

⁶Ibid.

stem from our "mortal struggle for survival of our way of life in the world."⁷ The advent of the age of space with the need for accelerated advances in technology and the shrinking world have also caused a surge of interest in our needs for a greater number of more creative individuals.

In his book, Guiding Creative Talent,⁸ Torrance presents a very clear picture of why educators should be extremely interested in the process of creativity and its development in our school youth. It is self-evident that a problem does exist for educators, since a universally accepted aim of education is to develop a complete, functioning personality of each individual. Man learns naturally through a creative process of combining and reorganizing known facts with the current situation to find solutions to his problems. No individual can develop to his maximum capacity if he is continually encouraged, forced, or ignored into submerging his inherent creative nature. Man's most valuable resource in coping with life is his creative nature. It is the duty of the school to see that each individual know, understand, use, and appreciate his creative potential.

The creative person is certainly a unique individual. This is not so surprising when it is aphoristic that each individual is unique. Hence, the basis is laid for Meams' theory that every individual is creative in his own way. Each of us has some sort of gift, be it large

⁷J. P. Guilford, "Traits of Creativity," Creativity and Its Cultivation, Harold H. Anderson, editor (New York: Harper & Brothers, 1959), p. 142.

⁸Torrance, op. cit., pp. 2-7.

or small.⁹ Other writers do not agree with this universal approach to creativity.¹⁰ Only if one considers creativity in its very broadest sense can it be said that each individual is creative.

Therefore it has been the attempt of a great many studies to characterize the individual who expresses his creativity more openly. A typical creative individual, if such exists, can be said to persist in certain traits which set him apart from the "average" individual. Lowenfeld has given various criteria or attributes of creativity which significantly differentiate the creative individual from his less- or non-creative counterpart.

1. The creative individual has a unique sensitivity to problems. His is the ability to recognize missing elements of a situation and to formulate given data into novel problematic situations.
2. The creative individual possesses a unique fluency of ideas. In a situation, such as a "brain-storming session," it is the creative individual who comes up with more than his share of ideas.
3. The creative individual is quite flexible in adapting to new situations with little or no loss of effort.
4. One of the more commonly known characteristics of the creative individual is his originality of ideas.
5. Other important characteristics of the creative individual are his ability to redefine and rearrange objects in a new way, the ability to abstract, an ability for synthesis, and a unique coherence of organization.¹¹

These are basically the same findings of an independent study conducted by J. F. Guilford.¹²

⁹Hughes Mearns, Creative Power: The Education of Youth in the Creative Arts (second edition; New York: Dover Publications, Inc., 1958), p. 164.

¹⁰Erich Fromm, "The Creative Attitude," Creativity and Its Cultivation, Harold H. Anderson, editor (New York: Harper & Brothers, 1959), p. 45.

¹¹Viktor Lowenfeld, "Current Research on Creativity," NEA Journal, 47:538-540, November 1958.

¹²Guilford, op. cit., pp. 145-152.

Knowing that a definite problem concerning creativity does exist for educators, it is natural to ask if creativity can actually be taught or cultivated in the classroom. According to an old cliché, gifted children should be taught only by gifted teachers. At the present time too little is known about learning processes to add that creative teachers make for creative students. There is no evidence that one characteristic in a teacher makes for given characteristics or qualities in a student.¹³ On the other hand, Mearns has this to say about the creative teacher, that is the teacher who cultivates the creative potential of his students:

"The peculiar mark of the creative teacher—as different from all other business of man—is not his learning ability alone but his ability to transform others by the contagion of his own creative powers."¹⁴

So it is that if the teacher is to cultivate creativeness in the individual students he must first recognize, use, and have faith in his own creative ability.

Zirbes,¹⁵ in Spurs to Creative Teaching, considers it a well-established assumption that creativity can be cultivated in youth and to a certain degree in adults. No one as yet knows a method of teaching for creativity in any particular classroom situation, but education can do a great deal in promoting and encouraging creative performance.¹⁶

¹³Getzels, op. cit., p. 131.

¹⁴Mearns, op. cit., p. 267.

¹⁵Laura Zirbes, Spurs to Creative Teaching (New York: G.P. Putnam's Sons, 1959), p. 3.

¹⁶Getzels, op. cit., p. 123.

A basic problem faced in teaching for creativity is that more concern is shown for "cramming into his (the student's) mind than drawing out of it."¹⁷ To develop creativity a student must have the opportunity to express his judgments and opinions. The individual becomes creative, or not, according as he meets with satisfaction and reward in his attempt to express himself.¹⁸ Creativity can become a habit.¹⁹

In each of the foregoing hypothetical explanations of creativity one can note certain common elements. There exists a felt need and an attempt is made to fulfill this need, from experiences, in a new and successful way.

¹⁷Janet E. Forslund, "An Inquiry into the Nature of Creative Teaching," Journal of Education, 143:73, April, 1961.

¹⁸Clarence Leuba, "A New Look at Curiosity and Creativity," The Journal of Higher Education, 29:134, March, 1958.

¹⁹Ibid.

GOALS AND SUGGESTED PROBLEMS

Goals

In teaching geometry, as in teaching any subject, a prerequisite is the establishment of certain goals or objectives of the course. Each teacher will undoubtedly have a different interpretation of the specific objectives of his or her class, in accordance with the type of class and the long range objectives of the students. But there are certain general objectives for a study of geometry in the high school that every teacher should have in mind when making an outline for the course.

In 1908, W. E. Bond briefly listed the aims for the teaching of geometry, as follows:

1. The pupil should acquire an accurate, thorough knowledge of geometrical truths.
2. The pupil should develop the power of original, logical, geometrical reasoning.
3. The pupil should acquire a habit of thought which will give him a practical sagacity; which will develop his judgment, increase his resourcefulness, and fit him to cope more successfully with the many and varied problems of his after life; which will teach him to take a many-sided view of things, so that if the avenue of attack is blocked he shall be able to promptly, cheerfully, and successfully attack from another quarter.¹

Bond considers the first of these aims to be the least important. This is in accord with the statement by Hoag and Loflin that facts may be unimportant, but the ability to discover them is of prime importance.²

¹W. E. Bond, "The Aims in Teaching Geometry and How to Attain Them," The Mathematics Teacher, 1:30, September, 1908.

²Jessie Mae Hoag and Zeke L. Loflin, "Goals in Geometry," The Bulletin of the National Association of Secondary-School Principals, 38:14, May, 1954.

In a study by Brown, it was found that although geometry teachers did consider it important to present a knowledge of the facts and principles of geometry, they considered it to be more important to "develop the habit of clear thinking and precise expression."³

Geometry is an ideal place to introduce to students the idea and ability of abstraction.⁴ There is a great deal of intuitive appeal in geometry which enables the teacher to choose examples that are easily understood by the student. Also, geometry is one portion of mathematics in the high school that has great intellectual depth.

Thus it is that a prime objective of teaching geometry may be to develop in students the ability to sense the existence of problematic situations in an abstract world; then to consider solutions to the problem in a logical manner until a clear, concise attempt has been hypothesized, tested, and accepted as a solution to the problem.

In a study by Brown,⁵ five hundred teachers of high school geometry, selected at random from a register of the National Council of Teachers of Mathematics, were asked for their opinions of various reasons for teaching geometry in the high school. Listed below are the reasons most frequently ranking in the category of the five most important reasons for teaching geometry. Following each reason is the percentage of teachers who included it as one of the five most important.

³Kenneth E. Brown, "Why Teach Geometry?," The Mathematics Teacher, 43:105, March, 1950.

⁴Robert R. Christian, "A New Role for High School Geometry," The Mathematics Teacher, 53:433, October, 1960.

⁵Brown, op. cit., pp. 103-106.

1. To develop the habit of clear thinking and precise expression. (94%)
2. To give a knowledge of the facts and principles of geometry. (48%)
3. To prepare for the study of science and advanced mathematics. (42%)
4. To develop the inquiring or questioning attitude of mind. (41%)
5. To develop the ability to analyze a complex situation into simpler parts. (40%)
6. To develop mental habits and attitudes that are needed in life situations. (36%)

In a follow-up of this study, students were asked to rate the reasons for studying geometry in their order of importance. It is interesting to note that "To develop the habit of clear thinking and precise expression," although considered by far the most important by the teachers, was seldom mentioned by the students.⁶

The goals of creativity deal mainly with the personality of the individual and his outlook on life. Creativity is, in many ways, an abstract idea. As such, the goals of creativity are basically abstract ideas. The main objective of creativity itself is to enable the individual to express his true self in all his work, freely and completely.

The goals of teaching for creativity are to encourage this freedom of individual expression. From a review of the literature it is apparent that the goals of creativity are not as easily exemplified as are those of teaching geometry. It can be seen, though, that there are certain things that a teacher must accomplish with his or her students in order to cultivate creativeness in their lives.

No place in the literature reviewed was the writer able to find the goals of teaching creativity listed or exemplified as in the case of

⁶Brown, *op. cit.*, p. 105.

the goals of geometry. Thus the goals of creativity as given below are the writer's ideas on the subject. The basis for the following goals lies in the fact that there are certain characteristics found to differentiate known creative students from their less- or non-creative counterparts. It is the belief of the writer that the development of these characteristics in individuals constitute the goals of teaching for creativity.

First, the student must realize and be encouraged in freedom of expression in the classroom. Some efficiency may be lost by this freedom, but an over-emphasis on efficiency in the classroom can kill creative thinking.⁷ By encouraging the free expression in the classroom, the teacher is accomplishing two purposes. The student is finding a certain degree of success with his ideas and is rewarded in the sense that he is not rejected for being different and having "far out" ideas. He will soon come to recognize that nonconformity is not totally undesirable.

Second, the student must be able to learn and "discover" for himself. The student should not be encouraged in thinking of the teacher as an encyclopedia with all the answers. To a certain degree, the teacher should be hesitant about giving direct answers to the questions of his students. The student should be given the least amount of guidance by the teacher which will enable the student to "discover" the answer for himself.

⁷Louis J. Rubin, "Teaching for Creativity," California Journal of Secondary Education, 35:490, December, 1960.

Third, in this process of "discovering for himself" the student should learn the methods of brainstorming. The criticism of ideas should be withheld, with the emphasis on the quantity of ideas, rather than on the quality. The student should recognize that the wilder the idea the better, as the ideas can be combined and improved later in the process.

Brainstorming sessions can do much to help cultivate creativeness in the students. Brainstorming can accomplish a very important goal of creativity; that is, to develop in the student the inquiring nature to ask such questions as "What if - - -," "How - - -," "Why - - -." But the student needs to learn a bit of self-discipline in this process. It is imperative that the student not lose contact with reality. The power of concentration must be enhanced so that the student will be able to perceive intuitively.

Fourth, the student needs encouragement and a recognition and belief in his individual worth and importance. The student needs acquaintances of wider knowledge and experience to aid him in the clarification of his goals and purposes in life.

Fifth, and last, but no at all the least important, is the need for the teachers to be able to recognize and have faith in his or her own abilities, especially their creative abilities.

There are many similarities in the goals of teaching geometry and in the goals of teaching for creativity. These common goals are the ones with which this study is concerned, making it possible for the classroom teacher of geometry to accomplish the goals of both, simultaneously.

The following objectives are common to both creativity and geometry.

If a teacher can cultivate these abilities and concepts in his or her students, the major goals of both geometry and creativity can be accomplished simultaneously.

1. The student must develop the ability of clear, concise thinking. He must be able to express his ideas in a complete, precise manner.

2. The ability to "discover" for one's self in terms of original ideas must be developed. As a student progresses through a class, and his educational career in general, he must come to depend less and less on the teacher to supply all the answers. He must be ready, willing, and able to construct, test, reconstruct, and retest his own concepts until he finds a solution to his problem.

3. The student must develop the ability to abstract from one situation to another. He must be able to use the material he learns in one situation, and the procedures he uses to "discover" this material, in life situations in general. His methods of logical thinking and expression should carry over into all he does.

4. The abilities of analysis and synthesis should become second nature to the student in his search for further knowledge. Complex situations should not be any more difficult and discouraging than simple problems. The knowledge of bits of information should make it clear what new concepts can be developed and proven by the process of synthesis.

5. The student should recognize, use, and appreciate his abilities. The prospect of being different and "odd" should not be discouraging. In fact, it should at times come to be a stimulant to attain higher goals.

The accomplishment of these goals, along with the recognition by the teacher of the worth and importance of each individual, and the encouragement of the individual in his strivings, should, in the opinion of the writer, be significant in cultivating individual creativeness in the classroom.

Problems

The traditional method of proof for theorems and corollaries in geometry is a very highly structured and formal proof. Usually the student makes a statement of the given data, what is to be proven, and a

brief analysis of the procedure he intends to use. Next follows the presentation of the proof in a series of statements and justifications of the statements.

Except for very short and simple proofs, this method tends to be quite long and many times vague concerning the thought processes of the student. The student needs to have a great many theorems, corollaries, axioms, and postulates memorized and at his command so that he may provide the justifications for the statements of the proof.

Ness⁸ suggests a very good method of proof which the writer considers an excellent procedure for cultivating the habit in students of "clear thinking and precise expression." Ness has used this method of proof in his geometry classes and estimates that they learned as much or more than the students from previous years using the traditional method of proof.⁹

Following is an exhibition of a relatively simple proof using the traditional method of proof and the method suggested by Ness which will hereafter be referred to as the "flow diagram" method. In this presentation it is clear how easily the thought processes of the student can be seen in the use of the flow diagram method.

Anyone who has had a course in high school geometry is well acquainted with the traditional method of proof, and hence it needs no explanation here as to the procedures and methods used.

⁸Harold M. Ness, Jr., "A Method of Proof for High School Geometry," The Mathematics Teacher, 55:567-569, November, 1962.

⁹Ibid., p. 568.

As can easily be seen from this presentation, the traditional method of proof can be a long and drawn-out procedure for even a relatively simple proof. To present the statements and reasons in a satisfactory manner, the student must have the previous theorems memorized, quite often at the expense of understanding.

A note of explanation is in order for the flow diagram method of proof. First, it will be noticed that the statements having no arrows leading into them are the data which can be derived from the hypothesis of the theorem. The proof is obviously much shorter and at first sight may seem incomplete. On closer observation one can see that all arrows leading into the statement $\angle EBC = \angle DCB$, sum up the steps 3-5 of the traditional method of proof. The thought process by which the student arrived at this statement is more obvious and not so formalized as in the traditional method. A proof of this nature can be summed up quite readily when one needs to make a brief statement of the proof.

The main disadvantage to be found with this method is the possibility of difficulty in remembering theorems which have been previously proven. Ness noted this among students who were acquainted with the method after having used the traditional method for some time.¹⁰ It is the opinion of the writer that a great deal of this difficulty would be overcome if the students were introduced to the flow diagram earlier in the year. Hence, there would be less dependence on the procedures of the traditional form.

Another difficulty which may be encountered with the flow diagram

¹⁰Ness, op. cit., p. 568.

procedure is the inability to make the proof very artistic and appealing to the eye. Here again, with a little acquaintance and practice in the method and procedures, this could be greatly overcome.

The preceding problem, worked out by the flow diagram method, accomplishes many of the combined goals mentioned in the preceding section. To be able to make a flow diagram of a proof such as that presented on page 19, the student must be able to think quite clearly and express his thoughts in a concise manner. Through this method the student's ability to synthesize is certainly challenged and developed. He must be able, without the aid of quoted theorems, to have the basic concepts of previously proven theorems at hand if his flow diagram is to proceed in a logical, orderly manner. For the same reason, the student must be able to analyze a situation very carefully and thoroughly.

The realization of the goals of creativity and geometry, as stated in the previous section, is sometimes very subtle. Often a specific problem or method of solving a problem places emphasis on the establishment of one goal.

Consider, for example, the flow diagram method of proof. The emphasis here is on the abilities of analysis and expression. But to find a solution of proof to a theorem, the student must have a certain amount of insight to "new" ways of proof, since the flow diagram method does not emphasize any given procedure. Hence, the student's ability to "discover for himself" an original proof is encouraged. To be able to diagram the proof, the student's abilities of analysis and expression must certainly be functioning to full capacity. The student's ability of abstraction will be broadened, as will his realization and use of his

creative abilities, just by the fact that he has succeeded in an original expression of his knowledge.

Thus, to realize one goal of creativity is to realize all five goals, specifically mentioned in the preceding section, because of their interrelatedness and inseparability.

Another theorem which might be considered at this point is directly connected to the theorem stated on page 19 concerning the bisectors of an isosceles triangle. If not properly cautioned, the student may take it for granted that, since the angle bisectors of an isosceles triangle are equal, any triangle having two equal angle bisectors is isosceles.

This, in fact, happens to be the case. It most certainly is intuitively obvious. But to present a rigorous proof of this theorem is quite another matter. To present this problem to a class of students can accomplish many purposes. First, in the search for a proof of this theorem the student is likely to call on a great deal of, if not all, his knowledge of geometry and thus review and reinforce his understanding tremendously.

But more important, the student will undoubtedly call upon all his creative powers to seek original approaches to a solution. There are numerous approaches which can be made to this problem. It may be that none of the students will arrive at a satisfactory solution. But the student will have many approaches which he tries and follows through to completion or until he can see that the approach will not ultimately arrive at a solution.

A yet different approach may be made to the proof of the problem

suggested above. This is the method of proof used by the SMSG mathematics. The method involves a paragraph approach to the proof. This is sometimes very difficult to do if one is to stress conciseness and clarity. But this is just the thing the creative student should develop. A paragraph approach to the proof of a theorem helps the individual to develop the ability to express new ideas in a complete, concise manner.

Following is a paragraph proof of the converse of the theorem presented on page 19:

Theorem. Any triangle having two equal interior angle bisectors is isosceles.

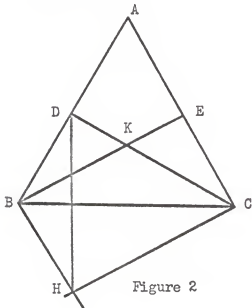


Figure 2

Given: BE bisects $\angle ABC$
 CD bisects $\angle ACB$
 $BE = CD$

To prove: $\triangle ABC$ is isosceles

Proof:

In considering angles $\angle ABC$ and $\angle ACB$, there are only three possibilities. Either angle $\angle ABC$ is less than, equal to, or greater than angle $\angle ACB$. If $\angle ABC = \angle ACB$, then $\triangle ABC$ is isosceles and the theorem is proven. Hence, assume that $\angle ABC$ is less than $\angle ACB$. Then $\angle CBE$ is less than $\angle BCD$ and CE is less than ED .

To complete the proof, draw a line through C , parallel to BE , and a line through B , parallel to CE , intersecting at H and forming parallelogram $BECH$. Join D and H .

Now, $CH = BE = CD$. Then $\triangle CDH$ is isosceles, and $\angle CHD = \angle CDH$.

$BH = CE$ less than ED which implies $\angle BHD$ is less than $\angle EDH$.

Add the angles at D and H . $\angle BHC$ is greater than $\angle BDC$ or $\angle BEC$ is greater than $\angle BDC$.

Consider triangles $\triangle BDK$ and $\triangle CKE$. $\angle BEC$ greater than $\angle BDC$ implies $\angle KCE$ less than $\angle KBD$, since $\angle DKB = \angle EKC$. Or $\angle ACB$ is less than $\angle ABC$, since $\angle KCE$ and $\angle KBD$ are halves of $\angle ACB$ and $\angle ABC$, respectively. $\angle ABC$ greater than $\angle ACB$ is a contradiction of the original assumption that $\angle ABC$ is less than $\angle ACB$.

By a similar argument, the assumption that $\angle ABC$ is greater than $\angle ACB$ leads to a contradiction.

Hence: $\angle ABC = \angle ACB$; $AC = AB$; and triangle ABC is isosceles by definition of an isosceles triangle.

As was stated previously, this is not a simple proof. But by giving the student an opportunity to wrestle with such a problem, the teacher can understand the particular abilities and weakness of the individual in respect to the ability to "come up with" ideas. It seems to the writer that this problem would be particularly good for a brainstorming session. As was stated earlier in this study, the emphasis is on the quantity of ideas in a brainstorming session. By receiving the viewpoints of many students on a problem of this nature, the class as a whole can discuss the ideas and find at what points the methods break down in logical reasoning, thus encouraging their own abilities to think logically and express ideas in a concise manner.

Many times it seems the teacher tends to pass over such problems as angle trisection because they cannot be solved. But here again is a type of problem that can be used to develop the brainstorming techniques so important to the individual's creativeness. Through the discussion of a problem such as angle trisection the student can develop his insight to problematic situations. The student will undoubtedly come up with all sorts of ways the problem "might be" solved. In classroom discussion the ideas can be considered and it can be determined at just what point the various techniques break down in logical reasoning. This will certainly help the individual to develop his own abilities in checking through his reasoning procedures to see what mistakes he makes. A problem of this nature also helps the student in understanding that there

are problems in mathematics which cannot be solved, which is a common misconception among many students of mathematics.

Students need to realize that there is more than one correct way to solve a problem in geometry. Wiseman¹¹ makes some very good suggestions along this line.

Take, for example, the theorem which states:

If two points are equidistant from the ends of a straight line segment, then the two points are on the perpendicular bisector of the line segment.

More often than not, this theorem would be solved using a figure such as that given in Figure 3 below. But if the theorem is stated in another equivalent manner, a completely new method of proof becomes obvious. The student's ability to transfer his knowledge and understanding from one situation to another is encouraged. Consider the statement of the theorem in this manner:

If two circles intersect and there exists a common chord, then their line of centers is the perpendicular bisector of the common chord. (See Figure 4.)

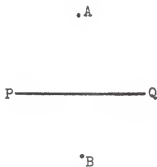


Figure 3

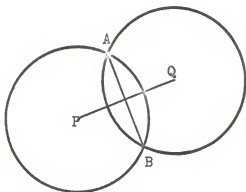


Figure 4

¹¹John D. Wiseman, Jr., "Some Related Theorems on Triangles and Circles," The Mathematics Teacher, 54:14-16, January, 1961.

There are many more problems of this nature that can be stated in various ways to give the student a realization of the relationships between the different components of geometry. Many theorems in plane geometry are adaptable to use in solid geometry.

The central idea in all such problems is to help the student understand and appreciate the complexity and yet the simplicity of geometry. Many of these ideas can be carried over into life itself. As W. E. Bond so clearly stated, it should be the aim of a course in geometry to teach the student to "take a many-sided view of things, so that if the avenue to attack is blocked he shall be able to promptly, cheerfully, and successfully attack from another quarter."¹²

¹²Bond, loc. cit.

CONCLUSIONS

It was found that there is a vast amount of literature available on the nature of creativity. Many studies have been made concerning the characteristics of individuals known to possess high creative abilities. But there was a shortage of literature dealing with the problem of actually developing these characteristics in the "average" individual. Although it is generally agreed that all individuals have some degree of creative ability, it appears that there is not enough concern with the realization of these abilities.

In regard to the goals and methods of teaching geometry, the literature was quite abundant, since any individual who has ever written on the teaching of geometry has his own ideas as to the goals of geometry and how they should be attained. As a result of the review of the literature, the following objectives were adopted for this report as the goals of teaching geometry:

1. The development of the habit of clear thinking and precise expression.
2. The development of a knowledge and appreciation of the facts and principles of geometry and the preparation of the student for the study of science and advanced mathematics.
3. The development of the inquiring or questioning attitude of mind.
4. The development of the ability to analyze a complex situation into simpler parts.
5. The development of a habit of thought which will give the student a practical sagacity; which will develop his judgment, increase his resourcefulness, and fit him to cope more successfully with the many and varied problems of his after life; which will teach him to take a many-sided view of things, so that if the avenue of attack is blocked he shall be able to promptly, cheerfully, and successfully attack from another quarter.

The goals of creativity, as used in this study, were the characteristics found by various studies to be common to those individuals

known to possess high degrees of creative ability. The actual goals of creativity as used were:

1. The encouragement of freedom of expression in the classroom and a realization by the student of the importance of this freedom.
2. The encouragement of the ability of the student to "discover" for himself and the lessening dependence of the student upon the teacher to provide the answers.
3. The encouragement of the methods of brainstorming in solving problems, that is, the quantity of original ideas is more important than the quality of ideas, since they can be critically evaluated later.
4. The encouragement of the individual's recognition of his own worth and importance in creativity.
5. The belief of the teacher in his own abilities of creativity.

When these characteristics, or goals, were compared with the goals of teaching geometry it was found that certain goals or individual characteristics are desirable for the attainment of the objectives of both creativity and geometry. The combined goals as presented in this report were:

1. The student must develop the ability of clear, concise thinking. He must be able to express his ideas in a complete, precise manner.
2. The ability to "discover" for one's self in terms of original ideas must be developed. As a student progresses through a class, and his educational career in general, he must come to depend less and less on the teacher to supply all the answers. He must be ready, willing, and able to construct, test, reconstruct, and retest his own concepts until he finds a solution to his problem.
3. The student must develop the ability to abstract from one situation to another. He must be able to use the material he learns in one situation, and the procedures he uses to "discover" this material, in life situations in general. His methods of logical thinking and expression should carry over into all he does.
4. The abilities of analysis and synthesis should become second nature to the student in his search for further knowledge. Complex situations should not be any more difficult and discouraging than simple problems. The knowledge of bits of information should make it clear what new concepts can be developed and proven by the process of synthesis.
5. The student should recognize, use, and appreciate his abilities. The prospect of being different and "odd" should not

be discouraging. In fact, it should at times come to be a stimulant to attain higher goals.

The suggestions made to attain the goals of geometry and creativity consist of a flow diagram method of proof which allows more individual freedom. A theorem is suggested which at first seems very obvious, but is not easy to prove and again encourages originality in search for a proof. Two other types of problems are mentioned which show the relationships between different aspects of geometry. One of these is the problem of angle trisection and the other is a problem concerning the relationships between certain theorems concerning triangles, which can be stated in terms of circles.

It is the conviction of the writer that any methods or specific problems which allow individual freedom of expression, based on original discovery, can be used to realize the combined goals of creativity and geometry if these goals are recognized and accepted throughout the period of instruction. Every problem and method that is used by the instructor must be carefully conceived and analyzed with these objectives in mind so that no procedure is used without a definite idea of what role it is to play in the realization of the combined goals.

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SUGGESTED GOALS FOR CULTIVATING INDIVIDUAL
CREATIVENESS IN GEOMETRY AT THE
HIGH SCHOOL LEVEL

by

DAVID L. ROSS

B. S., Kansas State University, 1962

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of

requirements for the degree

MASTER OF SCIENCE

School of Education

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1963

This study was mainly concerned with the meaning of creativity and its goals in relation to the goals of a course in high school geometry. It was the purpose of this study to (1) make a general review of the available literature on the meaning, goals, and methods of creativity and the literature on the goals and methods of teaching geometry; (2) make a comparison of the goals of creativity and the goals of geometry and attempt an integration of these findings into their common factors; and (3) make basically personal suggestions of the types of lesson planning and classroom procedures which will enable the classroom teacher to present material so as to realize the goals of teaching geometry and teaching for creativity, thus cultivating individual creativeness in the classroom.

For the purposes of this study, creativity was considered to be "the establishment of a relationship between accumulated information and the current situation, such that the individual adds something new to what already exists for himself."

This study was limited to library research with no classroom experimentation of the principles or suggestions used.

During the last ten to fifteen years there has been a marked increase in concern for the development of creativeness in our students. Creativity should definitely be of great concern to educators today, if they are to encourage the development of every individual to his fullest capacity.

Many studies have been made to determine the unique characteristics of the creative individual. Many of these studies have operated under the assumption that creativity can be developed in all individuals,

especially in the young child and to a more limited degree in adults. But many of the studies have lacked an explanation of just what it is that can be developed.

Such was the nature of this study to determine the factors which are characteristic of creative individuals and can be developed in each student, keeping in mind at the same time the goals of geometry. The common goals as found by this study are:

1. The ability of clear thinking and precise expression
2. The ability to discover and express originality
3. The ability of abstraction
4. The abilities of analysis and abstraction
5. A recognition and appreciation of the above abilities, with no fear of being "different"

A number of problems and methods of proof have been suggested which should enable the teacher to accomplish these goals, and help the student recognize, use, and appreciate his creative talents. Among these suggestions are a method of proof explained by Ness in The Mathematics Teacher. This method consists of a "flow diagram" procedure which makes it easier to follow the thought processes of the student and makes it quite necessary for the individual to have a clear understanding of all the geometric principles used in the proof, rather than a memorization of theorems and postulates.

Other suggestions included the use of such problems as angle trisection to encourage the understanding and use of brainstorming techniques. Problems showing the relationships between theorems concerning triangles and circles were used to help the student in understanding relationships and to develop his abilities of abstraction from one situation to another.

In conclusion, it was found that there was a great deal of literature available in both the areas of creativity and geometry. But the emphasis of the literature dealing with creativity seemed to lie with an analysis of the nature of creativity in individuals already known to possess high degrees of creative ability. There definitely needs to be more research regarding the cultivation of potential creative abilities in the "average" individual. It is the conviction of the author that as long as the teacher realizes and accepts the goals of creativity and the need for their development in each individual, any procedure which encourages individual freedom based on original discovery can be used to advantage in realization of the combined goals of creativity and geometry.

The results of this study have been of great value to the writer and have perhaps helped others who read the study to recognize the problem and encourage them to pursue the development of individual creativeness in their own classes.