FREE FIELD CALIBRATION OF ACCOUSTICAL DEVICES

by

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NOMENCLATURE

a	equivalent spherical radius
A_0	constant
A	constant
B	adiabatic bulk modulus
B_0	constant
C	electrical capacitance
c	speed of sound
d	distance
E	voltage
f	frequency
F_{air}	force due to acoustic pressure
$\overline{G}$	body forces
h	scale factor
H	electric field strength, reciprocity constant
i	$\sqrt{-1}$
I	current
k	angular wave number $k = \omega / c$
K	spring constant
ℓ	length of wire winding
L	electrical inductance
m	mass
M_0	V/p
M_s	I/p
p_0	constant mean pressure

NOMENCLATURE (cont.)

p_0	constant mean pressure
p	acoustic pressure $p = p' - p_0$
p'	instantaneous pressure
q	charge
$\dot{q}$	current
r	radial spherical coordinate
R	function of r, radial length, electrical resistance
R_m	air resistance coefficient
R'_m	air resistance coefficient
S	condensation $S = (\rho' - \rho) / \rho$
S_σ	p/I
S_s	p/V
t	time
T	function of time
U	electroacoustic transducer
$\overline{V}$	particle velocity
x	displacement
$\dot{x}$	velocity
Y	admittance matrix
Z	acoustic impedance
Z_x	acoustic impedance at point x
$\overline{Z}$	impedance matrix
α	constant

NOMENCLATURE (concl.)

Δ	Laplacian	$\Delta = \nabla \cdot \nabla$
θ	phase angle	
λ	wave length	
ρ	constant mean density	
ρ'	instantaneous density	
ϕ	velocity potential	
ω	radial frequency	

INTRODUCTION

The purpose of this report is to examine in detail the theory, assumptions, and limitations of a free field method of calibrating electroacoustical transducers [1]. By the term electroacoustical transducer is meant a microphone or loudspeaker. A microphone is calibrated when the relation between acoustic pressure and induced open circuit voltage is known. A loudspeaker is calibrated when the relation between driving current and resulting acoustic pressure some distance from the loudspeaker is known. By free field is meant a space in which acoustic waves are not deflected in any manner.

THEORY

In the absence of sources and sinks the general equation of continuity for a fluid is [2]

$$\mathrm{div}\,(\rho'\,\overline{V}) + \frac{\partial \rho'}{\partial t} = 0 \qquad (1)$$

The general hydrodynamical equation for an ideal fluid is

$$\overline{G} - \frac{1}{\rho}\,\mathrm{grad}\,p' = \frac{\partial \overline{V}}{\partial t} + \frac{1}{2}\,\nabla v^2 - \overline{V} \times \mathrm{curl}\,\overline{V}\,. \qquad (2)$$

For air the body forces, $\overline{G}$, may be neglected. For a spherical source in a free field curl $\overline{V}$ is zero. Also, since V is small, V^2 may be ignored.

With these facts and assumptions in mind (2) is

$$-\frac{1}{\rho}\ \text{grad}\ p' = \frac{\partial V}{\partial t}. \tag{3}$$

Since $\text{curl}\ V = 0$, it is reasonable to assume that

$$\overline{V} = +\ \nabla\phi. \tag{4}$$

It is also true that $\nabla p' = \nabla p$. With these two substitutions (3) is

$$-\frac{1}{\rho}\ \nabla p = \frac{\partial}{\partial t}\ \nabla\phi\ .$$

From this it is seen that

$$p = -\ \rho\frac{\partial\phi}{\partial t}\ . \tag{5}$$

It is true that

$$S = \frac{\rho' - \rho}{\rho}\ ,$$

therefore it is true that

$$\rho' = \rho S + \rho.$$

Substitution of this into (1) gives

$$\text{div}(\rho S\overline{V} + \rho\overline{V}) + \frac{\partial}{\partial t}(\rho S + \rho) = 0.$$

It is now assumed that S is small; consequently $\rho S\overline{V}$ is neglibible compared with $\rho\overline{V}$. Remembering that ρ is a constant it is seen that

$$\rho\ \text{div}\ \overline{V} + \rho\frac{\partial S}{\partial t} = 0,$$

which reduces to

$$\text{div}\overline{V} + \frac{\partial S}{\partial t} = 0.$$

Substitution of $\nabla\phi$ for $\overline{V}$ yields

$$\Delta\phi + \frac{\partial S}{\partial t} = 0. \qquad (6)$$

Now a relation between p and S is needed. The bulk modulus of elasticity of a fluid is defined as [4]

$$B = - dp' / \frac{d\,vol}{vol} .$$

However, the density increases as the volume decreases and

$$\frac{d\,vol}{vol} = - \frac{d\,\rho'}{\rho} = - \frac{\rho' - \rho}{\rho} = - S.$$

Now the bulk modulus is

$$B = \frac{d\,p'}{S} .$$

But dp' is merely the acoustic pressure p. Therefore

$$p = SB.$$

The velocity of sound may be written as [2]

$$c^2 = \frac{B}{\rho} .$$

Now the acoustic pressure may be written as

$$p = S\rho c^2 .$$

Solving this for S and substituting from (5) for p it is true that

$$S = - \frac{1}{c^2} \frac{\partial\phi}{\partial t} .$$

Substitution of this into (6) gives

$$\Delta\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = 0 \qquad (7)$$

Equation (7) is the wave equation in terms of the velocity potential function. Once ϕ has been found, the acoustic pressure and particle velocity follow from (5) and (4), respectively.

Radiation from a Spherical Source

In what follows, a spherical sound source is assumed so that

$$\phi = \phi\ (r, t).$$

Thus for our purposes

$$\Delta = \frac{1}{h_1\ h_2\ h_3}\ \frac{\partial}{\partial r}\left[\frac{h_2\ h_3}{h_1}\ \frac{\partial}{\partial r}\right] .$$

The scale factors, h_i, may be computed from

$$h_i = \left[\left(\frac{\partial x}{\partial x_i}\right)^2 + \left(\frac{\partial y}{\partial x_i}\right)^2 + \left(\frac{\partial z}{\partial x_i}\right)^2\right]^{1/2}$$

The coordinate transformation is

$$\begin{aligned} x &= r \sin\theta \cos\phi \\ y &= r \sin\theta \sin\phi \\ z &= r \cos\phi . \end{aligned}$$

For purposes of identification note that

$$\begin{aligned} x_1 &= r \\ x_2 &= \theta \\ x_3 &= \phi . \end{aligned}$$

From this it follows that

$$h_1 = 1$$
$$h_2 = r$$
$$h_3 = r \sin \theta .$$

Now the Laplacian, Δ, is

$$\Delta = \left[\frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} \right] .$$

Application of this to (7) gives

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{2}{r} \frac{\partial \phi}{\partial r} - \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} = 0 . \tag{8}$$

Now solve (8) by the method of separation of variables.

Assume that

$$\phi(r, t) = R(r)\, T(t).$$

Substituting this into (8) it is seen that

$$T \frac{\partial^2 R}{\partial r^2} + \frac{2}{r} T \frac{\partial R}{\partial r} - \frac{1}{c^2} R \frac{\partial^2 T}{\partial t^2} = 0.$$

Multiplying by c^2 and dividing by TR gives

$$c^2 \left[\frac{1}{R} \frac{\partial^2 R}{\partial r^2} + \frac{2}{rR} \frac{\partial R}{\partial r} \right] = \frac{1}{T} \frac{\partial^2 T}{\partial t^2} = - \omega^2 . \tag{9}$$

In (9) ω^2 is a constant which will turn out to be the angular frequency associated with the sound propagation.

From (9) it is seen that

$$\frac{1}{T} \frac{\partial^2 T}{\partial t^2} = - \omega^2 \tag{10}$$

$$\left[\frac{c^2}{R} \frac{\partial^2 R}{\partial r^2} + \frac{2}{r} \frac{\partial R}{\partial r} \right] = - \omega^2. \tag{11}$$

It can be verified by direct substitution that a solution of (10) is

$$T = A_0 \; e^{\pm i\omega t} \; . \tag{12}$$

It can be similarly verified that a solution of (11) is

$$R = \frac{B_0}{r} \; e^{\pm i \frac{\omega r}{c}} \; . \tag{13}$$

Now

$$\phi \, (r, t) = \frac{B_0}{r} \; e^{\pm i \frac{\omega r}{c}} \; A_0 \; e^{\pm i \omega t}$$

A convenient way to write this is

$$\phi = \frac{A}{r} \; e^{+ i (\omega t \pm \frac{\omega r}{c})} \; . \tag{14}$$

But note that

$$\lambda f = c$$

$$f = \frac{c}{\lambda}$$

$$\frac{\omega}{2\pi} = \frac{c}{\lambda}$$

$$\frac{\omega}{c} = \frac{2\pi}{\lambda}$$

$$\frac{\omega}{c} = k. \tag{15}$$

With this substitution (14) is

$$\phi = \frac{A}{r} \; e^{+ i (\omega t \pm kr)}$$

Only the outward bound wave is retained since the calibration will be done in a free field.

$$\phi = \frac{A}{r} \; e^{+ i (\omega t - kr)} \; . \tag{16}$$

Substitution of this expression for ϕ into (5) gives

$$p = - \rho \, i \omega \, \phi. \tag{17}$$

Substituting for ϕ from (16) into (4) gives

$$\overline{V} = \frac{\partial}{\partial r} \frac{A}{r} e^{+i(\omega t - kr)} .$$

Taking the indicated partial derivative and simplifying gives

$$\overline{V} = -\hat{n} \left[ik + \frac{1}{r} \right] \phi . \tag{18}$$

Of course $\overline{V}$ is directed radially outward from the source.

Now the specific acoustic impedance is

$$Z = \frac{-\rho i \omega \phi}{-ik \left[1 - \frac{i}{kr} \right] \phi} .$$

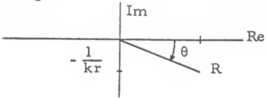
This reduces to

$$Z = \frac{\rho \omega}{k} \frac{1}{1 - \frac{i}{kr}} .$$

Substituting for $\frac{\omega}{k}$ from (15) gives

$$Z = \rho c \frac{1}{1 - \frac{i}{kr}} . \tag{19}$$

Now change the form of the denominator of (19)



$$R = \left[1 + \frac{1}{k^2 r^2} \right]^{1/2}$$

$$R = \left[\frac{k^2 r^2 + 1}{k^2 r^2} \right]^{1\;2} \tag{20}$$

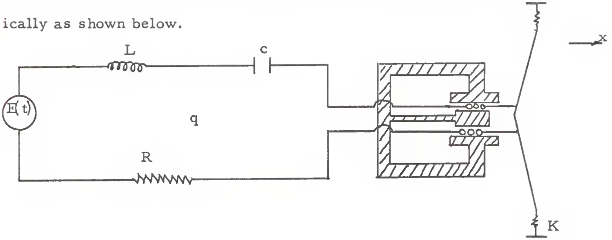
$$\theta = \tan^{-1} \frac{1}{kr} \tag{21}$$

With this terminology (19) is

$$Z = \frac{\rho c k r}{(1 + k^2 r^2)^{1/2}} e^{+i\theta} . \tag{22}$$

Moving Coil Electrodynamic Transducer

In order to make the proofs of certain electroacoustical reciprocity theorems more concrete consider in particular moving coil transducers. Such a transducer may be represented schematically as shown below.



From this sketch it is seen that the electrical equation of motion is

$$L\ddot{q} + R\dot{q} + \frac{1}{c}q = E(t) - H\ell\dot{x}\,.$$

The corresponding mechanical equation is

$$m\ddot{x} + R_m\dot{x} + Kx = -\ H\ell\dot{q} - \text{Fair}.$$

Written together in matrix form these equations are

$$\begin{bmatrix} L & 0 \\ 0 & m \end{bmatrix}\begin{Bmatrix} \ddot{q} \\ \ddot{x} \end{Bmatrix} + \begin{bmatrix} R & H\ell \\ H\ell & R_m \end{bmatrix}\begin{Bmatrix} \dot{q} \\ \dot{x} \end{Bmatrix} + \begin{bmatrix} \frac{1}{c} & 0 \\ 0 & K \end{bmatrix}\begin{Bmatrix} q \\ x \end{Bmatrix} = \begin{Bmatrix} E(t) \\ -\text{Fair} \end{Bmatrix}. \tag{23}$$

Now determine what Fair is. Substituting ϕ from (16) into (17) gives

$$p = \frac{(-\rho i\omega A)}{r} e^{+i(\omega t-kr)} . \tag{24}$$

From this it is seen that

$$|\bar{V}| = \frac{(-i\rho\omega A)}{rZ} e^{+i(\omega t-kr)} .$$

Therefore at the surface of a loudspeaker of equivalent spherical radius "a" it is true that

$$\dot{x} = \frac{(-i\rho\omega A)}{aZ_a} e^{+i(\omega t-ka)} .$$

Whence

$$(-i\rho\omega A) = Z_a \dot{x} a\, e^{-i(\omega t-ka)} . \tag{25}$$

Substituting this into (24) yields

$$p = \frac{Z_a \dot{x} a}{a} e^{-i(\omega t-ka)} e^{+i(\omega t-ka)}$$

for the acoustic pressure at the loudspeaker surface. This simplifies to

$$p = Z_a \dot{x} .$$

Assume that the acoustic pressure is constant over the surface of the loudspeaker; therefore

$$\text{Fair} = -4\pi a^2 \dot{x}.$$

Now define

$$R'_m = R_m + 4\pi a^2 Z_a .$$

With this substitution (23) is

$$\begin{bmatrix} L & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{q} \\ \ddot{x} \end{Bmatrix} + \begin{bmatrix} R & H\ell \\ H\ell & R'_m \end{bmatrix} \begin{Bmatrix} \dot{q} \\ \dot{x} \end{Bmatrix} + \begin{bmatrix} \frac{1}{c} & 0 \\ 0 & K \end{bmatrix} \begin{Bmatrix} q \\ x \end{Bmatrix} = \begin{Bmatrix} E(t) \\ 0 \end{Bmatrix} . \qquad (26)$$

Consider only steady state voltage of the form

$$E(t) = \mathrm{Re}\,(E_o \; e^{+i\omega t})$$

and steady state output of the form

$$\begin{Bmatrix} q \\ x \end{Bmatrix} = \mathrm{Re}\; e^{+i\omega t} \begin{Bmatrix} \bar{c} \\ \bar{\alpha} \end{Bmatrix} .$$

With these two substitutions (26) may be written as

$$[\bar{Z}] \begin{Bmatrix} q \\ x \end{Bmatrix} = e^{+i\omega t} \begin{Bmatrix} E_o \\ 0 \end{Bmatrix} .$$

This may be represented as

$$\begin{Bmatrix} q \\ x \end{Bmatrix} = e^{+i\omega t} \; [Y] \begin{Bmatrix} E_o \\ 0 \end{Bmatrix} . \qquad (27)$$

Note that $(\bar{Z})$ is symmetric, (Y) is symmetric.

For future convenience, stipulate that for transducer 1 it is true that

$$(Y_1) = \begin{bmatrix} \bar{a} & b \\ b & e \end{bmatrix} , \qquad (28)$$

and for transducer 2, it is true that

$$(Y_2) = \begin{bmatrix} \bar{A} & B \\ B & D \end{bmatrix} . \qquad (29)$$

For future convenience also note that for an impressed current

$$I(t) = \mathrm{Re}\,(I_0 \; e^{+i\omega t})$$

the mechanical equation of motion, from (26) is

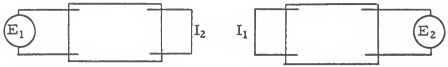
$$m\ddot{x} + H\ell I_0 e^{+i\omega t} + R'_m \dot{x} + Kx = 0 . \qquad (30)$$

Reciprocity

Now prove three reciprocity theorems for a free field moving coil transducer electroacoustic system. In the sketches below the boxes represent the electroacoustical system. There are two transducers involved in these theorems. They are connected to each other only acoustically. The subscript 1 refers to the first transducer while the subscript 2 refers to the second transducer.

Theorem 1.

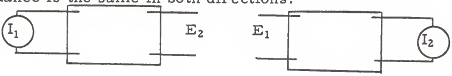
In a linear electroacoustic system the short circuit transfer admittance is the same in both directions.



$$\frac{I_2}{E_1} = \frac{I_1}{E_2}$$

Theorem 2.

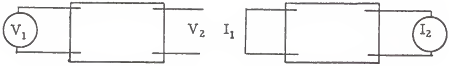
In a linear electroacoustic system the open circuit transfer impedance is the same in both directions.



$$\frac{E_2}{I_1} = \frac{E_1}{I_2}$$

Theorem 3.

In a linear electroacoustic system the open circuit transfer ratio in one direction is equal to the short circuit transfer ratio in the other direction.



$$\frac{V_2}{V_1} = \frac{I_1}{I_2}$$

Now prove Theorem 1. Let two reversible transducers face each other at a distance d. Let transducer 1 be driven by a voltage $E_0 \, e^{+i\omega t}$.

By (27) we have

$$\begin{Bmatrix} \dot{q}_1 \\ \dot{x}_1 \end{Bmatrix} = i\omega \, (Y_1) \begin{Bmatrix} E_0 \\ 0 \end{Bmatrix} e^{+i\omega t} \; .$$

Making use of (28) this gives

$$\dot{x}_1 = i\omega b \, E_0 \, e^{+i\omega t} \; .$$

Now substitution from (25) into (24) gives

$$p = \frac{Z_a \, \dot{x} \, a}{r} \, e^{-ik(r-a)} \; . \tag{31}$$

Therefore the acoustic pressure at the face of transducer 2 is

$$p_d = \frac{Z_{a_1} \, i\omega b E_0 \, a_1}{d} \, e^{+i\left[\omega t - k(d-a_1)\right]} \; . \tag{32}$$

Now the force driving transducer 2 is

$$Fair_d = \alpha \, 4 \pi a_2^2 \, p_d \; . \tag{33}$$

With (27) as a model it is true that

$$\begin{Bmatrix} \dot{q}_2 \\ \dot{x}_2 \end{Bmatrix} = i\omega (Y_2) \begin{Bmatrix} 0 \\ \text{Fair}_d \end{Bmatrix} .$$

Making use of (29) it is seen that

$$\dot{q}_2 = i\omega \, B \, \text{Fair}_d,$$

and finally

$$\dot{q}_2 = \frac{i\omega B \alpha 4\pi a_2^2 \, Za_1 \, i\omega bE_0 \, a_1 \, e^{+i[\omega t - k(d-a_1)]}}{d} . \qquad (34)$$

Similarly if the same voltage is now applied at transducer 2 the current induced at transducer 1 is

$$\dot{q}_1 = \frac{i\omega b \, \alpha \, 4\pi a_1^2 \, Z_{a_2} \, i\omega BE_0 \, a_2}{d} e^{+i[\omega t - k(d-a_2)]} . \qquad (35)$$

Now from (34) and (35) it is seen that $|\dot{q}_1| = |\dot{q}_2|$ if

$$a_1 \; |Z_{a_2}| = a_2 \; |Z_{a_1}| .$$

By (22) this is

$$\frac{a_1 \rho c k a_2}{\sqrt{1 + k^2 a_2^2}} = \frac{a_2 \rho c k a_1}{\sqrt{1 + k^2 a_1^2}} ,$$

which reduces to

$$\frac{1}{\sqrt{1 + k^2 a_2^2}} = \frac{1}{\sqrt{1 + k^2 a_1^2}} .$$

With (15) in mind, note that

$$\sqrt{1 + k^2 a_2^2} \doteq 1$$

$$\sqrt{1 + k^2 a_1^2} \doteq 1 .$$

Therefore $|\dot{q}_1| = |\dot{q}_2|$ as required.

Now prove Theorem 2. Let the two transducers face each other at a distance d as before.

Let transducer 1 have an impressed current $I_0 e^{+i\omega t}$. By (3) it is seen that

$$m_1 \ddot{x}_1 + H_1 \ell_1 I_0 e^{+i\omega t} + R'_{m_1} \dot{x}_1 + K_1 x_1 = 0.$$

Assuming an output of the form of the input it is seen that

$$\dot{x}_1 = \frac{-i\omega I_0 H_1 \ell_1 e^{+i\omega t}}{-\omega^2 m_1 + i R'_{m_1} \omega + K_1} .$$

In view of (31) the acoustic pressure at the face of transducer 2 is

$$p_d = \frac{Z_{a_1} a_1}{d} \frac{-i\omega I_0 H_1 \ell_1 e^{+i[\omega t - k(d-a_1)]}}{-\omega^2 m_1 + i R'_{m_1} \omega + K_1} . \qquad (36)$$

Now the driving force at transducer 2 is

$$Fair_d = \alpha 4\pi a_2^2 p_d .$$

The mechanical equation of motion of transducer 2 is

$$m_2 \ddot{x}_2 + R'_{m_2} \dot{x}_2 + K_2 x_2 = Fair_d$$

from which it is seen that

$$\dot{x}_2 = \frac{i\omega Fair_d}{-\omega^2 m_2 + i R'_{m_2} \omega + K_2} . \qquad (37)$$

But $V_2 = - H_2 \ell_2 \dot{x}_2$, therefore

$$V_2 = \frac{- H_2 \ell_2 i \omega \alpha 4\pi a_2^2 Z_{a_1} a_1(- i\omega I_0 H_1 \ell_1 e^{+i[\omega t - k(d-a_1)]})}{(-\omega^2 m_2 + i R'_{m_2} \omega + K_2) \quad d(-\omega^2 m_1 + i R'_{m_1} \omega + K_1)} . \qquad (38)$$

Similarly with the same impressed current at transducer 2 the induced voltage at transducer 1 is

$$V_1 = \frac{-H_1 \ell_1 i \omega \alpha 4 \pi a_1^2 Z_{a_2} a_2 (-i \omega I_0 H_2 \ell_2) e^{+i[\omega t - k(d-a_2)]}}{(-\omega^2 m_1 + i R'_{m_1} \omega + K_1) d (-\omega^2 m_2 + i R'_{m_2} \omega + K_2)} . \quad (39)$$

Looking at (38) and (39) it is seen that $|V_2| = |V_1|$ if $a_2 |Z_{a_1}| = a_1 |Z_{a_2}|$ which has been shown to be the case approximately. Therefore $|V_2| = |V_1|$ as required.

Now prove Theorem 3. Let the two transducers be positioned as before. Impressing a voltage $E_0 e^{+i\omega t}$ on transducer 1 it is seen by (33) and (32) that

$$\mathrm{Fair}_d = \alpha 4 \pi a_2^2 \frac{a_1}{d} Z_{a_1} i \omega b E_0 e^{+i[\omega t - k(d-a_1)]}$$

In view of (36) and the fact that $V_2 = -H_2 \ell_2 x_2$ it is true that

$$V_2 = \frac{-H_2 \ell_2 i \omega \alpha 4 \pi a_2^2 a_1 Z_{a_1} i \omega b E_0}{(-\omega^2 m_2 + i R'_{m_2} \omega + K_2) d} e^{+i[\omega t - k(d-a_1)]} .$$

Now

$$\frac{|V_2|}{|V_1|} = \frac{H_2 \ell_2 i \omega \alpha 4 \pi a_2^2 a_1 |Z_{a_1}| i \omega b}{-\omega^2 m_2 + i R'_{m_2} \omega + K_2} . \quad (40)$$

With a current $I_0 e^{+i\omega t}$ impressed on transducer 2, the acoustic pressure at the face of transducer 1 is, using (36) as a guide

$$P_d = \frac{a_2 Z_{a_2} \quad i\omega I_0 H_2 \ell_2 \quad e^{+i[\omega t - k(d-a_2)]}}{d(\omega^2 m_2 + i R'_{m_2} + K_2)} \quad . \tag{41}$$

Now with (27) as a model it is seen that

$$\begin{Bmatrix} q_1 \\ x_1 \end{Bmatrix} = e^{+i\omega t} (Y_1) \begin{Bmatrix} 0 \\ Fair_d \end{Bmatrix}$$

and with (28) this gives

$$\dot{q}_1 = i\omega b \, Fair_d \, .$$

The proper substitutions yields

$$\dot{q}_1 = \frac{I\omega b\alpha 4\pi a_1^2 \; a_2 Z_{a_1} (-i\omega I_0 H_2 \ell_2)}{d(-\omega^2 m_2 + i R'_{m_2} + K_2)} e^{+i[\omega t - k(d-a_2)]}$$

Now

$$\frac{|q_1|}{|I_2|} = \frac{i\omega b\alpha 4\pi a_1^2 \; a_2 |Z_{a_1}| i\omega H_2 \ell_2}{d(-\omega^2 m_2 + i R'_{m_2} + K_2)} \quad . \tag{42}$$

Looking at (40) and (42) it is seen that

$$\frac{|V_2|}{|V_1|} = \frac{|\dot{q}_1|}{|I_2|}$$

if $a_2 |Z_{a_1}| = a_1 |Z_{a_2}|$ as already shown to be the case approximately.

Reciprocity Constant

In what follows, a quantity known as the reciprocity constant will be of importance. Consider one reversible electromagnetic transducer.

First place the transducer in a plane wave sound field. The sound pressure at the transducer surface is p, the induced open circuit voltage is V. Define

$$M_0 = V/p.$$

Using the same transducer as a source drive it with a current $I_0 \sin\omega t$, and measure the acoustic pressure p_2 at any distance d. Define

$$S_0 = p_2 / I_0 \sin\omega t.$$

The reciprocity constant, H, is defined as

$$H = \frac{|M_0|}{|S_0|} .$$

Now find H in terms of basic parameters. Assume that the transducer has an equivalent spherical radius a.

From what has been done previously it is seen that, (37),

$$V = - H\ell \frac{i\omega 4\pi a^2}{-\omega^2 m + i R'_m \omega + K} p .$$

Therefore

$$|M_0| = \frac{+ H\ell\, i\omega\, 4\pi a^2}{-\omega^2 m + i\omega R'_m + K} .$$

By (36)

$$p_2 = -\frac{a}{d} Z_a \frac{i\omega H \ell I_0}{-\omega^2 m + i R'_m \omega + K} e^{+i[\omega t - k(d-a)]} .$$

Therefore

$$|S_0| = \frac{i\omega a H \ell |Z_a|}{d(-\omega^2 m + i\omega R'_m + K)},$$

and

$$H = \frac{H \ell i \omega 4\pi a^2}{-\omega^2 m + i\omega R'_m + K} \frac{d(-\omega^2 m + i\omega R'_m + K)}{i\omega a H \ell |Z_a|} .$$

This reduces to

$$H = \frac{4\pi a d}{|Z_a|} ,$$

or

$$H = \frac{4\pi a d \sqrt{1 + k^2 a^2}}{\rho c k a} .$$

But

$$\sqrt{1 + k^2 a^2} = 1$$

$$k = \frac{2\pi}{\lambda} .$$

Now

$$H = \frac{4\pi a d \lambda}{\rho c a 2\pi} ,$$

or

$$H = \frac{2 d \lambda}{\rho c} . \qquad (43)$$

The quantity H is seen to be independent of the design factors of the transducer.

For future reference define two more quantities. Place the transducer in a plane wave sound field. The sound pressure at the transducer surface is p, the induced current is $I(t)$.

Define

$$M_s = I/p.$$

Drive the transducer with a voltage $V(t)$; the acoustic pressure at some distance d is p_2. Define

$$S_s = p_2 / V.$$

Consequences of Reciprocity

Consider two reversible, transducers U and U' facing each other, separated by a distance d. U is at point 0, U' is at point D.

Drive U with a current $I(t)$. The acoustic pressure at D is $S_0 I(t)$. Therefore the induced voltage at D is

$$V' = M_0' \; S_0 \, I(t).$$

Drive U' with a current $I'(t)$. The acoustic pressure at 0 is $S_0' \; I'(t)$. The induced voltage at 0 is

$$V = M_0 \, S_0' I'(t).$$

It has been shown previously that, Theorem 2,

$$\frac{V'}{I'} = \frac{V}{I}.$$

Therefore

$$M_0' \; S_0 = M_0 \; S_0' . \qquad (44)$$

Drive U with a current $I(t)$. The acoustic pressure at D is $S_0 \, I(t)$. The induced current at U' is

$$I'(t) = M_s' \; S_0 \; I(t).$$

Drive U' with a voltage $V'(t)$. The acoustic pressure at O is $S'_s V'(t)$. The induced voltage at U is

$$V(t) = M_0 S'_s V'(t).$$

By Theorem 3

$$\frac{V(t)}{V'(t)} = \frac{I'(t)}{I(t)} ,$$

therefore $M_0 S'_s = M'_s S_0$. (45)

Drive U with a voltage $V(t)$. The acoustic pressure at D is $S_s V(t)$. The induced voltage at U' is

$$V'(t) = M'_0 S_s V(t).$$

Drive U' with a current $I'(t)$. The acoustic pressure at O is $S' I'(t)$. The induced current at U is

$$I = M_s S'_0 I'(t).$$

By Theorem 3

$$\frac{I(t)}{I'(t)} = \frac{V'(t)}{V(t)} ,$$

therefore $M_s S'_0 = M'_0 S_s$. (46)

Drive U with a voltage $V(t)$. The acoustic pressure at D is $S_s V(t)$. The induced current at U' is

$$I'(t) = M'_s S_s V(t).$$

Drive U' with a voltage $V'(t)$. The acoustic pressure at O is $S'_s V'(t)$. The induced current at U is

$$I(t) = M_s S'_s V'(t).$$

By Theorem 1

$$\frac{I'(t)}{V(t)} = \frac{I(t)}{V'(t)} ,$$

therefore $M'_s S_s = M_s S'_s$. (47)

In summary

$$\begin{aligned} M'_0 \; S_0 &= M_0 \; S'_0 \\ M_0 \; S'_s &= M'_s \; S_0 \\ M_s \; S'_0 &= M'_0 \; S_s \\ M'_s \; S_s &= M_s \; S'_s \; . \end{aligned}$$

By algebraic manipulation

$$\frac{S_0}{M_0} = \frac{S'_0}{M'_0} = \frac{S_s}{M_s} = \frac{S'_s}{M'_s} \quad . \tag{48}$$

This last equation says that the ratio M_0 / S_0 is independent of the configuration of the transducer. Therefore this ratio may be computed exactly by assuming an equivalent spherical transducer.

FREE FIELD CALIBRATION

Outline of Steps

Now that the proper foundation has been laid, consider the actual calibration of transducers. Three transducers will be considered; U to be calibrated, U_x a reversible transducer, U_y a sound generator.

Step 1

The transducers U_y and U are placed facing each other at a known distance d. With U_y as the source there is an unknown acoustic pressure, p_2, at the face of U. Record V, the induced open circuit voltage on U. Now $p_2 = V/M_0$. Without changing U_y in any manner whatsoever replace U with U_x. Record V_x, the induced open circuit voltage on U_x. Then $p_2 = V_x/M_0^x$. By equating the two acoustic pressures it is seen that

$$M_0^x = M_0 \frac{V_x}{V} . \tag{49}$$

Step 2

Place U where U_y was; U_y is no longer needed. Drive U_x with a measured current I_x. The unknown acoustic pressure at the face of U is p_1; measure V_1, the induced voltage on U. Now $p_1 = S_0^x I_x$, therefore $V_1 = M_0 S_0^x I_x$. Or

$$M_0 = V_1 / S_0^x I_x . \tag{50}$$

But $H = |M_0| / |S_0|$, and in view of (48) and (43) it is true that

$$\frac{M_0^x}{S_0^x} = \frac{2d\lambda}{\rho c} ,$$

or

$$M_0^x = \frac{2d\lambda S_0^x}{\rho c} . \tag{51}$$

Equations (50) and (51) yield

$$M_0^x \; M_0 = \frac{2 d \lambda \, S_0^x}{\rho \, c} \quad \frac{V_1}{S_0^x \, I_x} \; .$$

Substituting for M_0^x from (49) gives

$$M_0 \; \frac{V_x}{V} \; M_0 \; = \frac{2 d \lambda \, V_1}{\rho \, c \, I_x} \; ,$$

or

$$M_0 \; = \left[\frac{2 d \lambda}{\rho \, c} \quad \frac{V \; V_1}{V_x \; I_x} \right]^{\frac{1}{2}} . \qquad (52)$$

Equation (52) gives the calibration of U as a microphone in terms of fundamental parameters and measured electrical quantities.

To obtain the calibration of U as a speaker proceed as follows:

$$\frac{M_0}{S_0} = H = \frac{2 d \lambda}{\rho \, c}$$

$$S_0 = M_0 \; \frac{\rho \, c}{2 d \lambda}$$

$$S_0 = \left[\frac{\rho \, c}{2 d \lambda} \quad \frac{V \; V_1}{V_x \; I_x} \right]^{\frac{1}{2}} \qquad (53$$

Equation (53) gives the calibration of U as a speaker. Now the transducer U is calibrated completely.

Sample Calibration

In order to illustrate the procedure outlined above a 2-1/2-inch electromagnetic loudspeaker was calibrated. The three speakers used have a Burstein-Applebee Stock No. 22B182.

The following page shows the data collected. The current I_x was not recorded. It was sent through a 14 ohm resistor, and v_x the voltage drop across the resistor was recorded. The voltmeter used has a full-scale reading of 3.0 volts at a dial setting of 1.0. The observed voltages were left in relative form since only ratios of them are used. The units used are as explained in References [3] and [4]. The values $\rho c = 39.3\ \text{g/cm}^2\ \text{sec}$ and $c = 34,550$ cm/sec were computed as explained in [3]. Now compute M_0 by (52) for the four frequencies shown on the data sheet. In what follows, subscripts refer to frequency. Using the equation

$$\lambda = \frac{c}{f} = \frac{3.455 \times 10^{+4}}{f},$$

compute

$$\lambda_{501} = 6.9 \times 10^{+1}\ \text{cm}$$
$$\lambda_{2001} = 1.728 \times 10^{+1}\ \text{cm}$$
$$\lambda_{4010} = 8.62\ \text{cm}$$
$$\lambda_{4711} = 7.34\ \text{cm}$$

Whence

$$2d\lambda_{501} = 8.19 \times 10^{+3}\ \text{cm}^2$$
$$2d\lambda_{2001} = 2.05 \times 10^{+3}\ \text{cm}^2$$
$$2d\lambda_{4010} = 1.023 \times 10^{+3}\ \text{cm}^2$$
$$2d\lambda_{4711} = 8.70 \times 10^{+2}\ \text{cm}^2 .$$

TRANSDUCER CALIBRATION

Step 1		Step 2	
$U_y \xrightarrow{d} U$		$U_x \xrightarrow{d} U$	
U_x			
CPS	501	CPS	501
V	1.25 at 10 volts	R_x	14.0 ohms
V_x	1.28 at 10 volts	v_x	1.26 at 10 volts
d	59.4 cm	I_x	.9 at $1 = v_x/R_x$
		V	1.02 at 1
CPS	2001	CPS	2001
V	.35 at 10	R_x	14.0 ohms
V_x	.42 at 10	v_x	.94 at 30
d	59.4 cm	I_x	2.08 at 1
		V	.32 at 10
CPS	4010	CPS	4010
V	.31 at 10	R_x	14.0 ohms
V_x	.28 at 10	v_x	1.02 at 30
d	59.4 cm	I_x	2.19 at 1
		V	.31 at 10
CPS	4711	CPS	4711
V	.18 at 10	R_x	14.0 ohms
V_x	.20 at 10	v_x	.88 at 30
d		I_x	1.888 at 1
		V	.3 at 10

Temp. 76.5° F
Press. 28.67 in/hg
Date 3/2/63
By R.S.G.

Now

$$\frac{2d\lambda \times 10^{-7}}{\rho c} = \frac{2d\lambda \quad 10^{-7}}{.393 \times 10^{+2}} = 2.54 \times 10^{-9}(2d\lambda),$$

therefore

$$\frac{2d\lambda_{50\,I} \times 10^{-7}}{\rho\, c} = 2.08 \times 10^{-5}$$

$$\frac{2d\lambda_{20\,0\,I} \quad 10^{-7}}{\rho\, c} = 5.21 \times 10^{-6}$$

$$\frac{2d\lambda_{40\,10} \quad 10^{-7}}{\rho\, c} = 2.60 \times 10^{-6}$$

$$\frac{2d\lambda_{47\,II} \quad 10^{-7}}{\rho\, c} = 2.21 \times 10^{-6}$$

The following computations are self explanatory.

$$\left(\frac{V}{V_x}\right)_{50I} = \frac{12.5}{12.8} = .977$$

$$\left(\frac{V}{V_x}\right)_{20\,0\,I} = \frac{.35}{.42} = .834$$

$$\left(\frac{V}{V_x}\right)_{4010} = \frac{.31}{.28} = 1.108$$

$$\left(\frac{V}{V_x}\right)_{47\,II} = \frac{.18}{.20} = .900$$

$$(V_1)_{50I} = 1.02 \text{ volts} = 1.02 \times 10^{+8} \text{ abvolts}$$

$$(V_1)_{20\,0\,I} = 3.20 \text{ volts} = 3.20 \times 10^{+8} \text{ abvolts}$$

$(V_1)_{4010} = 3.10 \text{ volts} = 3.10 \times 10^{+8}$ abvolts

$(V_1)_{4711} = 3.00 \text{ volts} = 3.00 \times 10^{+8}$ abvolts

$(I_x)_{501} = .9 \text{ amp} = 9.0 \times 10^{-2}$ abampere

$(I_x)_{2001} = 2.08 \text{ amp} = 2.08 \times 10^{-1}$ abamp

$(I_x)_{4010} = 2.19 \text{ amp} = 2.19 \times 10^{-1}$ abamp

$(I_x)_{4711} = 1.888 \text{ amp} = 1.888 \times 10^{-1}$ abamp

$$\left(\frac{V_1}{I_x}\right)_{501} = 1.133 \times 10^{+9}$$

$$\left(\frac{V_1}{I_x}\right)_{2001} = 1.54 \times 10^{+9}$$

$$\left(\frac{V_1}{I_x}\right)_{4010} = 1.415 \times 10^{+9}$$

$$\left(\frac{V_1}{I_x}\right)_{4711} = 1.59 \times 10^{+9}$$

In terms of practical units (52) is

$$M_0 = \left[\frac{2d\lambda \times 10^{-7}}{\rho c} \quad \frac{V_1}{I_x} \quad \frac{V}{V_x}\right]^{\frac{1}{2}},$$

therefore

$$(M_0^2)_{501} = (2.08 \times 10^{-5})(1.133 \times 10^{+9})(.977) = 2.30 \times 10^{+4}$$

$$(M_0^2)_{2001} = (5.21 \times 10^{-6})(1.54 \times 10^{+9})(.834) = 6.69 \times 10^{+3}$$

$$(M_0^2)_{4010} = (2.60 \times 10^{-6})(1.415 \times 10^{+9})(1.108) = 4.07 \times 10^{+3}$$

$$(M_0^2)_{4711} = (2.21 \times 10^{-6})(1.59 \times 10^{+9})(.900) = 3.02 \times 10^{+3},$$

and

$$(M_0)_{501} = 1.518 \times 10^{+2} \text{ volts/dyne/cm}^2$$

$$(M_0)_{2001} = 8.18 \times 10^{+1} \text{ volts/dyne/cm}^2$$

$$(M_0)_{4010} = 6.38 \times 10^{+1} \text{ volts/dyne/cm}^2$$

$$(M_0)_{4711} = 5.50 \times 10^{+1} \text{ volts/dyne/cm}^2.$$

To obtain the calibration as a speaker use

$$S_0 = M_0 \frac{\rho c}{2d\lambda\, 10^{-7}} .$$

This gives

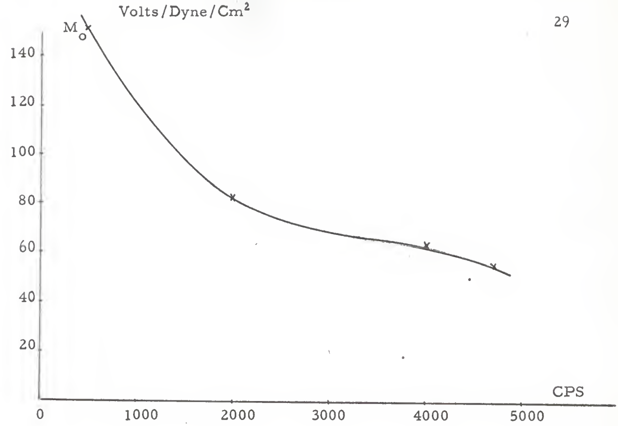
$$(S_0)_{501} = \frac{1.518 \times 10^{+2}}{2.08 \times 10^{-5}} = .729 \times 10^{+7} \text{ dyne/cm}^2\text{/amp}$$

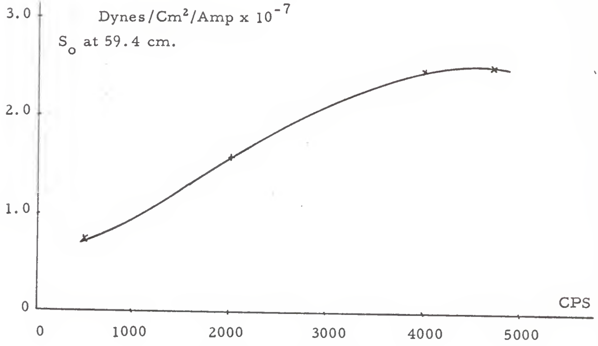
$$(S_0)_{2001} = \frac{8.18 \times 10^{+1}}{5.21 \times 10^{-6}} = 1.57 \times 10^{+7} \text{ dyne/cm}^2\text{/amp}$$

$$(S_0)_{4010} = \frac{6.38 \times 10^{+1}}{2.60 \times 10^{-6}} = 2.45 \times 10^{+7} \text{ dyne/cm}^2\text{/amp}$$

$$(S_0)_{4711} = \frac{5.50 \times 10^{+1}}{2.21 \times 10^{-6}} = 2.49 \times 10^{+7} \text{ dyne/cm}^2\text{/amp}$$

These two calibration curves are shown plotted on the following page.





CALIBRATION CURVES

Assumptions

In this section the assumptions underlying the theory are considered in order to point out the limitations of the method. The asumptions are

a) the acoustic medium has negligible body forces,

b) the particle velocities are small,

c) flow in the medium is irrotational,

d) the medium condensation is small compared with unity,

e) volume changes in the medium are adiabatic,

f) the medium is unbounded during calibration,

g) the sound generators are spherical,

h) cavitation does not take place at the face of the sound generator,

i) the electrical devices used in the calibration are linear and passive,

j) α is the same for transducers U and U_x ,

k) $\sqrt{1 + k^2 a_1^2} = \sqrt{1 + k^2 a_2^2}$

ℓ) acoustic pressure is constant over the transducer face.

All of these assumptions are implicit in the calibration technique. Assumptions a), b), c), d), e), f), h), and i) must be satisfied; these assumptions merely restrict the range for which transducers may be calibrated by the free field method. Assumptions g), j), and ℓ) are interrelated. Assumptions g) and j) will be satisfied in effect provided that during calibration the transducers are kept far enough apart that they appear to each other to be spherical. Assumption ℓ) is satisfied provided if, in addition, the wave length of the acoustic wave is large compared with the size of the transducer.

In lieu of this restriction on the wave length, the same effect will be realized if the transducer cone is rigid, i. e., no vibrations are allowed in the cone. In consideration of assumption k) note that $k = \omega / c$. Thus assumption k) is satisfied provided that $\omega << c$ allowing $k^2 a^2$ to be ignored with respect to unity. In lieu of this restriction on ω assumption k) is satisfied provided $a_1 = a_2$. Thus for calibration over a wide frequency range the transducers U and U_x must have stiff cones of the same size.

Advantages

The advantage of this method is that only electrical measurements need be made. Since electrical measurements are relatively easy to make, this is a very important advantage. Of course a ruler is needed to measure the distance d.

Disadvantages

The big disadvantage of this method is that an anechoic chamber is required. If an anechoic chamber is not available, and only a few calibrations are to be done, it would be advantageous to use some other calibration scheme. Also, if a wide frequency range is to be used, U and U_x must have rigid cones of the same size.

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FREE FIELD CALIBRATION OF ACCOUSTICAL DEVICES

by

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____
Major Professor

This report examines in detail the theory, assumptions, and limitations of a free field method of calibrating electroacoustical transducers. This method is due to MacLean [1].

The necessary reciprocity theorems are proved for moving coil electroacoustical transducers. A sample calibration is done.