THERMAL ANALYSIS OF HEAT EXCHANGERS FOR EARTH SATELLITES

by

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TABLE OF CONTENTS

INTRODUCTION

Radiation heat transfer applications in space technology have generated renewed interest in this field. The absence of convection in the operating environment of a space vehicle has directed attention to radiation, as the primary or exclusive mechanism of heat transfer for disposing of waste heat from power plants, electronic equipment or other sources in the space vehicle.

The surfaces which are required for this purpose are often huge, since the Stephan-Boltzman law sets an upper limit to the heat that can be transferred per unit surface area in unit time. Knowledge of heat exchangers for radiation heat transfer is, therefore, a very contemporary problem. Other space-oriented applications are (1) the capture of solar energy to maintain a steady or given temperature of the satellite, and (2) the conversion of solar energy into other useful forms.

In order to improve the techniques of radiative heat transfer for a space vehicle, the use of extended surfaces is an important consideration.

In the first part of the report, the primary consideration has been confined to rectangular radiating fins which transfer waste heat to the atmosphere. For the purpose of analysis the length of each fin has been divided into three parts. Assuming steady state (input equals output), temperatures of the three parts of the fin were calculated. From symmetry, temperature of the corresponding parts of the two fins will be same.

Fins not only augment the heat transfer by radiation, but also provide protection for the base surface against bombardment by space particles.

Depending upon the radiation and conduction properties of the material and on geometrical parameters and configuration, the use of radiation fins may result either in an increase or in a decrease in heat transfer.

It is evident that there will be a large number of parameters which will determine the heat transfer performance of radiating fins, more than usually found in the convective case. Consequently, analysis of finned radiating surfaces, in many cases must be carried out with a specific application in mind.

In part two of the report consideration has been given to the thermal control of a satellite of cylinderical configuration.

The temperature of a satellite must be controlled to satisfy the requirements of internal instruments. A man cannot survive more than 5°C. increase above normal body temperature, and most transistor networks are operable only between about 0°C. and 60°C. Some batteries will operate efficiently only within a range of about 40°C.

The control requirements for a vehicle are influenced by its outer-surface temperature. Surface temperatures, in turn, are determined by thermal radiation characteristics of the surface, the external environment, vehicle and orbit geometries, internal power generation, and conduction paths to the shell.

One side of the satellite is acted upon by solar radiation, and there is also some internal heat generation. The other two sides of it radiate heat to the space, though there are other kinds of input to the satellite, such as radiation from the earth due to its temperature and radiation from clouds, dust particles, and the earth itself, which is called albedo. For simplicity in calculation and because their magnitude is small relative to direct solar radiation, both albedo and radiation from earth are neglected in these calculations.

All the temperatures for the different sides of the satellite are calculated for steady state by varying the emissivities from 0.1 to the black body conditions.

NOMENCLATURE

A	Area
Ai	Area of surface i
Ac	Crossectional Area
Bi-j	Fraction of the energy emitted by surface i and absorbed by surface j
BTU	Unit of heat called British thermal unit
$B_{(x)}$ = $B_{(y)}$	Combined radiation flux (emitted and reflected) leaving a position x or y per unit time and unit area
E	Emissive power of the surface
Fi-j	Configuration factor or shape factor of the surface i with respect to surface j
g	Rate of incident energy per unit area from external source
h	Spacing between plates
$H_{(x)}$	Radiant energy arriving at x per unit time and area
K	Conductivity. BTU per hour per feet per degree
L	Length of the cylinderical satellite
L	Plate length
Q	Heat. BTU
q	Heat. BTU
R	Radius of the cylinderical satellite
$r_{s,p}$	Distance of sun from earth
T	Temperature absolute
t	Half thickness of the fin
T^*	Ratio of temperature of fin to the base temperature

W	Depth of fin
ε_i	Emissivity of the surface i
ρ_i	Reflectivity of surface i
τ_i	Transmissivity of surface i
σ	Stephan Boltzman constant = .1714 x 10^{-8}
ϕ	Angle with the normal
γ	Distance between the two surfaces
ρ	Density of the material
x	Distance between the two surfaces in the x direction
γ	Gap spacing ratio h/L
β	Dimensionless combined flux $B/\varepsilon\sigma T^4$

SOLAR RADIATION

The major external source of heat to a satellite is the thermal radiation from the sun. Since the satellite is at a great distance from the sun, it can be assumed that the solar flux at the satellite is essentially parallel waves of electromagnetic radiation. Thus for a unit area that is perpendicular to a radius vector from the sun, the radiant flux (outside the earth atmosphere) is inversely proportional to the square of the distance from the sun and is given by reference (17) as:

$$\frac{Q}{A} = \sigma \, \varepsilon_s \, f_{s-p} \, T_s^4$$

$$= \sigma \, \varepsilon_s \left[\frac{r_s}{r_{s,p}} \right]^2 T_s^4$$

Assuming $\varepsilon_s = 1.0$, $r_s = 2.2836 \times 10^8$ ft., $r_{s,p} = 4.90 \times 10^{11}$ ft. and $T_s = 10,360^\circ$ R (black body), the equation above gives a flux of 428 BTU/hr ft^2, which agrees with the value in Reference (10). Many estimates of this flux have been published, and these estimates vary from 420 to 440 BTU.

Another source of heat is the planetary reflection falling on the satellite. Though it is not negligible in actual practice, in this calculation it has been omitted. This input is not constant and varies from season to season and with the weather.

The third kind of heat input source is the internal heat generation, which can vary with the kind and amount of internal instrument.

CONFIGURATION FACTOR

When heat is transferred by radiation from a completely enclosed black body to the enclosing black body, the Stefan-Boltzman equation is used in its usual form:

$$q = \sigma A (T_1^4 - T_2^4) \quad ----------- \quad (1)$$

In many cases of heat transmission by radiation one body is not enclosed by another. Thus all of the radiation from one body is not intercepted by other body, except in the case of infinite parallel planes.

Therefore, for radiant heat transfer between two black bodies which do not directly intercept all of the radiation of each other, another term, $F_{c,i}$ configuration or shape factor must be placed in the Stefan-Boltzman equation, which then takes the form:

$$q = \sigma A F_{c,i} (T_1^4 - T_2^4) \quad ---------- \quad (2)$$

If radiation takes place between two infinitesimal surfaces dA_1 and dA_2 which are small in comparison to their distance apart, the configuration factor can be determined by assuming the Lambert cosine and the square of the distance laws to hold. The Lambert cosine law states that the radiation from a given area in a direction at an angle to the normal to the surface is proportional to the cosine of the angle; whereas, the square of the distance law states that the intensity of radiation from a point source decreases with the square of the distance from the point source.

GENERAL EQUATIONS OF RADIATIVE HEAT TRANSFER FROM ONE SURFACE TO ANOTHER

When two surface elements dA_a and dA_b located upon surfaces a and b are situated so that they can see one another, each will radiate energy to the other, and a net exchange of energy from the hotter to the cooler will result. The net amount of energy will be determined by the surface properties of the two areas, the absolute temperature and the geometry of the two surfaces.

When each of the two surfaces is black, the net radiant exchange is given by

$$dq_{1-2} = \left\{E_{b_1} - E_{b_2}\right\} \frac{\cos \Phi_1 \cos \Phi_2 \, dA_1 \, dA_2}{\pi r^2} \quad --- (3)$$

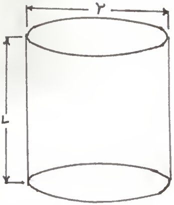
The net rate of radiation between two finite surfaces A and B can be obtained by integrating the equation (3) above over both areas, which gives

$$dq_{1-2} = \left\{E_{b_1} - E_{b_2}\right\} \int_{A_1} \int_{A_2} \frac{\cos \Phi_1 \cos \Phi_2 \, dA_1 \, dA_2}{\pi r^2} \quad - (4)$$

The double integral can be written as A_1F_{1-2} or A_2F_{2-1}, where F_{1-2} is the shape factor and represents the fraction of the total radiant energy leaving A_1 and which is intercepted by A_2, whereas F_{2-1} is the fraction of energy leaving A_2 and reaching A_1. According to the reciprocity theorem

$$A_1 F_{1-2} = A_2 F_{2-1}$$

Fig. 1



For the case of a cylinderical satellite, the shape factor between surface 1 and 2 is given by Reference (11).

$$F_{1-3} = \frac{\pi}{2}\left[L^2 + 2r^2 - \sqrt{L^2 + 2r^2 - 4r^4}\right] \text{ - - - - - - - - - - (5)}$$

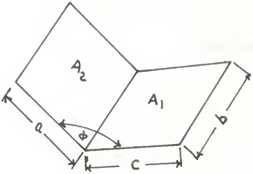
When the surfaces 1 and 3 are a distance L apart,

$$F_{2-3} = \frac{1}{4rL}\left[\sqrt{L^4 + 2L^2 \times 2r^2 - L^2}\right] \text{ - - - - - - - - - (6)}$$

Ref. (2)

For the case of two inclined plates

Fig. 2



Where $L = C/b$

$N = a/b$

$$F_{1-2} = \frac{1}{\pi L}\left[L \tan^{-1}\left(\frac{1}{L}\right) + N \tan^{-1}\frac{1}{N} - \sqrt{N^2 + L^2}\tan^{-1}\left(\frac{1}{N^2 + L^2}\right)\right]$$

$$- \frac{1}{4}\log_e\left[\left[\frac{(1 + L^2)(1 + N^2)}{1 + N^2 + L^2}\right]\left\{\frac{L^2(1 + L^2 + N^2)}{(1 + L^2)(L^2 + N^2)}\right]^{L^2}\right.$$

$$x\left\{\frac{N^2 \quad (1 + L^2 + N^2)}{(1 + N^2)(L^2 + N^2)}\right\}\Bigg] \qquad \text{- - - - - - - - - - - - - - - - (7)}$$

Ref. (12)

The shape factor for the case of two parallel plates at a distance C apart and having dimensions of a x b, and when $x = b/C$ and $Y = a/C$

$$F_{1-2} = \frac{2}{\pi xy}\left[\log_e\left[\frac{(1 + x^2)(1 + y^2)}{1 + x^2 + y^2}\right]^{\frac{1}{2}} + y\sqrt{1 + x^2}\,\tan^{-1}\frac{Y}{\sqrt{1 + x^2}}\right.$$

$$\left. + x\sqrt{1 + y^2}\,\tan^{-1}\left(\frac{x}{1 + y^2}\right) - y\tan^{-1} y - x\tan^{-1} x\right] \text{ - - - } (8)$$

$$\text{Lim } x \to \infty \qquad F_{1-2} = \sqrt{1 + \frac{1}{y^2}} - \frac{1}{y}$$

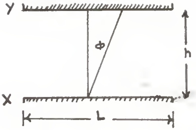
$$\text{Lim } y \to \infty \qquad F_{1-2} = \sqrt{1 + \frac{1}{x^2}} - \frac{1}{x^2}$$

$$\lim_{\substack{x \to \infty \\ y \to \infty}} \qquad F_{1-2} = 1$$

MATHEMATICAL APPROACH TO RADIATION PROBLEM

Mathematically the difference between the radiation problem and the conduction and convection problem is that the radiation problem must often be formulated as integral equations, while the convection and conduction problems are most frequently formulated as differential equations. The solution of the integral equation is somewhat difficult.

Fig. 3



In the figure are two parallel plates each of length L and extends indefinitely in the direction normal to the plane of the paper. The surfaces of both plates radiate and reflect in a diffuse manner.

The combined radiation flux per unit time per unit area leaving an area dA_x is designated as B(x). This flux is composed of two parts, i. e., direct emission and reflection.

$$B(x) = \varepsilon \sigma T^4 + \rho H(x) \quad \text{-------------} \quad (9)$$

where ρ H(x) is the reflected amount of radiant flux which arrives at x. Since there are two unknowns in equation (9), i.e., B and H, the energy that comes to x is related to the energy that leaves Y from the upper plate.

The energy leaving an area dAy from the Y plate is $B_{(y)}dA_y$ and of this amount $B(y)dAydF_{y-x}$ -------------- (10)

arrives at position X on the lower plate where dF_{y-x} is the shape factor of dA_x with respect to Y. By applying the reciprocity relation $dF_{y-x}\, dA_y = dF_{x-y}\, dA_x$, equation (10)

$$B_{(y)}\, dA_x\, dF_{x-y} \qquad (11)$$

Energy reaching X per unit area is $B(y)dF_{x-y}$. Since X receives energy from all positions of Y on the upper surface, therefore, the total is obtained by integrating equation (11).

$$H(x) = \int_{Y=-\frac{L}{2}}^{Y=L/2} B(y)\, dF_{x-y} \qquad (12)$$

but according to Jacob (Ref. 8)

$$dF_{xy} = \frac{1}{2}\, d\,(\mathrm{Sin}\,\phi) \qquad (13)$$

$$H_{(x)} = \frac{h^2}{2} \int_{-L/2}^{L/2} B(y) \frac{1}{\left[(y-x)^2 + h^2\right]^{3/2}}\, dy \qquad (14)$$

Substituting for H(x) into equation (9) we get

$$B(x) = \varepsilon \sigma T^4 + \frac{\rho h^2}{2} \int_{-L/2}^{L/2} B(y) \frac{1}{\left((y-x)^2 + h^2\right)^{3/2}}\, dy \qquad (15)$$

In the dimensionless form by substituting $h/L = \gamma$

$$B(x) = 1 + \frac{\rho \gamma^2}{2} \int_{-\frac{1}{2}}^{\frac{1}{2}} B(y) \frac{1}{\left[(y-x)^2 - y^2\right]^{3/2}} \, dY \quad ---- \quad (16)$$

Since the net flux at any position is ($q = B - H$)

Then $$\frac{q}{\varepsilon \sigma T^4} = \frac{1 - B\varepsilon}{\rho} \quad ---------- \quad (17)$$

Which gives variation of with X

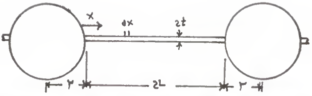
$$\frac{Q/L}{\varepsilon \sigma T^4} = \frac{1 - \varepsilon \int_{-\frac{1}{2}}^{\frac{1}{2}} \beta \, dX}{\rho} \quad -------------(18)$$

The more detailed solution is given in Ref. (14).

According to most texts the assumption is made that the energy H is the same for all points of X.

Similarly for the case of a straight fin (see sketch) which exchanges energy with its base when both fin and base are black.

Fig. 4



By applying the principle of conservation of energy for the steady state condition for a small element dx

$$(dQ\ cond)_{net} + (dQ\ rad)_{net} = 0 \quad ------------(19)$$

or $$(dQ\ cond)_{net} = \frac{d}{dx}\left[- ktw \frac{dT}{dx} \right] dx \text{ - - - - - - - - - } (20)$$

Where W is the width normal to the plane, from base energy transferred to the element is

$$(\sigma T_b^4 A_1)\ F_{1-x} + (\sigma T_b^4 A_2)\ F_{2-x} = \sigma T_b^4\ Wdx\ (F_{x-1} + F_{x-2}) \text{ --} (21)$$

By taking into account the external radiation g(x) per unit area

$$(dQ\ rad)_{net} = \left[\sigma T^4 - \sigma T_b^4\ (F_{x-1} + F_{x-2}) - g(x) \right] (Wdx) \text{ - - -} (22)$$

or $$\frac{d^2T}{dx^2} = \frac{\sigma}{kT}\left[T^4 - T_b^4\ (F_{x-1} + F_{x-2}) \right] - \frac{g(x)}{kT} \text{ - - - - } (23)$$

Substituting in equation (23) $Q = T/Tb$ and $X = X/L$

$$\frac{d^2Q}{dX^2} = N_c\left[Q^4 - (F_{x-1} + F_{x-2} \right] - \gamma X \text{ - - - - - - - - } (24)$$

Where $N_c = \frac{L^2 \sigma T_b^3}{kt}$, $X = \frac{g(x)\ L^2}{ktT_b}$

For the case when g(x) is zero and the boundary conditions

$$T_{(o)} = T_b\ , \quad \left.\frac{dT}{dx}\right)_{x = L} = 0$$

The solution by the digital computer for different values of N and L is given in Reference (15).

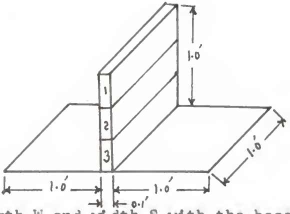
HEAT TRANSFER FROM A RADIATING FIN

In this problem various combinations of fin height, width, and spacing between the fin have been studied to determine a suitable fin for the radiation heat transfer from a space vehicle.

The problem is solved by the lump parameter method by dividing the fin into a number of small pieces and assuming that the temperature of each part of the fin is uniform. By making the energy balance, the temperature of each part of the fin is calculated by trial and error. The problem would have been solved more accurately with the help of a digital computer allowing the assumption of more lumps. Due to the limitation of time, the values presented were calculated with a desk calculator.

First the temperature along a single fin is calculated assuming that there is only one fin, and it exchanges heat by radiation and conduction with the base of the fin, and also radiates heat to the atmosphere.

Fig. 5



The fin is of length W and width S with the base length of 2C. Fin base and fin are assumed to be black bodies, and there is no loss of heat by convection. The fin is assumed to be radiating to space. The length of the fin is divided into N pieces, and each one of the N pieces is assumed to have uniform temperature along its length. For calculation, the temperature of each piece is taken at the center point of the respective piece. It is also assumed that there is no heat loss from the sides and the end of the fin.

The energy balance of each of the N pieces has been made by equating the energy input to energy output for the respective piece as follows:

$$2 \sigma A_1 T_1^4 = A_{1c} K_1 \frac{\Delta T}{\Delta X/2} + 2 A_B \sigma T_o^4 \times F_{B-1} \quad \text{- - - -} \quad (25)$$

or

$$2 \sigma A_1 T_1^4 = A_{1c} K_1 \frac{T_o - T_1}{\Delta X/2} + 2 A_B \sigma T_o^4 \times F_{B-1} \quad \text{- - - -} \quad (25a)$$

$$2 \sigma A_2 T_2^4 = A_{2c} K_2 \frac{T_1 - T_2}{\Delta X} + 2 A_B \sigma T_o^4 \times F_{B-1} \quad \text{- - - -} \quad (25b)$$

$$2 \sigma A_n T_n^4 = K A_{nc} \frac{T_{n-1} - T_n}{\Delta X} + 2 A_B \sigma T_o^4 \; F_{B-n}$$

Where n is the number of the pieces on lumps.

Using the temperatures of the single fin as an approximation, the input and output for the respective parts of the fin are equated. The energy input to the fin is by conduction and radiation from the base and from the fins on both sides. The output is the heat transfer by radiation. Because of symmetry, temperatures on the corresponding parts of two fins are assumed to be equal.

Using this set of temperatures as approximate temperatures, the temperatures are solved again, and this procedure is repeated until the temperatures satisfy the heat balance equations. It should be noted that all the temperatures are for steady state.

Mild steel is used as the fin material. The dimensions of the fin are as shown in figure (5). Dividing the fin length into three equal parts, we get shape factors of each part with respect to the base as follows:

$$F_{B-3} = 0.120$$

$$F_{B-2} = 0.050$$

$$F_{B-1} = 0.030$$

For surface three, equating input equal to output we get

$$2\sigma A_3 T_3^4 = A_{c_3} K \frac{T_o - T_3}{\Delta x/2} + 2A_B \sigma T_o^4 \times F_{B-3} \text{ - - - - - - } (26)$$

All terms are known in the above equation except T_3, the temperature of surface at the center point. By trial and error we get $T_3 = 585.6^\circ$ R, using K = 26.3.

Similarly for parts 2 and 1.

$$2\sigma A_2 T_2^4 = A_{c_2} K \frac{T_3 - T_2}{\Delta X} + 2A_B F_{B-2} \sigma T_B^4 \text{ - - - - - - } (27)$$

$$2\sigma A_1 T_1^4 = A_{c_1} K \frac{T_2 - T_3}{\Delta X} + 2A_B F_{B-1} \sigma T_B^4 \text{ - - - - - - } (28)$$

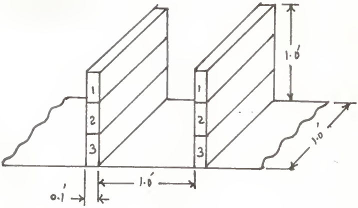
Solving for T_2 and T_1 by trial and error we get

$T_2 = 554^\circ$ R and $T_1 = 525.6^\circ$ R

For a pair of fins it is necessary to know the shape factor F_{11}, F_{12}, F_{13} .... ext. From tables in Reference (12), we can determine the shape factors for parallel plates directly opposite each other.

For the configuration shown

Fig. 6



the shape factors are:

$$F_{1-1} = 0.08$$

$$F_{12-12} = 0.15$$

$$F_{123-123} = 0.20$$

$$A_{12} F_{12-12} = A_1 F_{1-12} + A_2 F_{2-12} \text{ - - - - - - - - - - - - - } (29)$$

$$= A_1 F_{1-1} + A_1 F_{12} + A_2 F_{2-1} + A_2 F_{2-2} \text{ - - - } (30)$$

From geometry $A_1 = A_2 = A_3$ and $F_{1-1} = F_{2-2} = F_{3-3}$

$$\text{or } A_1 F_{1-1} = A_2 F_{2-2} \text{ - - - - - } (a)$$

$$\text{and } A_1 F_{1-2} = A_2 F_{2-1} \text{ - - - - - } (b)$$

Substituting a and b in (30) we get

$$A_1F_{1-2} = \frac{A_{12} F_{12-12} - 2A_1F_{1-1}}{2} \text{ - - - - - - - - - - - - - - - -} (31)$$

$$F_{1-2} = F_{12-12} - F_{11} \text{ -} (32)$$

Similarly

$$A_{123}\ F_{123-123} = A_1\ (F_{1-1} + F_{1-2} + F_{1-3}) +$$

$$A_2\ (F_{2-1} + F_{2-2} + F_{2-3}) + A_3\ (F_{3-3} + F_{3-2} + F_{3-1}) \quad (33)$$

$$A_1\ F_{1-3} = \frac{A_{123}\ F_{123-123} - A_1\ (3F_{1-1} + 4F_{12})}{2}$$

making an energy balance for each of the three surfaces for the case of a gray surface with diffuse radiation.

$$2\sigma \varepsilon_3 A_3 T_3^4 = A_{c3}\ K\ \frac{T_o - T_3}{\Delta X/2} + 2A_B \sigma B_{B-3}\ T_o^4 + A_1 \sigma T_1^4\ B_{13}$$

$$+ \sigma A_2 T_2 B_{2-3} + \sigma A_3\ T_3^4\ B_{3-3} \quad (34)$$

$$2\sigma \varepsilon_2 A_2 T_2^4 = A_{c2}\ K\ \frac{T_3 - T_2}{\Delta X} + \sigma \Big[2A_B\ B_{B-2}\ T_o^4 + A_1\ T_1^4\ B_{1-2}$$

$$+ A_2 T_2^4 B_{2-2} + A_3\ T_3^4\ B_{3-2} \Big] \quad (35)$$

$$2\sigma \varepsilon_1 A_1\ T_1^4 = A_{c-1}\ K\ \frac{T_2 - T_1}{\Delta X} + \sigma \Big[2A_B\ B_{B-1}\ T_o^4 + A_1\ T_1^4\ B_{1-1}$$

$$+ A_2\ T_2^4\ B_{2-1} + A_3\ T_3^4\ B_{3-1} \Big] \quad (36)$$

For the case of black surfaces it is given by

$$2\sigma A_3 T_3^4 = A_{c3}\ K\ \frac{T_o - T_3}{\frac{\Delta X}{2}} + 2A_B\ F_{B-3}\ \sigma T_o^4 + A_1 \sigma (T_1^4\ F_{1-3}$$

$$+ T_2^4\ F_{2-3} + T_3^4\ F_{3-3})$$

$$2\sigma A_2 T_2^4 = A_{c-2}\ K\ \frac{T_3 - T_2}{\Delta X} + 2\ A_B\ F_{B-2}\ \sigma\ T_o^4 + A_1 \sigma$$

$$(T_1^4\ F_{1-2} + T_2^4\ F_{2-2} + T_3^4\ F_{3-2})$$

$$2\sigma A_1 T_1^4 = A_{c-1} K \frac{T_2 - T_1}{\Delta X} + 2 A_B F_{B-1} \sigma T_o^4 + A_1 \sigma$$

$$(T_1^4 F_{1-3} + T_2^4 F_{2-3} + T_3^4 F_{3-3})$$

Solving for the temperatures T_1, T_2, and T_3 by trial and error we get

$$T_3 = 586.9^\circ \text{ R}$$

$$T_2 = 557.5^\circ \text{ R}$$

$$T_1 = 530.5^\circ \text{ R}$$

The net heat transfer to the atmosphere is calculated as follows:

Heat transfer from surface 1 $= 2\sigma T_1^4 A_1 (1 - F_{1-1} - F_{1-2} - F_{1-3} - F_{1-B}) = 2 \times .1714 \times \left(\frac{530.5}{100}\right)^4 \times 1/3\,(1 - .08 - .07 - .0405 - .09) = 65.0 \frac{BTU}{hr}$

Similarly from surface 2 $= 69.6 \frac{BTU}{hr}$

Similarly from surface 3 $= 56.0 \frac{BTU}{hr}$

Heat transfer from the fin base $= A_B \sigma T_B^4 (1 - 2F_{B-1} - 2F_{B-2} - 2F_{B-3}) = 1 \times .1714 \times 10^{-8} \times (600)^4 \times (1 - .4) = 133.2 \frac{BTU}{hr}$

Total heat transfer per fin $= 65 + 69.6 + 56 + 133.2$

$= 323.8 \frac{BTU}{hr}$

Heat transfer per foot of fin base $= 323.8 \times \frac{1}{1.1} = 294.36 \frac{BTU}{hr\ ft}$

It has been assumed in the above calculations that the heat losses from the edges and the tip of the fin are negligible.

Also all the calculations are based on surface emissivities of 1.0 (Black body conditions).

Heat losses from various fins configurations have been calculated, varying the distance between the fins, the height of the fins, and the thickness of the fins. The results are tabulated below and shown plotted in Figures (8-10).

Table 1. Fins of constant height of 1 foot and constant depth of 1 foot with .1' thick.

Heat transfer without fin = 222.13 $\frac{BTU}{hr\ ft}$

Spacing	1.0'	.5'	0.10'	5.0'
BTU/ft of fin base per hour	294.4	317.0	239.55	244
BTU/lbm of fin	6.605	3.88	0.978	9.12

Table 2. Constant height of 1 foot and constant depth of 1 foot with constant spacing of 1 foot.

Thickness	0.025'	0.050'	0.10'
BTU/ft of fin base per hour	258	285	294.4
BTU/lbm of fin	21.60	12.35	6.605

Table 3. Constant height of 1 foot and constant depth of 1 foot with constant spacing

Spacing = .5'

Fin thickness feet	.025'	.05'	.10'
BTU per ft of fin base	233	280.5	317
BTU/lbm	10.0	6.30	3.88

Table 3a. Spacing = .1'

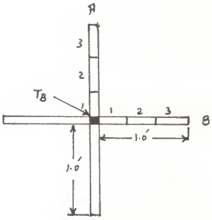
Fin thickness feet	.025'	0.5'	.10'
BTU per ft of fin base	231	246	239
BTU/lbm	2.36	1.51	0.978

Table 4. Constant height of 3.6" or .3' feet with depth of 1 foot and constant thickness of .25"

Fin spacing feet	0.60'	0.30'	0.15'
BTU per ft of fin base	226	278	189.3
BTU/lbm hr.	45.7	29.1	9.5

Example

Fig. 7



In this example a fin of configuration shown in the figure has been solved. To check the accuracy, the results are compared with the results given in Reference (7), which were obtained with a digital computer.

Surfaces A and B with an included angle of 90°, are radiating to space. Their common edge is the base surface of the fin at a constant temperature of 600° R.

Solving for T_1, T_2, and T_3 as we did previously, we find

$$T_1 = 544^{\circ}\ R$$

$$T_2 = 475.5^{\circ}\ R$$

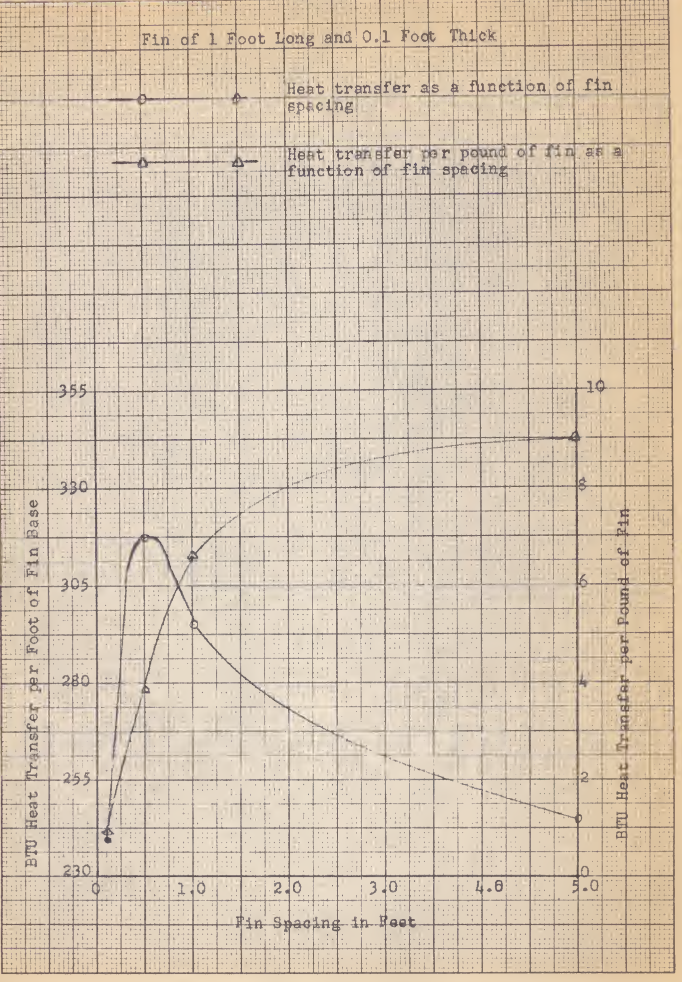
$$T_3 = 428^{\circ}\ R$$

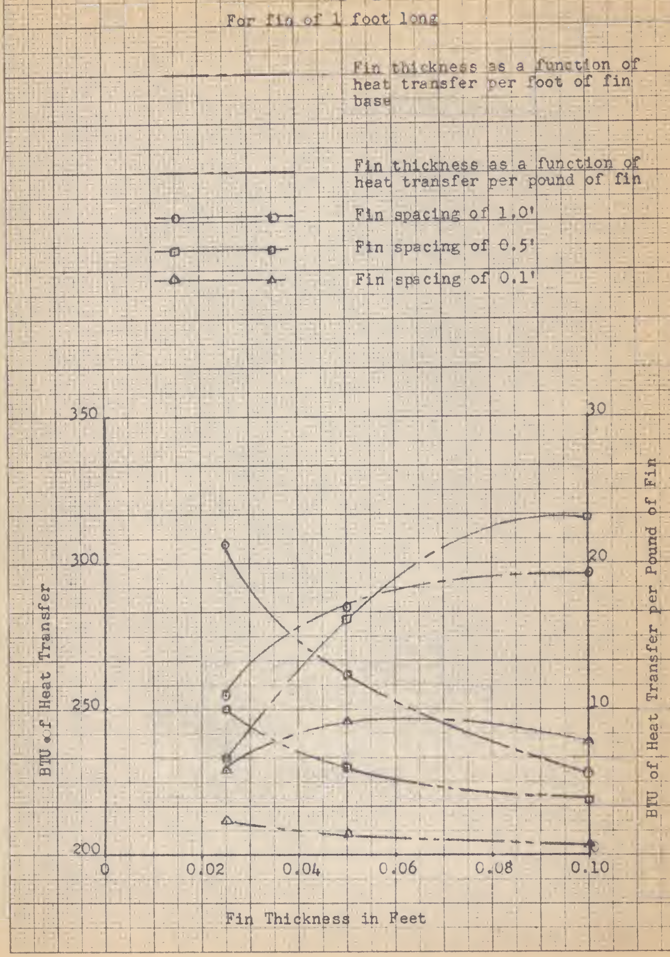
Table 5.

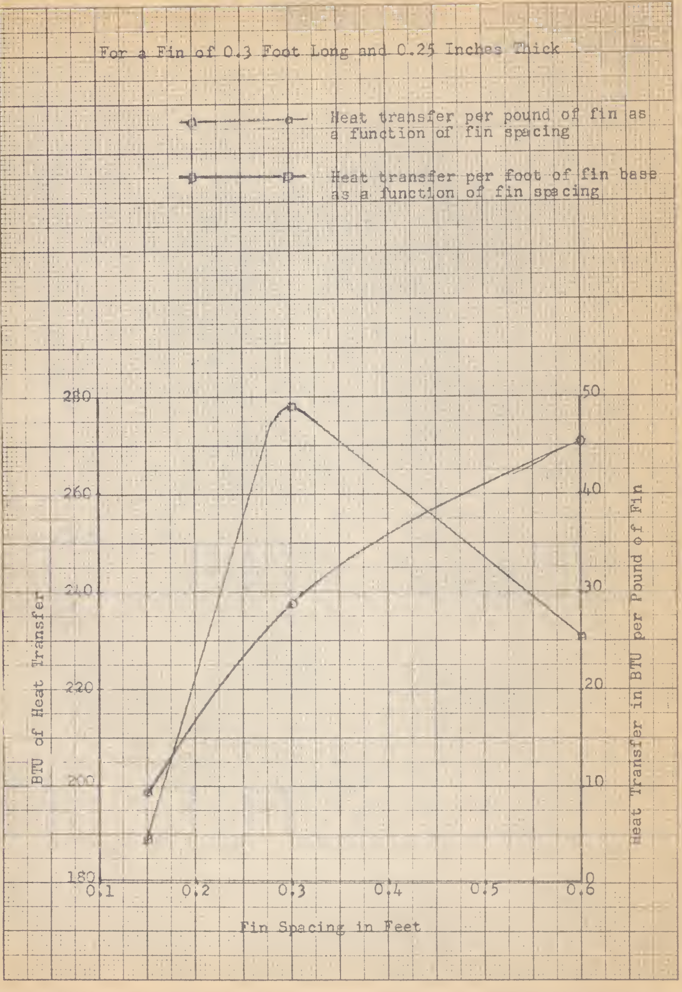
	Calculated	Exact from Reference (6)	Percentage error
T_1^*	0.9067	0.925	1.945
T_2^*	0.7910	0.850	6.95
T_3^*	0.7130	0.805	11.40

Average error for the whole fin = 6.765 %

Fig. 8


Fin of 1 Foot Long and 0.1 Foot Thick
Heat transfer as a function of fin spacing
Heat transfer per pound of fin as a function of fin spacing
BTU Heat Transfer per Foot of Fin Base
BTU Heat Transfer per Pound of Fin
355
330
305
280
255
230
10
8
6
4
2
0
0
1.0
2.0
3.0
4.0
5.0
Fin Spacing in Feet


For fin of 1 foot long
Fin thickness as a function of heat transfer per foot of fin base
Fin thickness as a function of heat transfer per pound of fin
Fin spacing of 1.0'
Fin spacing of 0.5'
Fin spacing of 0.1'
350
300
250
200
30
20
10
0
0.02
0.04
0.06
0.08
0.10
BTU of Heat Transfer
BTU of Heat Transfer per Pound of Fin
Fin Thickness in Feet


For a Fin of 0.3 Foot Long and 0.25 Inches Thick
Heat transfer per pound of fin as a function of fin spacing
Heat transfer per foot of fin base as a function of fin spacing
280
260
240
220
200
180
BTU of Heat Transfer
50
40
30
20
10
0
Heat Transfer in BTU per Pound of Fin
0.1
0.2
0.3
0.4
0.5
0.6
Fin Spacing in Feet

TEMPERATURE OF A SATELLITE

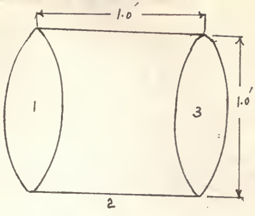


Fig. 11

Satellite is assumed to be of cylinderical configuration as shown in the above figure (12), with surface facing normal to the radiation from sun of intensity 400 BTU per foot^2 per hour.

Case I

Surfaces 1 and 3 are assumed to be black. Surface 2 is a reflecting and reradiating wall, such that there is no heat loss from surface 2. Radiation from sun impinges on surface 1, which in turn radiates internally to surface 3.

Energy leaving surface 1 and reflected back to one

$$E_1\left[F_{1-2}\,F_{2-1} + F_{1-2}\,F_{2-2}\,F_{2-1} + F_{1-2}\,F_{2-2}^2\,F_{2-1} + \cdots\right] \quad \text{-- (37)}$$

$$= \frac{E_1\,F_{1-2}\,F_{2-1}}{1 - F_{2-2}} \quad \text{- - - - - - - (38)}$$

Energy leaving 3 and falling on 1 by direct radiation and by reflection from 2

$$E_3\left[F_{3-1} + F_{3-2}\,F_{2-1} + F_{3-2}F_{2-2}\,F_{2-1} + F_{3-2}\,F_{2-2}^2\,F_{2-1} + \cdots\right] \quad \text{- -(39)}$$

$$= E_3\left[F_{3-1} + \frac{F_{3-2}\,F_{2-1}}{1 - F_{2-2}}\right]$$

For a satellite with

$A_1 = A_3 = \quad ft^2$

$A_2 = 2 \quad ft^3$

$F_{1-3} = 0.652$ - - - from Ref. (11)

$F_{1-2} = 0.348$

$F_{2-3} = .309$ - - - from Ref. (2)

$F_{2-2} = .517$

$F_{2-1} = .174$

Similarly energy leaving 3 and reflected back to 3

$$= \frac{E_3 F_{3-2} F_{2-3}}{1 - F_{2-2}} \quad (40)$$

Energy leaving 1 and falling on 3 by direct radiation and by reflection from 2

$$= E_1 \left[F_{1-3} + \frac{F_{1-2} F_{2-3}}{1 - F_{2-2}} \right] \quad (41)$$

For equilibrium, Radiant energy leaving 3 = Radiant energy coming to 3.

$$2A_3E_3 = \frac{E_3 F_{3-2} F_{2-3}}{1 - F_{2-2}} + E_1 F_{1-3} + \frac{F_{1-2} F_{2-3}}{1-F_{2-2}} \quad (42)$$

For surface 1,

$$2A_1E_1 = 400A_1 + \frac{E_1 F_{1-2} F_{2-1}}{1-F_{2-2}} + E_3 \quad F_{3-1} + \frac{F_{3-2} F_{2-1}}{1 - F_{2-2}} \quad (43)$$

Since $E_1 = \sigma T_1^4$ and $E_3 = \sigma T_3^4$

In these two equations all terms are known except T_1 and T_3. Solving equations (42) and (43) simultaneously we get

$T_1 = 638^\circ$ R $\qquad\qquad T_3 = 510^\circ$ R

For the case when there is heat generation of 100 watts inside as point source

$T_1 = 675^\circ$ R $\qquad\qquad T_3 = 585^\circ$ R

Case II

Surfaces 1 and 3 are black, while surface 2 is grey with $\tau = 0$ and $\rho_2 = \varepsilon_2 = .5$.

Energy leaving 1 and falling back on 1 by reflection from surface 2 =

$$\frac{E_1 F_{1-2} \rho_2 F_{2-1}}{1 - F_{2-2} \rho_2} A_1 \text{ - - - - - - - } (44)$$

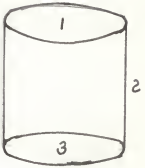


Fig. 12

Energy leaving 3 and falling on 1 by direct radiation and reflections from surface 2 $= E_3 A_3 \left[F_{3-1} + \frac{F_{3-2} \rho_2 F_{2-1}}{1-F_{2-2} \rho_2} \right]$ - - (45)

Energy leaving 2 and falling on 1

$$= \frac{E_2 A_2 F_{2-1}}{1 - F_{2-2} \rho_2} \text{ - } (46)$$

Energy leaving 2 and falling on 3

$$= \frac{\varepsilon_2 E_2 F_{2-3} A_2}{1 - F_{2-2} \rho_2} \text{ - } (47)$$

Energy leaving 1 and falling on 3

$$= E_1 \left[F_{1-3} + \frac{F_{1-2} \rho_2 F_{2-3}}{1-F_{2-2} \rho_2} \right] \text{ - - - - - - - - - - - - - - - - } (48)$$

Energy leaving 3 and reflected back to 3

$$= \frac{A_3 E_3 F_{3-2} \rho_2 F_{2-3}}{1 - F_{2-2} \rho_2} \quad \text{------------------} \quad (49)$$

Energy emitted by 1 and absorbed by 2

$$= \frac{E_1 F_{1-2} \varepsilon_2 A_1}{1 - F_{2-2} \rho_2} \quad \text{------------------} \quad (50)$$

Energy emitted by 3 and absorbed by 2

$$= \frac{E_1 A_3 F_{3-2} \varepsilon_2}{1 - F_{2-2} \rho_2} \quad \text{------------------} \quad (51)$$

Energy emitted by 2 and absorbed by 2

$$= \frac{E_2 \varepsilon_2 F_{2-2} A_2}{1 - F_{2-2} \rho_2} \quad \text{------------------} \quad (52)$$

Setting the incoming energy equal to the outgoing energy for each of the three surfaces, we obtain three equations in three unknowns, E_1, E_2, and E_3. These can be solved very easily, and from E we obtain the temperatures by the relation $E = \sigma T^4$.

Solving for T_1, T_2, and T_3 we obtain

$T_1 = 620^\circ$ R

$T_2 = 452^\circ$ R

$T_3 = 463^\circ$ R

For the case when there is heat generation which is assumed as a point source of 100 watts:

$T_1 = 629^\circ$ R

$T_2 = 502^\circ$ R

$T_3 = 513^\circ$ R

Case III

Surfaces 1 and 3 are black, and surface 2 has zero emissivity and zero conductivity, but transmissivity and reflectivity equal .5. Surface 1 is normal to solar radiations of intensity 400 BTU/hr ft^2.

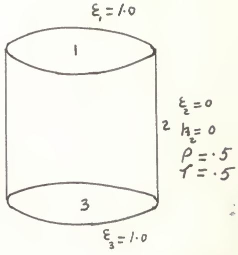


Fig. 13

Making an energy balance and solving for the temperatures, we obtain

$$T_1 = 605^\circ R \qquad T_3 = 432^\circ R$$

When there is heat generation of 100 watts inside the satellite assumed as a point source, we obtain

$$T_1 = 622^\circ R \qquad T_3 = 472^\circ R$$

Case IV

When all the three surfaces are black with solar radiations falling on surface 1, the temperatures obtained from the heat balance equations are:

$$T_1 = 605^\circ R$$

$$T_2 = 427^\circ R$$

$$T_3 = 434^\circ R$$

With heat generations of 100 watts

$$T_1 = 619^\circ R$$

$$T_2 = 470^\circ R$$

$$T_3 = 472^\circ R$$

Case V

All surfaces are gray. When all three surfaces are non-black, the radiation analysis becomes very complicated. In this case Gebhart's method was used as explained in Reference (6). Writing the equations in terms of B_{ij}, which is fraction of energy emitted by i and absorbed by j, we obtain a set of equations:

$$B_{13} = F_{1-3}\,\varepsilon_3 + F_{1-1}\,\rho_1\,B_{13} + F_{1-2}\,\rho_2\,B_{23} + F_{1-3}\,\rho_3\,B_{33}$$

$$B_{23} = F_{2-3}\,\varepsilon_3 + F_{2-1}\,\rho_1\,B_{13} + F_{2-2}\,\rho_2\,B_{23} + F_{2-3}\,\rho_3\,B_{33}$$

$$B_{33} = F_{3-3}\,\varepsilon_3 + F_{3-1}\,\rho_1\,B_{13} + F_{3-2}\,\rho_2\,B_{23} + F_{3-3}\,\rho_3\,B_{33}$$

These three equations in three unknowns, B_{12}, B_{23}, and B_{33}, can be solved very easily.

In the same way we can solve for B_{11}, B_{21}, B_{31}, B_{12}, B_{22}, and B_{32}.

It is assumed that each one of the three surfaces is grey and diffuses with uniformly incident and emitted radiation. It is also assumed that there is no emitting and absorbing media. By keeping $\rho_2 = \varepsilon_2 = .5$ as constant and varying ρ_1 and ρ_3 between .1 and .7, the values of different B'x is tabulated as follows:

Table 6. ρ_2 = .5 (Constant)

	$\rho_1=\rho_3=0.1$	$\rho_1=\rho_3=0.3$	$\rho_1=\rho_3=0.5$	$\rho_1=\rho_3=0.7$
B_{13}	.450	.380	.310	.205
B_{23}	.361	.314	.276	.201
B_{33}	.128	.140	.1415	.1160
B_{11}	.131	.141	.142	.1165
B_{21}	.3675	.331	.261	.203
B_{31}	.455	.379	.299	.205
B_{12}	.486	.471	.553	.683
B_{22}	.266	.324	.433	.617
B_{32}	.373	.458	.547	.667

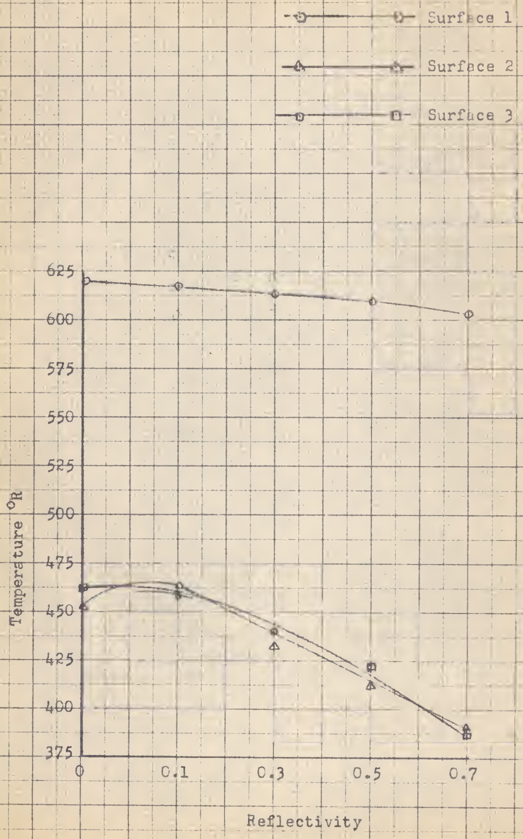
Using these values of B's, the input and output to each of the three surfaces are obtained with surface 1 facing normally to the solar radiation of intensity 400 BTU per square foot per hour. All these values are calculated by keeping ρ_2 equal to 0.5 a constant. Solving the heat balance equation, we obtain:

Table 7. Temperatures of the three surfaces of satellite when surface 2 has constant reflectivity of .5 and varying the reflectivities of surface 1 and 3.

	$\rho_1=\rho_3=0$	$\rho_1=\rho_3=0.1$	$\rho_1=\rho_3=0.3$	$\rho_1=\rho_3=0.5$	$\rho_1=\rho_3=0.7$
T_1^o R	620	618	613	611	603
T_2^o R	452	462	432	416	390
T_3^o R	463	460	441	424	388

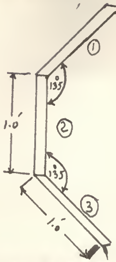
Fig. 11

Satellite Temperature as a Function of Reflectivity



Temperature of a Saucer Shaped Satellite

Fig. 15



Part A. All three surfaces are black. Surfaces 1, 2, and 3 are each 1 foot square with an included angle of 135°. The back or left side of each of the three surfaces is facing the solar radiation of intensity 400 BTU per foot square per hour. Surface 2 is normal to the solar radiation, while surfaces 1 and 3 are facing the solar radiations at an angle of 45°.

$F_{1-2} = F_{2-3} = F_{3-2} = F_{2-1} = .0535$ - - From Ref. (12)

$F_{1-3} = F_{3-1} = .1165$

Since surfaces 1 and 3 are symmetrical, we can therefore assume their temperatures equal.

The heat balance for surface 1 is

$$2A_1T_1^4\sigma = A_1 \times 400 \times .707 + 0.535\sigma T_2^4 + .1165\sigma T_3^4$$

For surface 2 it is

$$2A_2T_2^4\sigma = 400 \times A_2 + 2 \times 0.0535\sigma A_3\ T_1^4$$

Solving the two equations simultaneously for T_1 and T_3

$$T_1 = T_3 = 548^\circ\ R$$

$$T_2 = 583^\circ\ R$$

Part B. Surfaces 1, 2, and 3 are grey. Using Gebhart's method, the system is solved for B's for two cases, i.e., $\rho_1 = \rho_2 = \rho_3 = .3$ and .7.

Table 8.

	B_{13}	B_{23}	B_{33}	B_{11}	B_{21}	B_{31}	B_{12}	B_{22}	B_{32}
$\rho_1=\rho_2=\rho_3 =.3$	.082	.0393	.00353	.00353	.0393	.082	.0387	.00125	.0392
$\rho_1=\rho_2=\rho_3 =.7$	.0356	.0175	.0036	.0036	.0185	.0356	.0175	.0013	.0175

Surfaces 1, 2, and 3 are assumed to be ¼" thick and made of aluminum. In calculating the temperatures, conduction is also taken into consideration. To simplify the calculations, the average temperature of surfaces 1 and 3 is taken at the mid-point (at point a and a^1), while for surface 2 it is taken at point b and b^1, which is equal by symmetry.

$$\text{Heat flow by conduction} = -KA\frac{dt}{dx}$$

Assuming $\Delta T = 5^\circ$ E, i.e., between (b-a). Surfaces are painted black and having conduction equal to 118 BTU per hour per degree per foot.

$$2E_1 = 400 \times .707 + \frac{1}{48} \times \frac{118}{.75} \times 5 + .0535E_2 + .1165\, E_3$$

$$2E_2 = 400 + 2 \times \frac{1}{48} \times \frac{118}{.75} \times 5 + 2 \times .0535 \times E_1$$

Solving for E_1 and E_2 and T_1 and T_2 we get T_1 T_2, which is impossible, because T_2 is supposed to have higher temperature than T_1.

By assuming $\Delta T_{(b-a)} = 8^o$

$$2E_1 = 400 \times .707 - \frac{118}{48} \times \frac{8}{.75} + .1165E_3 + .0535E_2$$

or $E_1 - .0284E_2 = 164$ - - - - - - - - A

$$2E_2 = 400 + 2 \times \frac{118}{48} \times \frac{8}{.75} + 2 \times .0535E_1$$

or $E_1 - 18.7E_2 = 3250$ - - - - - - - B

Solving A and B simultaneously for E_1 and E_2 and then T_1 and T_2, we get

$$T_2 = 570^o \text{ R}$$

$$T_3 = T_1 = 562^o \text{ R}$$

Our assumption of $\Delta T = 8^o$ is true.

Similarly by solving for T_1, T_2, and T_3, when the surfaces are grey and diffuse, we get

Table 9.

	$\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1.0$ $\rho_1 = \rho_2 = \rho_3 = 0$	$\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0.7$ $\rho_1 = \rho_2 = \rho_3 = 0.3$	$\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0.3$ $\rho_1 = \rho_2 = \rho_3 = 0.7$
$T_1 = T_3^o$ R	562	560	556
T_2^o R	570	566.5	559

CONCLUSION

It can be seen from Figures (8-10) that for a fixed fin thickness and length, there is an optimum fin spacing at which the heat transfer is maximum. For the case of a one ft. long fin and 0.1 ft. thick fin, the optimum fin spacing comes out to be 0.5 ft. If it becomes necessary to get maximum heat transfer per pound of fin, as well as the overall maximum output, then the most optimum size is at one ft. spacing.

For the case of 0.3 ft. long fin with a thickness of ¼ inches and one ft. wide, the maximum heat transfer per pound of fin base is when the fin spacing is at 0.3 ft., while the maximum output per pound of fin is when the fin spacing is 0.6 ft. So the most optimum spacing at which both heat transfer per ft. of fin base as well as per pound of fin, comes out to be at a fin spacing of 0.47 ft.

Similarly, by varying the fin thickness as well as the fin spacing, the most optimum size and spacing at which to get maximum yield for least weight is when the fin spacing is one ft. and the fin thickness is 0.1 ft.

For the case of a cylinderical satellite, the curves show that there is very little change in the temperature of surface, with the increase in the reflectivity from zero to a maximum of 0.7. For the case of surface 2, the temperature is maximum when the reflectivity is 0.1, and it decreases very sharply with the increase in reflectivity. Whereas, for the case of surface 3, the temperature decreases as the reflectivity increases, and vice versa.

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THERMAL ANALYSIS OF HEAT EXCHANGERS FOR EARTH SATELLITES

by

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AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Mechanical Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1962

In Part One of this report, various combinations of fin height, width, and spacing between the fin have been studied to determine an optimum fin for the radiation heat transfer to space. The problem is solved by the lump parameter method. Temperatures at various points of the fin are calculated by trial and error until the temperatures satisfy the heat balance equations.

Fins are compared with each other on the basis of heat transfer per foot of fin base and also on the basis of heat transfer per pound of fin material.

For the case when the fins are one foot high and 0.1 ft. thick, the maximum heat transfer per foot of fin base is when the fin spacing is 0.5 ft. On BTU per pound basis, the maximum heat transfer is when fin spacings are one foot apart.

For 0.3 ft. high fins and ¼ inch thick, the maximum heat transfer is for 0.3 ft. spacing, and on BTU per pound basis, the maximum occurs with a spacing of 0.6 ft.

In Part Two of the report, temperatures of a cylinderical satellite have been calculated for various surface emissivities, when the input is only solar radiations and when there is heat generation as well as solar radiations.

It has been determined that as the reflectivity of the surfaces of the satellite increases, the temperatures of the surfaces decrease very sharply.