

# Public Infrastructure, Congestion, and Fiscal Policy

John Bennett, Brunel University

Phillip Lawler, University of Wales Swansea

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## Abstract

A macro model is developed incorporating the productive effects of public expenditure, but also allowing for congestion. The Pigouvian tax rate to correct for the distortion caused by congestion is found and the optimal level of public expenditure is characterized.

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Corresponding author: John Bennett, Department of Economics and Finance, Brunel university, Uxbridge, Middlesex, UB8 3PH, United Kingdom.

Tel. (+44) 1895 816201; fax (+44) 18895 203384; e-mail john.bennett@brunel.ac.uk

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## 1 Introduction

By the late 1980s, the search for explanations for the slowdown in the rate of productivity growth in the US and other OECD countries had stimulated considerable interest in the macroeconomic role of public infrastructure. In particular, Aschauer (1989) who modeled the aggregate production function with public expenditure as an input and undertook an empirical investigation for the US, concluded that the size of the ‘core’ infrastructure (roads, airports, sewers, etc.) had a significant impact on productivity. However, Aschauer’s results have been hotly debated (see Fernald, 1999) and the inclusion in macro models of a productive role for public expenditure has remained relatively uncommon. Exceptions include Lindbeck and Nandakumar (1990) and Baxter and King (1993). See also the early contributions of Grossman and Lucas (1974) and Barro (1981).

When public expenditure is included as an argument of the production function in the macroeconomics literature it is usually treated as a pure public good. However, this approach fails to take into account that the core infrastructure is the classic case in which congestion may occur. Empirical evidence indicates, for example, that congestion on US roads has held back productivity significantly since about 1973 (Fernald). In this paper we therefore develop a simple macroeconomic model in which the productive effects of public expenditure are allowed

for, but the possibility of congestion is also recognized: public expenditure provides an impure public good.

For simplicity, we restrict the analysis in this paper to perfect competition. Insofar as there is congestion in the use of public infrastructure, firms overuse it; that is, goods output exceeds the welfare-maximizing level. We therefore determine the tax rate that will correct for this distortion, as well as characterizing the optimal level of public expenditure.

## 2 The Model

Consider a closed non-monetary economy for a single time period, with perfectly competitive goods and labor markets. The goods market is in long-run equilibrium, with zero profit. The representative household has a Cobb-Douglas utility function,

$$u = C^\delta(1 - L)^{1-\delta}, \quad 0 < \delta < 1, \quad (1)$$

where  $C$  is the quantity it purchases of the representative good,  $L$  is the time it spends working, and 1 is its time endowment. Using the good as *numéraire*, the household's budget constraint is

$$C = wL - T, \quad (2)$$

where  $w$  is the real wage rate and  $T$  is a lump-sum tax.

Maximizing (1) subject to (2) yields the goods demand and labor supply functions

$$C = \delta(w - T) \tag{3}$$

$$L = \delta + \frac{(1 - \delta)T}{w}. \tag{4}$$

The government buys a quantity of the representative good  $G$ , which it transforms freely, at a constant proportionate rate, into infrastructure. We therefore refer to  $G$  interchangeably as ‘government expenditure’ and ‘infrastructure.’ We shall consider the possibility that the government discourages overuse of infrastructure by Pigouvian taxation. The analysis is developed in terms of a per unit tax rate  $t$  on labor input, but a tax on output would have equivalent effects. Assuming that the government’s budget is balanced,

$$G = T + twL \tag{5}$$

There are  $N$  firms producing the representative good, where  $N$  is large. The representative firm uses input  $l$  of labor services to produce output  $y$ , according to

$$y = \left( \frac{G}{Y^\theta} \right)^\epsilon l, \quad 0 < \epsilon < 1, 0 \leq \theta \leq 1, \tag{6}$$

where  $Y$  is the aggregate output of the good and  $\epsilon$  and  $\theta$  are parameters.  $Y = Ny$ , but since the firm is ‘small’ it treats  $Y$  as a parameter. From the perspective of the firm (i.e., given  $G$  and  $Y$ ), equation (6) exhibits constant returns.  $(G/Y^\theta)^\epsilon$  is a shift parameter and is our representation of the flow of services from the stock of infrastructure  $G$ . A greater value of  $(G/Y^\theta)^\epsilon$  entails more services for the firm and therefore a higher labor productivity. For example, if the government improves the transport infrastructure, the firm may transport its output using less labor time.  $\epsilon$  is the elasticity of the flow of services from the infrastructure with respect to the amount of the infrastructure  $G$ . We assume that  $0 < \epsilon < 1$  to represent the idea that, in terms of the firm’s labor productivity, there are diminishing marginal returns to improvements in the infrastructure.

The parameter  $\theta$  is our measure of the extent to which the infrastructure is prone to congestion. If  $\theta = 0$ ,  $(G/Y^\theta)^\epsilon$  reduces to  $G^\epsilon$  and so the firm’s labor productivity is independent of aggregate production. Alternatively, suppose  $\theta = 1$ , so that (6) becomes  $y = g^\epsilon l$ , where  $g \equiv G/Y$ , the infrastructure per unit of output in the economy. If, say, aggregate output  $Y$  doubles, with  $G$  held constant, the firm’s labor productivity falls. The amount of infrastructure would have to be doubled in order to restore the firm’s level of labor productivity. We regard  $\theta = 1$  as representing a high level of congestion in the use of infrastructural services. Similarly, if  $0 < \theta < 1$ , the economy is regarded as being subject to milder congestion. We exclude the case of  $\theta > 1$  because it has no obvious intuitive justification.

Aggregating (6) over the  $N$  firms, and taking into account that the labor market clears, the aggregate production function is

$$Y = \left( \frac{G}{Y^\theta} \right)^\epsilon L, \quad (7)$$

where  $L = Nl$ . (7) can be rewritten,

$$Y = G^{\frac{\epsilon}{1+\epsilon\theta}} L^{\frac{1}{1+\epsilon\theta}}.$$

Thus, paralleling endogenous growth models, if  $\theta < 1$  aggregate production displays increasing returns to scale with respect to  $\{G, L\}$ . However, there are decreasing returns for the firm with respect to  $l$ , which, together with the fact that the firms do not co-ordinate their activity, enables a competitive equilibrium to obtain (Klibanoff and Morduch, 1995).

Each firm is a price-taker and an infrastructure service-taker. In long-run equilibrium the cost per worker equals the average product:

$$w(1+t) = \left( \frac{G}{Y^\theta} \right)^\epsilon. \quad (8)$$

The system of equations (3), (5), (8) and (7), plus the goods market equilibrium

condition  $Y = C + G$ , reduces to

$$Y = (1 - \delta)G + \frac{\delta}{1 + t} \left( \frac{G}{Y^\theta} \right)^\epsilon \quad (9)$$

The right-hand side is decreasing in  $Y$  and so the solution is unique;  $dY/d\epsilon > 0$  and  $dY/d\theta < 0$ .

### 3 Optimal Fiscal Policy

If  $\theta > 0$  a decision by a firm to expand its output imposes a negative externality on all other firms. Since, in the absence of government intervention, there is no mechanism to internalize this external effect, the individual firm's output level is too high. As a consequence, the equilibrium achieved by the economy is inefficient, so corrective action may be taken in the form of a Pigouvian tax.

**Proposition 1** *For any given level of  $G$ , the optimum tax rate  $t$  is  $\hat{t} = \theta\epsilon$*

**Proof.** Differentiate (1)-(5) and (8)-(7) totally. Holding  $G$  constant, we obtain  $du/dt = [(t - \theta\epsilon)\delta u/C(1 + t)](dY/dt)$ . From (9),  $dY/dt \neq 0$ . Therefore  $\hat{t} = \theta\epsilon$  is the f.o.c. At  $t = \hat{t}$ ,  $d^2u/dt^2 = [\delta u/C(1 + t)](dY/dt) < 0$ . ■

With labor the only factor that can be varied at the discretion of the individual firm, a tax on labor input and a tax on output can have the same effects. Thus, equivalent to the tax rate  $\hat{t}$  on labor, a per unit tax rate  $\hat{\tau}$  on output may be

imposed, where

$$\hat{\tau} = \frac{\theta\epsilon}{1 + \theta\epsilon}. \quad (10)$$

The optimal tax rate  $\hat{t}$  (or  $\hat{\tau}$ ) is increasing in  $\theta$ , the congestion parameter, and in  $\epsilon$ , the elasticity of the flow of infrastructural services with respect to the amount of infrastructure  $G$ . If there is no congestion ( $\theta = 0$ ) there should be no tax. In the presence of congestion, however, the greater is the potential effectiveness of the services (as measured by  $\epsilon$ ), the more the use of the infrastructure should be taxed. When  $\epsilon$  is large, congestion has a greater opportunity cost and so should be taxed at a higher rate. Note that in order to set  $t = \hat{t}$ , the government does not need to know the propensity to consume  $\delta$  (this property holds for a more general utility function). Also,  $\hat{t}$  is independent of  $G$  and  $g$  (though this is the result of our assumption that  $\epsilon$  and  $\theta$  are constants).

**Proposition 2** *If  $t = \hat{t}$ , the optimum value of  $g$  is*

$$\hat{g}(\hat{t}) = \frac{\epsilon}{1 + \theta\epsilon} \quad (11)$$

**Proof.** Similar to Proposition 1. For given  $t$ ,  $du/dG = [\delta u/C(1+t)]\{[\epsilon Y - (1+t)G]/G + (t - \theta\epsilon)dY/dG\}$ . For  $t = \hat{t}$ , (11) is therefore the f.o.c. The s.o.c. is  $dY/dG < (1 + \theta\epsilon)/\epsilon$  (the effect on output of a marginal increase in infrastructure is not ‘too’ strong), which we assume is satisfied. ■



From (11),  $\hat{g}(\hat{t})$  is increasing in  $\epsilon$  and decreasing in  $\theta$ . If  $t$  is set optimally, the optimum ratio of  $G$  to  $Y$  is increasing in the potential effectiveness of infrastructural services, but decreasing in the size of the congestion parameter. Like  $\hat{t}$ ,  $\hat{g}(\hat{t})$  is independent of  $\delta$ .

Proposition 2 specifies the optimum value of  $g$  when  $t$  is set optimally. More generally, for any given value of  $t$ , the optimum value of  $g$  is

$$\hat{g}(t) = \frac{[1 + (1 + \mu)t]\epsilon}{[1 + (1 + \mu)\theta\epsilon](1 + t)} \quad (12)$$

where  $\mu = (w/L)(\partial L/\partial w)$  for constant  $u$ ; i.e.,  $\mu = (1 - L)\delta/L$ . It is evident that if  $t \neq \hat{t}$  the value of  $\hat{g}(t)$  is dependent on consumer preferences. Furthermore, gives  $d\hat{g}(t)/dt > 0$ : the more that the externalities are discouraged, the greater is the proportion of  $Y$  that the government should devote to infrastructure.

To identify explicitly the general equilibrium effects of an increase in infrastructural spending, we differentiate (5), (7) and (9) totally, and use (3) to obtain, for any  $\{t, g\}$ ,

$$\frac{dY}{dG} = \frac{\delta\epsilon + (1 - \delta)(1 + t)Y^{\theta\epsilon}}{\delta\theta\epsilon g + (1 + t)Y^{\theta\epsilon}} \quad (13)$$

In the equivalent model with no infrastructure,  $\epsilon = 0$  and  $dY/dG = 1 - \delta$ . With infrastructure, however, since  $\epsilon > 0$ , we find from (13) that, given that  $g < 1$ ,  $dY/dG > 1 - \delta$ . In fact, because labor productivity is increasing in  $G$ , it is

possible that  $dY/dG > 1$ . For example,

$$\frac{dY}{dG} \Big|_{g=\hat{g}(t)} = \frac{(1+\mu)(1+t)}{1+(1+\mu)t} > 1. \quad (14)$$

An implication is that at  $\hat{g}(t)$  a small increase in  $G$ , through its effect on labor productivity, would raise consumption  $C$ . However, this would, as a second-order effect, reduce welfare: the induced rise in labor supply would have a disutility that outweighed the utility of increased consumption.

## 4 Concluding Comments

Our model brings together the macroeconomic modelling of the effects of government expenditure with the approach taken to modelling congestion in the welfare economics literature (e.g., Cornes and Sandler, 1996). Unlike the latter, we separate consumers and firms as decision-makers and emphasize macroeconomic factors such as national income and the multiplier. Further development of the model would allow for the labor input that is necessary to transform tax revenue into infrastructure, and for consumption of infrastructural services by households. There is also scope to develop a dynamic formulation along the lines of endogenous growth theory.

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