SETTLEMENT ANALYSIS
OF HIGHWAY EMBANKMENTS

by

ROBERTO A. ESPINDOLA

B. S., Universidad de Guayaquil, 1966

A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Civil Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1968

Approved by

Wayne W Williams
Major Professor

ACKNOWLEDGEMENTS

I wish to express my appreciation to Dr. Jack B. Blackburn, Head of the Civil Engineering Department at Kansas State University, who accepted my application for graduate study and thus made possible the ambitions since my early youth of receiving an American education and of obtaining a Master of Science degree in Civil Engineering.

Also, and in a special manner, I wish to thank my advisor, Professor Wayne W. Williams, for his helpful advice and kind cooperation, in the completion of this report.

My sincere gratitude to all the professors of the Civil Engineering Department for their worthy teaching, that gave me a better understanding of my profession, and to all the persons that in one way or another contrihuted to complete this modest work.

To my parents

TABLE OF CONTENTS

LIST OF FIGURES

I. INTRODUCTION

In all engineering projects, the basic equation is action vs reaction. In most cases, the action and reaction are accurately predicted from theory derived from observations and research findings and from the correct application of engineering principles. In others, for example Soil Mechanics, the theory and the correct application of engineering principles are not yet well established. Reaction is thus designed to exceed the action by an amount based on experience, in order to assure a state of equilibrium. The ratio of reaction to action is designed to exceed 1.0 and is called a safety factor against failure.

In Soil Mechanics the need for a safety factor is due principally to the extreme variability of physical properties of soil masses. The strength and deformation characteristics, and other engineering properties, of a given soil, will vary appreciably from location to location and may be altered significantly with time for a given location. Soil properties are also affected by reworking during construction operations. Highway embankments are examples of soil masses where soil properties have been altered by reworking during construction, and for which safety factors for slope stability and for settlement must be determined to be adequate to prevent failure.

In addition to determining safety factors for embankments, the underlying soil, on which the embankment is placed, must also be studied to determine that it has adequate load bearing capacity to support the embankment. The bearing capacity of the underlying soil can be evaluated by two possible consequences: a) deformation due to volume change, and b) deformation due to rupture or shear.

Deformation, or settlement, of an embankment may result from volume change, or consolidation in the embankment material, in the underlying soil, or in both. Rupture, or shear, of the embankment can also occur in the embankment, in the underlying soil, or in both. Even with small loads these effects are present. The embankment may not fail, but will settle a small amount and, if the load is increased beyond the bearing capacity of the soil, the soil will fail by shear deformation.

These two effects, settlement, or consolidation, and shear, sre always present in embankments and in soils underlying embankments. If properly designed, the embankment soil will have sufficient strength so that consolidation is neglegible and the load applied to the underlying soil will not exceed the shear strength of the soil.

I. A. Statement of the Problem

The increasing complexity of highway design, in which many or all route crossings are grade-separated, has resulted in a large increase in the number of highway embankments which serve as approach fills for overpass structures.

The resulting loads on the soils underlying these approach fills cause consolidation of compressible layers of the foundation soil. While this consolidation cannot be considered a failure it does result in fill settlement which, when it occurs adjacent to bridge abutments, results in a lower roadway elevation on the approach fill than that on the bridge deck surface. Such an abrupt change in road elevation can become a serious hazard to traffic with continuing consolidation or settlement of the approach fill.

I. B. Purpose of the Study

The purpose of this report is to present the general theory of consolidation and its application to the estimation of the settlement of highway embankments adjacent to unyielding walls such as bridge abutments.

I. C. Scope of the Study

This is an analytical study of the subject based on research by others concerning settlement of fills. An annotated review of the related literature is presented as an appendix.

This application of the theory of consolidation for predicting settlement in fills behind retaining structures is limited to an unyielding wall. The wall is not assumed to settle, being founded on hard stratum, and also, the wall is assumed not to suffer overturning moments, either toward the approach fill or away from it. The retaining structure is assumed to be rigid enough to prevent deformations within its structural parts due to the action of acting forces.

II. REVIEW OF THE LITERATURE

Soil has been used as a foundation and construction material since the earliest days of recorded history. By the time scientific methods of analysis and design became known, man had been constructing buildings, bridges, dams, canals, roads, and other types of structures for centuries. Earth work and foundation engineering developed during this time from practical rules, tradition and experience, and was more "art" than science.

Other sciences helped in the dcvelopment of earth science. For example in Mechanics, Hookes law was applied in relation with the relationship of stress-strain of soil masses.

In 1776, Coulomh, (1) made efforts to obtain values for earth pressure on retaining walls, dcveloping the classical formula $s=c+\tan\phi$ still much used today. Coulomb considered the limit equilibrium of a sliding wedge of a soil behind a retaining wall. He assumed that failure occurred along a plane surface, and his method was hased on comparing several possible failure planes. The plane of failure was the one for which the ratio of resisting force to driving force was a minimum and was equal to or less than 1.0. Coulomb did his work at a time when trigonometric functions were not yet in use, and because of that, all his results were cxprcssed as ratios.

In Coulomb's time empirical methods were used in pile driving and in port construction such as cofferdams, bulkheads, locks and other structures. Military purposes demanded increasingly complex engineering structures. These factors, and the desire for more scientific explanation of physical phenomena during the renaissance period, led to the establishment of the scientific method of investigation. Engineering, as we know it today, is based on the natural laws of mathematics and physics, as they were originally developed in the eighteenth century.

Some of the basic concepts of Soil Mechanics were developed, prior to the existence of Soil Mechanics, in the earth sciences. Early contributions were made by Rankine in 1857 (2) in earth pressure calculations in his classical work concerning stresses in an earth mass and the concept of active and passive earth pressure.

Rankine studied the state of stress within a loose granular mass with zero cohesion. His analysis was based on the assumption that the slightest deformation of the soil is sufficient to bring into play its full frictional resistance and immediately to produce an "active state" if the soil tends to expand parallel to its surface, and "passive state" if it tends to compress parallel to its surface. His solutions were expressed in trigonometric formulas, and they were later extended by Resal (3) in 1910 to include cohesive soils.

Another problem which was studied was the settlement of soils due to loads applied at the surface. Boussinesq (4), in 1885, solved the problem of a concentrated load applied to the surface of a semi-infinite, elastic solid; that is, a load applied to a mass of elastic material, limited by a horizontal plane surface, and extending to an infinite distance in all directions below that plane. The soil beneath a horizontal surface may be considered to represent a semi-infinite solid when its depth is large in comparison to the dimensions of the loaded surface area. The stress analysis follows Hookes law and its results give the stress at a point of depth z from the surface, and a horizontal distance r from the point of application.

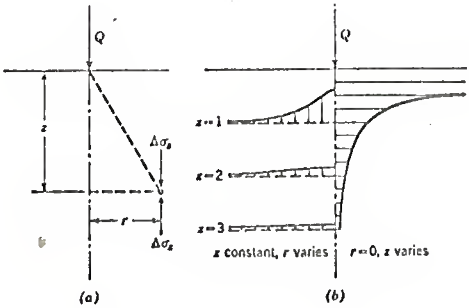


Figure 1. Stress produced at a point in a semi-infinite mass by a concentrated vertical load acting at the surface. a) definition of depth, distance. b) variation of the vertical stress with r and z.

The formula developed by Boussinesq for the normal stress, is

$$\Delta\sigma_z = \frac{3Q}{2\pi R^2} \quad \frac{2z^2}{R^3} \qquad \text{(II.1)}$$

A complete analysis of the stress distribution in the soil mass requires six combinations of stresses. Three normal stresses and three tangential stresses.

Within the past fifty years the discipline of earth science developed in great extent. The earth sciences did not have a definite position in Civil Engineering until the advent of the 20th century, when many engineers felt the necessity of studying soil behavior due to foundation soil failures caused by excessive loading by buildings, dams, road embankments, etc. Also increasing knowledge in other disciplines provided the necessary tools to begin treating soil as an engineering material in construction projects.

Mohr (5), in 1900, investigated the failure criteria for the comhination of applied stresses on an elastic body. His investigations are fundamental in the analysis of stresses in soil. The theory of Mohr is based in the postulate that a material will fail when the shearing stress, on a plane along which failure is presumed to occur, is a unique function of the normal stress acting on that plane. This investigation provided the basic theoretical knowledge which helped to develop the science called Soil Mechanics.

In 1925, Terzaghi (6) originated the name Soil Mechanics to apply to problems dealing with properties of soil as an engineering material, which differed from the previous conventional mechanics of solids by the predominant influence which is exercised on the mechanical properties of soils by the action of the water in the voids of the soil. He is for this reason called the father of Soil Mechanics.

A rigorous mathematical solution of the process of consolidation was first published by Terzaghi in 1923 (7), where he gives the solution of the problems encountered with the rate of consolidation of elay layers. In presenting an explanation of consolidation it is shown that pore water pressure influence other properties of soils such as the shearing strength of clays. Having developed the theory, and having recognized the significance of effective stress as the essential parameter affecting the strength and compression of soils, Terzaghi realized that the shearing resistance of saturated clays would be dependent on the time rate of loading. As an example, for a given laboratory specimen size the rate of application would govern the degree to which excess pore pressure would be dissipated during laboratory shear testing. It is thus shown that the shearing strength of clay is greatly increased when the pore pressure is dissipated by drainage during loading.

Terzaghi in 1932 (8) predicted that the strength of saturated clays subject to rapidly applied shear stress would be independent of the applied normal stress if no drainage occurred.

The basic equation of the consolidation process that relates the rate of change of the excess pore water pressure μ, in respect to time, to the amount of water which is expulsed from the voids of the pores of a clay prism, during the same time interval, is:

$$\frac{\delta \mu}{\delta t} = \frac{K}{M_v \gamma \omega} \frac{\delta^2 \mu}{\delta z^2} \quad , \quad \text{or}$$

$$\frac{\delta \mu}{\delta t} = C_v \frac{\delta^2 \mu}{\delta z^2} \tag{II.2}$$

$$C_v = \frac{K}{M_v \gamma \omega} = \text{coefficient of consolidation}$$

By deriving the differential equation II.2, Terzaghi in 1923 solved the problem of consolidation of clay layers.

Thus, the quantitative explanation of the consolidation process (Terzaghi 1923,1924,1925), the development of a suitable concept of soil structure (Terzaghi 1925,1941), and the realization of the significance of effective stress in controlling shear strength (Terzaghi 1929), permitted suitable modifications of the classical theories of soil action to the point where predictions regarding the behavior of foundations can be made with a degree of confidence not known before. The publication of Terzaghi's work opened the door for solution of foundation problems unsolvable up to that time. One such problem was the settlement of compressible layers of soil due to the action of loads.

From the application of the theory of consolidation developed by Terzaghi, and with the application of the theory of elasticity to loaded soil masses, it is possible to predict the settlement caused by loads that will occur in a certain time.

Jurgenson (9) in 1934 presented values of all the principal stresses acting within a soil mass, due to loads. In the case of settlement, only the vertical compressible stress is needed for computations. Jurgenson made his theoretical investigations in the early years of Soil Mechanics when the methods and applicability of this science to foundation engineering problems were not yet well defined. His work gives a synthetic theoretical analysis to apply the theories of elasticity to foundation problems by the use of charts. They give the value of the principal stress at any point in the foundation soil. For the case of settlement problems, a chart of vertical stresses in the foundation soil, due to a triangular load is presented. The stresses obtained in this way, are compared with the shearing strength of the supporting material as measured by the direct shear test, the cylinder test, and the "squeezing test". With the determination of the maximum shearing stress on the foundation soil, it is important to know where it occurs and whether it will cause failure. Only in the case where a progressive failure is possible, does the mere exceeding of maximum shearing strength lead to a slide.

In 1937, Palmer and Barber (10) conducted an experimental and theoretical study in order to determine the magnitude of experimental errors resulting from variables in testing apparatus and procedures developed by Terzaghi in that time, and to determine the agreement between deformation observed in tests with those computed in accordance with the theory of consolidation. They concluded that on the basis of their investigations it was possible to state definitely that the apparatus and testing procedures have been found to be satisfactory.

In the same year, Palmer and Barber (11) conducted another important research study in the principles of Soil Mechanics that should govern the design of fills. The authors show formulas and methods to obtain the analysis for the stability and supporting power of the undersoil. They conclude that as long as a cohesive supporting soil has a greater cohesion than the maximum shear stress imposed by the load of the fill, no further consideration of failure is necessary. If the contrary is the case, a further evaluation is necessary for bearing capacity analysis.

In December of 1938, Palmer (12) continued the work he had done with Barber in a study of retaining walls and bridge abutments. He indicated the limitations of the analytical method, based on the assumption of conditions of plain strain in the case of supporting soil under abutments, piers and retaining walls. He stated that the legitimate use of the formulas for plain strain, in connection with abutments and retaining walls problems, depended chiefly on the ratio of the length-to-width of the footing. He concluded by saying that the analytical method assuming plane strain conditions is applicable for all bridge piers with relative dimensions of length-to-width of 2.0 or greater, and for all values of ϕ ranging from zero to typical values for sand. The same analytical method is warranted for bridge abutments and retaining walls when the length-to-width ratio of the base is 3.0 or greater, and ϕ having any value ranging from 0 to 22. The author compared four different types of abutments, varying the length-to-width ratio, and comparing the resulting stresses σ_z (vertical) with the results using Newmark charts (13). For a given footing the results showed thst the difference in values of σ_z, as obtained by the two methods, increased with depth.

In 1940, Palmer and Barber (14) published the results of theoretical investigations dealing with the causes of the total settlement of an earth embankment, including the part of settlement due to lateral "displacement" within the fill material and in the foundation soil. They concluded that the total settlement of a highway fill is the result of four factors: settlement due to lateral displacement of the fill material; settlement due to lateral displacement of the foundation soil; settlement due to consolidation of the fill material; and, settlement due to consolidation of the foundation soil. The authors observed that it is a serious mistake to rely completely on any mathematical formula derived from theory and assumptions, without reference to observations in the field and to practical experience. When the problem deals with large earth masses, mathematical expressions can, at best, only indicate the general trend of physical occurrances.

In 1942, Newmark (15), published a simplification of the Boussinesq formulas to apply directly to the computation of stresses in foundation soils. From the charts the stresses are computed by counting the number of elements of area or blocks covered by a plan of the loaded area drawn to a proper scale and overlaid on the chart.

In 1942, at the Annual Meeting of the Highway Research Board, the Committee of Stress Distribution in Earth Mass suggested to the Department of Soil Investigation the idea of preparing a summary of both, the theories of stress distribution, and the methods of design for earth structures. Jacob Feld was a member of the Committee and in 1943, he prepared his first report (16) for the first part of this investigation. He describes the general considerations related to the design of bridge abutments. The main portion of his report covers "Forces and Resistances". In his second report, in 1944, Feld (17) covered the different types of materials and different methods of design according to different types of abutments. In 1945, Feld (18) completed his work with the last report for the Highway Research Board, covering other considerations in design abutments such as drainage details, types of abutment foundations, etc.

In 1952, Sloane (19) applied the general theory of consolidation to the prediction of settlement of fills. His intention was to present a method easy to comprehend and use for highway engineers. He presents a discussion of the mechanics of consolidation for the one-dimensional case. The author uses two methods to compute total settlement: the void ratio method; and, the compression index method. To obtain the load increase due to the embankment load, he used Newmark charts. The author did not find a significant difference between the two methods.

In 1957, Osterberg (20) published a simplification based on the Boussinesq formulas to apply for pressures due to embankment loads. He constructed a chart for which an influence value is given as a function of the embankment shape and dimensions.

In 1958, Webber (21) also published results of work based on the general theory of consolidation developed by Terzaghi. He used the void ratio method to predict the total settlement, and the general approach developed by Terzaghi to estimate time-settlement relationships. The purpose was to give a simplified explanation of the theory of consolidation in the particular case of settlement of highway embankments.

In 1959, Jones (22) published an article about bridge approaches based on failures in the Los Angeles area, relating them with the soil formation of the area. He gave the causes of failures and recommendations for the cases of open-end abutments and closed-end abutments according to foundation soil conditions. He stated that a decision must be made in accordance with an economic study of the various possible solutions.

In England, in 1967, Mc. Laren (23) investigated the effect of different types of back fill materials in the settlement of bridge approaches. The investigations were made in closed-end abutments founded on piles. The settlements were measured in different parts of the approach fills. The observed settlements were of the order of 1/8" because the underlying material was firm to hard red silty clay. These results showed that the embankment material had little effect on total settlement, when it is compacted according with established specifications.

In 1967, Perloff, Baladi and Harr (24) presented a theoretical method to obtain the magnitude and distribution of stresses due to embankment loads. They assumed also that the material of the foundation soil was homogeneous, isotropic, and elastic. They present their results as a function of the angle α of the embankment with the horizontal foundation soil, the height of the embankment, and the half-width of the top of the embankment. They show that the results obtained by this method in relation with the horizontal distribution of the vertical stress is more nearly uniform than is usually assumed.

III. THEORETICAL ANALYSIS

III. A. GENERAL THEORY OF CONSOLIDATION

The application of stresses on all materials causes strains. In some materials, a certain amount of time is required for the occurence of strain and a relationship between stress, strain and time exists. In purely elastic materials the theory of deformation is well known and stress-strain relationships may be accurately defined. This theory involves only two stress-strain constants: the modulus of elasticity and Poisson's ratio.

Consolidation theory is based on the assumption that the material acts as an elastic material but with the modification involving time, especially in all fine grained soil materials. Due to the complexities that arise both in the mathematical analysis and the difficulty of instrumentation for recording volume change and natural strain, three dimensional consolidation cannot be accurately determined. Karl Terzaghi greatly simplified the general problem by a rigid analysis of one dimensional consolidation. The applicability of his analysis to settlement problems has been verified and widely accepted.

Soil engineering problems in which data of stress-strain and stress-strain-time relationship are needed, are of two types:

A. Cases where stress is insufficient to overtax the strength of the soil in shear, but may be of a magnitude to cause serious displacements, or settlement,

B. Cases where the loads exceed the shearing stress of the soil termed stability problems.

It will be eonsidered more espeeially eases of type A in this report. The soil mass may be considered as a skeleton of solid particles enelosing a intereonneeted number of voids usually filled with gas, liquid, or a eombination of both. If a sample of soil is plaeed under a eonfining stress in sueh a way that its volume is deereased, there are three possible factors responsible for the deerease in volume:

A. Compression of the solid partieles.

B. Compression of water and air within the voids.

C. Eseape of water and air from the voids.

The idealized equilibrium equation, ignoring surfaee tension forces, may be expressed in the following form:

$P = P_s(A_s) + P_w(A_w) + P_g(A_g)$, where (III.1)

P = total applied load

P_s = pressure between granular particles

P_w = pressure in the pore water

P_g = pressure in the gaseous phase

A = eorrespondent area

The solid partieles and the water within the voids, are relatively ineompressible and a deerease in the volume of the mass is thus due, in saturated soils, to the escape of water from the voids sinee a state of saturation is assumed in eompressible soils, even though a small amount of eompressible gas within the voids may be present.

The elastie eompression, while negligible, of the samples whieh oecurs without a ehange in water content ean be expressed by the formula used by Yong and Winkerton (25)

$$\text{Settlement} = Pb\,\frac{1-\mu^2}{E}\,I_p \qquad \text{(III.2)}$$

where P = pressure applied

b = breadth or diameter of loaded area

μ = Poisson's ratio

E = elastic modulus

I_p = influence factor based on shape of the loaded area such that

$$I_p = \frac{1}{\pi}\left[\ell \log \frac{1+\sqrt{\ell^2-1}}{\ell} + \log(\ell+\sqrt{\ell^2+1}\right] \qquad \text{(III.2a)}$$

where ℓ = L/b

L = length of loaded area

The one dimensional compression process which involves, simultaneously, a slow escape of water and a gradual densification by compression and stress transfer is called consolidation. Initially in a saturated soil as stress is applied, a portion of the load is carried by the skeleton of the soil at point to point contacts as so called effective pressure and a part as excess hydrostatic pressure by the water in the soil pores termed simply pore pressure.

Considering the following figure 2, representing the portion of a curve of pressure vs. void ratio, for the pressure increment from p_1 to p_2 with point p_1 representing the unloaded condition the sample is at pressure and void ratio conditions represented by point A. The intergranular pressure is p_1 and the void ratio

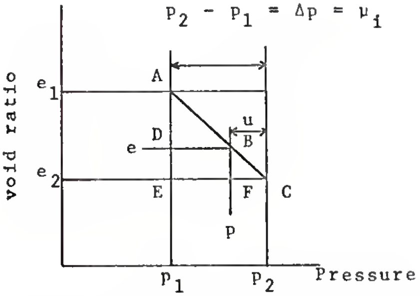


Figure 2. Pressure vs void ratio relationship for a typical pressure increment. According to Terzaghi theory.

is e_1. An instant after the load is increased, the total pressure acting on the sample is p_2, but the void ratio is still e_1, and the pressure is a carried by both pore pressure and effective pressure. As the initial void ratio is reduced to conditions indicated at e_2 and p_2 a transfer of the pressure from the pore water to effective pressure occurs due to the escape of pore water from the sample.

The increase in stress represented by point p_2, tends to produce a strain represented by e_1-e_2, but because of this hydrodynamic lag, the strain occurs over a period of time and the added pressure is carried by the water in the voids of the soil.

If the entire sample is hermetically sealed, pore pressure cannot dissipate, but in the consolidation device due to porous stones, and in the in situ soil mass, due to pervious layers of soil, the hydrostatic excess pressure is dissipated by drainage at the top and bottom of the sample. Thus, at the surface of the sample, at the time of instantaneous loading, the water pressure is zero, while the effective pressure is p_2, or equal to the applied load but this condition is reversed within a short distance of the face and the load is carried by the pore water. This high gradient at the surface is caused by rapid drainage of water from pores near the surface. Gradually, the void ratio decrease, the hydrostatic excess pressure decreases, and the intergranular pressure increases throughout the entire soil mass. This process is always in a more advanced state near the boundaries of the sample at any given time. The sample is said to be consolidating under the stress increase p_2-p_1, and the action continues until the hydrostatic excess pressure has become zero. At this point the effective pressure is p_2, and the sample is said to be consolidated under the stress p_2.

As a measure of the degree to which consolidation has progressed at any point within a consolidation sample, the ratio between a void ratio change attained at any time and the final void ratio change at the completion of consolidation is used. The ratio is:

$$U_z = \frac{e_1-e}{e_1-e_2} \tag{III.3}$$

This ratio is called consolidation and it is often expressed in percent. From figure 2:

$$p_2 = p_1 + u_i = p + u \tag{III.4}$$

The slope of the straight line curve of pressure vs void ratio, is negative, the expression of its numerical value being:

$$a_v = \frac{e_1-e_2}{p_2-p_1} = -\frac{de}{dp} = \frac{e_1-e}{p-p_1} \tag{III.5}$$

This stress-strain ratio is called coefficient of compressibility. It is numerically equal to the slope of the curve on the natural scale plot of the pressure vs void ratio relationship.

By use of figure 2 and equations III.4 and III.5, equation III.3 can be expressed:

$$U_z = \frac{e_1-e}{e_1-e_2} = \frac{(p-p_1)a_v}{(p_2-p_1)a_v} = 1 - \frac{u}{u_i} \tag{III.6}$$

It is evident that U_z equals zero at the instant the applied stress is increased to p_2, and that it increases to 100% as the void ratio decreases from e_1 to e_2. The effective pressure meanwhile, increases from p_1 to p_2 and the hydrostatic excess pressure decreases from u_i to zero.

The Terzaghi theory of consolidation is based on the following assumptions: (26)

1. Homogeneous soil.
2. Complete saturation of the voids of the soil.
3. Negligible compressibility of soil grains and water.
4. Action of infinitesimal masses not different from larger masses.
5. One dimensional compression.

6. One dimensional flow.

7. Validity of Darcy's law.

8. Constant values for certain soil properties.

9. The idealized relationship of pressure vs void ratio shown by Figure 2.

The first three assumptions have been discussed; the fourth, is of academic interest because the differential equations used, represent an infinite mass. The fifth and sixth assumptions are closely met in the laboratory; the seventh may be accepted, but the eight assumption introduces some errors. Assumption nine is justified to simplify analysis.

Recognizing that consolidation is directly dependent on the expulsion of pore water the governing equations are set up in terms of flow rate of pore water.

The fundamental expression for flow in a cube of unit size x, y, and z, related to time rate of change of volume, is:

$$k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} + k_z \frac{\partial^2 h}{\partial z^2} \quad dx\ dy\ dz \qquad \text{(III.7)}$$

This expression is dependent only on assumption 1 to 4 inclusive, and 7. For one dimensional flow, (assumption 6) the absence of gradient in the x and y direction, eliminates the first two terms of the parenthesis. The permeability k_z may be written k, and

$$k \frac{\partial^2 h}{\partial z^2} \ dx\ dy\ dz \qquad \text{(III.8)}$$

The volume of the element, is dx dy dz, the pore volume is $\frac{e}{1+e}$ dx dy dz, and since all changes in volume must be changes in pore volume, a second expression for the time rate of change of volume may be written:

$$\frac{\partial}{\partial t}\left(dx\ dy\ dz\ \frac{e}{1+e}\right) \tag{III.9}$$

Since $\frac{dx\ dy\ dz}{1+e}$ is the constant volume of solids, the above expression may be written

$$\frac{dx\ dy\ dz}{1+e}\ \frac{\partial e}{\partial t} \tag{III.10}$$

Equating this expression to expression III.8 above, and canceling dx dy dz, gives

$$k\frac{\partial^2 h}{\partial z^2} = \frac{1}{1+e}\ \frac{\partial e}{\partial t} \tag{III.11}$$

Only heads due to hydrostatic excess pressure will tend to cause flow in the case under consideration. Thus h in the above equation may be replaced by u/γ_ω, giving:

$$\frac{k}{\gamma_\omega}\ \frac{\partial^2 u}{\partial z^2} = \frac{1}{1+e}\ \frac{\partial e}{\partial t} \tag{III.12}$$

Equation III.4 shows that dp = - du, and substituting this value in equation III.5, gives the following expression of assumption 9:

$$de = a_v du \tag{III.13}$$

The substitution of this relationship into equation III.12 gives:

$$\frac{k(1+e)}{a_v \gamma_\omega} \quad \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t} \tag{III.14}$$

And if c_v is substituted for $k(1+e)/a_v\gamma_\omega$, formula III.14 becomes

$$c_v \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t} \tag{III.14a}$$

c_v is defined as the coefficient of consolidation. This equation relates the rate of change of the excess pore pressure u, with respect to time, and to the amount of water expelled from the voids of a clay prism during the same time interval.

The consolidation theory, the z coordinate is measured vertically, as seen in the following Figure 3, from the surface of the sample. The thickness of the sample is designated by 2H, the distance H being the length of the longest drainage path. In the

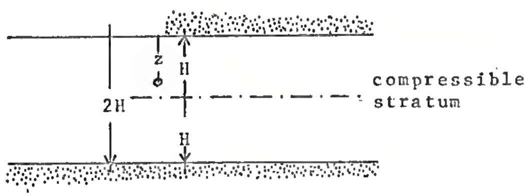


Figure 3. Illustration of layer thickness and drainage path in Terzaghi consolidation theory.

case of single drainage of the sample layer, the longest drainage path would be designated 2H.

The boundary conditions for this case of one-dimensional consolidation may be expressed as follows:

1. There is complete drainage at the top of the sample.
2. There is complete drainage at the bottom of the sample.
3. The initial hydrostatic excess pressure u_i is equal to the pressure increment p_2-p_1. The mathematical expressions for these three boundary conditions, are:

$$\begin{aligned} &\text{A.} \quad \text{When } z=0,\ u=0 \\ &\text{B.} \quad \text{When } z=2H,\ u=0 \\ &\text{C.} \quad \text{When } t=0,\ u=u_i \end{aligned} \tag{III.14b}$$

The case of main interest is when u_i is a constant, but solutions are possible when u_i varies with depth.

The next step is to obtain the solution of equation III.14a for the boundary conditions III.14b stated above. The mathematical solution of equation III.14a for the boundary conditions III.14b involves Fourier series expansion and is taken from Taylor (26).

Assuming that u is a product of some function of z, and some function of t, that is:

$$u = F(z)\cdot\phi(t) \tag{III.15}$$

Equation III.14a may be written:

$$c_v\phi(t)F''(z) = F(z)\phi'(t) \qquad \text{where:}$$

$\phi'(t)$ represents $\frac{\partial}{\partial t}\phi t$

$F''(z)$ represents $\frac{\partial^2}{\partial z^2}F(z)$ and

$$\frac{F''(z)}{F(z)} = \frac{\phi'(t)}{c_v\phi(t)} \tag{III.16}$$

Since the left member of this equation does not contain the variable t, its value cannot be altered by a change in t; therefore, if the equality is to be preserved, a change in t must not affect the value of the right member of the equation. A similar argument holds true if z is considered variable. Hence, each term must be equal to a constant. For convenience, call this constant $-A^2$. On this basis, the left member of the above equation, III.16, gives:

$$F''(z) = -A^2 F(z)$$

It is verified that the expression that satisfies this relationship, is:

$$F(z) = C_1 \cos Az + C_2 \sin Az$$

in which C_1 and C_2 are arbitrary constants.

The right member of equation III.16, gives:

$$\phi'(t) = -A^2 C_v \phi(t)$$

and the expression that satisfies this relationship, is:

$$\phi(t) = C_3 \varepsilon^{-A^2 c_v t}$$

in which ε, is the Naperian base, and C_3 is an arbitrary constant. Thus, equation III.15, becomes

$$u = (C_4 \cos Az + C_5 \sin Az)\varepsilon^{-A^2 c_v t} \qquad \text{(III.17)}$$

The remaining requirement is the satisfying of boundary conditions III.14b, The first condition is satisfied if $C_4=0$; this leaves

$$u = C_5(\sin Az)\varepsilon^{-A^2 c_v t} \qquad \text{(III.18)}$$

The second condition will be satisfied if $2AH=n\pi$, in which n is any integer; then

$$u = C_5 \sin \frac{n\pi z}{2H} \varepsilon^{-n^2\pi^2 c_v t/4H^2} \qquad \text{(III.19)}$$

The term C_5 is merely an arbitrary constant, and n can assume any integral value whatever. Therefore, a series of the form

$$u = B_1 \sin \frac{\pi z}{2H} \varepsilon^{-\pi^2 c_v t/4H^2} + B_2 \sin \frac{2\pi z}{2H} \varepsilon^{-4\pi^2 c_v t/4H^2} + \ldots\ldots$$

$$+ B_n \sin \frac{n\pi z}{2H} \varepsilon^{-n^2\pi^2 c_v t/4H^2} + \ldots\ldots$$

in which B_1, B_2, B_3,....B_n, are constants, will still be a solution. This series may be written in abbreviate form:

$$u = \sum_{n=1}^{n=\infty} B_n \sin \frac{n\pi z}{2H} \varepsilon^{-n^2\pi^2 c_v t/4H^2} \qquad \text{(III.20)}$$

The third boundary condition will be fulfilled, if the constants B_n in equation III.20, are determined, so that:

$$u_i = \sum_{n=1}^{n=\infty} B_n \sin \frac{n\pi z}{2H} \qquad \text{(III.21)}$$

This is a common type of Fourier expansion, and the constants may be readily determined by the use of the following relationship, which appear in practically all lists of definite integrals:

$$\int_0^{\pi} \sin mx \sin nx \, dx = 0, \text{ and}$$

$$\int_0^{\pi} \sin^2 nx \, dx = \frac{\pi}{2}$$

where m and n are unequal integers. With change of variable, from x to $\pi z/2H$, the expressions become:

$$\int_0^{2H} \sin \frac{m\pi z}{2H} \sin \frac{n\pi z}{2H} \, dz = 0 \text{ , and} \qquad \text{(III.22)}$$

$$\int_0^{2H} \sin^2 \frac{n\pi z}{2H} \, dz = H \qquad \text{(III.23)}$$

If both sides of equation III.21 are multiplied by $\sin(\frac{n\pi z}{2H})dz$ and integrated between 0 and 2H, all terms in the series, except the n^{th} term will assume the form of equation III.22, and vanish; the n^{th} term will be in the form of equation III.23 and will have a definite value. Thus:

$$\int_0^{2H} u_i \sin \frac{n\pi z}{2H} \, dz = B_n \int_0^{2H} \sin^2 \frac{n\pi z}{2H} \, dz = B_n H$$

because of formula III.23. From where:

$$B_n = \frac{1}{H}\int_0^{2H} u_i \sin\frac{n\pi z}{2H}\,dz \qquad (III.24)$$

when this value is placed in equation III.20, the solution becomes

$$u = \sum_{n=1}^{n=\infty}\left(\frac{1}{H}\int_0^{2H} u_i \sin\frac{n\pi z}{2H}\,dz\right)\left(\sin\frac{n\pi z}{2H}\right)\varepsilon^{-n^2\pi^2 c_v t/4H^2} \qquad (III.25)$$

This equation is perfectly general for the conditions assumed, and enables the hydrostatic excess u to be computed for a soil mass under any initial system of stress u_i at any depth z, and in any time t.

Equation III.25 may be written in more general form since time t, appears as a multiple of c_v/H^2, which is a constant for any given case. Let

$$T = \frac{c_v t}{H^2} \qquad (III.26)$$

An analysis of the units of the quantities involved shows that T is a dimensionless number, and it is called the "time factor". Substituting equation III.26 in equation III.25 gives the following form

$$u = \sum_{n=1}^{n=\infty}\left(\frac{1}{H}\int_0^{2H} u_i \sin\frac{n\pi z}{2H}\,dz\right)\left(\sin\frac{n\pi z}{2H}\right)\varepsilon^{-1/4n^2\pi^2 T} \qquad (III.27)$$

In particular, if the initial hydrostatic excess pressure, u_i is constant u_0, the above equation becomes:

$$u = \sum_{n=1}^{n=\infty} \frac{2u_0}{n\pi} (1 - \cos n\pi)(\sin \frac{n\pi z}{2H}) \varepsilon^{-1/4 n^2 \pi^2 T}$$

When n is even, $1-\cos n\pi$ vanishes

When n is odd, $1-\cos n\pi = 2$

Therefore it is convenient to let $n = 2m + 1$, in which m is an interger. The substitution of

$$M = \frac{1}{2} \pi (2m + 1) \tag{III.28}$$

is of considerable aid in the simplification of the equations that follow, since M appears frequently. After these substitutions are made, the equation for constant initial hydrostatic excess pressure becomes:

$$u = \sum_{m=0}^{m=\infty} \frac{2u_0}{M} (\sin \frac{Mz}{H}) \varepsilon^{-M^2 T} \tag{III.29}$$

In terms of consolidation ratio, defined by equation III.6, the expression becomes:

$$U_z = 1 - \sum_{m=0}^{m=\infty} \frac{2}{M} (\sin \frac{Mz}{H}) \varepsilon^{-M^2 T} \tag{III.30}$$

This equation is shown graphically in Figure 4.

The average degree of consolidation over the depth of the stratum at any time during consolidation process can now be

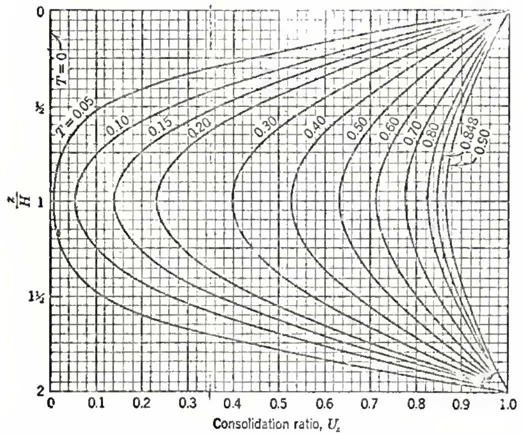
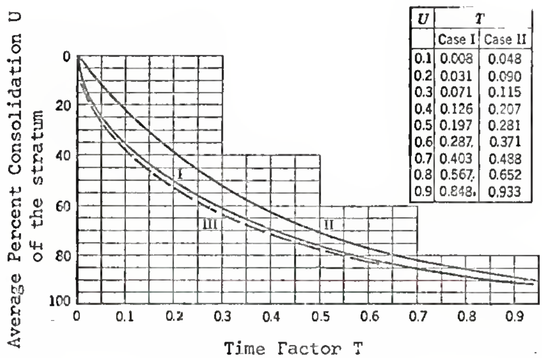


Figure 4. Consolidation as a Function of Depth and "time factor" T.



U	T	
	Case I	Case II
0.1	0.008	0.048
0.2	0.031	0.090
0.3	0.071	0.115
0.4	0.126	0.207
0.5	0.197	0.281
0.6	0.287	0.371
0.7	0.403	0.488
0.8	0.567	0.652
0.9	0.848	0.933

Figure 5. Consolidation Curves According to Terzaghi theory.

determined, and is useful for practical purposes. The average initial hydrostatic excess, u_i, may be expressed:

$$\frac{1}{2H}\int_0^{2H} u_i \, dz$$

Similarly, the average hydrostatic excess u at any intermediate time t, is:

$$\frac{1}{2H}\int_0^{2H} u \, dz$$

The average consolidation ratio U, is the average value of U_z over the depth of the stratum. It is equal to the average value of $1 - \frac{u}{u_i}$, and it may be expressed:

$$U = 1 - \frac{\int_0^{2H} u \, dz}{\int_0^{2H} u_i \, dz} \qquad \text{(III.31)}$$

Substitution of the value of u given by equation III.27, and integrating, gives:

$$U = 1 - \sum_{m=0}^{m=\infty} \frac{2\int_0^{2H} u_i \sin \frac{Mz}{H} \, dz}{M\int_0^{2H} u_i \, dz} \varepsilon^{-M^2 T} \qquad \text{(III.32)}$$

In the special case of constant initial hydrostatic excess u_0, the equation becomes:

$$U = 1 - \sum_{m=0}^{m=\infty} \frac{2}{M^2} \varepsilon^{-M^2 T} \tag{III.33}$$

Equation III.33 is represented by curve I of figure 5.

Equation III.30 contains only abstract numbers. Values of U_z may be obtained by assigning values of z/H and T to allow the determination of family of curves shown in Figure 4.

Figure 4 presents the entire picture of the theoretical process of consolidation. As noted before, consolidation proceeds most rapidly at the drainage faces, and least rapidly at the center of the layer due to the longer escape route for the pore water.

Considering, for example, the curve for a T value equal to 0.1 which represents conditions after the lapse of a definite time, measured from the beginning of the consolidation process. This definite time, (in days or years), in a natural layer, is a constant multiple of T and T is a constant, depending on the values of c_v and H, as shown by the formula:

$$t = T \frac{H^2}{c_v}$$

At this time, and at a depth equal to 1/10 the height of the layer, consolidation is 65% complete, whereas at the center of the layer, is only 5% complete.

With time the consolidation U_z, at every point increases, and finally, after an infinite time, U_z = 100% at all depths, the hydrostatic excess pore pressure is zero, and all the applied pressure is carried by solids within the soil.

The relationship between time and the average state of consolidation over the height of the stratum expressed by equation III.33 is shown in Figure 5, curve I.

In the application of consolidation theory to the prediction of settlements, only the average consolidation need be considered. For this purpose formula III.32 is used, and with the modification of assuming constant initial hydrostatic excess pore pressure u_0, it is transformed to equation III.33, for simplifying the computations.

Therefore values of U_z representing consolidation at various points (as shown in Figure 4) are used much less than values of U, representing the average consolidation of the stratum as a whole.

Three examples of variable u_i are presented in Figure 6. In the upper portion of the figure, u_i diagrams for these cases are shown:

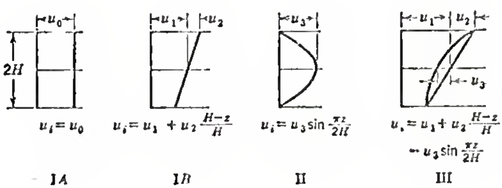


Figure 6. Variation of initial hydrostatic excess pressure u_i.

Diagram IA, represents the constant u_i case already described and for which curve I, of Figure 5 applies.

Diagram IB represents the case of linear variation of initial hydrostatic excess. To obtain the consolidation for this case, the linear expression given below the diagram must be substituted in equation III.32. It is interesting to see that the result of this case (as for IA) is equation III.33. Thus consolidation curve I applies for any case of linear variation of initial hydrostatic excess.

Diagram II shows sinusoidal variation of initial hydrostatic excess. The substitution of the sinusoidal function, given below the graph, into equation III.32, gives an expression represented by curve II, in Figure 5.

Actual cases may be closely approximated by combination of cases I and II. For example, diagram III, represents a typical case, and its u_i relationship may be expressed as a linear relationship minus a sinusoidal relationship.

Consolidation curve III is not greatly different from curve I. This similarity of curves, the extremely approximate nature of the main assumption in the Terzaghi theory, and the questionable assumption of one dimensional drainage, are all factors that lead to the generally accepted conclusion that curve I is an adequate representation of typical cases in nature.

The described boundary conditions, and Figure 4 shows that the analysis applies to a stratum with drainage at both, top and bottom surfaces. There are many clay stratum in nature with drainage at one surface only. All such occurences of single drainage may be considered to be the upper half of a case of double drainage, the actual strata extending to the depth at which $z=H$ and the other half being a fictitious mirror image. For all such cases, curve I of Figure 5 is an acceptable representation of the consolidation function. The most important difference between single and double drainage, is that the stratum thickness equals H in single drainage, whereas it equals 2H in double drainage.

III. B. MODIFICATION WHICH PERMIT THE USE OF GENERAL THEORY OF CONSOLIDATION FOR PREDICTING SETTLEMENT IN FILLS.

The consolidation of compressible soils should be well understood by any foundation and highway engineer since consolidation problems in the construction of embankments, bridge abutments, and other highway structures are common. The settlement of these structures would not be serious, if the movements were uniform, but differential settlements occur which can develop additional stresses not considered during design of considerable magnitude with the consequent failure of the structure.

Differential settlements in the case of concrete roads, founded on embankments cause serious cracking in the pavement slab, and differential settlement at bridge abutments cause cracking at the union of the two structures, with consequent continuous repair and danger to traffic operations.

These type of failures may be analyzed by the theory of consolidation developed by Karl Terzaghi described in section III. A.

The settlement analysis may be divided into three parts:

A. Determination of the characteristics of the subsurface soil strata, by drilling, sampling and testing.

B. Analysis of stress distribution and magnitude under the linear, trapezoidal soil embankment load.

C. Analysis of the data from the soil investigation, to predict settlement and stability of the soil within and under the highway embankment.

Part A, above, is often neglected, but is a very important step in the analysis, since parts B and C are based upon the results obtained. The general requirements for part A are:

1. A carefully designed statistical boring plan, designed to provide samples truly representative of the vertical and horizontal variation of the soil strata.
2. A carefully prepared report showing these variations and the basic characteristics of each of the soil materials encountered.
3. Accurate testing of the soil samples to provide data for consolidation analysis, to allow an accurate classification of each soil material, to measure soil properties that will allow prediction of changes in the soil with time, and to measure soil density for the calculation of the unit weight of the soil.

Since the settlement of highway fills due to consolidation is the main theme of this report, consideration of the modifications necessary in the theory, will be considered in more detail later.

The application of the theory of consolidation to settlement of a highway fill requires the recognition of the following errors in the basic theory:

A. The laboratory consolidation test is a greatly scaled down model test, and is thus subject to modeling effects.

B. The assumption of one-dimensional flow of the pore water is unwarranted when considering relatively narrow highway embankments.

C. Secondary consolidation is ignored in the general consolidation theory. In organic soils, the effect of secondary consolidation is greater than the one of primary consolidation.

D. Plastic lag due to the adsorbed water is not considered.

E. All the constituents are considered incompressible and air is considered absent from the voids of the soil.

F. Instantaneous application of the load is assumed.

G. Lateral or vertical displacement of the soil due to unloading of nearby areas, having no surcharge, is not considered.

The factors listed above were already described in section III. A. Those listed in A, B, E, and G introduce errors on the magnitude of the settlement while the others are related only to the rate of consolidation. For these latter factors, modification of the general theory of consolidation for application to settlement of highway fills is no different than the modification of the general theory to any type of structure, and must be based on common sense and experience. For these reasons, the assumptions from where the theory of consolidation is based, will not be considered as part of this study and it is assumed that the theory of consolidation is sufficiently applicable and accurate to be used for the prediction of settlement of highway fills.

With all these considerations, the most important parts of the study are: 1) the stress distribution through the soil resulting from the load of the embankment, 2) the void ratio-effective pressure relationship, 3) percent consolidation - time relationships.

The settlement of a structure may be caused by the combined effects of vertical consolidation, and lateral and upward displacement due to lateral pressures and to shearing stress. The settlement component due to lateral and vertical displacement is of practical importance mainly in the case of soils with small shearing strength, which can be displaced like viscous fluid. (27)

In 1940, Palmer and Barber (14) in a theoretical research related to highway embankments to determine the causes of settlement due to the action of embankment loads, conclude that part of the total settlement, is due to lateral displacement of the soil within and underneath the embankment.

To remedy this, Tschebotarioff (27) states that this displacement of soft clays, with small shearing resistance, can be avoided by the use of sheet piling on the sides of the structure, and that in all other soils, the settlement component due to vertical displacement usually predominate.

Following the general procedure, the pressure analysis consists of determination of the values of the pressure before and after loading, or pressures p_1 and p_2. The increment of pressure from p_1 to p_2 is called Δp. Deformation due to volume change occurs when the soil is stressed from its state of equilibrium in nature, to a value p_2 dependable on the acting load, in this case, due to the highway embankment load. Both, the initial intergranular pressure p_1 and the final intergranular pressure p_2, will vary with depth, but the average values of each, may be obtained, and the averages will be used as representative values p_1 and p_2 in the final steps of analysis. (26)

When no horizontal variation exists, the total vertical pressure before loading, at any depth below ground surface is dependent only on the weight of the overlying material. The weight of overlying material or overburden stress is calculated, and the effective pressure must be used, that is, the pore water pressure must be deducted from the total overburden pressure (28).

For calculating the overburden stress, the following formula can be used:

$$\gamma_{effective} = \gamma_{dry}(1 + \frac{w}{100}) - 62.4, \text{ where} \qquad \text{(III.34)}$$

γ_{dry} = dry unit weight of soil, in p.c.f.

w = moisture content, in %

62.4 = unit weight of water in p.c.f.

At this point it is necessary to take a close look at the consolidation test data of each layer or stratum, and of prime importance is the plot of void ratio e vs log. of effective stress p, for each compressible stratum or sample. Generally a soil sample, representative of a soil stratum, can show that the soil is in one of the following states:

A. Under Consolidated. These stratum have not completely consolidated or settle under their present overburden stress. Or in other words, their present overburden pressure is greater than the maximum vertical stress that the soil has ever been subjected in the past.

B. Normally Consolidated. These have completely consolidated or settle under their present overburden stress, and have never been subject in the to a vertical stress greater than the present.

C. Over Consolidated. These soils have been in the past subjected to a greater vertical stress than their present overburden stress.

The state of consolidation in which the stratum is, is determined by an empirical procedure developed by A. Casagrande. (29)

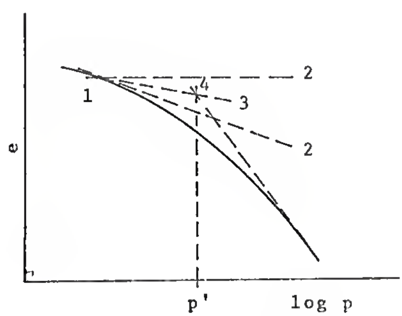


Figure 7. Maximum Past Consolidation Pressure Determination According to A. Casagrande.

1. Pick up the point of maximum curvature of the e-log. p curve.
2. Construct a tangent to the curve and a horizontal line through this point.
3. Bisect the angle formed by the outer portion of these lines, from the point of tangency.
4. Extend the straight portion of the e-log. p curve to its intersection with the bisection.
5. From this intersection, construct a vertical line downward to the log p axes.
6. Note the preconsolidation pressure p' indicated by the intersection of the vertical line with the log p axes.

Once the maximum consolidation stress has been estimated for each stratum, a comparison can be made against the present overburden pressure to determine if the stratum is under, normally or over consolidated.

If the present stress is within $\pm$ 0.3 t.s.f. of the indicated preconsolidation pressure, the soil is considered normally consolidated. The value of e taken for calculations, is taken as the one corresponding to the present overhurden stress P_0.

For under consolidated soils, the initial void ratio, e_0, is determined from the sample and a field consolidation curve is constructed.

If the soil is only slightly over-consolidated ($p_0+1.0\geq p'$) e_0 is determined as for a normally consolidated soil. When the soil is heavily overconsolidated (say $p_0+1.0<p'$) it is

necessary to correct the laboratory e-log. p curve, and determine e_0 from the corrected curve at stress p_0 by a method developed by Karl Terzaghi (30) that simplifies calculations by a graphical construction shown in the following figure (8) and description:

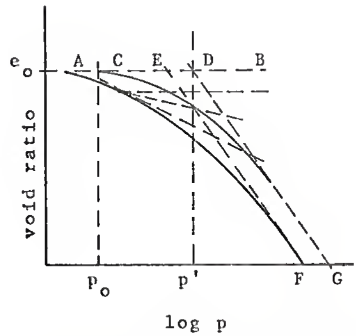


Figure 8. Correction for Determination of Void Ratio Change for Overconsolidated Soils, According to Terzaghi.

1. Draw a horizontal line, AB, through the point of initial void ratio e_0.
2. Determine present overburden stress P_0, and max. past consolidation pressure p'
3. Draw vertical lines through p_0 and p' intersecting AB at C and D.
4. Draw line DG parallel to EF, from intersection of line EF and line AB.
5. Starting at C, sketch a stress-void ratio curve of similar trend and curvature of the laboratory curve, and tangent to line DG.

The final void ratio, e_f for settlement analysis is determined from laboratory curve AF at pressure p_f, for under and normally consolidated soils. For over consolidated soils (slightly or heavy) the e_f is taken from the corrected curve CG, at pressure p_f.

The procedure used to estimate settlements using the general theory of consolidation, relates the effective pressures p_1 and p_2 to the corresponding void ratio change in a e-log. p curve. The increase in stress in the soil due to the dead load of the structure is thenestimated and the change in the corresponding void ratios of the compressible layer is found from the void ratio - log. effective pressure relationship obtained from the laboratory consolidation test.

The physical dimensions of the proposed embankment must be known to obtain the magnitude of the acting load p_2 that cause the settlement.

The added load or pressure per unit area that produces consolidation, is known as the consolidation pressure or consolidation stress. At the instant of application, the consolidation pressure p_2 is carried by the water in the voids of the soil, as seen in the preceding section. The application of Terzaghi's Theory of Consolidation to settlement problems in earth masses is commonly referred to as Terzaghi's Analogy.

Of prime importance in the calculation of consolidation under a highway embankment is the determination of effective stress actually applied to the soil materials by the surcharge.

The equations expressing the stress conditions within a soil mass caused by a superimposed load on the surface of an elastic, isotropic, homogeneous mass assumed to extend infinitely were developed by Boussinesq. (4) From Boussinesq equations, various procedures have been developed to simplify the application of these formulas.

The original expression of Boussinesq for the vertical normal stress at any point in the soil mass due to the action of a point load applied at the surface is:

$$\sigma_z = \frac{3Q}{2\pi z^2} \left[\frac{1}{1 + (\frac{r}{z})^2}\right]^{5/2} \qquad \text{(III.35)}$$

The vertical pressure on the compressible layers is maximum directly below the center of the loaded area and decreases in all directions from this point. (28) This basic equation has been simplified both graphically and mathematically, and values of σ_z resulting from various kind of loads can be obtained from various tables and charts.

In the theory of elasticity, in semi-infinite elastic solids, the strain produced by a composite state of stress is equal to the sum of the strains produced by each one of the stresses individually. This is known as the principle of superposition. (31)

L. A. Palmer, (12) in 1938, used this procedure to obtain formulas for stresses due to different kind of loads.

In practice, a complete solution can only be obtained for a relatively few simple shapes of loaded area, such as a point, a line, a rectangle, a square, or a circle, under load distribution which are either simple or vary in a uniform manner. Some of these have been calculated by L. Jurgenson, (9) and according to him, the pressure bulb for the case of triangular loading is shown in the following figure:

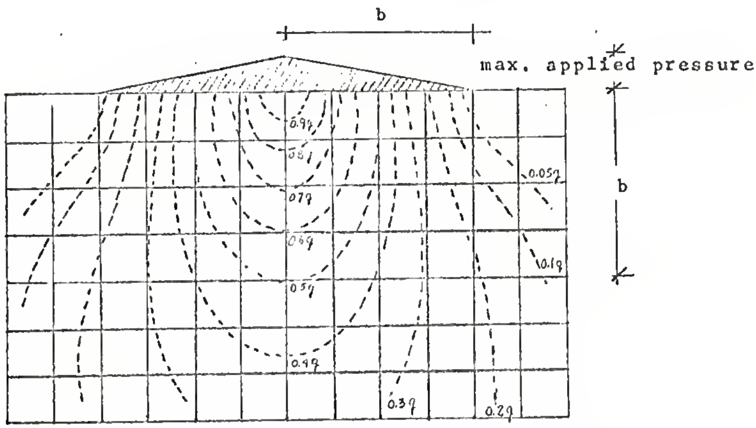


Figure 9. Bulb of Pressure Under Strip with Triangular Load.

On each horizontal plane below the embankment load, there is a symmetrical bell shaped distribution of pressure due to the general decrease in stress as the distance r of Boussinesq's formula increases as measured from the load.

If a line is drawn from the loaded point at an angle to the axis of symmetry it will cut horizontal planes at a series of points at which the vertical stress decreases as the square of the distance of the plane from the loaded surface. Those points which have the same vertical stress can be joined by curves, and the resulting family of iso-stress surfaces, have a ovate shape and was first called a "pressure bulb," by K. Terzaghi. These are shown in the figure with dashed lines.

Knowing the unit weight of the fill γ, the pressure distribution at the base line can be represented by a triangle of height $q = \gamma H$. H is the height of the triangle at the point of the cross section of the embankment where the stress calculations are desired. The value of H is easily obtained, since the shape of the proposed embankment is known. The value of q at this point is the maximum applied pressure, and with this value the magnitude and location of the maximum vertical stress, σ_z, is obtained from the figure. This figure is quite convenient to use to estimate settlement of embankments.

A normal section of an embankment may be considered as the difference between two superimposed triangles with a common angle at the top as shown in Figure 10. The stress distribution beneath such an embankment may be obtained by subtracting the stress due to the smaller triangle from those of the larger triangle. (28)

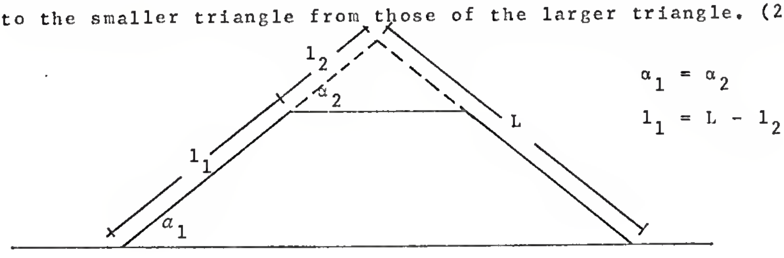


Figure 10. Illustration of the Procedure Developed by Jurgenson for Vertical Stress Distribution Under Trapezoidal Load.

An influence chart method has been suggested by Osterberg (20), based on the original formulas developed by Boussinesq. Osterberg integrated Boussinesq equation for a single concentrated vertical load on a semi-infinite, isotropic, homogeneous mass and obtained formulas for stresses for various shapes and load distributions. He presented the solution given the values of the stresses as a function of "influence values."

In 1942, N. Newmark (15) published charts for easy application of load conditions to obtain the values of vertical pressures. These charts, developed by Newmark, were used by R. L. Sloane (19) assuming the cross section of the embankment as a rectangle of equivalent area.

In 1958, Ray Webber (21) also used Newmark charts (15) to compute vertical stresses due to embankment loads and considering the fill foundation having dimensions of a rectangle one side of which is equal to the average width of the fill, and the other side equals 4 times depth of compressible soil. The large rectangle is analyzed as a 1/4 section, by the theory that the maximum load is on one corner of a 1/4 rectangle. Each of these dimensions is A and B, and the coefficients m and n relating this rectangle to Newmark chart, are:

$$m = \frac{A}{\text{depth to midpoint of layer from ground surface}}$$

$$n = \frac{B}{\text{depth to midpoint of layer from ground surface}}$$

Reeently, Perloff, Baladi and Harr (24) developed a method based on their own theoretieal and experimental research to give stresses within and under elastie eontinuous embankments as a funetion of the slope angle α, and the ratio of the half width of the top of the embankment L, to the embankment height, H.

Irregardless of the form of obtaining the eonsolidation pressure p_2, the procedure for calculating the total settlement of a strueture is the same (28). It can be following the void ratio method, or the eoeffieient of compressibility method, as shown later.

The degree of eompressibility of a soil is sometimes expressed by the eompressibility eoeffieient, a_v.

$$a_v = -\frac{\Delta e}{\Delta p} 10^{-3} \text{ in } \frac{cm^2}{gm}$$

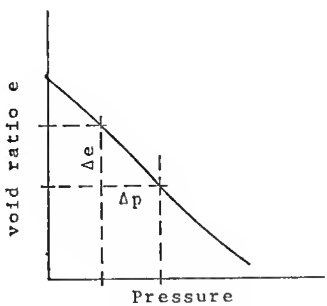


Figure 11. Void Ratio-Pressure Curve for an Undisturbed Sample of Clay.

From the figure 11 the coefficient a_v represents the slope of the straight line section of the void ratio-pressure curve. The following formula is used to compute settlement, based on Figure 12.

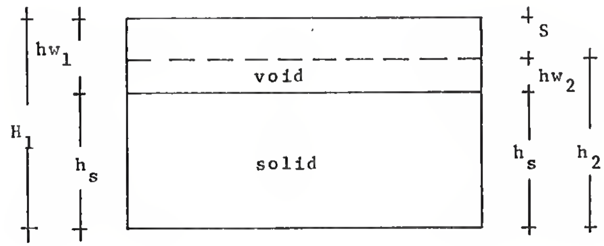


Figure 12. Illustration of Computation of the Settlement S of a Fully Saturated Clay, Subjected to Compression.

$$S = hw_1 - hw_2 = \frac{hw_1 - hw_2}{hs + hw_1} H_1 =$$

$$\frac{hw_1/hs - hw_2/hs}{1 + hw_1/hs} H_1 = \frac{e_1 - e_2}{1 + e_1} H_1 =$$

$$\frac{\Delta e}{1 + e_1} H_1 = \frac{a_v}{1 + e_1} \Delta p \; H_1 \times 10^{-3} \qquad \text{(III.36)}$$

Another coefficient, the modulus of volume change, m_v, is used such that:

$$m_v = \frac{a_v}{1 + e_1} \text{ in } cm^2/gm$$

For practical purposes, when estimating settlement, it is more convenient to take m_v, in square cm./Kg, so that:

$$m'_v = m_v \times 10^3, \text{ and}$$

$$m'_v = \frac{S}{\Delta p H_1} \text{ in } cm^2/Kg$$

$$S = m'_v \, \Delta p \; H_1 \qquad \text{(III.36a)}$$

Another way of calculating settlements, is by use directly of the void ratio in which the relation between change in thickness and change in void ratio of an element of a layer of soil which is laterally confined, may be developed by the following reasoning:

$$(Vt)_1 - (Vt)_2 = \Delta Vt = \Delta Vv = (Vv)_1 - (Vv)_2$$

Substituting $Vv = V_s e$

$$\Delta Vt = Vs(e_1 - e_2) = Vs\Delta e$$

$$Vs = \frac{Vt}{1 + e}$$

$$\Delta Vt = Vt \frac{\Delta e}{1 + e}$$, and in cases of no lateral strain:

$$\Delta H = H \frac{\Delta e}{1 + 3} \qquad \text{(III.37)}$$

$(Vt)_1$ = Total volume before loading.

$(Vt)_2$ = Total volume after loading.

ΔVt = Change in total volume due to loading.

ΔVv = Change in voids volume due to loading.

$(Vv)_1$ = Total volume of voids before loading.

$(Vv)_2$ = Total volume of voids after loading.

Vv = Voids volume.

Vs = Solids volume.

e = Original void ratio.

e_1 = Void ratio before loading.

e_2 = Void ratio after loading.

Δe = Void ratio change due to loading.

Vt = Total volume.

ΔH = Change in stratum thickness.

H = Original stratum thickness.

Ray Webber (21) in his analysis cited previously uses the data of the e-log. p curve from consolidation test to predict fill settlement using the void ratio method.

Once the total expected amount of settlement is known by either one of the methods mentioned above, the rate of settlement, or percent consolidation-time relationship, is important for its practical applications in building the embankment.

III. C. FURTHER MODIFICATION REQUIRED TO PERMIT THE USE OF THE GENERAL THEORY OF CONSOLIDATION FOR PREDICTING SETTLEMENT IN FILLS.

In section III.B, the stress distribution of an embankment was explained and the procedure to predict settlement of highway fills acting freely above the foundation soil, based on the general theory of consolidation. This general theory has to be further modified as the embankment approaches permanent unyielding structures. Figure 13 shows three sections of embankment approaching a rigid wall. First, a section A considered free of effects from the structure, second, a section B adjacent to the rigid and unyielding retaining structure, and C, the structure consisting of an abutment for a bridge or similar structure. In a vertical plane there are basically three types of soil:

1. The fill material of the embankment.
2. The compressible material of the foundation under the embankment.
3. The unyielding hard foundation stratum.

In section III.B, of the report, the magnitude of the total settlement S, of the fill due to consolidation of material A2 was predicted, in this part of the report, the behavior of parts B1 and B2, are considered.

Assuming the same physical dimensions of the highway embankment, or the same type of load, and the same kind of dimensions of the compressible soil 2, the same settlement would be expected immediately adjacent to the rigid structure.

In most of the cases, that continuous magnitude of settlement, does not exist, and the differential settlement observed between the roadway and the bridge surface, occurs at some distance away from the wall.

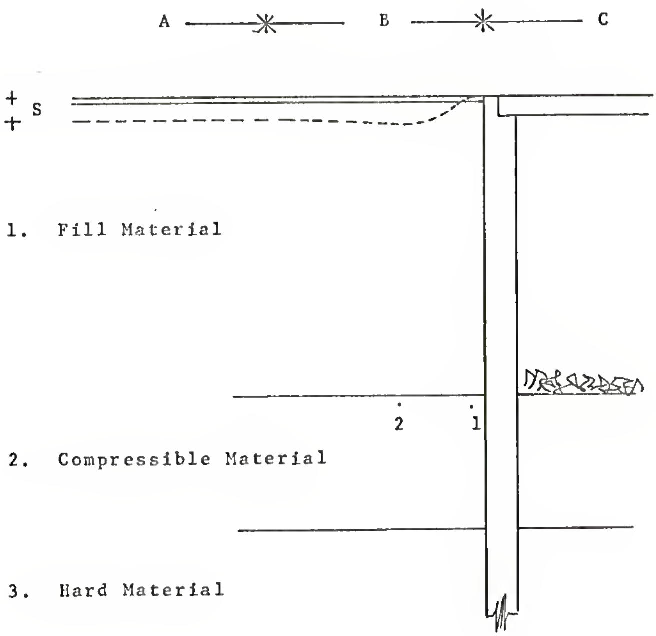


Figure 13. Retained Structure Founded on Hard Stratum.

The compressible stratum B2 has the same depth of fill superimposed and should yield the same amount of settlement as that in section A. This however is not found to be the case for the following reasons:

Assuming the fill material homogeneous, the differential settlement has to obey one constraint: part A of the embankment settles as a free body following the phenomena already described in the preceding sections, but part B of the embankment does not yield the same load to the compressible soil B2, that part A does, because there exists a shearing resistance to sliding along the face of the wall. This shearing force is due to three components;

1. The weight of the embankment
2. The frictional characteristics of the soil
3. The unit cohesion of the clay materials.

The reason that the differential settlement occurs over some distance from the wall, is due to the shearing resistance existing over a zone of influence in section B. The shearing resistance existing over the wall diminishes the vertical stress due to the embankment load on section B2. Point 1 in the figure will receive less pressure than point 2, because part of the uniform longitudinal trapezoidal load has been transferred to the vertical wall. Theoretically, at a point immediately against the wall, the total trapezoidal load will be carried entirely by the wall.

The road surface is depressed as shown by the broken line, having a more compressed part in its part of maximum curvature because of the action of dynamic forces that overcompact the fill material by a semi-impact loading of the vehicular traffic in the area of differential settlement.

The general theory of consolidation is applicable in this area but because the stress distribution to which the theory is closely related, varies due to the load transfer from the compressible soil to the wall, the settlement is less and the theory of consolidation seems to fail. A different effect is found to occur when the structure is founded on piles, as shown in the following figure 14:

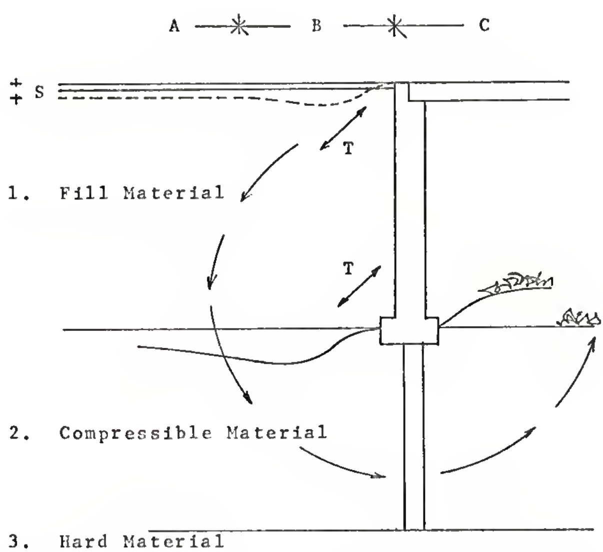


Figure 14. Retained Structure Founded on Piles

Leonards (32) in 1962, states that shearing stresses along the back of the wall and the rear piles, prevent surface settlements in the zone adjacent to the walls, however, a short distance away from the wall, the settlements tend to increase to a greater amount than farther from the wall. This type of movement is due to plastic deformations of the underlying clay which tend to occur with the lateral consolidation of the adjoining clay, carrying lighter vertical loads between and beyond the piles under the structure.

The curvature of the fill surface indicates that horizontal tensile stresses T are induced in the fill close to the wall. The exact magnitude and distribution of the lateral pressure, continues Leonards, of the plastic clay along individual piles and between pile rows are difficult to determine since no direct relevant measurements have as yet been performed. The impact loading cited in the first case also operates in this type of structure and greatly magnifies the problem due to greater differential settlement.

It is the opinion of the author of this report, that the general theory of consolidation, based on assumptions of one dimensional compression, that the same friction existing in the walls of the consolidation device exists in the field for this particular case of settlement of embankments against retaining structures. Taylor (26) points out the fact that the simple type of compression holds except for minor variations caused by side friction in the laboratory consolidation test.

In summary it is recognized that the general theory of consolidation is adequate and yields results well within the limits for this type of construction work. The stress distribution on the other hand is usually not accurately determined near structures and the effects of plastic flow (or lateral movement by shear) is not considered by any of the references cited in open type structures. It is recognized that in most cases the field information necessary for the required analysis is either not obtained prior to design of the embankments.

IV. ILLUSTRATION OF THE FEASIBILITY OF USING THE ANALYSIS DEVELOPED IN III C, ABOVE, FOR PREDICTING SETTLEMENT IN APPROACH FILLS.

When the approaches of a bridge abutment are founded on soil containing compressible layers, the approach fill will settle, as described in section III B, up to a point where the zone of influence of the shearing resistance between the soil and the retaining wall, reduces the effective pressure on the compressible layer, as seen in section III C.

Whenever a rigid unyielding retaining structure is abutted with an approach fill founded on compressible stratums of soil, differential settlement will occur between the fill surface and the structure if time for the fill to settle is not permitted before the retaining structure is built. One part of the embankment can be analyzed by the theory of consolidation, but the other part cannot, because the transfer of load from the compressible stratum of soil to the supporting shearing resistance of the wall is not considered in the general theory of consolidation and the solution of the problem of this differential settlement has to be founded outside the general theory of consolidation in the area adjacent to the wall.

Referring to Figure 13, in section III C, the settlement of section A can be analyzed by the conventional consolidation analysis with sufficient accuracy by careful determination of the compressibility of the various layers of soil, and the stress distribution under the fill.

The analysis of settlement in section B of the embankment is more complex since it involves a combination of consolidation and shear. Osterberg (20) suggests that the stress is reduced by some 50% in this section with a consequent reduction in settlement.

At the boundary with section A, pure consolidation controls the settlement while at the boundary with the structure C, the settlement, or lack of settlement, is controlled by shear and consolidation in the following general relationship:

1) The shearing resistance between the soil and the structure, transmits a part of the load from the foundation soil to the foundation of the structure.
2) The stress at the boundary, between the fill and the foundation soil is thus reduced by a varying and up to this time unknown amount.
3) Consolidation occurs in the foundation soil exactly the same as in section A but with the reduction in stress less consolidating pressure results in less settlement.

The relationships shown in Figure 14 are much more complex since the openings through the foundation of the structure and the lack of surcharge in area C, allows plastic flow of the material resulting in greater settlement at some point away from the wall than that found in the general fill area A. In this case the movement of the soil laterally through the structure is pure shear and settlement can not be analyzed at this time.

As the plastic flow occurs, tension in the soil occurs near the wall which minimizes shear between the wall and the soil thus resulting in a progressive failure that causes the depressed area to occur closer and closer to the structure until stability from shear is achieved.

As in the case of Figure 13, this failure condition has not been analyzed by careful measurement and calculations of actual existing stresses under the embankment.

V. CONCLUSIONS

As stated in section III and IV, the analysis required to predict settlement of highway embankments, follows three separate studies:

1) An appropriate sampling plan and correct testing of the foundation soil, including shearing resistance and consolidation.
2) Estimation of the stresses that the foundation soil is actually subjected to.
3) Computation of the settlement by the theory of consolidation based on the above data and an estimate of the plastic flow calculated.

It is obvious that each of these three factors contribute to the accuracy of the results. In all cases the extreme variability of the soil must be considered and analysis must be hased upon this variability.

One difficulty of predicting settlements is based in that the analytical approach is based on theory developed for elastic, homogeneous materials, rather than soil, and that the data on the soil compressibility is obtained by empirical procedures, or derived from tests on small specimens which have been removed from the ground and transported to a laboratory for testing.

The stress distribution on the soil mass, the results of consolidation, and the effect of shearing resistance within the soil and between the soil and the structure, are the three main factors in predicting settlement of embankments founded on compressible soil. To predict settlement of highway embankments, the general theory of consolidation is as applicable as in any other structure with only the necessity of an accurate computation of stresses due to the embankment trapezoidal load.

The effects of the complex shear and consolidation combination can be minimized by the following suggestions taken from Jones (22).

1) Providing a waiting period prior to applying the surfacing.
2) Preloading the site of the embankment before paving.
3) Providing sand drains within the foundation compressible soil to speed consolidation.
4) Enlarging the structure to reach the abutments with less fill height, and consequently subjecting the compressible layers to less superimposed load.
5) Removal of the compressible material.
6) Providing a section of flexible pavement to avoid the cost of continuous repairs which may be replaced by permanent paving when stability is achieved.
7) Providing a long approach slab to minimize the differential settlement.

In all these cases, the general theory of consolidation has a direct and fundamental application. The consolidation process is accelerated by adding load or facilitating the water of the compressible soil to escape. It can be noted that the most economical solution is allowing settlement and stabilization of the embankment with time. In most cases, as noted in 6 above, a temporary pavement of asphalt could be applied through section B, thus allowing use of the facility with the least possible cost and greatest ease of repair after stability of the fill is achieved.

Finally, it is necessary to show graphically what is happening to sciences, in general, and to Soil Mechanics related to settlement of highway embankments, in particular. This appreciation is taken from a lecture given by Dr. Jack Blackburn, in the Civil Engineering Department of Kansas State University.

A time $t_2 - t_1$ has elapsed since the discoveries of Terzaghi's work. Curve A represents the improvements of theoretical Soil Mechanics, that at time t_1 had almost no gap y, with its applications or curve B. On time t_2, the gap y_1 of the application of theory to practice, is bigger and it seems that it continues to grow. The applicability of principles made on t_1, are almost the same after a time t_2. That is happening in settlement analysis. Today are being applied the same principles made back in 1925, and there has not been any improvements, as far as applicability of new principles of Soil Mechanics related to settlement analysis is concerned.

That is why a small difference exists between calculated settlements and observed settlements. It has been said that these differences do not affect the results in most of the cases, but they can be improved, specially, when cases in nature deviate from the assumptions of Boussinesq and Terzaghi.

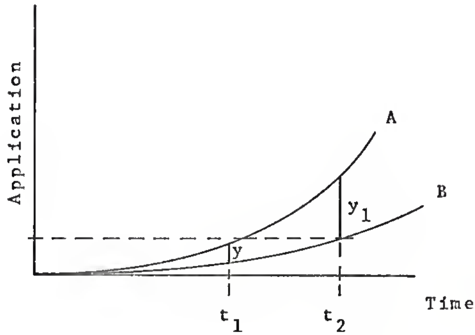


Figure 1. Application from Theory to Settlement Problems During Time.

VI. AREAS OF FUTURE RESEARCH

It is obvious that a program of research to collect data for the determination of the relationship of consolidation and shear in and under embankments approaching rigid structures is urgently needed. The effects of plastic flow through open types of bridge abutments should be determined by careful instrumentation of structures during the process of construction.

An economic evaluation of the effects of these failures and the remedial measures suggested is needed.

VII. LIST OF REFERENCES

1. Coulomb, C. A., "Experiments on the application of the rules of maxima and minima to some problems of stability related to architectural works," Vol. 7, Memoirs, Royal Academy of Sciences, Paris, 1776.

2. Rankine, W. J. M., "On the stability of loose earth," Vol. 147, Transactions, Royal Society of London, 1857.

3. Resal, Jean, La Pousce des Terres, deuxieme partie: theorie des terres coherentes, Beranger, Paris, 1910.

4. Boussinesq, J., Applications des potentiels a letude de l'equilibre et du mouvement des solids elastiques, Gauthier-Vallaris, Paris, 1885.

5. Mohr, O., "The elastic limit and the failure of a material," Vol. 44, Zeitschrift, Vereins Deutsche Ingenieure, 1900.

6. Terzaghi, Karl, Erdbaumechanik, F. Deutecke, Vienna, 1925.

7. Terzaghi, Karl, "The calculation of the coefficient of permeability of clays on the basis of hydrodynamic stress phenomena, Vol. 132, Sitzungsberichte der Wiener Akademie der Wissenschaften, Series of Mathematics and Natural Sciences, Vienna, 1923.

8. Terzaghi, Karl, "Bearing capacity of shallow foundations," Proceedings, 1st International Congress of Bridge and Structural Engineering Association, Paris, 1932.

9. Jurgenson, Leo, "The application of theories of Elasticity and Plasticity to foundation problems," Journal, Boston Society of Civil Engineers, 1934.

10. Palmer, L. A. and Barber, E. S., "A theory of soil consolidation and testing of foundation soils," Vol. 18-19, Public Roads, U. S. Bureau of Public Roads, 1937.

11. Palmer, L. A. and Barber, E. S., "Principles of soil mechanics involved in fill construction," Vol. 17-19, Public Roads, U. S. Bureau of Public Roads, 1937.

12. Palmer, L. A., "Principles of soil mechanics involved in the design of retaining walls and bridge abutments," Vol. 18-19, Public Roads, U. S. Bureau of Public Roads, 1938.

13. Newmark, N., "Simplified computations of vertical pressures in elastic foundations," Circular 24, University of Illinois Engineering Experiment Station, 1935.

14. Palmer, L. A. and Barber, E. S., "The settlement of earth embankments," Vol. 20-21, Public Roads, U. S. Bureau of Public Roads, 1940.

15. Newmark, N., "Influence charts for the computation of stresses in elastic foundations," Bulletin 338, University of Illinois Engineering Experiment Station, 1942.

16. Feld, Jacob, "Abutments for small highway bridges, Part 1," Vol. 23, Proceedings, Highway Research Board, 1943.

17. Feld, Jacob, "Abutments for small highway bridges, Part 2," Vol. 24, Proceedings, Highway Research Board, 1944.

18. Feld, Jacob, "Abutments for small highway bridges, Part 3," Vol. 25, Proceedings, Highway Research Board, 1945.

19. Sloane, R. L., "Settlement analysis for high fills on compressible foundation soil," Vol. 95, Road and Streets, 1952.

20. Osterberg, J. O., "Influence values for vertical stresses in a semi-infinite mass due to an embankment loading," Vol. 1, Proceedings, 4th International Conference on Soil Mechanics and Foundation Engineering, London, 1957.

21. Webber, Ray, "Description of a method of predicting fill settlement using voids ratio," Bulletin 173, Highway Research Board, 1958.

22. Jones, Ch., "Smoother bridge approaches," Civil Engineering, American Society of Civil Engineers, 1959.

23. Mc.Laren, D., "Settlement behind bridge abutments," RRL Report LR76, Road Research Laboratory, London, 1967.

24. Perloff, W. H., Baladi, G. Y., and Harr, M. E., "Stress distribution within and under long elastic embankments," Highway Research Record 181, Highway Research Board, 1967.

25. Yong, R. N., and Warkentin, B., Introduction to Soil Behavior, Macmillan, 1966.

26. Taylor, D., Fundamentals of Soil Mechanics, John Wiley & Sons, 1948.

27. Tschebotarioff, G., Soil Mechanics, Foundation, Earth Structures, McGraw-Hill, 1951.

28. Road Research Laboratory, Soil Mechanics for Road Engineers, Her Majesty's Stationery Office, London, 1952.

29. Casagrande, A., "The Structure of Clay and Its Importance in Foundation Engineering," Vol. 19, Journal, Boston Society of Civil Engineers, 1932.

30. Terzaghi, K., "Relation of Undisturbed Sampling to Laboratory Testing," Transactions, American Society of Civil Engineers, 1943.

31. Terzaghi, K., Theoretical Soil Mechanics, John Wiley & Sons, 1943.

32. Leonards, G. A., Foundation Engineering, McGraw-Hill, 1962.

APPENDIX

9* Jurgenson, Leo., "The Application of Theories of Elasticity and Plasticity to Foundation Problems," Journal, Boston Society of Civil Engineers, 1934.

ABSTRACT

Statement of the Problem

In analyzing the stability of foundations, it is neccssary to know the values of the maximum shear stress to compare it with the shearing strength of the soil, to prevent shearing failure.

Purpose of the Study

The study was done early in the age of Soil Mechanics when the methods and applicability of this science was not well defined to foundation problems. This "classical" paper of Soil Mechanics tries to give a synthetic procedure to apply the theories of elasticity and plasticity to foundation problems. For the case of settlement of highway embankments, a chart with values of maximum vertical stress due to triangular load is given.

Scope of the Study

The study covers a resume of the laboratory and theoretical research done by the author. The study gives, first, the determination of shearing stresses, and second, the determination of the shearing strength of the underground materials.

For the first part, the theoretical computation involving the theory of elasticity used by Carothers, is used to obtain the formulas shown in this study for different kinds of loads. For the second part, or determination of strength, the author gives three types of tests: Direct shearing test, Cylinder test, and Squeezing test. Then, a comparison is made between the three types, and a summary of test results is shown in a graph. Several examples are shown for different types of loads and foundations, and in the appendix are shown the mathematical deriations of the formulas used in different cases, and the graphs and diagrams, based on the formulas, that give the stresses.

Experimental Procedures

Design of Experiment

The method of applying the theory of elasticity to foundation problems in analogous to the use of the same theory in Structural design. In designing a truss, we first find the stresses in a

*Corresponds to number 9 in the List of References.

given member and then compare them with the strength. The same procedure can be followed in many problems of foundation engineering. We first find the stresses that a given system of external forces produces in the ground, and second, determine the strength of the material to see if it can resist the imposed stresses without causing excessive deformations. The present study will be limited to the question of deformations caused by shear.

Distrotion or change in shape is caused by shearing stresses. Depending upon the magnitude of the shearing stress it may be only a slight deformation proceeding very slowly or it may be a rapid slide resulting in considerable deformation. The magnitude and the rate of a gradual distortion are very difficult to analyze. It is much easier to deal with the question of ultimate resistance, i.e., the resistance of failure by sliding. We can do this by comparing the stresses produced in the ground by given external loads with the ultimate shearing resistance of the soil. The analysis thus resolved itself into two moving parts: first, determination of the shearing stresses, and second, determination of the shearing strength of the under ground materials. When computing the stresses it is of advantage to consider separately the stresses due to external loads and the stresses due to the weight of the overburden soil.

Procedures and Apparatus

For the determination of stresses, formulas based on the theory of elasticity for the cases which occur most recently, are developed by Carothers (not shown in the study) and the different stresses are shown in tables.

Tables and diagrams show the distribution of stresses for the following types of loading: uniformly loaded long footing, uniformly loaded circular footing, long footing with triangular loading and a terrace loading. If two or more systems of external loads are to be superimposed, the stress components n_z, n_x, n_{xz} due to each separate system must be added and the resultant principal shearing stress $S = 1/2\sqrt{(n_z-n_x)^2 + 4S_{xz}^2}$. Shearing stresses shown in the diagrams are the principal shearing stresses, i.e., the maximum shearing stress at the given point. The tables contain all stress components, the principal stresses and their direction.

For the determination of shearing strength, the author gives three types of tests. In the Direct Shearing test, the sample is inserted between two gratings, and the force H is measured which produces a shearing failure of the material. A vertical force V, is acting on the upper grating to provide a normal pressure that can be varied.

As we want to find the shearing strength of the day in its natural state, care has to be taken to prevent further consolidation of the material during the test. The force V should not be excessive, the gratings, impervious, and the test must be run quickly.

In the Cylinder Test, a cylinder of clay is subjected to compression to failure. The failure plane is forced into the middle of the sample and away from local stress concentrations at the ends. The distrihution of stresses on the failure plane, is therefore, at least as uniform as the material itself. The intensity of the maximum shearing stress is one half of the principal stress. In a relatively imperious material (condition of plasticity $2C = \sqrt{4S^2+(n_z-n_x)^2}$)the planes of maximum shear should be the failure planes. Their directions are at 45° to the principal planes. In a pervious material (condition of plasticity $1/2\sqrt{(n_z-n_x)^2+4S^2} = 1/2(n_z+n_x)\sin\phi$), the rupture planes are at $45° + 1/2\phi$ to the plane of maximum principal stress. In the case of cohesionless sand, the cylinder test has to be performed under lateral pressure.

The Squeezing Test can be used for relatively impervious clays, a thin sample of material is placed on a rigid box, open on two opposite sides, and pressures applied through a rigid top plate so that the material is squeezed out on the open sides.

Conclusions

If the computed maximum stress is higher than the maximum shearing strength, it not necessarily means that a danger of slide exists. Only inside the isoshear strength line, does the stress exceed the strength. This danger zone is confined and surrounded on all sides by material that still has a reserve of resisting capacity. The material in the plastic zone will yield somewhat and will transmit to the adjoining material that part of the load which it cannot resist itself. The theoretical stress diagram is then no longer correct, as the plastic region will actually be larger than the area inside the maximum isoshear line of maximum strength. If the external loads are further increased, the plastic zone is again extended until failure occurs when the ultimate resistance of the under ground is exceeded.

Hence, in addition to finding the maximum shearing stress, we also have to consider where it occurs and to what possible consequences it may lead.

Materials of low permeability, such as fat clays, derive their total strength from pressures to which they have been consolidated hefore we apply our external loads. Not until the clay begins to adjust itself to the newly imposed pressures and begins to loose excess water, do the imposed stresses begin to contribute to its strength. The strength of the material is increasing with time at a rate which can be analyzed by the theory of consolidation.

Referring to the three methods to obtain the shearing strength of the undersoil, in the shearing test, the sample can fail along the single surface which offers the least resistance.

This test is very sensitive to any local defect, non-uniformity of material, or stress concentration. In the Cylinder Test, the failure plane is far away from the ends where the forces are applied, and the stress distribution is more uniform. In the Squeezing Test, one single sliding plane cannot lead to failure as the material must shear along a number of planes. Since natural materials are never quite uniform, this is a desirable feature as the test reveals the average strength of the material and not its strength along the weakest plane. The cylinder test is the simplest to make and requires no complicated apparatus.

The stresses computed by the Theory of Elasticity do not hold beyond the point where the maximum shearing stress reaches the shearing resistance of the material. If the pressure be increased beyond this limit, a plastic zone will develop in the ground. The stresses at this semi-plastic stage are rather complicated and solutions are more easily obtained for the fully plastic state when the ground has reached its ultimate resistance and failure occurs.

10* Palmer,C.A., and Barber,E.S.,"The Theory of Soil Consolidation and Testing of Foundation Soil," Vol. 18-19, Public Roads, U.S. Bureau of Public Roads, 1937.

ABSTRACT

Statement of the Problem

In 1937, the testing apparatus and procedures used in the theory of soil consolidation were not used with confidence. The innovations on soil mechanics because of the investigations and publications of Karl Terzaghi were young, and the observation between the new laboratory procedures and field problems demanded investigations to compare the results and accept these procedures.

Purpose of the Study

The purpose of this study is to determine the magnitude of experimental errors resulting from variables in testing apparatus and procedures furnished by K. Teraghi, and to determine the agreement between deformations observed in tests, with those computed in accordance with the theory of consolidation on which the tests are based.

Scope of the Study

The study is based on a review of literature, and theoretical work done by the authors. The study covers a description of the compression device designed by Terzaghi and general explanation of the consolidation theory, and the determination of the coefficient of permeability and compressibility. Part 1 explains the application of theory of soil consolidation, subdivided into formulas or various types of loaded areas, formula for rate of settlement, time-consolidation relations, and computations of various types of pressure distribution and boundary conditions. Examples illustrate the procedure followed.

Part 2 develops the theory of consolidation and Boussinesq analysis for stress distribution in soils.

Experimental Procedures

Design of the Experiment

The study is pure theoretical and shows the obtention of formulas and how to apply them in each particular case by examples. Five types of load areas and five cases of pressure distribution in soil strata are considered.

Procedures and Apparatus

For part 1, the procedure followed in applying the derived formulas to practical cases. The manner of applying the test data

*Corresponds to number 10 in the List of References.

in practice, varies, depending upon the pressure distribution conditions beneath the loaded area, the thickness of the compressible layer, and whether it is free to drain both, at top and bottom of layer.

For part 2, the stresses in the undersoil for different types of loads are obtained integrating Boussinesq equation. The general theory of consolidation is developed mathematically by Fourier expansion series.

The only apparatus involved in the study is the consolidation device designed by K. Terzaghi, and a description of the method to use for the device is explained.

Analysis of Data

For the boundaries conditions of porous layers at the top and bottom of the compressible layer, equation $t = N_1(D_0/2)^2/1{,}400C$ in years, is used. If the drainage of the compressible layer is achieved by one boundary only, the formula to use is: $t = N_1 D_0 2/1{,}400C$ in years. From a chart of void ratio-pressure, the average value of e corresponding to the void ratio of the top and bottom of the layer can be obtained, being these voids ratios the correspondents to the pressures at the top and bottom of the compressible layer. After the load is applied, another value of e can be obtained relating these void ratios with the applied pressures, and the average value of void ratio after load is applied is e'. With these data, the settlement of the layer is computed by the formula

$$S = \frac{e_1 - e_2}{1+e_1} D_1, \text{ being}$$

e_1 = initial void ratio or the average prior to the application of the load, e.

e_2 = final void ratio or the average after the application of the load, e'.

D_1 = thickness of compressible stratum.

If there were no voids whatsoever in the material composing the compressible stratum, its thickness would be

$$D_0 = \frac{1}{1+e_1} D_1$$

To compute the pressure below a spot load, use is made of the equation:

$$p_2 = \frac{kP}{Z^2}, \text{ where}$$

$$k = \frac{3}{2\pi(1+\frac{r^2}{z^2})^{5/2}}$$

p = the spot load

p_2 = vertical pressure at a distance r from the spot load, and at a depth z.

For a relatively narrow loaded area of great length take a vertical cross section of the loaded strip at a point on the vertical center axis; pz is computed from the equation:

$$pz = \frac{p}{\pi}(\alpha + \sin\alpha)$$

α = twice the angle, in radians, whose tangent is half the width of the strip divided by the vertical distance z to the point in question.
p = the load per unit area of the footing

To compute the maximum vertical pressure, pz, at any point below a circular footing and on the center line:

$$pz = p(1-\cos^3\beta), \text{ where}$$

p = load per unit area of the footing

β = the angle whose tangent is $\frac{r}{2}$

The value of the void ratio is reduced by the surface loading from e to e', with an average of e+e'/2. From tables given, with this value of e", a value of c, coefficient of consolidation is obtained. Applying the equations of $t = N_1(D_0/2)^2/1{,}400C$ and $t = N_1 D_0^2/1{,}400C$, the time required for any percent consolidation is obtained. For a percent consolidation, values of N are given in tables according to the drainage conditions, type of load area and pressure distribution. For the obtention of stresses due to loads, Boussinesq analysis applies.

Conclusions

The Bureau of Public Roads and MIT were involved in research of performing compression tests. On the basis of the Bureau's investigation, experience in practical use of the test data and reports, it is now possible to state definitely that the apparatus and testing procedure, have been found satisfactory, to present a practical method of estimating that part of the total settlement of soil caused by the loss of water forced vertically out of saturated compressible soil strata in certain soil profiles.

11* Palmer, L.A., and Barber, E.S., "Principles of Soil Mechanics Involved in Fill Construction," Vol. 17, Public Roads, U.S. Bureau of Public Roads, 1937.

ABSTRACT

Statement of the Problem

The "stability of the fill" and the "supporting power" of the undersoil are important factors in design. Fills fail, when deformation in the fill or in their foundation exceed those permissible in the design of the particular structure.

Purpose of the Study

The purpose of this study is to present a concise method and to show formulas to estimate the stability of fills and supporting ability of the foundation soil.

Scope of the Study

The study is based on theory developed by the authors, and is subdivided in two possible cases where the stability of the fill and the supporting power of the undersoil are main considerations in the design of fills:

1. Fills on good undersoil, and
2. Fills on questionable undersoil.

The study covers the distribution of stress $S_{max.}$ in the foundation material, due to an embankment load with $a=b$, and the application of the "ϕ circle method" for stability analysis. The distribution of stress is based in Boussinesq formulas, and the magnitude of $S_{max.}$ is presented in the form of a chart with isoshear lines. The "ϕ circle method" is applicable from a table with the values of the angles of the slope of the fill and the "stability number". For the case of fill founded on a soil with questionable supporting power, Prandtl's formula is used to compute q, the bearing capacity of the foundation soil.

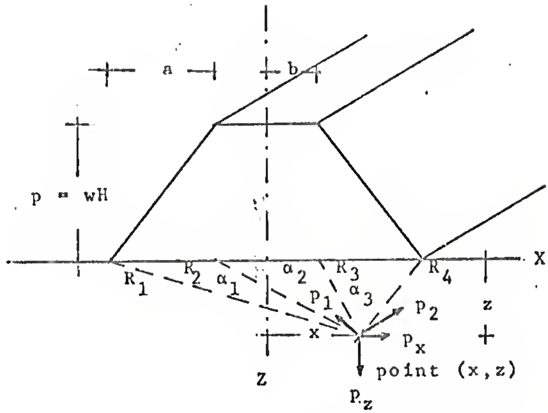
Experimental Procedures

Design of the Experiment

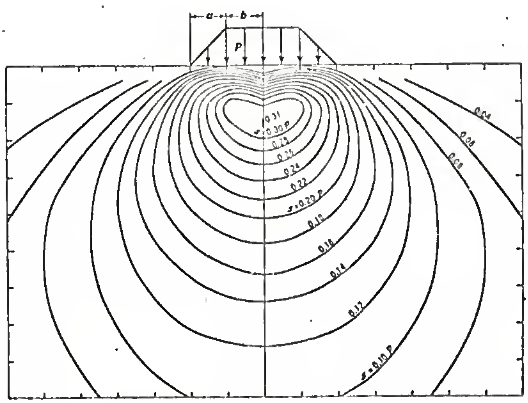
The analysis of the supporting power of the undersoil is made by comparison of the shearing strength of the undersoil with the maximum shearing stress imposed by the fill (shown

*Corresponds to number 11 in the List of References.

below) and given by a chart of isoshear linear values for S_{max}



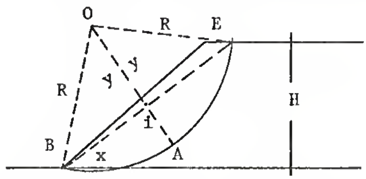
as seen below. Where the imposed stress is greater than the shearing strength of the undersoil material, a plastic state developes and the supporting power of the undersoil q is obtained by applying Prandtl's formula. The values of Prandtl's formula are taken from samples of the undersoil and from the physical dimensions of the embankment.



For stability analysis, the "ϕ circle method" is used. Knowing the angles of the slope of the embankment with the horizontal and the angle of internal friction of the undersoil, values of the angles x and y are obtained from tabulated values of the method. With this values, a correspondent "stability number" is obtained this "stability number" is given in function of the cohesion of the fill material c, the safety factor F, the effective unit weight w of fill material and the height of the fill H. Assuming a safety factor, the maximum height of the fill can be obtained for the conditions of the angle i and ϕ.

Procedures and Apparatus

On the "ϕ circle method", with a given angle i, and ϕ, the values of the angles x and y and the "stability number c/Fwh are obtained. With the values of x and y, the chord BE can be drawn.



From the points B and E, the radial lines R form the known angle y with the center O, which is on the perpendicular bisector of BE. With R as radius and O as center describe the arc BAE, which is the most dangerous circle. The above construction is permissible when the most dangerous circle passes through the toe of the slope, which is the case when N is negative or zero. $N = 1/2 (\cot x - \cot y - \cot i + \sin\phi \csc x \csc y)$; N, is the ratio of the distance from the top of the slope to the intersection of the dangerous arc with the ground surface to the vertical height, H.

Analysis of Data

The shearing strength of the foundation material is obtained by application of Coulomb's formula $S = c + pn \tan \phi$. When a = b, the greatest value of S_{max}, according to the chart of isoshear lines, is 0.31p, and occurs at the centerline of the embankment at a depth z = 3/2a. For a = 2b, the value of Smax is 0.30p, also in the centerline of the embankment and at a depth z = 0.96a.

For any type of fill, within the range of a=b to a=2b, the greatest value of Smax is approximately 0.3p. If the cohesion of the undersoil is less than the value of Smax, it is not necessary follows that the danger of failure exists. If the undersoil has a uniform strength of 0.3p, it is seen from the chart of isoshear lines that only inside the Smax = 0.3p curve, is the strength exceeded by the stress. This danger zone is confined and is surrounded on all sides by material having a reserve of resisting capacity. The material in the plastic zone may yield to some indeterminate extent and transmit to the adjoining material that part of the load which if cannot resist itself. Under those conditions, a plastic zone having been developed, the isoshear lines for Smax no longer are correct and Prandtl's formula is used to obtain the supporting or leaning capacity of the undersoil.

$$q = (c \cot\phi + w'b \cot\Lambda)\left[\frac{1+\sin\phi}{1-\sin\phi}e^{\pi\tan\phi}-1\right] ,$$

where
q = bearing capacity of the undersoil
c = cohesion of the undersoil
ϕ = angle of internal friction of the undersoil
w' = effective unit weight of undersoil
b = half width of top of embankment plus half the slope.
$\Lambda = 45^\circ - \phi/2$
e = base of natural or Naperian logarithms.

With the formula, $F = q/wH$, can be obtained the correct safety factor in function of q and H. Or in other words, the height of the fill required for an assumed factor of safety is determined by the above formula.

Conclusions

A cohesive supporting soil is considered as safe if its greatest shearing stress does not exceed the cohesion corresponding to the maximum allowable deformation. In such a case no further consideration need be given to the problem of bearing capacity of the supporting soil.

12* Palmer, L. A., "Principles of Soil Mechanics Involved in the Design of Retaining Walls and Bridge Abutments," Vol. 18-19, Public Roads, U. S. Bureau of Public Roads, 1938.

ABSTRACT

Statement of the Problem

In a large number of earth pressure and foundation problems, the stresses found by the method based on elasticity are independent of elastic constant and are classified as problems of plane strain or plain deformation. These are problems involving two dimensions, and in their solution an analysis is made of the stresses in a vertical cross section of the earth embankment or supporting soil under a foundation, having this method its limitations.

Purpose of the Study

The limitations of the analytical method based on the assumptions of the conditions on plane strain are indicated in this paper in the case of the supporting soil under embankments, piers and retaining walls. The object of this paper is to make available for practicing engineers the method based on elasticity as a theoretical and practical approach to the study of earth problems. The legitimate use of the formulas for plane strain in connection with abutment and retaining wall problems, depend chiefly on the ratio of length to width of the footing. There have been apparently no rules as to procedures where this ratio is small. Furthermore, for a rectangular footing whose length is not great in comparison to its width, there are apparently no formulas for stresses other than the vertical one, p_z, and the lateral ones p_x and p_y to be found in currently reported literature. It is interesting therefore to compute p_z for a relatively long footing, both by the formula as applicable for the case of plain strain and by the formula for the case evolving three dimensions.

Scope of the Study

In this study formulas will be derived for stresses in the supporting earth below:

1) Symmetrical fill

2) Bridge abutment subjected to a rotating movement of such a nature that a trapezoidal load distribution is transmitted by the abutment.

*Corresponds to number 12 on the List of References

Experimental Procedures

Design of the Experiment

The study is done by applying the formulas developed by Carothers for stresses in earth masses.

Recourse will be had to well known methods of the superposition of loading systems in the derivations. Further application of the principles of plane strain will be made in the analysis of the three types of earth pressure against retaining walls: a) earth pressure at rest; b) active earth pressure; and, c) passive earth pressure. The method based on plastic equilibrium will be used in deriving an expression for the bearing capacity of soil supporting a bridge abutment and finally, there will be indicated by direct computation the extent of application of the method based on plane strain in determining earth stress below loaded rectangular areas of different sizes.

The active and passive earth pressure in the earth back of the retaining walls are determined by using the analytical method of Coulomb.

The computation of p_z, vertical pressure is computed both by the formula as applicable for the case of plane strain and by the formula involving three dimensions. The comparison is shown in a table.

Analysis of Data

The maximum shearing stress in the part below the base of an abutment subjected to a rotating moment producing a maximum vertical pressure p_o at the toe is p_o/π. If p_o/π is less than the cohesion of the undersoil, it will not fail under this stress. However, if p_o/π exceeds c, it is necessary to determine the factor of safety q/p_o against failure, obtaining q from the formula,

$$q = \frac{2c}{\tan\alpha \sin^2\alpha} + \frac{wa}{2\tan\alpha}\left[\frac{1}{\tan^4\alpha} - 1\right] + \frac{wd}{\tan^4\alpha}, \text{ where}$$

q = supporting power of undersoil
c = unit cohesion
w = unit weight
$\alpha = 45° - \phi/2$
d = depth of surcharge
a = width of the base of the abutment.

The active and passive earth pressure in the earth back of the retaining walls, are

$$p_h(\text{active}) = p_v \tan^2(45°-\phi/2) - 2c\ \tan(45°-\phi/2)$$
$$p_h(\text{at rest}) = Kp_v$$
$$p_h(\text{passive}) = p_v \tan^2(45°+\phi/2) + 2c\ \tan(45°+\phi/2)\text{, where}$$

ϕ = angle of internal friction of embankment material
c = unit cohesion of embankment material
K = coefficient of earth pressure at rest, of the embankment material
p_h = horizontal pressure
p_v = vertical pressure.

Conclusions

1) The analytical method, assuming plane strain conditions, is applicable for all bridge piers with ratio of the dimensions of the base, of length to width ratio, of 2 or greater.

2) The same analytical method is warranted when bridge abutments and retaining walls when the ratio is 3 or greater; ϕ having any values from 0° to 22°.

For a given footing, it is seen in the table of comparison of results, that the difference in values of p_z as computed by the two methods increases with depth.

14* Palmer, L.A. and Barber, E.S., "The Settlement of Earth Embankments," Vol. 20-21, Public Roads, U.S. Bureau of Public Roads, 1940.

ABSTRACT

Statement of the Problem

One of the most important considerations in the design of highway embankments, is the amount of settlement that may be expected. It is possible for a high fill to subside several feet due to "displacement" of the supporting soil without its failure.

Purpose of the Study

The purpose of the study is to present a method to estimate the total settlement of all earth embankment, including the part of settlement due to lateral "displacement" within the fill material and also, in the foundation soil.

Scope of the Study

The study covers a theoretical research of the subject and proper theoretical analysis of the principal aspects related to settlement of earth embankments.

1. Determination of C, ratio of stress deformation
2. Determination of V, downward "displacement" of soil within the embankment.
3. Determination of V', downward "displacement" of soil underneath the embankment. Settlement due to lateral "displacement" $S_L = V + V'$.
4. Determination of S_c, settlement due to consolidation, within and under the embankment.

Experimental Procedures

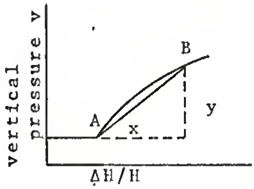
Design of the Experiment

The study is based on derivation of formulas to obtain the expression for lateral displacement of the fill material and for the foundation soil. This formulas are dependable on values given in charts due to the variability of dimensions of earth embankments and the depth of the compressible layer. The study applies the difference in void ratio formula to obtain the settlement due to consolidation of the fill material and the foundation soil. Also, the triaxial test device is used to obtain C, the modulus of deformation.

*Corresponds to number 14 in the List of References.

Procedures and Apparatus

Using the triaxial test, a graph of unit decrease in height $\Delta h/h$ vs vertical pressure v, is obtained, and C, the ratio of stress deformation is computed. The vertical load is applied through a plunger by an hydraulic machine and is increased at a constant rate of 0.05"/min. An automatic recording device gives the complete vertical load vs. change in height curve for the entire test. Point A, is the state of deformation where consolidation takes place due to volume (height) change. Point A is located when v = 1, or vertical = lateral pressure. Beyond A, indicates distortion without volume change or S_L. The modulus of deformation is taken as the slope of the secant line drawn from A, where 1=v, to B, determinable from the conditions of the particular problem. This is attained knowing v-1, and the ordinate of point B is taken to use in the formula C = y/x. Once C is obtained, we use charts to know the values of the ratios b/B and z/B and obtain the values to substitute on S_L, both, within and under the highway embankment. To obtain the value of S_L, settlement due to consolidation, the void ratio method is used.



Analysis of Data

For the estimation of S_L within the embankment,

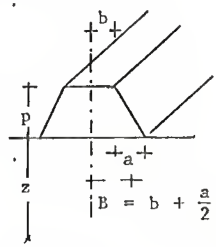
$$V = S_L = -\frac{3w}{8C} H^2 (1-k') \text{ is used, where}$$

H = the height of the fill
w = unit weight of the fill material
C = modulus of deformation
k'= is the ratio of lateral to vertical pressure at bottom of the fill, usually is taken = 0, to be on the safe side.

To obtain S_L, underneath the highway embankment,

$$S_L = \frac{p}{C}(b+\frac{a}{z})F, \text{ is used, where}$$

p = pressure at surface and on centerline
C = modulus of deformation of layer
F = factor from chart, for z/B, b/B and μ
z = depth of layer causing settlement



To obtain S_c, it is applied the formula:

$$S_c = \frac{e_i - e_f}{1+e_i} D, \text{ where}$$

e_i = initial void ratio of soil
e_f = final void ratio of soil due to applied pressure
D = thickness of compressible layers

This formula is used to obtain both, the settlement within the embankment due to its own weight, or underneath the embankment due to the weight of embankment plus overburden pressure.

Conclusions

The total settlement of a highway fill is the sum of four factors: a) settlement due to displacement of the fill material; b) settlement due to displacement of the soil underneath the fill; c) settlement due to consolidation of the fill material, and d) settlement due to consolidation of the soil underneath the fill.

It is a serious mistake to rely completely on any mathematical formula derived from theory and assumptions without reference to observation in the field and to practical experience. Where the problem deals with large earth masses, mathematical expressions can at the best only indicate the general trend of physical occurrences. The theoretical expressions or formulas in the paper, would be expected to be more precise for relatively small deformations.

Recommendations for Further Research

The authors require the need of correlation of field observations with theoretical and laboratory studies.

16* Feld,Jacob.,"Abutments for Small Highway Bridges, Part 1," Vol. 23, Proceedings, Highway Research Board, 1943.

ABSTRACT

Statement of the Problem

For highway engineers it is very important to determine the magnitude and distribution of forces acting on bridge abutments, and the type of abutment to design in each particular case.

Purpose of the Study

The objective of the report is to put in the hands of highway engineers available and useful information in the design of bridge abutments.

Scope of the Study

The study is based on a literature review of previous work pertinent to the problem and a theoretical analysis made by the author based on his own experience and knowledge. The first part of this report, contains a general outline of all factors related to the design of bridge abutments. The main portion of the report includes a discussion of "Forces and Resistances" subdivided in the following considerations:

1. Active Earth Pressure.
2. Passive Earth Resistance.
3. Foundation Pressures.
4. Bearing Value of Piles.
5. Uplift Resistance of Piles.
6. Lateral and Pull-out Resistance of Soils.
7. Lateral Resistance of Piles.
8. Ice Pressure and Uplift.
9. Scour of water and silt.

Experimental Procedures

Design of the Experiment

There is no experiment or research involved; the paper summarizes the important considerations to take into attention in the design of abutments. For example, for "Active Earth Pressure," the author presents ten rules to follow to obtain the correct Active Earth Pressure for each particular case. These are:

1. Horizontal component.
2. Vertical component of lateral pressure.

*Corresponds to number 16 in the List of References.

3. Pressure of wet materials.
4. Pressure of submerged soils
5. Drainage
6. Pressure due to earthquake
7. Variation of pressure due to temperature and age
8. Pressure due to surface loading, with point of application of resultant
9. Pressure of cohesive soils.

For the calculation of "Passive Earth Resistance," the author gives recommendations of:

1. horizontal component of pressure
2. vertical component of pressure
3. location of the resultant of passive earth pressure.

For the "Foundation Pressures," it is recommended that the foundation be treated separately in the analysis, and the design of an abutment be considered in two parts, the retaining wall and the foundation.

The rest considerations are listed, and they are not given or subdivided into important factors in design.

Analysis of Data

The horizontal component of lateral pressure Ea, for vertical walls and horizontal fills, is:

$$Ea = \frac{1}{2}wH^2 tg^2(45° - \frac{\phi}{2}) \text{, or}$$

$$Ea = \frac{1}{2}wH^2 ka \text{ , where:} \qquad 1$$

w = average unit weight of the fill, in lbs/cu. feet
H = height of fill, in feet
ϕ = angle of internal friction

For cohesive soils: $Ka = 0.75 - \frac{q}{wH}$

Ka = hidrostatic pressure ratio to be used for formula 1

For passive earth pressure, $Ep = \frac{1}{2}wH^2 tg^2(45 + \frac{\phi}{2})$

$$Ep = \frac{1}{2}wH^2 Kp \text{, and}$$

$$Kp = 45 + \frac{\phi}{2}$$

For bearing value of piles:

$$P = \frac{R}{F} = \frac{ewh}{S+K} \times \frac{w + n^2 M}{w + M} \quad \text{where:}$$

R = dinamic pile resistance
P = allowable load
F = factor of safety
w = weight of striking part of hammer
m = weight of pile

h = height of fall of hammer
S = penetration of pile per blow
k = 1/2 rebound of pile eap
n = eoeff. of restitution
e = effieieney of hammer

Discussions

As Mr. V. T. Boughton, editor, Eng. News Record said, "there is no relation in the study, about vibration due to loads in general, and the ones of railroads, in partieular, to the design of the abutments."

Mr. A. W. Buskell do not agree with the 10% variation on pressure due to slope filled baekfill and also, he sees the formula for piling as complieated; he states the values of the formula as unreliable. Prof. R. G. Hennes, Univ. of Washington, said that item 3, Foündation Pressures, leaves the designer with the following questions unanswered:

1. How should allowable base pressures be determined?
2. What laboratory tests should be made?
3. What field tests should be made?
4. How are tests results to be applied to design?

17* Feld, Jacob, "Abutments for Small Highway Bridges, Part 2," Vol. 24, Proceedings, Highway Research Board, 1944.

ABSTRACT

Purpose of the Study

The author continues the work done by him in 1943, to cover other considerations in the design of bridge abutments.

Scope of the Study

This part covers Part II, or "Material and Stresses" and "Types of Abutments - Method of Design". Under "Materials and Stresses," are described the applicable characteristics of various abutment types of Masonry (dry and mortar rubble, ashlar, brick, mass concrete and precast units), Reinforced Concrete, Concrete sheeting and piles, Steel sheeting and piles, Cribbing and Arch structure materials. Specification references: special precautions and practical recommendations are listed for each material. Typical uses are illustrated by diagrams. This part of the report is not of immediately use or related to the settlement analysis of highway embankments, so the second part is described more specifically. Under types of abutments and methods of design, gravity walls, cribbing, counterforted walls, cantilever walls, sheeting, open frames, hollow boxes, filled boxes, anchored bulkheads and braced bulkheads are described as types, with variations encountered, with outlines of methods of analysis. Special formulas are derived for unusual types and recommendations made for approximate and accurate design solutions.

Experimental Procedures

Design of the Experiment

There is no experiment involved, only methods and recommendations of designing types of abutments according to the several conditions that may be encountered in each practical case.

The abutment or wing wall is considered as a structure of two parts.

1. Foundation, a horizontal placed unit, and
2. Wall, a vertical placed unit.

The wall unit is loaded by:

1. Bridge reaction
2. Weights of fill resting on the back
3. Its own weight
4. Lateral earth pressure

*Corresponds to number 17 in the List of References.

The foundation unit is loaded by:

1. Reaction at the base of the wall
2. Weights of the fill resting on the upper surface outside of the wall limits.
3. Its own weight
4. Lateral earth pressure on the rear.

The subsoil below the foundation unit must also be investigated and design corrections made, if internal stability of the undersoil will not exist at any depth, or along any surface, under the resultant loadings from the foundation unit.

According to conditions of the site, and to design considerations the ahutments can be:

Gravity walls.- are the ones that take its own weight to secure stability. The resultant force is within the middle third at each horizontal section. These walls may have recessed niches in front or in the rear. The latter are respectively known as butressed and counterforted gravity walls. Design investigations for gravity walls must check:

1. Maximum compressive stresses at all sections.
2. Sufficiency of shear resistance at the junction of the wing wall to the face wall.
3. Necessity for tension resistance to prevent rotation cleavage of the wing wall from the face wall, about vertical and horizontal axes.

Cribbing.- Is designed as a gravity section on the assumption that all the earth fill within the limits of the framework acts with the crib units on a monolithic mass. However, less than 50% of the fill is carried by the sills of the cribbing.

If open face cribbing is considered, the type of cribbing used must be heavy granular soil, and have freedom from any effect of water. Consolidation of the fill and the loss through the face will tip the crihbing forward, so that some allowance must be made for such expected loss in batter. Cribbing made up of headers and stretchers only, without a framework in the fill parallel to the face, must be so detailed that the imbedded headers are positively held and supported in position. The danger of distortion from consolidation of the fill must be avoided.

Counterforted walls.- They are used where the volume of necessary masonry is reduced hecause of economical reasons or where minimum load reactions on the foundations are necessary. If the counterforts are spaced close enough, the fill can be considered as an integral part of the abutment. The additional resistance so obtained must be considered in the light of the added form cost, less the volume of concrete saved. For heights below 20', the design of the face wall as a reinforced concrete slab spanning vertically between the footing and a beam at a bridge seat level, which beam frames into counterforts or butresses under the bridge reactions, has been found economical. If butresses are used, the volume of excavation is considerably reduced. For concrete girder spans, the butress can be designed as a beam carrying the bridge reaction as an end thrust and spanning between the footing and the bridge girder.

Cantilever walls.- This may be used as abutments with bridge reactions carried either by individual butresses or by the entire stem as a wall. In the opinion of the writer, in the latter case the buckling resistance of the stem must be investigated as a plate loaded along its lower edge and supported at the two (or more) bridge reactions, as axial loading and also supporting the earth pressure as a transverse loading. The stem of the cantilever wall is designed as a cantilever slab, with the additional axial loading from the bridge reactions and also the horizontal resistances developed at the sole plates. The concrete sections are under combined stress, and may be designed without regard to any live load hridge reactions.

Special attention must be given to the details in the contact zone between bridge girders and abutment seat. The fact that the abutment was designed as a contilever wall does not guarantee the necessary conditions to permit it to act as such. If the bridge seat details prevent the freedom of movement necessary for cantilever action, the stem of the wall will become a slab supported at two ends and will develop cracks with possible failure, unless sufficient reinforcement for such action has been provided.

Sheeting.- The piling carries the bridge reaction and also retain the fill. The amount of added resistance in a cantilever sheet resulting from the vertical loading is not known, and is disregarded.

Any formula derived for stresses of sheeting must be based on assumptions of the shape that the sheeting takes under load.

Hollow boxes.- Or cellular framed abutments are box shaped masonry pockets buried in the road fill. Since the road approach is carried by the box or frame, this type is actually a partially buried viaduct approach span also serving as an abutment. The type should only be considered where the bridge seat is over 30' above the ground level. The box acts as a gravity wall and is designed as such. The sides of the box, usually braced internally by a rectangular system of streets and beams must be designed to transmit exterior earth pressure.

Filled boxes.- They are "U" type gravity abutment and wing wall groups where the using walls are tied together by structural members buried in the fill. The approach road is placed on the fill. Special effort must be expended in placing the fill to avoid overloading the buried ties and to speed up the consolidation. Walls are designed for the lateral pressure of the fill placed inside the box, considering the surcharge from the road.

Anchored Bulkheads.- Are sheetpiling walls which depend upon buried anchors for stability. The Baumann method is a graphical determination of moments and pressures from an assumed shape of the deflecting sheeting. The location of the anchorage must be beyond the possible surface of fracture of the soil behind the sheeting.

Conclusions

Prof. Gregory P. Tschebotarioff discusses the study presented by J. Feld, saying: "Dr. Feld report does not go into the question of the relationship between the movement of a rigid wall and the pressure distribution against it.

18* Feld, Jacob., "Abutments for Small Highway Bridges, Part 3," Vol. 25, Proceedings, Highway Research Board, 1945.

ABSTRACT

Purpose of the Study

The purpose of this report is to continue with the work done by the author in the two previous years, about considerations in design of bridge abutments.

Scope of the Study

This study covers: Part D: Types of foundations; Part E: Bearing and drainage details; Part F: correlation with soil profile studies and investigations; and G: Correlation with previous studies of abutment designs.

Experimental Procedures

Design of the Experiment

Under Part D: Types of foundations, are described the basic limitations of foundation design and the practical applications of recently developed ideas, as well as of the heritage of experience, to the proper structural dimensioning of an abutment foundation. Settlement, rotation and translation of a foundation results from:

1. Elastic compression of the structure.
2. Elastic deflection of the structure.
3. Compression and shear deformation of the soil immediately adjacent to the structure.
4. Consolidation of the soil body.
5. Shear deformation of the soil body.

The types of foundations are:

1. Spread footings
2. Pile footings
3. Column footings
4. Processed fills.

Under Part E: Bearing and drainage details, are described the various methods used for connecting the bridge structure to the abutment with provisions for transfer of vertical and horizontal reactions, including vibration and temperature variations of the reactions. Under Drainage Details are described the necessary precautions in construction and backfill procedures to assure that the acting loads on the abutments will closely approximate the design assumptions. The details are:

*Corresponds to number 18 in the List of References.

1. Abutment seat details
2. Backfill materials and compaction
3. Expansion joints
4. Stabilization of backfill for road hase
5. Drainage methods

Under part F: Correlation in the soil profile studies and investigations, are discussed thc necessary collection of physical and geological data which arc used on the basis for design assumptions.

1. What types of soil and at what depths will they be excavated?
2. Where is the ground water level normally, and how high may it go?
3. What is the nature of the soil on which the foundation will rest and what uniformity do local conditions indicate in the soil layers when cut horizontally?
4. What types of soil underly the foundation level, at least to a depth of 10' below such level?
5. Are there any signs of local slips in the soil body, and is the bed rock bclow the soil fairly level?
6. Is the local material easily compacted as backfill, or can be other material be economically hrought to the site (with the cost of disposing the excavated material taken into considcration)?

Under part G: Correlation with previous studies of abutment designs, are listed the previously published reports on the standardization of abutment designs as well as several references to the standards, usually empirical in nature, set by various authorities.

19* Sloane, R.L., "Settlement Analysis for High Fills on Compressible Foundation Soil," Vol. 95, Roads and Streets, 1952.

ABSTRACT

Statement of the Problem

Among the many problems confronting the highway engineer in the design and construction of the modern superhighway is the problem of estimating the settlement of high approach fills for overpasses or bridges due to consolidation of soft foundation soil upon which the fills may rest.

Purpose of the Study

To present a method easy to understand and easy to use. To estimate settlement on high fills on compressible foundation soil.

Scope of the Study

The study is based on theory, and as illustration, it gives an example to estimate the total settlement. A theoretical comparison in obtaining the total settlement is made using the void ratio method and the compression index method. It gives a brief discussion of the mechanics of consolidation for one dimensional case. The last part of the article deals with discussion of results and also, gives some measures to improve the effect of settlement of fills.

Experimental Procedures

Design of the Experiment

The analysis consists of three distinct parts. The first part consists in determining the soil profile and the properties of the soil encountered in the site. The second part consists of the analysis of the subsurface pressures within the compressible layer or layers induced by the weight of the overlying soil and the weight of the fill. The third part consists of making use of one-dimensional consolidation theory and data from the first two parts to give the final settlement results. The second and third parts of the analysis are presented by considering a typical problem and following a complete analysis through. The following steps are done in the illustrative example:

*Corresponds to number 19 in the List of References.

1. Soil profile with its measures and descriptions.
2. Location of water table.
3. Location of previous material.
4. Laboratory e-logp curves for each strata.
5. Specific gravity of each stratum.
6. Calculation of submerged unit weight (based on e-logp graph and $\gamma_b = \frac{G-1}{1+e}\gamma\omega$)
7. Calculation of soil pressures before surface loading.
8. Determination of pre-loading pressure condition.
9. Calculation of pressures increase due to fill. This is based in Newmark charts.
10. Estimation of total amount of settlement. This is done by two methods: void ratio method, and compression index method.
11. Time settlement predictions.

Analysis of Data

To obtain the submerged unit weight, γ_b, a trial and error method is used. An assumed value of γ_b is taken, with this value, the pressure at any depth is computed; with this pressure, and looking in the e-logp chart, we obtain a value of e; substituting this value of e, into the formula $\gamma_b = \frac{G-1}{1+e}\gamma\omega$ we obtain a value of γ_b that is again involved in the procedure until no significant difference is seen between the assumed value and the one obtained.

In using Newmark chart, the pressure at the desired depth (σ_z) is equal to the product of the unit surface load (q) and a function of the factors "m" and "n". These factors are simply the ratios of the dimensions of the loaded area to the depth (z) below the surface at which pressures are desired. The curves in the chart give the values of the functions of "m" and "n". It should be noted that the pressure is the one at depth z below one corner of the loaded area, therefore to compute pressures below the center of a loaded area it is necessary to consider the area, split up into four rectangular areas. To compute the ultimate settlement the voids ratio method uses the formula

$$\frac{2H_1}{1+e_1}(e_1-e_2)$$ where:

H_1 = is the length of the longest vertical drainage path

e_1, e_2 = void ratios before and after loading.

The compression index method uses the formula:

$$\frac{2H_1}{1+e_1}\frac{p_2-p_1}{\frac{1}{2}(p_2+p_1)}\, 0.435\, Cc,$$ where:

Cc is the numerical value of the slope of the e-logp curve.

H_1 is as before.

e_1 initial void ratio before loading.

p_1,p_2 pressures on the stratum before and after loading; or in the e-logp curves, the pressures that give e_1 and c_2.

Conclusions

The estimation of settlement using the two methods are closely related and the differences arc negligible. The results, presented in a time-settlement chart, gives us the time data to decide:

A - If the settlement will occur during construction period

B - How much additional fill is necessary to continue with the profile grade between road and bridge.

C - If it is necessary to provide sand-drains to accelerate the process of consolidation.

20* Osterberg, J.O.,"Influence Values for Vertical Stresses in a Semi-infinite Mass due to an Embankment Loading," Vol.1, Proceedings, 4th International Conference on Soil Mechanics and Foundation Engineering, London, 1957.

ABSTRACT

Statement of the Problem

Computation of vertical normal stresses in soil masses constitutes an important task in estimating settlements due to surface loadings. In this case, due to embankment loadings.

Purpose of the Study

To present a quick way in obtaining the vertical stresses due to embankment loadings and to use the information in estimating settlement of embankments.

Scope of the Study

Formulas giving stresses for various shapes and load distributions are obtained from integrating the Boussinesq solutions for stresses due to a single concentrated vertical load on a semi-infinite homogeneous isotropic mass. A chart giving "influence values" for vertical stresses for an embankment type loading is presented here, with examples of application by simple superposition. A brief review of literature pertinent to the problem is summarized.

Experimental Procedures

Design of the Experiment

The influence chart is based on the equation for a triangular load of infinite extent developed by Newmark, 1942, as shown in Fig. 1. By superposition, the loading in Fig. 2 is obtained. The vertical stress at the location shown in a function of a/z and b/z, and the unit load q. From this relationship an "influence chart" is constructed which simplified the computation of vertical stress beneath embankment loadings.

Analysis of Data

The stresses given by the chart, are the vertical stress directly under the vertical force of a portion of an embankment of infinite extent. Vertical stress for any point in the foundation can be found by superposition. For stresses under a corner such as under the vertical face of an embankment ending abruptly against a wall, the stresses are one half of those given in the chart.

*Corresponds to number 20 on the List of References.

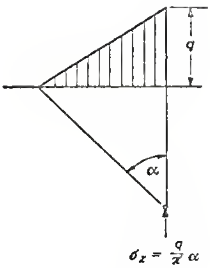


Fig. 1

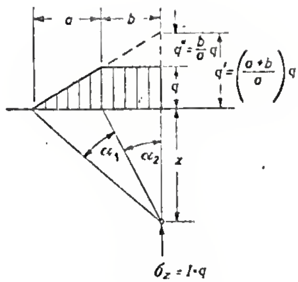
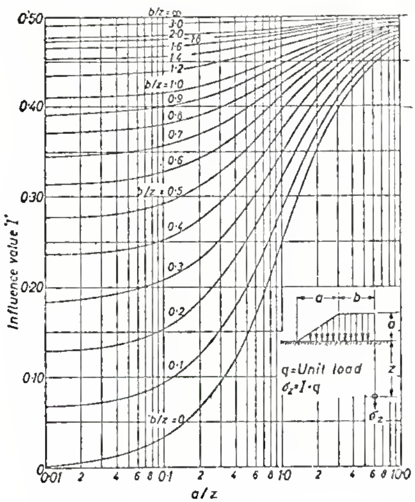


Fig. 2



Influence Chart for Vertical Stress Embankment

Loading - Infinite Extent. Boussinesq Case

21* Webber, Ray.,"Description of a Method of Predicting Fill Settlement Using Voids Ratio," Bulletin 173, Highway Research Board, 1958.

ABSTRACT

Statement of the Problem

The application of some principles of soil mechanics to engineering projects are not fully understood by engineers. In our case, the relation of the theory behind settlement of fills, is not fully reached by highway engineers.

Purpose of the Study

The purpose of this report is to give a simplified explanation of the theory of consolidation and related computations for estimating fill settlement, using the voids ratio method.

Scope of the Study

The following work has been selected from the publications of soil authorities, and is a review of the procedure done in the past about the subject. Also, the author joints his experiences with this procedure, to present this continuity study of predicting settlement of fills.

The paper covers:

1. Estimate of total settlement using voids ratio relation.
2. Estimate of rate of settlement
3. Review of several important factors related to sampling procedures affecting the accuracy of results
4. Testing and reporting
5. Example of computations

Experimental Procedure

Design of the Experiment

The problem is solved in a step-by-step method, with comments on each step, and reviewing the theory of soil mechanics behind each step. The author takes existing formulas and theoretical recommendations of studies done in the past with his own experience and applies them to the subject.

Conclusions

Based on theory of consolidation, the process of settlement occurs when the voids, fill with water, carry the initial stress, and during a time t, this stress is transfered or is supported by

*Corresponds to number 21 in the List of References.

the soil particles due to dissipation of excess pore pressure by the escape of water through the voids of the soil mass. Theoretically the settlement will stop when no water is in the voids of the soil, and because of this basic principle that the soil is saturated (assumption not always correct), we can estimate the stage of settlement at any time of the process of consolidation relating it with the change of void ratio. In our particular problem, the change in void ratio due to the fill load, gives us the clue to estimate the settlement of the fill.

22* Jones,Ch., "Smoother Bridge Approaches," Civil Engineering, American Society of Civil Engineers, 1959.

ABSTRACT

Statement of the Problem

Surface irregularities at ends of bridges, a highly undesirable feature on some highways, reduce driver comfort and safety and adversely affect the durability of roads and structures. Furthermore, where traffic is heavy, surface patching may be hazardous. Continuing settlement of approaches, which require repeated maintenance patching, is particularly undesirable.

Purpose of the Study

The purpose of the study is to obtain solutions and improvements when differential settlement occurs between the road and bridge surface. Also, the paper gives recommendations on where to use open-end vs. close-end abutments.

Scope of the Study

The study covers the investigation of four freeways in the Los Angeles area. The area studied is in part an allovial plain some 30 miles in width between granite mountains on the north and ocean in the south. Roads cutting across the area in various directions were selected for the study.

Experimental Procedures

Design of Experiment

Most of the bridges examined had concrete decks with concrete approaches, and most had concrete aprons about 12' long resting on paving notches. The general requirement for approach fills was 90% relative compaction except for the upper 2', where it was 95%. For the usual highway separation structure, approach fills averaged 20' in height and about 200' in base width. In the case of railway grade-separation overpasses, approach fills were 30' in height.

Procedures and Apparatus

The procedure used was investigating the existing bridges and knowing the subsoil conditions and design type for each case, and giving the solution that could have been made in order to prevent differential settlements.

*Corresponds to number 22 in the List of References.

Conclusions

The investigation revealed that more approach patching was required for closed-end abutment bridges than for open-end structures. In the case of open-end bridges, the approach fills are usually completed before the bridges are built.

During the construction of the bridge, there is time for the underlying ground to become consolidated by the weight of the completed approach fill. Also, grading and road compacting equipment may be employed to secure good compaction of the fill.

In the case of closed-abutment bridges, the approach fills are completed after the completion of the bridge and its abutments. To keep within specified time limits and to get the road open to traffic as quickly as possible, approach pavements are normally laid soon after the approach grading is completed. This allows little time for consolidation of the underlying strata on which the fill rests. Furthermore, good compaction of fill requires special equipment, time, effort, money, careful inspection, and frequently, imported fill. Poor workmanship may not show up until months after the contract is completed. The surface irregularities that develop at the ends of bridges are caused by differential settlement between the bridge and the approach. The approach settlement may be due in part of the settlement of the ground beneath the approach fill and in part to consolidation of the fill itself.

It was found that on San Bernadino freeway, much of which lies in course granular material, there was no approach patching at the end of bridges. On the Santa Ana freeway, with is further removed from the mountains, the material changes from granular to cohesive as one proceeds from Los Angeles to and through Orange County. On this road 20% of the structures examined had approach patching at one or both ends. On the Hollywood freeway, which crosses rolling terrain, there are quite a few clay-filled depressions. On this road 60% of the structures examined had patched approaches. On the freeway, which extends toward the ocean and passes through areas of silt and soft clay, 70% of the structures examined had approach patching. Contrary to expectations, little difference was found in the amount of approach surface patching for structures on piles or compared with those on spread footings. 40% of the structures on piles and 38% of those on spread footings had approach patching. Except in one instance, differential settlement at the end of bridges did not exceed 6".

In the majority of cases a single patch job took care of the situation for a period of years but in some cases frequent patching had been needed to maintain a smooth road.

Among the measures that may prove useful in reducing surface irregularities on bridge approaches are the following, here listed in order of increasing cost:

1. Use an open-end construction rather than a closed abutment type, particularly where the underlying soil is poor.
2. Specifying early construction of approach fills at the ends of the bridge in order to allow as much time as possible for consolidation before the approaches are paved.
3. Use of good backfill material such as sand, gravel or crushed rock, with a high shear value.
4. In the case of asphalt roads, extending the asphalt surface across the bridge deck.
5. Surcharging the approach fills.
6. Making the approach apron slab about 30' in length to eliminate suden grade breaks by bridging depressions near the ends of the bridge.
7. Removing of underlying material near the ends of the bridge and replacing it with good material before fill is constructed.
8. Lengthening the bridge to reduce the height of the approach fills.

23* Mc.Laren,D., "Settlement Behind Bridge Abutments," RRL Report LR76, Road Research Laboratory, London, 1967.

ABSTRACT

Statement of the Problem

A differential settlement occurs frequently between bridges and their approach embankments, causing a serious problem in maintenance and safety.

Purpose of the Study

The investigation was done to know the performance of different materials when used for filling behind bridge abutments.

Scope of the Study

This report describes the investigations in which a medium clay fill was used to compute the construction approach embankments to a bridge on the M1 Motorway. The investigation was carried in one bridge, with close abutments. The walls were of 20' high approx., and the base of 20' wide. The bridge carries two bounds of traffic; 3 lanes or 36' in one way, and 36', and another 3 lanes, in the other way. The span of the bridge is 40' approx. The site of investigation was the southern underbridge of the M1/A46 interchange at Enderby, Leicestershire. This interchange provides Leicester's main southern connection with the M1 Motorway. The construction formed part of contract G, of the M1 extension from Crick to Leeds.

Experimental Procedures

Design of Experiment

The experimental procedure involved observations on the compaction of the clay fill and measurements of the settlement of the subsoil and the road surface. These measurements enable the settlement occuring within the fill, after construction was completed, to be evaluated.

Settlement measurements were made on the subsoil beneath the approach embankments and a few feet clear of the abutment wall, and in the final road surface, to assess the magnitude of any settlement which resulted from movements occuring within the fill material after the road was completed. Observations were made also during the placing of the fill, to determine how closely the state of compaction and moisture content of the

*Corresponds to number 23 in the List of References medium clay conformed to the requirements of the M.O.P. (Ministry of Transport Specifications), for road and hridge work. The measurement devices were placed on three different parts of the embankment to know the different effect of settlements in these parts:

1. Compaction observations.
2. Settlement of Road Surface.
3. Settlement of Subsoil.

Procedures and Apparatus

The observed settlements were recorded with a multi-point mercury filled settlement gauge. The rod type gauge consisted of a 6" diameter borehole augered through the compacted fill, and installed at a depth corresponding to the original ground level. This type of gauge consisted of a solid steel rod 3/4" in diameter which was set into a core of concrete 1 foot thick placed at the bottom of the borehole. The rod was made up of screwed sections of 3' in length and was isolated from the surrounding soil by an outer sleeve of 2-1/2" diameter rigid P.V.C. pipe.

Conclusions

The settlement which have occurred within the fill behind the bridge abutments since the completion of the road pavement have been small: 1/8". The compression of the subsoil, caused by construction of the embankments, was largely completed before the road was surfaced and this has resulted in only a small amount of settlement at the approaches to the bridge. This has had no appreciable effect on the riding quality of the road. These embankments, which were well compacted in accordance with the requirements of the M.D.T. Specifications, have performed as well as a number of bridge approaches investigated on the M-4 Motorway (Maidenhead By-pass) where good quality hoggin and sandy gravel fills were used and found to be satisfactory.

The investigation has shown that a medium clay soil can readily be employed as a satisfactory fill material for constructing approach embankments behind bridge abutments. The likely costs including extra cost of compaction involved when placing common fill behind bridge abutments have been examined, and are shown to be small in relation to the substantial expense which would be incurred by importing special fill materials for use in these areas.

24* Perloff, W.H., Baladi, G.Y., and Harr, M.E., "Stress Distribution Within and Under Long Elastic Embankments," Highway Research Record 181, Highway Research Board, 1967.

ABSTRACT

Statement of the Problem

The problem considered herein is the determination of the distribution of stresses within and under a long elastic embankment continuous with the underlying material, resulting from the self-weight of the embankment.

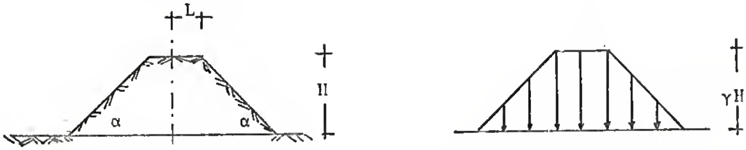
Purpose of the Study

The purpose of the study is to present charts to compare the vertical normal stress σ_y within and under embankments that are continuous with its foundation soil and the results with the vertical stresses resulting from the usual assumption of normal loading approximation, that is, stresses proportional to the embankment height applied normal to the foundation soil, as shown below. Despite numerous attempts to determine the distribution of stresses within and under an embankment, no closed form solution is presently available. It is the objective of this paper to present such a solution.

Scope of the Study

The study is the result of the theoretical investigations and research done by the authors. The research described herein was supported in part by the Ohio Department of Highways in conjunction with the U.S. Bureau of Public Roads, and in part by the Joint Highway Research Project, Purdue University, in conjunction with the U.S. Bureau of Public Roads.

The study gives a short review of the literature about the subject and they are called "approximations of the real problem." The embankment is shown schematically in the figure. It is assumed that the embankment and the foundation material with which it is continuous are composed of homogeneous, isotropic, linear elastic material. Further, the embankment is assumed to be



*Corresponds to number 24 in the List of References.

sufficiently long so that plane strain conditions apply. The shape of the symmetric cross section is defined by the slope angle α, and the ratio of the half width of the top of the embankment L, to the embankment height H.

Experimental Procedures

Design of the Experiment

The solution is obtained by transforming the region of the embankment where the solution is unknown, into a half-spare where the solution can be found. Application of the Cauchy integral formula to the boundary conditions permits determination of the stresses. An outline of the method is given in an appendix, and the results are shown in the body of the study by graphs.

A comparison between the values of vertical stress due to normal loading approximation vs. the values obtained are given in the form of charts and graphs; also, values are obtained for vertical normal stress, σ_y, horizontal normal stress σ_x, horizontal and vertical shear stress T_{xy}, and maximum shear stress Zmax.

Conclusions

The vertical normal stress produced in the foundation material below the elastic embankment are generally smaller than computed for the normal loading approximation.

The stress distribution due to the normal loading approx. is independent of Poisson's ratio μ. The stresses due to the elastic embankment are dependent upon μ. However, the vertical stresses are insensitive to its magnitude; changing μ from 0.3 to 0.5, changes the vertical stress at a point by less than 5%.

The L/H ratio has a pronounced effect on the distribution of vertical stress. As L/H decreases, the stress decreases; a smaller L/H ratio produces a more rapid dissipation of stress with depth.

The stress distribution is much more uniform under the elastic embankment than is ordinarily assumed. The difference becomes especially apparent as the L/H ratio of the steeper embankments decreases. Moreover, the magnitude of the stress under the central zone of the elastic embankment is less than that shown by curves representing normal loading approx. For $\alpha = 45°$ and $L/H = 0$, σ_y equal 65% of that usually assumed. The difference between those curves becomes less pronounced near the central portion of the embankment, as L/H increases. However, near the outer edge of the embankment, the stresses are still significantly larger on a proportional basis than indicated by the normal loading approx.

Thus, for embankments with moderate L/H ratios, the normal load approximation leads to larger estimates of differential settlement, assuming one dimensional compression, than would be computed by the method presented herein.

For horizontal normal stress σ_x, it is shown that the maximum horizontal stress occurs within the body of the embankment and decreases with increasing depth. In the foundation material in the vicinity of the elastic embankment, $\sigma_x/\sigma H$ is less than half of that usually assumed. As the embankment becomes narrower $L/H \leq 1$, the stress is actually negative at some points below the centerline. That is, the embankment causes a reduction in horizontal stress at these points. The stresses due to an elastic embankment are smaller than those due to normal loading approximation. In fact, in the vicinity of the embankment it is more than five times as large under the centerline and twice as large under the toe. The normal loading approx. does not produce negative horizontal stress at any depth.

A change in Poisson's ratio from 0.3 to 0.5 changes the stresses at shallow depths below the central portion of the embankment by as much as a factor of three. The difference decreases as the L/H ratio increases. The influence of μ is less pronounced below the toe than below the centerline. Respect horizontal and vertical shear stress, the study shows that they exist within the body of the embankment, in area increasing to a value in excess of $0.2\sigma H$ at the base near the toe of the slope. However, the maximum value of horizontal shear stress (approx. $0.3\sigma H$) occurs below the base of the embankment.

T_{xy} is affected markedly by the magnitude of Poisson's ratio. However, the effect observed depends upon the position of the point considered, relative to the base of the embankment. In the zone below the embankment to a depth of y/H equal approx. two or three, the shear stresses in the incompressible material ($\mu=0.5$) are less than for the case in which $\mu=0.3$. At greater depths the reverse is true. The shear stress determined from the normal loading approx. is less than that for either μ above a depth factor of approximately 3 to 5, and more at greater depths. The magnitude of this effect depends upon the horizontal location considered.

The normal loading approximation assumes that there is no shear stress at the base of the embankment; in this method this assumption is not reasonable. The horizontal shear stress is zero at the centerline, as required by summetry, and reaches a maximum near the toe of the slope. The magnitude of the maximum and its location depend upon α and the embankment shape. As L/H decreases for a given α, the maximum $T_{xy}/\gamma H$ increases and moves closer to the toe of the slope. The magnitude of the increase is slight for $\alpha=15°$, but becomes more evident as α increases.

The magnitude of the maximum shear stress Tmax, transmitted from the embankment to the foundation material is approximately $0.25\gamma H$ at the base of the embankment in the vicinity of the toe.

However, the largest shear stress, $0.33\gamma H$, occurs beneath the centerline at $y/H = 1.8$. Also, within the embankment, the maximum shear stresses, Tmax, is larger near the top than in the mid depth region, and that they increase again as depth increases.

The stresses Tmax, due to normal loading approx. are less than that produced by elastic embankment in a shallow zone below the embankment, but larger shear stresses at depth.

The study shows the conclusions between the results of the theoretical study and observed "in site stresses". In the case of a built up embankment, it is likely that the embankment material will exhibit significantly different mechanical properties from the foundation material. For a cut-down slope, the assumption of homogenity in the two zones may be more nearly justified. The non-linearity in the mechanical response of most natural materials will undoubtedly also influence the results. However, the feature which may be most significant, at least in the case of build-up embankments, is the fact that they are constructed in layers rather than instantaneously. Thus, when the topmost lift is placed on an earth embankment, the upper material does not undergo strain due to elastic deformation of the embankment resulting from the stress imposed by the entire mass, rather the strains are due only to the increment of stress imposed by this layer. The results indicate that the horizontal distribution of vertical stress is more nearly uniform than is usually assumed. Thus differential settlements compacted using the normal loading approximation will be larger than those determined using the stress distribution presented in this study. The horizontal vertical shear stresses created in the foundation material by the embankment are found to be significantly higher at shallow depths for the elastic embankment than for the normal loading approximation.

SETTLEMENT ANALYSIS OF HIGHWAY EMBANKMENTS

by

ROBERTO A. ESPINDOLA

B. S., Universidad de Guayaquil, 1966

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Civil Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1968

SETTLEMENT ANALYSIS OF HIGHWAY EMBANKMENTS

by

ROBERTO A. ESPINDOLA

ABSTRACT

Settlement of embankments by the general process of consolidation is expected in all highway construction projects, due to weight of the embankment and the compressibility of the foundation soil.

This settlement is not a failure of the embankment but can be of a magnitude that will cause cracking of the roadway surface. This is a serious problem on approach fills to a structure since the structure is founded on an unyielding foundation and differential settlement occurs between the fill and the abutment.

This report analyzes settlement of highway embankments by the general theory of consolidation, based on the work of other authors, and specifically studies the application of the general theory of consolidation to highway embankment approaches to bridge structure. This study is limited to the case of an unyielding structure; that is, a structure that will not settle and one that resists overturning moments.

An appendix contains the abstracts of the pertinent articles published in the past in the area of this study of settlement of highway embankments.

The theory of consolidation is applicable to the problem of predicting the total amount of settlement of an embank-

ment and the time over which it will occur. Application of the theory to the section of the embankment adjacent to a rigid structure, is difficult due to the distribution of stresses applied to the supporting soil since part of the weight of the embankment is carried by the wall.

General design and construction procedures to minimize the effects of differential settlement are given, based on modifications to the general theory of consolidation. Suggestions for further research in this area conclude the report.