CONTINUITY IN PRESTRESSED CONCRETE BEAMS

by

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SYNOPSIS

The main purpose of a continuity analysis in prestressed concrete beams is to achieve benefits in strength and economy. Secondary moments produced by a prestressing force take an important role in the analysis of continuous prestressed concrete beams. Continuity produces a decrease in moments and hence reduction of section for the same load and span in comparison with that of simple prestressed concrete beams. The prestressing cable is located in such a way that stresses produced at any section remain within the allowable limits. Usually, computation of moments due to applied loading involves an integration process. However, in this study of continuous prestressed concrete beams, equivalent loads are computed from the secondary moments through a differentiation process. From the equivalent loads, final moments which give the eccentricity of pressure line, are computed. The displacement of prestressing cable and pressure line involves the principle of linear transformation and concordancy of cable. Two typical problems of beams having variable moments of inertia and subjected to fixed and variable loading conditions are discussed as an illustration of location of cable and displacement of pressure line.

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INTRODUCTION

The basic idea behind continuity in prestressed concrete beams is to achieve benefits in strength and economy. By introducing continuity in prestressed concrete beams, the load carrying capacity of a continupus beam is increased because the prestressing cable at intermediate support contributes to resisting moment. Also, continuity results in a reduction of the amount of concrete and steel. Because of this property of continuous construction it is possible to provide smaller section for the same loading. This study treats the effect of continuity on the design of prestressed concrete beams.

The actual economy of materials and cost of construction resulting from the use of continuity is greatly influenced by design criteria, magnitudes of the spans involved, type of structure under consideration, type of loading and available methods of prestressing.

Continuity could be used effectively and economically in long span prestressed concrete structures, particularly bridges. With the increasing development and use of limit design and partial prestressing in concrete structural design, it is expected that the economy achieved through the use of continuity will be extended to moderate span and perhaps short span structures in the future.

Continuous prestressed spans frequently have depth to span ratios of the order of 1 to 30 for prismatic member and as little as 1 to 80 for minimum depth of members having variable depths. The greater rigidity of continuous prestressed members also results in less vibration from moving or altering loads.[2]

Continuous prestressed concrete structures are more rigid and for this reason shallower members can be used on long spans without incurring any excessive deflection. The over-all structural stability and resistance to longitudinal and lateral loads is normally improved through the introduction of continuity.

> "In prestressed concrete, the same cable for positive moment is bent over to the other side to resist the negative moment, with no loss of overlapping. In addition, continuity in prestressed concrete saves end anchorages otherwise required over the intermediate supports, thus resulting in further economy and convenience. Owing to the variation of moment along the beam, the concrete section and amount of steel can be varied accordingly. The peaks of negative moments can be reinforced with nonprestressed steel, thus reducing the amount of prestressing steel."[3]

In continuous prestressed concrete beams, the moments induced by the prestressing force at any section depends on the eccentricity of prestressing cable and magnitude of secondary moment at that section. The line of center of compressive forces, called the line of pressure or C-line, usually lies above or below the profile of the centroid of the cable and has the same general shape. The reason for this is that the tension in the cable creates elastic deformations which are resisted by the restraint at the supports.

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2. "Prestressed Concrete Design and Construction", J. R. Libby, The Ronald Press Company, New York, 1961, p. 171.

3. "Design of Prestressed Concrete Structures", T. Y. Lin, John Wiley & Sons, Inc., New York, 1955, p.286.

PROBLEMS DUE TO CONTINUITY

Continuity creates some problems in design and construction of prestressed concrete structures. Continuous structures are necessarily indeterminate structures. The additional complications which result from the use of indeterminate prestressed concrete structures would be unfamiliar and unliked by contractors. The layout of continuous prestressed concrete beams involves many practical difficulties such as adjusting the shape of tendon trajectory, stressing and threading of tendons, grouting the cable hose etc. Moreover, the economy achieved through the use of continuous construction is significant for long span structures only, but it is not as significant for moderate spans or small spans.

There is a large variation in prestressing force due to friction between the cable and sheath during prestressing, which results in loss of prestress. This loss is very serious in longer and highly curved cables. Although such loss can be minimized by using relatively straight cables in haunched beams or by providing overtensioning.[4]

It is easier to achieve continuity in cast-in-place construction, but continuity in precast construction involves the difficulty of stressing and placing cap cables which would require large number of anchorage devices. Moreover, handling and transporting long precast beams is inconvenient and involves a certain possibility of damage. In such cases the precast beams are made of smaller convenient length equal to the length

4. "Prestressed Concrete", Gustov Magnel, Concrete Publications Limited, 14 Dartmouth Street, Westminster, S.W., London

of span and then continuity is achieved during construction.

Secondary stresses are of primary importance in the analysis of continuous prestressed concrete beams. Secondary stresses due to prestressing could be serious if they are not properly controlled or allowed in the design. The problems of secondary moments and frictional losses are considerably important, hence they are discussed under separate headings.

SECONDARY MOMENTS IN CONTINUOUS BEAMS

In design of continuous prestressed concrete beams, the moments induced by prestressing force are called secondary moments. The moments given by the eccentricity of prestressing force are called primary moments.

The magnitude and nature of secondary moments can be illustrated by considering two span continuous prismatic beam as shown in Fig. 1. The beam is prestressed with straight tendon which has a force F and eccentricity e. When the beam shown in Fig. 1(a) is prestressed, it bends upwards. The bending of the beam would tend to lift the beam at all points except at the end supports. In this condition, the beam would tend to deflect away from the central support by the amount:

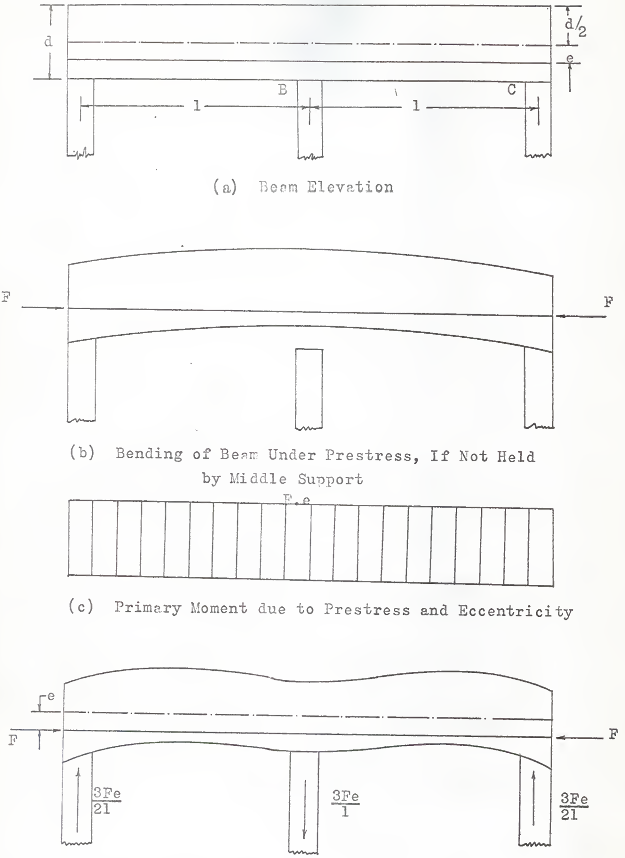
$$= \frac{F.e.(2l)^2}{8EI} = \frac{F.e.l^2}{2EI} \text{ ....(1)}$$

where, F denotes the effective prestressing force, e the eccentricity of prestressing force, l the length of each span, E the modulus of elasticity for concrete, and I the moment of inertia of the section.

In order to keep the beam in contact with the middle support, a reaction force acting downward must be exerted on the beam to hold it in place, as shown in Fig. 1(d). The deflection at the center due to induced downward reaction R_b is:

$$= \frac{R_b(2l)^3}{48EI} = \frac{R_b l^3}{6EI} \text{ ....(2)}$$

The deflections given by Eq.(1) and (2) should be equal in magnitude and opposite in direction. Equating Eg.(1) and (2) yields:



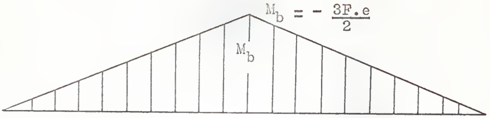
(a) Beam Elevation

(b) Bending of Beam Under Prestress, If Not Held by Middle Support

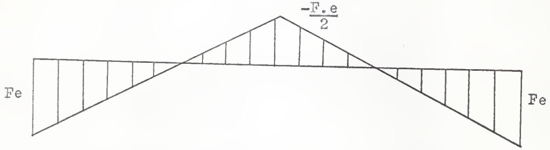
(c) Primary Moment due to Prestress and Eccentricity

(d) Reactions Exerted to Hold Beam in Place

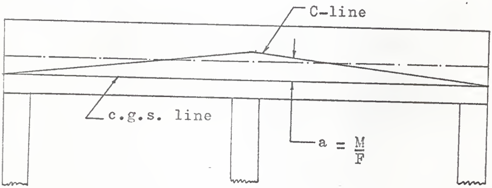
Fig.1. ANALYSIS OF SECONDARY MOMENTS



(e) Secondary Moment Diagram due to Downward Reaction R_b



(f) Resulting Moment Diagram



(g) Deviation of C-line from C.G.S. line

Fig.1. ANALYSIS OF SECONDARY MOMENTS(Cont'd)

$$R_b = \frac{3F.e}{l} \quad \text{..................(3)}$$

This reaction R_b produces upward reactions at the end supports, the sum of which should be equal to the downward force applied at the middle support. Thus, in continuous beam prestressing force induces reactions. These reactions produce moments which are called secondary moments, as shown in Fig. 1(e). To resist these moments, the pressure line must move to a distance a from the center of gravity of steel as shown in Fig. 1(f), such that the internal resisting moment will be equal to the external moment. It follows that

$$a = M/F$$

in which, a denotes vertical distance between the pressure line and the c.g.s. line and M is the secondary moment at the section under consideration.

In this example, secondary moments are positive and are favorable at the middle support because they should be deducted from the negative bending moments due to the loads. However, they are unfavorable at midspan because they should be added to the positive bending moment due to the loads.

It is noted from the previous example that one of the effects of prestressing is the creation of secondary reaction which cause linear moment diagram shown in Fig. 1(e). The combination of the moment diagrams shown in Fig. 1(c) and 1(e) results in an actual moment shown in Fig. 1(f). This shows the nature of secondary moments.

The secondary moments are not negligible. Their magnitude and sign depend on the position of the cable. The secondary moment make the design and analysis of continuous prestressed concrete beams complex. Its complexity increases with the increase of number of redundants in indeterminate structures.

In most area of structural design, the term secondary moment denotes undesirable moments. However, in the design of continuous prestressed concrete beams, secondary moments are not always undesirable and can be very helpful as explained in previous discussion.

FRICTIONAL LOSSES

Frictional resistance is developed between the cable and the concrete when prestressing cables are tensioned. This produces a loss of prestress. The frictional force $\mu F/R$ and the loss of prestress dF/dx in an infinitesimal length dx are equal in magnitude and opposite in direction. Thus, they can be equated as follows:

$$\frac{dF}{dx} = -\frac{\mu F}{R}$$

or $$\frac{dF}{F} = -\frac{\mu dx}{R}$$

where, u denotes the coefficent of friction, R the radius of curvature of tendon and $de = dx/R$ is the angle subtended at the center of curvature of length dx.

Substitution of dx/R by de in the above equation yields:

$$\frac{dF}{F} = \mu de \quad \text{...............(4)}$$

At the time of prestressing, the magnitude of initial prestressing force is Fi. After the loss of prestress, the effective prestressing force becomes equal to F. The effective prestressing force, F, is the active force throughout the life time of structure. F will always be smaller then Fi.

Integrating Eq.(4) between these limits yields:

$$\int_{Fi}^{F} \frac{dF}{F} = -\mu \int_{o}^{e} de$$

or $$F = Fi.e^{-\mu\theta} \quad \text{............(5)}$$

The frictional losses mentioned above depend on the following factors:[5]

5. "Continuous Prestressed Concrete Beams", Fritz Leonhardt, ACI Proceedings, 1952, vol. 48, p.45-34.

(1) Friction coefficent between the prestressing cable and the cable channel. This depends on: (a) surface properties such as rust formation on the cable or sheathing prior to prestressing, (b) tightness of the cable casings against penetration of mortar when concreting, (c) pressure of prestressing cables against the casings, and (d) hardness difference between the prestressing steel and its casing.

(2) Pressure created by the normal reaction due to change of angle of prestressing units.

(3) In multiple prestressing units which are not prestressed simultaneously, the bending of prestressed and unprestressed wires.

(4) Deviation of the channels from the required position.

Frictional forces can be minimized by applying initial prestressing force greater than that required and then reducing the force to the required tension before anchoring the wires. Proper measures for reducing the friction without disturbing the bond between the concrete and the cables include the use of (a) sliding provisions in the cable channels, (b) concentrating the prestressing steel in a few cables and arranging the cables in horizontal layers, instead of circular arrangement, (c) using the straight prestressing cables, and (d) changing cross section of girder such that girder's center of gravity line occupies desirable position with respect to axis of prestressing cable.

ANALYSIS OF CONTINUOUS BEAMS BY ELASTIC THEORY

In the analysis and design of continuous prestressed concrete beams the following assumptions are generally made:[2]

(1) The concrete acts as an elastic material within the range of stresses permitted in the design.

(2) Plain sections remain plain after bending.

(3) The effects of each cause of moments can be calculated independently and superimposed to attain the result of the combined effect of the several causes.

(4) The effect of friction on prestressing force is negligible. But where it is appreciable, it should be taken into account.

(5) The same tendons run through the entire length of the member.

(6) The eccentricity of prestressing force is small in comparison with the span and hence, the horizontal component of the prestressing force can be considered uniform throughout the length of the member.

(7) Axial deformation of the member is assumed to take place without restraint.

> "Tests on continuous prestressed concrete beams have shown that the elastic theory can be applied with accuracy within the working range. Since there is little or no tensile stress in the beam under working loads, there are no cracks, and the beam behaves as a homogenuous elastic material, more so than an ordinary reinforced-concrete beam which usually is cracked in certain portions."[3]

Several methods are available for the analysis of continuous prestressed concrete beams. However, the selection of a particular method is determined by the designer for the problem under consideration. The moment distribution method is considered the easiest method for analysing prismatic beams. For nonprismatic beams, theorm of three moments or conjugate beam method can be used.

The usage of moment distribution requires to convert the end eccentricities, curvature, and abrupt changes in slope of prestressing tendons into equivalent end moments, equivalent uniform and equivalent concentrated loads. The fixed end moment resulting from the equivalent loads are distributed according to usual procedure of moment distribution. In general, equivalent loads for parabolic tendons are uniform loads and equivalent loads for straight slope tendons are concentrated loads.

The following are the important features in the analysis of continuous prestressed concrete beams: (a) secondary moments, (b) moment at continuous support, (c) equivalent loading, (d) linear transformation and concordancy of cables, (e) location of cable, and (f) location of pressure line. The nature and behaviour of secondary moments have been discussed previously. The other features will be discussed in the following headings.

METHOD OF EQUIVALENT LOADING

Fig. 3(a) shows a two span continuous beam prestressed by a parabolic tendon having an eccentricity, e_1, at each support. The tendons deflect downward parabolically between supports through a total vertical displacement equat to e_t. From the geometry of parabola, the tangent to the parabolic tendon at the support is equal to:

$$\tan e = \frac{2.e_t}{1/2} = \frac{4\ e_t}{1} \quad \ldots\ldots\ldots\ldots\ldots(6)$$

Since the curvatures are assumed to be small,

$$\tan e \cong \sin e \cong e$$

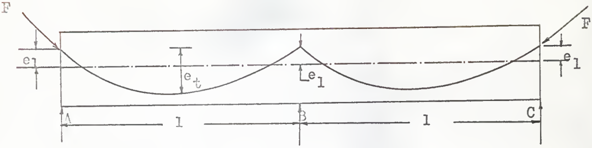
The vertical component of prestressing force at each end of each span is:

$$V_F = F \sin e \cong F \tan e \cong \frac{4\ F.e_t}{1} \ldots\ldots\ldots(7)$$

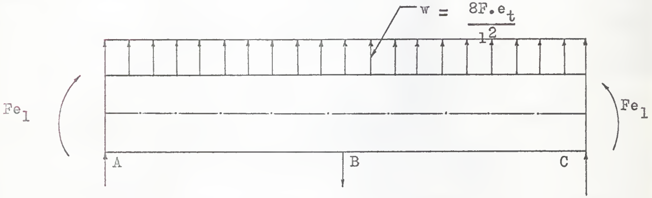
Therefore, the total vertical component of the prestressing force which acts on each span is equal to twice the force which acts at each end and the equivalent uniform load is:

$$w = \frac{8\ F.e_t}{1^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

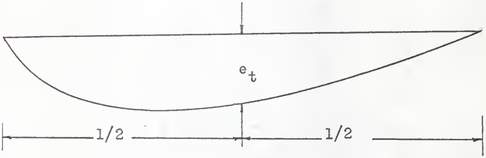
The vertical components of prestressing force which occur at the supports do not cause moments in the beam but pass directly through the supports and for this reason these forces are disregarded in the equivalent loading. The horizontal component of the prestressing force is eccentric by an amount equal to e_1 at each end of the beam and the equivalent loading must therefore include end moments equal to $F.e_1$. The equivalent loading is shown in Fig. 3(b).



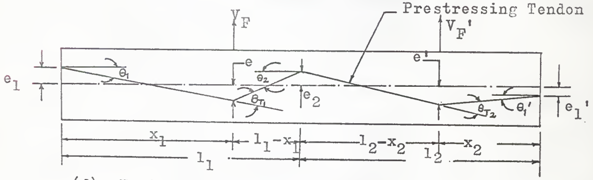
(a) Elevation of Beam



(b) Equivalent Loading for parabolic tendon



(c) e_t for Parabolic Tendon with Different End Elevation



(d) Equivalent Load for Straight Tendon

Fig.3. METHOD OF EQUIVALENT LOAD

When the curvature is not uniform over the entire span or when curves end on different elevations, the value of e_t to be used in Eq. (8) is the vertical distance at the center of span 1 from the curve to the chord which connects the ends of the curve as shown in Fig. 3(c).

> "It is usually sufficiently accurate to assume all curves are parabolic even though they may be circular or of other shape. Since the eccentricity is normally small in comparison to the span, the error which is introduced by this assumption is small."2

Fig. 3(d) shows a two span continuous beam prestressed by tendons having straight slopes and having different eccentricities at different levels. The equivalent vertical load, V_F, resulting from an abrupt change in slope of the tendons is computed as follows:

$$V = F.\sin\theta = \tan\theta \text{ ....(7)}$$

The value of tanθ is determined by the dimensions of the tendon trajectory as shown in Fig. 3(d).

$$\tan\theta_{T_1} = \theta_{T_1} = \theta_1 + \theta_2$$

$$= \frac{e_1 + e}{x_1} + \frac{e_2 + e}{l_1 - x_1} \text{ ....(9)}$$

Substitution of tanθ in Eq. (7) yields:

$$V_F = F\left(\frac{e_1 + e}{x_1} + \frac{e_2 + e)}{l_1 - x_1}\right. \text{ ....(10)}$$

In the same way, expression for equivalent load V_F' can be computed from the geometry of the figure as follows:

$$V_F' = F\left(\frac{e' - e'_1}{x_2} - \frac{e_2 - e)}{l_1 - x_1}\right. \text{ ....(11)}$$

where, V_F denotes the equivalent concentrated load in the span AB, V_F' the equivalent concentrated load in the span BC, e_1 the eccentricity of cable at exterior end in span AB, e the eccentricity of cable at angular bend in span AB, e_2 the eccentricity of cable in span BC at the extreme right end.

As an illustration, two problems showing the method of equivalent loading, computation of final moments at continuous support and position of pressure line will be treated.

Problem 1:

Compute the moments due to prestressing by using the equivalent loading method and show the location of pressure line for the prismatic beam shown in Fig. 4(a). Prestressing force is equal to 400 kips.

Solution:

Equivalent uniform load w is:

$$w = \frac{8F.e_t}{l^2} \quad \text{................(8)}$$

The value of e_t to be substituted in Eq.(8) for parabolic tendon having different end elevation is:

$$e_t = 0.60 + \frac{0.40 + 1.0}{2} = 1.30$$

Substitution of e_t in Eq.(8) yields:

$$w = \frac{8.400.(1.30)}{(100)^2} = 0.416 \text{ k/ft.}$$

The fixed end moments due to this uniform load are:

$$M^F_{AB} = -\frac{w.l^2}{12} = -0.416\,x(100)^2 = -346.66 \text{ k-ft.}$$

$$M^F_{BA} = +\frac{w.l^2}{12} = +346.66 \text{ k-ft.}$$

$$M^F_{BC} = -\frac{w.l^2}{12} = -346.66 \text{ k-ft.}$$

$$M^F_{CB} = +\frac{w.l^2}{12} = +346.66 \text{ k-ft.}$$

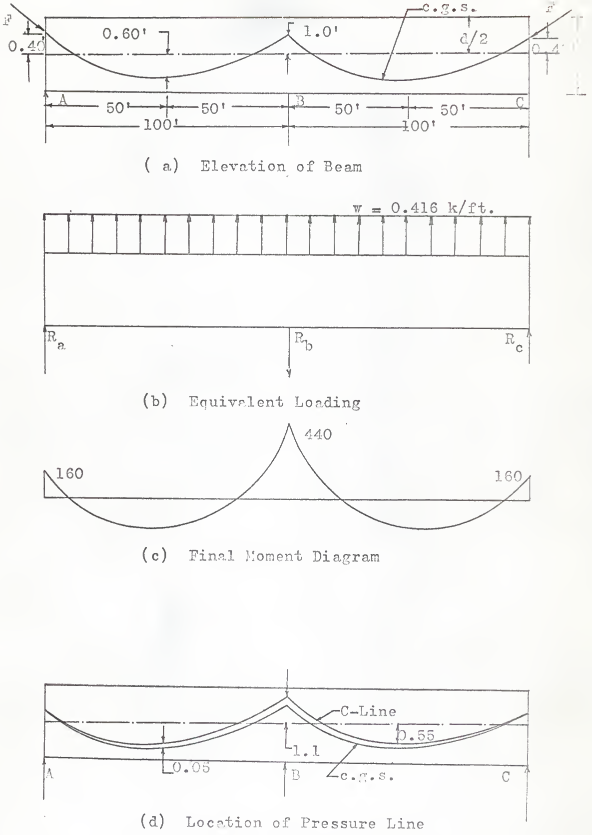


Fig. 4. ANALYSIS OF PARABOLIC TENDON

Overhang moments due to end eccentricities:

$M_A = -0.40 \times 400 = -160.00$ k-ft.

$M_C = +9.40 \times 400 = +160.00$ k-ft.

The moment distribution yields the moments shown in Fig. 4(c). The resulting moment at the support B is 440.0 k-ft. The eccentricity of pressure line at the support B is:

$$e = \frac{440}{400} = 1.10 \text{ ft.}$$

This shows that the pressure line is 1.1 ft. above the center of gravity of concrete, in other words, the pressure line is shown in Fig. 5(e).

The moment M_B as computed by moment distribution can be checked by using the theorm of three moments:

$$M_A \cdot l_1 + 2M_B (l_1 + l_2) + M_C \cdot l_2 = -\left(\frac{w \cdot l_1^3}{4} + \frac{w \cdot l_2^3}{4}\right) \text{ .....(14)}$$

here, $w = 9.416$ and $l_1 = l_2 = 100$ ft.

Substitution of value of w, l_1 and l_2 in Eq. (14) yields:

$$M_A + 4M_B + M_C = -2 \left(\frac{0.416 \times 100^2}{4}\right) \text{ ....(15)}$$

Substitution of values for M_A and M_B as found from end eccentricities, in Eq.(15) yields:

$4M_B = -2080 - 2(160) = -1760$

or $M_B = -440.0$ k-ft.

The actual moment at the center of the span AB as shown in Fig.4(c), is 220 k-ft. The eccentricity of pressure line at the center of span AB is:

$$e = \frac{220}{400} = 0.55 \text{ ft.}$$

In other words, the pressure line is (0.60–0.55) = 0.05 ft. above the c.g.s. line at the center of span AB. The same displacement of 0.05 ft. can be explained as:

$$\frac{50 \times 0.10}{100} = 0.05 \text{ ft.}$$

by using the linear transformation which will be discussed later on page 22.

Problem 2:

Compute the moments due to prestressing for the prismatic beam as shown in Fig.5(a), by applying the method of equivalent loads and show the location of pressure live for F = 600 kips.

Solution:

The equivalent concentrated loads are obtained by applying Eq. (10) and Eq.(11). Equivalent concentrated load for span AB, V_F', is:

$$V_F = 600 \left(\frac{0+1.40}{60} + \frac{0.60+1.40}{40}\right)$$

$$= \frac{600(8.80)}{120} = 44.0 \text{ kips}$$

Similarly, equivalent concentrated load for span BC, V_F', is:

$$V_F' = F\left(\frac{0.80+0}{50} + \frac{0.60+0.80}{50}\right)$$

$$= \frac{600 \times (2.2)}{50} = 26.4 \text{ kips}$$

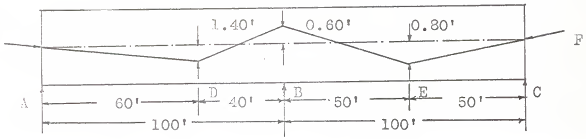
Fixed end moments are given by:

$$M^F_{AB} = \frac{W.a.b^2}{l^2} = \frac{44 \times 60 \times (40)^2}{(100)^2} = -422.0 \text{ k-ft.}$$

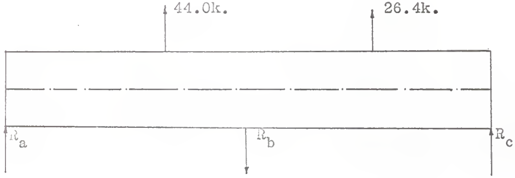
$$M^F_{BA} = \frac{W.b.a^2}{l^2} = \frac{44 \times 60 \times (60)^2}{(100)^2} = 630.0 \text{ k-ft.}$$

$$M^F_{BC} = \frac{(26.4) \times 50 \times (50)^2}{(100)^2} = -330.0 \text{ k-ft.}$$

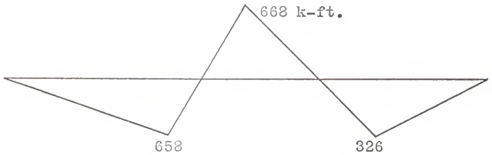
$$M^F_{CB} = \frac{(26.4) \times 50 \times (50)^2}{(100)^2} = 330.0 \text{ k-ft.}$$



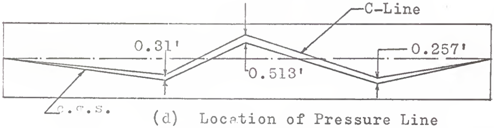
(a) Elevation of Beam



(b) Equivalent Load Diagram



(c) Final Moment Diagram



(d) Location of Pressure Line

FIG. 5. ANALYSIS OF STRAIGHT TENDON

The moment distribution yields the moments as shown in Fig. 5 (d). The moment at point D and E, as shown in Fig. 5(c) are:

at D: 658.0 k-ft.

at E: 326.0 k-ft.

The eccentricity of pressure line at support B is:

$$e = \frac{668}{600} = 1.113 \text{ ft.}$$

Therefore, the displacement of pressure line at B from c.g.s. line is:

1.113-0.60 = 0.513 ft.

The eccentricity of pressure line at point D is:

$$e = \frac{655}{600} = 1.09 \text{ ft.}$$

Therefore, the displacement of pressure line at D and E, with respect to c.g.s. line is:

at D: 1.40 - 1.09 = 0.31 ft.

at E: 0.80 - 0.543 = 0.257 ft.

The eccentricities at point B, D and E are shown in Fig. 5 (d).

LINEAR TRANSFORMATION AND CONCORDANCY OF CABLES

When the position of c.g.s. line or of C-line is moved over the interior supports of a continuous beam without changing the general shape of the line within the individual span, the line is said to be linearly transformed. The position of the c.g.s. line is moved only over the interior supports and not at the ends of the beam. In fact, a line can still be called linearly transformed if it is moved at the ends. But for design purposes, linear transformation without altering the position of c.g.s. line at end supports is more useful.

The C-line resulting from prestressing a continuous beam is a linearly transformed line from the c.g.s. line. In other words, in prestressing a continuous beam, the C-line gets deviated from c.g.s. line. This is due to the fact that secondary moments produce the deviation. This deviation between the two lines varies linearly between any two consecutive points. Another point is that, in a continuous beam, linear transformation of any tendon can be made without changing the location of C-line. This means that linear transformation of c.g.s. line does not affect the stresses in the concrete since the C-line remains unchanged. This shows that C-line is independent of the eccentricity of c.g.s line. This can be explained by an illustrative problem.

<u>Problem 3:</u>

A prestressed concrete beam is continuous over two spans with two linearly transformed c.g.s. lines as shown in Fig.8. Show that these c.g.s. lines give the same C-line. Prestressing force is equal to 400 kips.

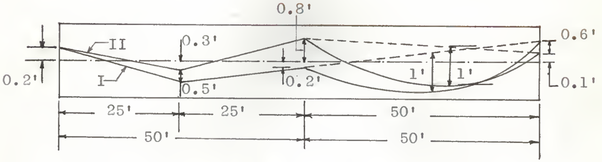


Fig.6. LINEAR TRANSFORMATION OF C.G.S. LINE

Solution:

If it can be proved that equivalent loading due to the first c.g.s. line on each span is equal to the equivalent loading due to the second c.g.s. line on the respective spans, then the C-line for both c.g.s. lines is the same. With this in mind, equivalent loads are computed for each case: Due to the first c.g.s. line:

Span AB:- Equivalent concentrated load is given by:

$$V_F' = F\left(\frac{e' - e_1'}{x_2} - \frac{e_2 - e'}{l_2 - x_2}\right) \quad \ldots\ldots\ldots\ldots\ldots(11)$$

$$= 400\left(\frac{0.8-0.2}{25} - \frac{0.2+0.8}{25}\right)$$

$$= \frac{400 \times 1.6}{25} = 25.6 \text{ kips.}$$

Span BC:- Equivalent uniform load is given By:

$$w = \frac{8F.e_t}{l^2} \quad \ldots\ldots\ldots\ldots\ldots(8)$$

$$= \frac{8 \times 400 \times (1.25)}{50^2} = 1.6 \text{ k/ft.}$$

Due to the second c.g.s. line:

Span AB:- Equivalent concentrated load is given by:

$$V_F = F\left(\frac{e_1 + e}{x_1} + \frac{e_2 + e}{l_1 - x_1}\right) \quad \ldots\ldots\ldots\ldots(10)$$

$$= 400 \left(\frac{0.2+0.3}{25} + \frac{0.8+0.3}{25}\right)$$

$$= \frac{400 \times (1.6)}{25} = 25.6 \text{ kips}$$

Span BC:- Equivalent uniform load is given by:

$$w = \frac{8 \times 400 \times (1.25)}{50^2} = 1.6 \text{ k/ft.}$$

From the above computations, it is noted that the corresponding equivalent loads on each span is the same for the first and the second c.g.s. line, hence it is proved that the C-line is the same for two linearly transformed c.g.s. lines.

The principle of linear tranformation is equally applicable to concordant or nonconcordant cables. This principle is particularly useful in designing continuous beams when it may be desirable to adjust the location of tendon in order to provide more protective cover over the prestressing tendons without altering the location of pressure line.

In continuous beam, if the C-line coincides with c.g.s. line, c.g.s. line is said to be concordant cable. If the C-line does not coincide with c.g.s. line, it is said to be noncordant cable. Concordant cable does not produce secondary moments. In a continuous beam external reactions will usually be induced by prestressing. These reactions produce secondary moments which shift C-line from c.g.s. line. In this condition, cable is called nonconcordant cable. It is possible that by chance or purpose, no reactions are induced in continuous beam in which case secondary moment does not exist and hence the cable is concordant. A concordant cable, while prestressed, tends to produce no deflection of beam over the supports hence no reactions will be induced. The C-line due to prestressing is itself a line linearly transformed from c.g.s. line. A concordant cable being easier in computation is preferable, but in practice, concordant cable is not found so frequently, unless desired specifically. The use of concordant or nonconcordant cable depends on the practical situation. The real choce of a location of c.g.s. line depends on the production of desirable C-line and satisfying the corresponding practical requirement and not on concordancy or nonconcordancy of cable. If the location of concordant cable falls outside the beam it can be linearly transformed to give a more practical location.

In continuous beam, final moment diagram, while plotted to suitable scale, is one location for concordant cable in that beam, provided that the beam is applied with constant prestress. From final moment diagram, any number of concordant cable can be found as proportional to final moment diagram and one most suitable concordant cable is selected. Combination of two or more concordant cables will result in another concordant cable by applying the principle of superimposition but the combination of concordant cable with nonconcordant cable will result in nonconcordant cable.

For any particular beam, the specific shape of the cable between two supports will result in specific pressure line for each condition of eccentricity at end supports. Alternation of eccentricity at one or both end supports or changing the basic shape of the trajectory between supports will shift the location of pressure line. Alternation of eccentricity of cable trajectory at interior supports will not affect the location of pressure line, if the eccentricities at the ends of beam are not changed and general shape of the trajectory is not changed.

ANALYSIS OF CONTINUOUS BEAMS BY CONJUGATE BEAM METHOD

A typical problem of the beam subjected to variable moments of interia is analysed by using conjugate beam method.

Problem 4:

Compute the moments due to prestressing for the beam shown in Fig. 7(a) and show the location of pressure line. Relative moments of interia is 1.0 for outermost 75 ft. of each span and 1.25 for the middle 50.0 ft. The center of gravity of the section is a straight line. Prestressing force is equal to 1000 kips.

Solution:

The M/I diagram due to eccentricies of prestressing force, the assumed M_b diagram due to secondary moments, and corresponding M_b/I diagram are shown in Fig. 7(b), (c), (f) and (g). Here, instead of finding equivalent loads, the aim is kept towards finding the secondary moments by using the conjugate beam method. The deflection at the middle support of the beam, due to the primary moment and secondary moment should be equal and opposite. The moment at the center of the beam AC, loaded by M/I diagram and M_b/I diagram, will be computed by conjugate beam method.

(1) The upward deflection due to $Fe/I = M/I$ diagram is δ_{Fe}:

Dimensions	Area	Distance of C.G. from B in ft.	Moments in k-ft.
(1) -700 x 60/2	-21000.0	60.0	-1260000.0
(2) -212 x 15	- 3180.0	32.5	- 103500.0
(3) -488 x 15/2	- 3660.0	35.0	- 128200.0
(4) -170 x 6.5/2	- 552.5	22.8	- 12600.0
(5) +480 x 185/2	+ 4440.0	6.16	+ 27250.0
	-23952.0		-1477050.0

The reaction R_a and R_c of conjugate beam are of opposite sign from the sign of loading. This shows:

$R_a = 23952.0$ kips

or $M_B = 23952 \times 100 - 1477050 = 918150.0$ k-ft.

By properties of conjugate beam, the moment at B is equal to deflection at B. This yields:

$\delta_{Fe} = 918150.0$ k-ft. ....(12)

(2) The downward deflection due to M_b/I diagram is δ_{M_b}:

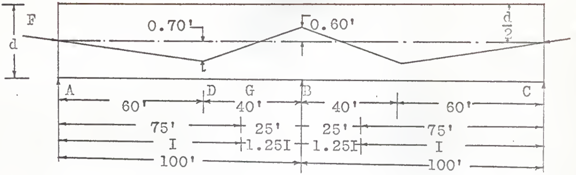
Dimensions	Area	Distance of C.G. from B in ft.	Moments in k-ft.
(1) 0.75 M_b x 75/2	28.2	50	1420.00
(2) 0.60 M_b x 25	15.0	12.5	187.50
(3) 0.20 M_b x 25/2	2.5	8.33	20.82
	45.7 M_b		1628.32 M_b

The reaction of conjugate beam is:

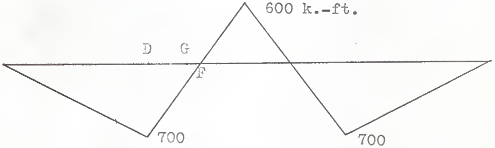
$R_a = R_c = -45.7\ M_b$

Taking moment about B for conjugate beam,

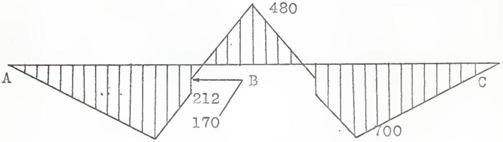
$M_B = -45.7\ M_b \times 100 - 1628.32\ M_b$



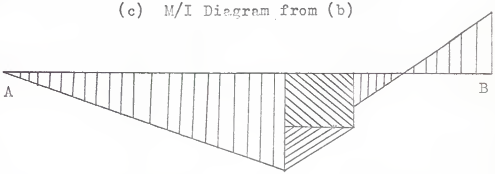
(a) Elevation



(b) Primary Moment Diagram

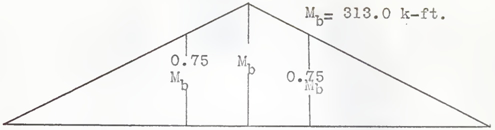


(c) M/I Diagram from (b)

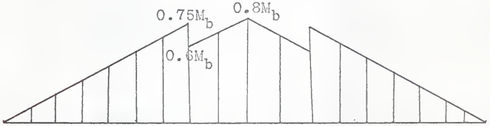


(d) Magnified Sketch for Span AB

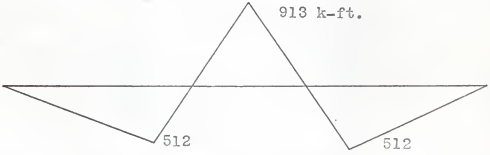
Fig. 7. CONTINUOUS BEAM WITH VARIABLE I



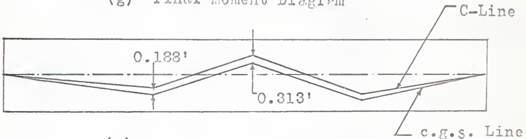
(e) Secondary Moment Diagram (M_b)



(f) M_b/I Diagram from (e)



(g) Final Moment Diagram



(h) Location of Pressure Line

Fig.7. CONTINUOUS BEAM WITH VARIABLE I (Cont'd)

or $M_B = -2942\, M_b$

Therefore, downward deflection at B, due to secondary moment M_b is:

$\delta_{M_b} = 2942\, M_b$ ....(13)

Equating Eq. (12) and (13) yields:

$918150 = 2942\, M_b$

or $M_b = 313.0$ k-ft.

The ordinates of moment diagram at various points can be found by refering Fig. 7(b), (c), and (g). The ordinate of moment diagram at point G. 25' away from B is found by calculating some distances as follows:

$$\frac{600}{700} = \frac{FB}{40-FB}$$

or $FB = 18.5$ ft.

or $DF = DB - FB = 40-18.5 = 21.5$ ft.

$DG = DB - GB = 40-25.0 = 15.0$ ft.

$GF = DF - DG = 21.5 - 15.0 = 6.5$ ft.

The moment at G is:

$$GH = \frac{6.5}{21.5} \times 700 = 212.0 \text{ k-ft.}$$

The moment at B. as shown in Fig. 7(b) is:

$0.60 \times 100 = 600.0$ k-ft.

Corresponding moments in M/I diagram, in Fig. 7(c) are:

at B: $\frac{600}{1.25} = 480.0$ k-ft.

at G: $\frac{212}{1.25} = 170.0$ k-ft.

The moments in secondary moment diagram:

at B: M_b

at G: $\frac{75}{100} M_b$

From Fig. 7(b)

primary moment at B: 0.60x1000 = 600.0 k-ft.

primary moment at D: 0.70x1000 = 700.0 k-ft.

Once the value of secondary moment, M_b, is found out, the resulting moment is the superimposition of Fig. 7(b) and Fig. 7 (f).

The resulting moment at B and D are:

at B: 600 - 313 = 913 k-ft.

at D: 600 - 0.60x313 = 512.2 k-ft.

The eccentricity of pressure line at B and at D, due to resulting moment at B and D is:

eccentricity at B: $e = \frac{913}{1000} = 0.913$ ft.

eccentricity at D: $e = \frac{512}{1000} = 0.512$ ft.

Therefore, displacement of C-line from c.g.s. line, at B, is:

0.913 - 0.60 = 0.313 ft.

and, displacement of C-line from c.g.s. line, at D, is:

0.70 - 0.512 = 0.188 ft.

It is interesting to note that if the cable shown in Fig. 7 (a) were linearly transformed in such a manner that eccentricity were zero at B and accordingly, eccentricity at A were $(0.70 - 0.60 \times \frac{60}{100}) = 1.06$ ft. at 60 ft. from A, the primary moment computed by the conjugate beam method would be equal to actual moment.

DESIGN OF CONTINUOUS BEAMS

Designing a continuous beam is essentially a procedure of trial and error. The following steps are as design guide:

(1) Assume the section and compute dead load moments.

(2) Compute the maximum and minimum moment due to given live load for various combination of dead load and live load.

(3) From the moments, compute the prestressing force, by priliminary design formula $F = \frac{M_T}{0.65\ h}$, where M_T is total momont and h is the height of section. This prestress force will be as guide to proceed for the design.

(4) Plot the top and bottom kern line limits, by taking distance from center line as r^2/y_t and r^2/y_b for top and bottom respectively. The kern limit is the zone where, if the pressure line falls, results in no tension. r is the radius of gyration and y_t, y_b are the distances from center of section to top and bottom fibers.

(5) With these moments and prestressing force, plot $a_G = \frac{M_G}{F_o}$ due to girder moment only, from top and bottom kern line. Plot $a_{max} = \frac{M_{max}}{F}$ with the help of M_{max} from the top kerm limits. Plot $a_{min} = \frac{M_{min}}{F}$ with M_{min} from bottom kern line. M_{mim} is the algibrically smallest moment. The ordinate of a_{min}, a_{max}, and a_G should be plotted upwards for negative moment and downward for positive moment. The area covered by these lines will be the zone of trial location of c.g.s. and location of pressure line due to prestress. In the trial c.g.s. zone, locate the prestressing cable and find the equivalent loading

due to prestressing force and check the stresses for the allowable limits.

A typical design problem of a continuous prestressed concrete beam subjected to variable moments of inertia is discussed here.

Problem 5:

Design a prestress continuous beam with two spans. Each span is 80' with straight haunch as shown in Fig. 8(a). The beam is designed for the design live load of 80 psf. Design the beam showing the location of cable and pressure line. Ultimate unit stress in concrete is 5000 psi.

Solution:

To start with, the depth of section is taken 14" from the end to the center of the span and then increasing by 14" at intermediate support. The width of beam is assumed as 12 inches. As the loading condition for bridge is varying, the beam is designed for girder moment, maximum moment for both spans loaded, and minimum moment for only one span loaded.

The fixed end moments for nonprismatic beam are calculated by using "Handbook of Frame Constants"(PCA Publication).

a_A = ratio of length of haunch at A to the length of span = 0

a_B = ratio of length of haunch at B to the length of span = 0.5

$r_A = \frac{h_A - h_C}{h_c}$ for rectangular section at end A = 0.0

$r_B = \frac{h_B - h_C}{h_c}$ for rectangular section at end B = 1.0

where, h_A, h_B, h_C is the deph of member at end A,B and at minimum section respectively. With these dimensions, entering the table

No. 52, page 22, the values of fixed end moments are like this:

$M^{R}_{AB} = 0.0597\ w.L^2$: $M^{F}_{BA} = 0.1390\ w.L^2$ (Uniform load)

$M^{F}_{AB} = 0.0033\ W_B\ .L^2$ $M^{F}_{BA} = 0.0321\ W_B.L^2$(Straight Haunch Load)

$M^{F}_{AB} = 0.0788$ P.L: $M^{F}_{BA} = 0.2371$ P.L. (Concentrated load)

w -Weight of straight portion of beam = $\frac{14}{12}$ .150 = 0.175 ksf.

W_B-Weight of straight haunch per lin. ft. at end B = $\frac{14}{12}$ .150

= 0.175 ksf.

w-Uniformly distributed live load = 80 psf. = 0.08

Fixed end moments due to straight portion of the beam:

$M^{F}_{AB} = 0.0597 \times 0.175 \times 6400 = 66.8$ k-ft.

$M^{F}_{BA} = 0.1390 \times 0.175 \times 6400 = 156.0$ k-ft.

Fixed end moments due to straight haunch of beam:

$M^{F}_{AB} = 0.0033 \times 0.175 \times 6400 = 3.72$ k-ft.

$M^{F}_{BA} = 0.0321 \times 0.175 \times 6400 = 35.8$ k-ft.

Fixed end moments due to live load:

$M^{F}_{AB} = 0.0597 \times 0.08 \times 6400 = 35.8$ k-ft.

$M^{F}_{BA} = 0.2371 \times 0.08 \times 6400 = 71.0$ k-ft.

Total dead load fixed end moments are:

$M^{F}_{AB} = 66.8 - 3.72 = 70.52$ k-ft.

$M^{F}_{BA} = 156 - 35.8 = 191.8$ k-ft.

Total live load fixed end moments are:

$M^{F}_{AB} = 70.52 - 30.6 = 101.12$ k-ft.

$M^{F}_{BA} = 191.8 - 71.0 = 262.8$ k-ft.

From Table 52, carry over factors and stiffness factors are:

$C_{AB} = 0.948$, $C_{BA} = 0.385$, $K_{AB} = 4.99$, $K_{BA} = 12.88$

As there is symmetry of the structure, there is no need to find relative stiffness factors and distribution factors at B are $\frac{1}{2}:\frac{1}{2}$. The fixed end moments at B, due to girder load, due to live load on both spans and live load on one span only, as calculated by moment distribution, are 259, 358 and 308 k-ft., respectively.

Moments due to freely supported action will constitute the moment due to straight portion and haunch.

Straight portion: $M_x = 7x - 0.0875\ x^2$

$M_{10} = 70 - 8.75 = 61.25$ k-ft.

$M_{40} = 140.0$

Haunch portion: $R_A = \frac{40 \times 0.175 \times 10}{2 \times 3 \times 100} = 0.466$ kip.

$M_{10} = 4.66$

$M_{50} = 23.30 - \frac{10 \times 0.0438 \times 10}{2 \times 3} = 22.57$

$M_{60} = 27.96 - \frac{20 \times 0.0875 \times 20}{2 \times 3} = 22.13$

$M_{70} = 32.62 - \frac{30 \times 0.1314 \times 30}{2 \times 3} = 12.92$

Freely supported moment due to live load:

$w = 0.08$ k/ft.: w.L = 6.4 k.

$M_x = 3.2\ x - 0.04\ x^2$

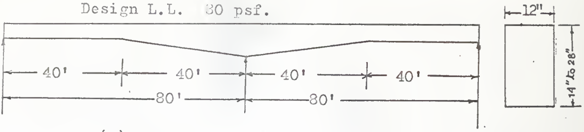
$M_{10} = 32 - 4 = 28.0$ k-ft.

$M_{40} = 64.0$ k-ft.

The actual moment on the beam due to dead load, and two live load conditions are shown in the fig. 8.

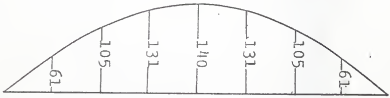
The prestressing force is found by applying the thumb rule formula, to start with, in the design.

$$F = \frac{M_T}{0.65\ h} \quad \text{..................(14)}$$

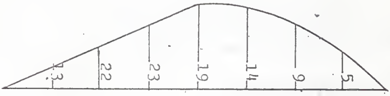


(a) Elevation

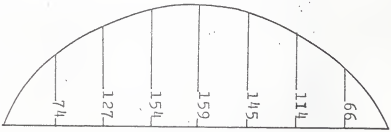
Cross-section



(b) Freely Supported Moment of Straight Portion

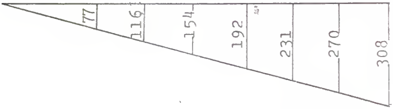


(c) Freely Supported Moment of Straight Haunch

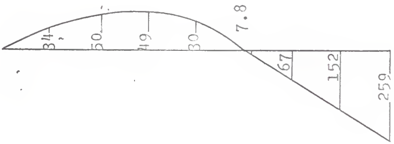


(d) Total Freely Supported Moment Due to Dead Load

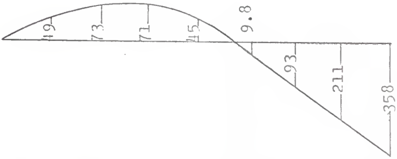
Fig. 8. CONTINUOUS BEAM WITH VARIABLE MOMENTS OF INERTIA



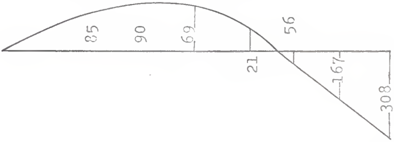
(i) Fixed End Moment Due to Live Load on Span AB Only



(j) Actual Moment Under Dead Load



(k) Actual Moment Due to Live Load on Both Spans



(l) Actual Moment Due to Live Load on Span AB Only

Fig.8. CONTINUOUS BEAM WITH VARIABLE MOMENTS OF INERTIA(Cont'd)

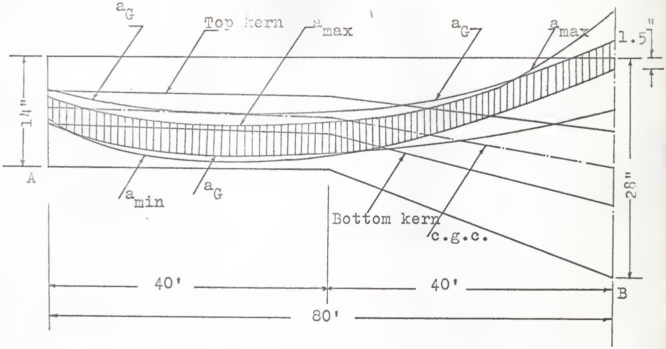


Fig. 9. LIMITING ZONE FOR TRIAL C.G.S. LOCATION

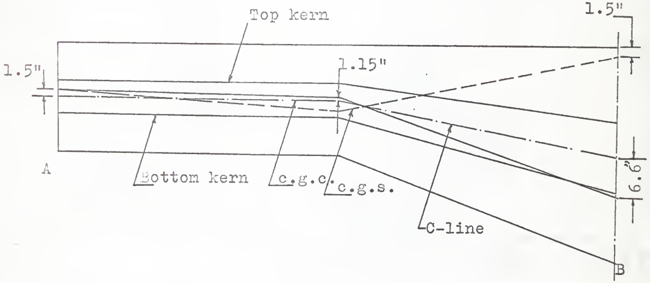


Fig. 10. LOCATION OF C-LINE UNDER FINAL CONDITION

where, F denotes the effective prestressing force, M_T the total moment at the section under consideration and h the height of the section under consideration.

Substitution of value of M_T and h at section B yields:

$$F = \frac{358 \times 12}{0.65 \times 28} = 236.0 \text{ kips}$$

While plotting trial c.g.s. location zone this will give corresponding ordinate of $\frac{358 \times 12}{236}$ = 18.2 inches, measured upward from bottom kern line. When this ordinate is plotted from bottom kern line, it will give the protective cover of the cable at section B as:

$$28 - 9.32 - 18.2 = 0.48 \text{ inch.}$$

However, to comply with the requirement for minimum cover, cable at section B in trial c.g.s. location, is kept 1.5 inches below the top of beam. Take F = 240 kips. The corresponding M_G/F_1, M_{mas}/F and M_{min}/F ordinates are computed as follows:

$$\frac{M_G}{F_1} = -1.54, -2.28, -2.22, -2.02, +0.36, +3.04, +7.0, +11.9$$

$$\frac{M_{max}}{F} = -2.76, -4.25, -4.45, -3.42, -1.06, +2.8, +8.35, +15.4$$

$$\frac{M_{min}}{F} = -2.46, -3.64, -3.55, -2.20, +0.49, +4.65, +10.5, +17.9$$

Considering the loss of prestress of 15 per cent, initial prestressing force, F_1, will be 240/0.85 = 282 kips. With the help of above ordinates, trial c.g.s. zone is plotted as shown in Fig. 9. From Fig. 9

eccentricity of prestress at end of A = 1.5 inches

eccentricity of prestress at the center of span = 5.5 inches

eccentricity of prestress at B = 11.5 inches or the relative height from the center line at the center of span is 5.5 inches.

The cable is located with the triangular bend at the center of span which indicates the equivalent load is point load.

$$\text{Total angle change} = \left(\frac{1.5 + 5.5}{12 \times 40} + \frac{5.5-5.5}{12 \times 40}\right) = \frac{18}{480}$$

Equivalent concentrated load, V_P, is:

$$V_P = 240 \times \frac{18}{480} = 9.0 \text{ kips}$$

Fixed end moments due to this equivalent loading are:

$M^F_{AB} = 0.0788 \times 0.08 \times 80 = 56.60$ k-ft.

$M^F_{BA} = 0.2371 \times 0.08 \times 80 = 171.0$ k-ft.

The moment distribution yields the moment of 224.60 k-ft. at Support B.

The effect of moment due to equivalent loading is to keep C-line above the center line, but the moment due to dead load has reverse effect. As the moment due to dead load is more, final effect would be to deviate the C-line below c.g.c. line.

net moment under dead load = +224.60 -258.6 = 34.0 k-ft.

$$\text{or eccentricity of C-line at B} = \frac{34.0 \times 12}{240} = 1.7 \text{ inches}$$

net moment at B under live load condition is

224.60 - 357.8 = -133.20 k-ft.

$$\text{or eccentricity of C-line at B} = \frac{133.20 \times 12}{240} = 6.6 \text{ inches}$$

Here also, live load moment is more than equivalent load moment, hence its effect will be to shift the pressure line below c.g.c.

Checking of the design stresses will be in this pattern:

(1) Under initial condition with full prestress and no live load

(2) Under final condition after losses have taken place and with full live load.

In the computations above, for finding the location of trial c.g.s. zone, ordinates due to maximum and minimum live load conditions were computed in terms of effective prestress, which takes place after losses from initial prestressing force. In fact, effective prestress is the prestressing force which is going to act for the life time of structure. However, for checking stresses under initial condition, initial prestressing force should be used.

Section properties:

At Section A:

Area of cross section , A= 14 x 12 = 168 sq.in.

Moment of inertia I = 2750.0 in^4

At section B:

Area of cross section, A = 28 x 12 = 336.0 sq.in.

$$I = 22000.0 \ in^4$$

(1) Stresses under initial condition at section B:

$$f = \frac{Fi}{A} \mp \frac{Fi\ .e.y}{I}$$

$$= \frac{264}{336} \mp \frac{264x1.7x14}{22000}$$

$$= -0.785 \mp 0.286$$

= -1071.0 psi, compression, at bottom

= -499.0 psi, compression, at top

(2) Stresses under final condition at section B:

$$f = \frac{F}{A} \mp \frac{F.e.y}{I}$$

$$= \frac{-240}{336} \mp \frac{240x6.6x14}{22000}$$

= -1725.0 psi, compression = -1725 psi, at bottom

= -295.0 psi, tension = -295.0 psi, at top.

Allowable compressive and tensile stresses with respect to the concrete of ultimate unit stress, f_c' of 5000 psi, are:

compressive stresses : $0.45 f_c' = 2250.0$ psi.

tensile stresses : $0.065 f_c' = 325.0$ psi.

This indicates that stresses developed under live load and dead load, while considering section B, are within the allowable limits, hence section B is O.K.

(1) Stresses under initial condition at center of span:

$$f = \frac{264}{168} \mp \frac{264 \times 1.2 \times 7}{2750}$$

$$= -1.56 \mp 0.80$$

= -2360 psi compressive, at top

= -760 psi compressive, at bottom

The stress at top is little more but near to allowable. Although, this stress is only for short period, i.e. only at the time of transfer; under live load, the stress will be under the allowable limits.

(2) Stresses under live load at the center of span:

$$f = \frac{-240}{168} \mp \frac{240 \times 1.15 \times 7}{2750}$$

$$= -1.43 \mp 0.7$$

= -2130 psi compressive, at top

= -730 psi compressive, at bottom

stresses are within allowable limits, hence section is O.K. Now, the stresses at the end A will be checked. As end A is free, it will not have any type of moment due to loads, but will have moment due to eccentricity of prestress.

(1) Stress under initial condition at end A:

$$f = \frac{-264}{168} \mp \frac{264 \times 1.5 \times 7}{2750}$$

$$= -1.57 \mp 0.84$$

= -2410 psi, compressive at bottom

= -730 psi, compressive at top

Stress at the bottom is little more but near to allowable limit, but is only for very short period, so its effect will not be predominant. Under final condition, the stresses are within the allowable limits as shown below:

(2) Stresses under final condition at end A:

$$f = \frac{-240}{168} \mp \frac{240 \times 1.5 \times 7}{2750}$$

$$= -1.43 \mp 0.765$$

= -2195 psi, compressive at bottom

= -665 psi, compressive at top.

CONCLUSIONS

Introduction of continuity in prestressed concrete beams results in an increase of strength. Most engineers are under the impression that continuous prestressed concrete structures are extremely difficult to design and analyse, due to secondary moments. One of the reasons is that many structural engineers are unfamiliar with this new method of design and construction. However, from this study, it was revealed that analysis of continuous prestressed concrete beams is not particularly complex and involves only the familiar principles used in the analysis of ordinary statically indeterminate structures.

Research on the performance of continuous prestressed beams has revealed that the assumptions of elastic theory do not introduce significant errors in normal applications. Special attention should be given if the cracking load of the beam is exceeded and if the effect of friction during prestressing is significant.

Design of continuous prestressed concrete beams by concordant cable is somewhat easier, because the C-line and c.g.s. line are identical which results in less computations. It is often desirable to start a design by assuming that the tendon trajectory is concordant cable. The tendon location can then be linearly transformed into nonconcordant cable.

With the same prestressing force, the section of the continuous prestressed concrete beam can be increased which would result in an increase of eccentricity of the cable. This will enable to balance the tensile stresses produced by applied load.

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APPENDIX I - BIBLIOGRAPHY

(1) "Design of Prestressed Concrete Structures", T. Y. Lin, John Wildy & Sons, Inc., New York, 1955.

(2) "Prestressed Concrete Design and Construction", J. R. Libby, The Ronald Press Company, New York, 1961.

(3) "Prestressed Concrete", Gustov Magnel, Concrete Publications Limited, 14 Dartmouth Street, Westminster, S.W., London.

(4) "Proceedings of First U.S. Conference on Prestressed Concrete", M.I.T., August 1951.

(5) "Continuous Prestressed Concrete Beams", Fritz Leonhardt, ACI Proceedings, 1953, Vol. 49, p. 617-636.

(6) "Design for Continuity in Prestressed Concrete Structures", A. L. Parme and G. H. Paris, ACI Prodeedings, 1952, Vol. 48, p. 45-64.

(7) "Practical Prestressed Concrete", Preston, McGraw-Hill Book Company, Inc., New York, 1953.

(8) "Prestressed Concrete", Vol.II, P. Guyon, John Wiley & Sons, Inc., New York, 1960.

(9) "Design of Prestressed Concrete Beams", W. H. Connolly, F. Dodge Corporation, New York, 1960.

(10) "Prestressed Concrete Structures", A. E. Komendant, McGraw-Hill Book Company, Inc., New York, 1960.

(11) "Strength of Continuous Prestressed Concrete Beams under Static and Repeated Loads", T. Y. Lin, ACI Proceedings, Vol. 51, p. 1037-1059.

(12) "Equivalent Load Method for Analysing Prestressed Concrete Structures", Robert B. B. Moorman, ACI Proceedings, Vol.48, p. 405-413.

APPENDIX II - NOTATIONS

a- Vertical distance at any section between C-line and c.g.s.

a_G- Ordinate due to girder moment, measured from kern limits.

a_{max}- Ordinate due to maximum moment, measured from top kern.

a_{mim}- Ordinate due to minimum moment, measured from bottom kern.

c.g.c.-Center of compressive forces or pressure line.

C-line-Center of compressive forces or pressure line.

c,g.s.-Center of gravity of steel.

F_i- Initial prestressing force.

F- Effective prestressing force.

f_e'- Ultimate unit stress in concrete, generally at 28 days old.

M_b- Secondary moment at section B.

r- Radius of gyration.

R- Radius of curvature of cable.

V_F- Equivalent concentrated load for straight tendon.

w- Equivalent uniform load for straight tendon.

y_t, y_b- Distance from center line of member to top or bottom fiber.

u- Coefficent of friction.

CONTINUITY IN PRESTRESSED CONCRETE BEAMS

by

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ABSTRACT

The basic idea behind continuity in prestressed concrete beams is to achieve benefits in strength and economy. By introducing continuity in prestressed concrete beams, the load carrying capacity of a continuous beam is increased. Secondary moments produced by a prestressing force take an important role in the analysis of continuous prestressed concrete beams. Continuity produces a decrease in moments and hence reduction of section for the same load and span in comparison with that of simple prestressed concrete beams.

The prestressing cable is located in such a way that stresses produced at any section remain within the allowable limits. Usually, computation of moments due to applied loading involves an integration process. However, in this study of continuous prestressed concrete beams, equivalent loads are computed from the secondary moments through a differentiation process. From the equivalent loads, final moments which give the eccentricity of pressure line, are computed.

The displacement of prestressing cable and pressure line involves.the principle of linear transformation and concordany of cable. The principle of linear transformation is particularly useful when it may be desirable to adjust the location of tendon in order to provide more protective cover over the prestressing tendons without altering the location of C-line. Linear transformation of c.g.s. line does not affect the stresses in the concrete since the C-line remains unchanged.

This shows that C-line is independent of the eccentricity of c.g.s. line. Design of continuous prestressed concrete beams by concordant cable is somewhat easier, because C-line and c.g.s. line are identical which results in less computations. It is often desirable to start a design by assuming that the tendon trajectory is concordant cable. The tendon location can then be linearly transformed into nonconcordant cable. Two typical problems of beams having variable moments of inertia and subjected to fixed and variable loading conditions are discussed as an illustration of location of cable and displacement of pressure line.