HIGH-SPEED HELICAL GEAR DESIGN

by

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SYMBOLS USED

a Thickness of arms, in.

C_p Center distance, in.

d_s Shaft diameter, in.

d_1 Operating pitch diameter of pinion, in.

d_2 Operating pitch diameter of gear, in.

e Error in action, in.

E_p Modulus of elasticity of pinion, psi.

E_g Modulus of elasticity of gear, psi.

F Face width, in.

F_a Acceleration load, lb.

F_1 Force required to deform teeth amount of effective error, lb.

F_2 Average force required to accelerate the masses when they are considered as rigid bodies, lb.

h Thickness of the rim of gear, in.

j Number of arms.

J Geometry factor.

K Load-stress factor.

K_f Surface condition factor.

K_h Hardness ratio factor.

K_ℓ Life factor.

K_m Load distribution factor.

K_o Overload factor.

K_p Elastic coefficient.

K_r Factor of safety.

K_s Size factor.

K_t Temperature factor.

m Effective mass, slugs.

m_1 Effective mass of pinion, slugs.

m_2 Effective mass of gear, slugs.

n_p Speed of pinion, rpm.

n_g Speed of gear, rpm.

N Number of teeth.

N_p Number of teeth on pinion.

N_g Number of teeth on gear.

p Circular pitch, in.

p_a Axial pitch, in.

p_n Normal pitch, in.

P_{at} Allowable power of a gear set based on tensile strength, hp.

P_{ac} Allowable power of a gear set based on surface durability, hp.

P_d Diametral pitch.

P_n Normal diametral pitch.

Q Ratio factor.

R_1 Pitch radius of pinion, in.

R_2 Pitch radius of gear, in.

S_{ac} Allowable contact stress, psi.

S_{at} Allowable tensile stress, psi.

S_s Shear stress, psi.

S_c Calculated contact stress, psi.

S_t Calculated tensile stress, psi.

T	Torque, in. lb.
T_f	Peak operating temperature, degree F.
V_t	Pitch line velocity, fpm.
W_d	Dynamic load, lb.
W_t	Transmitted tangential load at operating pitch diameter, lb.
ϕ	Pressure angle, degree.
ϕ_n	Normal pressure angle, degree.
ψ	Helix angle, degree.
ρ	Density, lb./cu.in.
μ_p	Poisson's ratio for pinion.
μ_g	Poisson's ratio for gear.

REVIEW OF THE LITERATURE

In 1939 Maleev (3) presented a helical gear design. He recommended some formulae to find the limiting load for wear and also the dynamic load. He derived a bending strength formula by considering the gear tooth as a cantilever beam.

In 1946 Buckingham (1) suggested a method to calculate the dynamic load by considering the weight of the gear. He found the effective mass of gears by using the polar moment of inertia and then dividing by the pitch radius squared. This work is the most authoritative study of dynamic loads that is available to the gear designer.

In 1954 Dudley (2) carried out a set of experiments and suggested some practical methods to design helical gears. He introduced high-speed helical gears. Dudley also solved a few numerical problems on high-speed helical gears.

In 1956 Shigley (5) designed a helical gear by considering the dynamic load as a series effect. He also introduced the helical gears which are used for high speed. He suggested a few standard tooth proportions for helical gears.

In 1961 Wellauer (10) presented three formulae to find the bending stress and power capacity of a helical gear set based on strength. He described the relation between calculated and allowable bending stresses. He considered all permissible factors which affect the strength of the gear teeth, and suggested various values for different types of factors.

In 1962 Tuplin (6) presented the selection of materials and the determination of manufacturing dimensions of gears for some specified purposes. The methods described by him are applicable to gears of any class of service and as far as it is consistent with that condition, they have been kept as simple as possible. Tuplin (7) stated that the maximum gear tooth loading is considerably higher than that corresponding to transmitted power because of the dynamic effect of unavoidable imperfections in the gear.

In 1964 Wellauer (11) showed how to predict surface durability of helical gears and presented some formulae to find surface contact stress and the relation between calculated and allowable contact stresses. He found the power capacity based on surface durability for helical gears.

In 1965 Wellauer (8) suggested a more fundamental and more accurate theoretical analysis of the stress system. This particularly applies to the strength of a gear tooth. He presented a better basic understanding and evaluation of the dynamic effects caused by speed.

In 1965 Wellauer (9) developed that the strength and profile durability (pitting) life of gear teeth is a fatigue phenomenon measured by stress level, contacting cycle, and mortality rates. The life of a gear power transmission depends upon mutually dependent survival of the principal elements--gears, shafts, and bearings.

INTRODUCTION

Most engineers prefer to use spur gears when power is to be transferred between parallel shafts because they are easier to design and manufacture. However, sometimes the design requirements are such that helical gears are a better choice. This may be true when the loads are heavy and speeds are high, or the noise level must be kept low. Although spur gears are ordinarily used in slow-speed applications, and helical gears are used in high-speed applications, spur gears may be used at very high-pitch line velocities if the noise requirements permit.

In the spur gears the line of contact is parallel to the axis of rotation; in helical gears the line of contact is diagonally across the face of the tooth. It is this gradual engagement of the tooth and smooth transfer of load from one tooth to another that gives helical gears the ability to transmit heavy loads at high speeds with low noise.

Gear Teeth in Action

The load-carrying capacity of any gear drive may be limited by any one or more of the following factors.

1. Excessive heat of operation
2. Breaking of the gear teeth
3. Excessive wear of the gear tooth surfaces.

In addition to these, excessive noise in operation may make a gear drive unsuitable for use even though none of the three foregoing factors is involved. To be satisfactory, gears must transmit power smoothly, with a minimum of vibration and noise, and must have reasonable length of useful life. Noise is relative rather than absolute and may be defined as unpleasant or objectionable sound (1).

Inadequate lubrication may also be a source of excessive heat, noise, and wear. At high speeds the major purpose of the oil is to act as coolant and carry away the frictional heat of operation. Here the lubrication may be a secondary factor. Conversely, too much oil at the mesh point of the gears may be squeezed from between the teeth.

It cannot be doubted that slow speeds will allow higher working stresses than high speeds, but it may be questioned whether a tooth with a velocity of 100 feet a minute is twice as strong as a tooth at 600 feet a minute, or four times as strong as the same tooth at 1800 feet a minute (1).

With the introduction of higher speed prime movers, such as steam turbines and electric motors, centrifugal pumps, fans, etc., it has been found by experience that after a pitch line velocity of the order of 5,000 feet a minute is reached, the load-carrying ability is practically constant for any higher speed (1).

When the helical gears are made accurate enough to operate satisfactorily at 5,000 feet a minute pitch line velocity, there seems to be little difference, except at critical speeds, in either quietness or load-carrying ability between that speed and a speed of 10,000 feet a minute (1).

Dynamic Loads and Influence of Fine Pitch and High Speed

It is apparent that for all gear drives, regardless of the extent of the actual error in action, there will be a speed at which the dynamic load is independent of the actual error in action. It may be that the dynamic load will reach a maximum value at some speed and then reduce with a further increase in speed. In such cases the gears must be strong enough to carry the loads through this maximum value without failure, in order to be able to operate at the higher speeds (1).

If the gears are to run at the higher speeds, and have a reasonable length of useful life, then design stresses should be higher than the endurance limit of the materials (1).

DESIGN

Selection of Gear Materials

Gears are commonly made of steel, cast iron, bronze, or phenolic resins. In many applications, steel is the only satisfactory material because it combines both high strength and low cost. Sometimes the gears are made of both plain carbon and alloy steels. In many cases the choice will depend upon relative success of the heat-treating department with the various steels (2).

Cast iron is a very popular gear material. It is easy to cast and machine, has good wearing characteristics, and transmits less noise than the steel. The cast-iron classes and the physical properties of steel shown in Table 1 and Table 2 are recommended by the American Gear Manufacturers Association (AGMA) (2).

Bronze may be used for gears when corrosion is a problem. It is also very useful where the sliding velocity is high.

Phenolic resin is widely used under the name of bakelite, or laminated sheets obtained by compressing layers of paper or canvas impregnated with phenolic resin. Also it is used to manufacture silent gears and bearing shells (3).

Number of Teeth

High load and high-speed spur gear pinions are manufactured with from 18 to 70 teeth. High-speed helical pinions have from 35 to 70 teeth. Naturally, the number of teeth on a pinion or gear cannot be a fraction (2).

Tooth-to-tooth Relationship

The relation between the normal circular pitch and the circular pitch (5) (Fig. 1) is given by

$$p_n = p \cos \psi \qquad (1)$$

$$p_a = \frac{p}{\tan \psi} \ . \qquad (2)$$

The relation between the normal diametral pitch P_n and the diametral pitch P_d is given by

Table 1. Properties of cast irons.

Class	Minimum tensile stress, psi	Minimum hardness on tooth portion, b.h.n.
20	20,000	-
30	30,000	175
35	35,000	185
40	40,000	200
50	50,000	215
60	60,000	220

Table 2. Physical properties of steel.

Materials	Specific weight, lb/cu in	Allowable tensile stress, psi	Modulus of elasticity, psi 10^6	Endurance limit, psi	B.h.n.
SAE 1010	.282	15,500	30.3	24,000	110
SAE 1020	.282	17,500	30.2	26,000	125
SAE 1030	.282	42,000	30.0	32,000	150
SAE 1040	.282	25,000	29.8	37,000	180
SAE 1050	.282	26,000	29.7	42,000	190
SAE 1095	.282	40,000	29.7	65,000	300
SAE 1120	.282	22,500	30.2	26,000	125

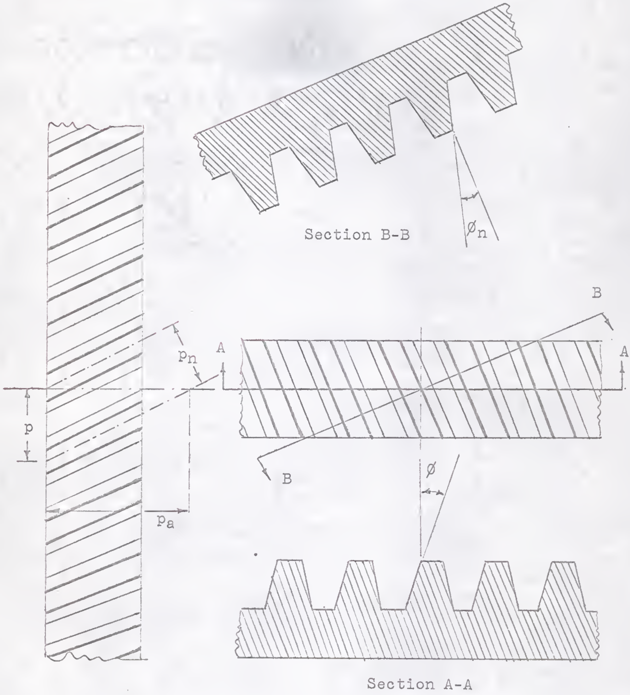

Fig. 1. Circular pitch relations and dimensioning of pressure angle $\emptyset$ and normal pressure angle $\emptyset_n$.

$$P_n = \frac{P_d}{\cos \psi} . \qquad (3)$$

Generally the normal pressure angle ϕ_n for helical gear is 20 degrees and the relation between ϕ_n and pressure angle ϕ is given by

$$\text{Cos } \psi = \frac{\tan \phi_n}{\tan \phi} . \qquad (4)$$

Helix Angle (ψ)

According to definition of the AGMA, the helix angle ψ is the angle between a tangent to a helix and an element of the cylinder along the pitch line. Helix angles of 15 to 23 degrees are more commonly used because they give low axial thrust load. The axial thrust increases with an increase of the helix angle (ψ) (3).

Face Width (F)

The face width of helical gears should be at least two times the axial pitch. For very high speeds the face width may be four or more times the axial pitch, but the accuracy of the teeth must be increased correspondingly (5).

Normal Diametral Pitch (P_n)

Normal diametral pitch of a helical gear presents the number of teeth per inch of normal pitch diameter.

The size of normal diametral pitch for which a cutter may be obtained (3) is from one to four, by increments of one-fourth; from four to six, by one-half; from six to 16, by one; and from 16 to 32, by two.

Bending Strength

The basic equation for the bending stress (11) in a gear tooth is

$$S_t = \frac{W_t\,K_o}{K_v}\;\frac{P_d}{F}\;\frac{K_s\,K_m}{J}\;. \qquad (5)$$

The overload factor K_o can be found from Table 3.

Table 3. Overload factor K_o.

Character of power source	Character of load on driven machine		
	Uniform	Moderate shock	Heavy shock
Uniform Electric motor turbine	1.00	1.25	2.00
Light shock Multicylinder I.C. engine	1.25	1.50	2.25
Medium shock Single-cylinder I.C. engine	1.50	1.75	2.50

The dynamic factor K_v is

$$K_v = \sqrt{\frac{78}{78 + \sqrt{V_t}}} \tag{6}$$

The face width F is

$$F \geq 2\ p_a \text{ to } 4\ p_a\ . \tag{7}$$

The size factor K_s can be found from Table 4.

Table 4. Size factor K_s.

Normal diametral pitch P_n	Size factor K_s
2.00	1.22
4.00	1.15
6.00	1.10
8.00	1.05
10.00	1.00
12.00	0.96
14.00	0.93
16.00	0.92
18.00	0.91

The load distribution factor K_m can be found from Table 5.

Table 5. Load distribution factor K_m.

Face width F, in.	Load distribution factor K_m	
	Precision gear	Less accurate gear
2.00	1.20	1.50
4.00	1.28	1.57
6.00	1.34	1.64
8.00	1.42	1.72
10.00	1.48	1.78
12.00	1.56	1.86
14.00	1.63	1.93
16.00 and over 16.00	1.70	2.00

The geometry factor J can be found from Table 6.

Table 6. Geometry factor J.

Helix angle ψ, degree	Geometry factor J
5.00	0.49
10.00	0.53
15.00	0.54
20.00	0.52
25.00	0.52
30.00	0.50
35.00	0.46

The relation between calculated and allowable stress is

$$S_t \leq \frac{S_{at}\ K_\ell}{K_t\ K_r} \quad . \tag{8}$$

The life factor K_ℓ can be found from Table 7.

Table 7. Life factor K_ℓ.

Required tooth contact cycle	Minimum hardness B.h.n.			
	450	350	250	160
10 to 1000	3.40	3.00	2.40	1.60
10^4	2.40	2.20	1.80	1.40
10^5	1.70	1.60	1.50	1.20
10^6	1.30	1.25	1.20	1.10
10^7 and over	1.00	1.00	1.00	1.00

The temperature factor K_t is

$$K_t = \frac{460 + T_f}{620} \quad . \tag{9}$$

The factor of safety K_r covers the unknowns in the various rating factors, the spread or scatter of material properties, and the concept of statistical reliability now widely used. The factor of safety can be found from Table 8.

Table 8. Factor of safety K_r.

Requirement of application	Factor of safety K_r
Highest reliability Probability of failure practically nil	2.00 or over
Commercial reliability	1.20
Failure frequency (per cent)	
1	1.00
20	0.80
30	0.70

Power Capacity (Based on Strength)

The power capacity of a gear set based on strength (10) can be calculated as

$$P_{at} = \frac{n_p\ d_1\ K_v}{126{,}000\ K_o}\ \frac{F}{K_m}\ \frac{J}{K_s\ P_d}\ \frac{S_{at}\ K_\ell}{K_r\ K_t} \quad . \tag{10}$$

Surface Durability

The load-carrying capacity of a helical gear is normally limited by the pitting resistance. This is commonly called the "durability capacity". Pitting is defined as surface fatigue failure of the material caused by repeated surface or sub-surface stresses that exceed the endurance limit of the material (11).

The recently developed AGMA gear-rating standard AGMA 212.02 provides the fundamental formula for determining the surface durability especially for helical and herringbone gears.

Since the calculated stress and allowable stress have a linear relation, the exact stress and location need not be determined.

The fundamental surface durability formula for gear teeth is

$$S_c = K_p \sqrt{\frac{W_t\ K_o}{K_v}\ \frac{K_s}{d_1 F}\ \frac{K_m\ K_f}{J}} \tag{11}$$

The elastic coefficient K_p is

$$K_p = \sqrt{\frac{1}{\dfrac{1-\mu_p{}^2}{E_p} + \dfrac{1-\mu_g{}^2}{E_g}}}\ . \tag{12}$$

The surface condition factor K_f depends on profile surface, finish residual stress, and plasticity effects (work hardening). Excessively deep hob scallops or shaper grooves, as well as surface heat-treating checks caused by improper flame or induction surface hardening, might singly or in combination require a K_f factor from 1.25 up to as high as 1.40. Gears that are generated by shaving or manufactured with reasonable care can be rated with a $K_f = 1.00$.

The relation between calculated and allowable contact stresses (11) is

$$S_c \leq S_{ac} \frac{K \; K_h}{K_t \; K_r} \quad . \tag{13}$$

The hardness ratio factor K_h depends on the ratio of B.h.n. of gear to B.h.n. of pinion, and it also depends on reduction gear ratio. It is obvious that two meshing gears, particularly with a high ratio, have a higher capacity when a hardness differential exists between gear and pinion. Even at low ratios a differential in hardness should exist between the gear and pinion to minimize the possibility of scuffing or scoring. The hardness ratio factor K_h can be found from Table 9.

Table 9. Hardness ratio factor K_h.

Single reduction gear ratio	Ratio: B.h.n. of gear to B.h.n. of pinion					
	1.20	1.30	1.40	1.50	1.60	1.70
2.00	1.003	1.004	1.005	1.006	1.007	1.008
4.00	1.008	1.010	1.013	1.016	1.018	1.020
6.00	1.012	1.018	1.022	1.025	1.030	1.034
8.00	1.017	1.024	1.030	1.036	1.042	1.048
10.00	1.021	1.030	1.040	1.047	1.054	1.081
12.00	1.026	1.037	1.048	1.058	1.067	1.075
14.00	1.030	1.044	1.057	1.069	1.079	1.090
16.00	1.035	1.051	1.065	1.079	1.091	1.130

Power Capacity (Based on Surface Durability)

The power capacity of a gear set based on surface durability (11) is

$$P_{ac} = \frac{n_p \; F}{126{,}000} \; \frac{J \; K_v}{K_s \; K_m \; K_f \; K_o} \left[\frac{S_{ac} \; d_1}{K_p} \; \frac{K_\ell \; K_h}{K_t \; K_r} \right]^2 . \qquad (14)$$

Dynamic Load (W_d)

There is an approximation to the Buckingham equation which may be used for rapid computations (5). This is

$$W_d = W_t + \frac{.05 \; F_1 \; V_t \; \cos \; \psi}{.05 \; V_t + \sqrt{F_1}} \qquad (15)$$

where

$$F_1 = F \; C \; \cos^2 \psi \; + W_t \qquad (16)$$

$$C = \frac{e}{c_1 \left[\frac{1}{E_p} + \frac{1}{E_g} \right]} . \qquad (17)$$

c_1 = 9.345 for 14 1/2 degree gears (normal pressure angle) and 9.000 for 20.0 degree gears (normal pressure angle).

Error in action, e, in., can be found from Table 10.

Table 10. Error in action, e, in.

Normal diametral pitch, P_n	Class 1	Class 2	Class 3
1.00	0.0048	0.0024	0.0012
2.00	0.0040	0.0020	0.0010
3.00	0.0032	0.0016	0.0008
4.00	0.0026	0.0016	0.0007
5.00	0.0022	0.0011	0.0006
6.00 and over	0.0020	0.0010	0.0005

There are a few changes in the fundamental Buckingham equation (15) for the dynamic load when applied to helical gears (1).

The changed equation is as follows.

$$W_d = W_t + \sqrt{F_a(2F_1 - F_a)} \qquad (18)$$

where

$$F_a = \frac{F_1\ F_2}{F_1 + F_2} \qquad (19)$$

$$F_2 = H\ m \cdot V_t^2 \cos^2 \psi \qquad (20)$$

and

$$H = c_2 \left[\frac{1}{R_1} + \frac{1}{R_2}\right]. \qquad (21)$$

$c_2 = 0.0012$ for 20.0 degree normal pressure angle gears, and 0.00086 for 14 1/2 degree normal pressure angle gears,

$$m = \frac{m_1\ m_2}{m_1 + m_2}. \qquad (22)$$

Limiting Load for Wear (W_w)

In helical gear design the wear load is almost always the ruling design factor, and hence usually determines the size of the gear. Gears in continuous service lose their usefulness because of a sudden failure (3). Wear occurs in five ways.

1. By pitting of the tooth surface by repeated compressive stresses.
2. By abrasion caused by foreign matter.
3. By scoring caused by sharp and projecting edges and rough surfaces.
4. By scuffing, which results from the use of an improper lubricant.

5. By seizing due to a complete failure of the lubrication accompanied by locally generated heat sufficient to weld the surfaces to each other (6).

The limiting load for wear W_w is the load beyond which wear is likely to be rapid. It is given by the expression

$$W_w = \frac{2\ R_1\ F\ Q\ K}{\cos^2\ \psi}\ . \tag{23}$$

The ratio factor Q is given by

$$Q = \frac{2\ R_2}{R_1 + R_2} \tag{24}$$

and the load stress factor K is given by

$$K = \frac{S_{ac}{}^2\ \sin\phi \left[\frac{1}{E_p} + \frac{1}{E_g}\right]}{1.4}\ . \tag{25}$$

Gear Blank Design

Gear blanks are produced by casting, forging, machining, and fabricating. When the pinion is small, it is frequently made integral with the shaft, thus eliminating the key as well as an axial-loading device (5).

In designing a gear blank, rigidity is almost always a prime consideration. The hub must be thick enough to maintain proper fit with the shaft and to provide sufficient metal for the key slot. This thickness must also be large enough so that the torque may be transmitted through the hub to the web or spokes without serious stress concentration. The hub must have length in order that the gear will rotate in a single plane without wobble. The arms or web and the rim must also have rigidity without excessive weight because of inertia effects (3).

The length of the hub should be at least equal to the face width, or greater if this does not give sufficient key length. The spokes may be designed with elliptical cross section, an H- or I-section, or any shape depending upon the stiffness and strength desired (6).

The hub length (shown in Fig. 2) is given by

$$L \geqq F \text{ to } 1.2\ F \tag{26}$$

and the hub diameter D is given by

$$D = 2\ d_s . \tag{27}$$

The Westinghouse-Nuttall formula for the rim thickness is given by

$$h = \frac{1}{P_n} \sqrt[3]{\frac{N}{2j}} \ . \tag{28}$$

PROBLEM

A turbine is geared to a 600-kw generator. The turbine and generator are running at 10,000 rpm and 1,200 rpm respectively. Center distance (the distance between the center of pinion and that of gear) should be about 17.00 inches. Assume the efficiency of the generator equal to 93.5 per cent.

SOLUTION

Selection of Materials

For protection against too much pinion wear, a medium-hard pinion will be matched with a low-hardness gear. This will have the advantage of giving some increase in load capacity and slightly lower coefficient of friction on the teeth. A pinion of 300 B.h.n. and gear of 200 B.h.n. will be considered. Corresponding materials are:

Pinion: Steel SAE 1095, allowable tensile stress 65,000 psi, B.h.n. 300, $E_p = 29.7\ 10^6$ psi.

Gear: Cast iron class 40, minimum tensile stress 40,000 psi, B.h.n. 200, $E_g = 16.0\ 10^6$ psi.

Center distance (Fig. 3)

$$C_p = R_1 + R_2 = \frac{d_1 + d_2}{2}$$

so that
$$\frac{C_p}{d_1} = \frac{(1 - d_2/d_1)}{2}$$

or
$$d_1 = \frac{2C_p}{(1 - d_2/d_1)} \quad \text{and} \quad \frac{d_2}{d_1} = \frac{n_p}{n_g}\ .$$

Let us consider $C_p = 17.00$ inches so that

$$d_1 = \frac{2(17.0)}{(1 + 10{,}000/1{,}200)} = 3.64 \text{ inches.}$$

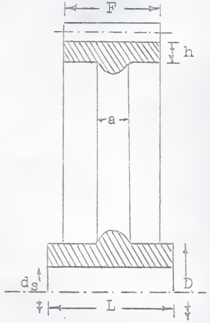

Fig. 2. Gear rim dimensions.

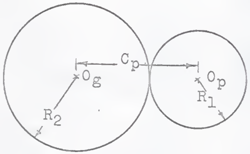

Fig. 3. Meaning of center distance.

Number of Teeth

About 35 to 40 pinion teeth should be desirable for quiet running and good wear resistance. Consider 37 teeth on the pinion, i.e., $N_p = 37.0$, so that

$$N_g = \frac{37(10,000)}{1,200} = 308.5.$$

To keep turbine speed on the high side, a ratio of 37 to 309 will be used.

The normal diametral pitch $P_n = 37/3.64 = 10.16$. Assume $P_n = 10.00$, so that modified pinion diameter $d_1 = 3.70$ inches and gear diameter $d_2 = 30.9$ inches, which gives center distance

$$C_p = \frac{(3.70 + 30.90)}{2} = 17.30 \text{ inches,}$$

which is permissible, and the circular pitch

$$p = \frac{\pi D}{N} = \frac{3.14(3.70)}{37.0} = 0.314 \text{ inch.}$$

Assume 20-degree normal pressure angle for pinion and gear, and 23-degree helix angle for both. Now the normal circular pitch

$$p_n = p \cos \psi = 0.314 \cos 23^\circ = 0.2894 \text{ inch,}$$

and the axial pitch

$$p_a = \frac{p}{\tan \psi} = \frac{0.314}{\tan 23^\circ} = 0.740 \text{ inch.}$$

The face width $F \geqq 4\, p_a = 4(0.740) = 2.960$ inches. Consider $F = 3.00$ inches.

Transmitted Tangential Load W_t

Efficiency of the generator is 93.5 per cent, so that the transmitted horsepower of a gear set should be greater than 600.0 hp. The horsepower that the gear must transmit =

$$\frac{600.0}{0.746(0.935)} = 860.0 \text{ hp. Since the hp} = \frac{W_t\ V_t}{33{,}000}, \text{ and}$$

$$V_t = \frac{2.0\ \pi\ d_1\ n_p}{2(12)} \text{ fpm} = \frac{2(3.14)(3.70)(10{,}000)}{2(12)} \text{ fpm} = 9707.9 \text{ fpm,}$$

so
$$W_t = \frac{860(33{,}000)}{9{,}707.9} = 2939.2 \text{ lbs.}$$

The diametral pitch $P_d = P_n \cos \psi = 10.0 \cos 23^\circ = 9.2050$.

The dynamic factor

$$K_v = \sqrt{\frac{78.0}{78.0 + \sqrt{V_t}}} = \sqrt{\frac{78.0}{78.0 + \sqrt{9{,}707.9}}} = 0.664.$$

Bending Stress

The bending stress formula (5) is

$$S_t = \frac{W_t\ K_o}{K_v}\ \frac{P_d}{F}\ \frac{K_s\ K_m}{J}, \text{ and } S_t \leq \frac{S_{at}\ K_\ell}{K_t\ K_r}.$$

$$S_t = \frac{2939.2(1.25)}{0.664}\ \frac{(9.205)}{3.0}\ \frac{(1.0)(1.24)}{0.52}$$

$$= 40.300 \text{ psi.}$$

Now
$$S_t \leq \frac{S_{at}\ K_\ell}{K_t\ K_r}.$$

Various factors are as follows:

Factors	Source	Value	Remarks
Overload factor Ko	Table 3	1.250	Uniform power source and moderate shock on driven machine
Dynamic factor K_v	Formula 6	0.664	--
Diametral pitch P_d	Formula 3	9.205	$P_n = 10.0$, $\Psi = 23^o$
Face width F, in.	Assumed	3.000	$F = 4\ p_a$
Size factor K_s	Table 4	1.000	$P_n = 10.0$
Load distribution factor K_m	Table 5	1.250	Precision gear and F = 3.00 inches
Geometry factor J	Table 6	0.520	Helix angle = 23^o
Factor of safety K_r	Table 8	1.200	Commercial reliability
Hardness ratio factor K_h	Table 9	1.040	B.h.n. ratio = 1.50
Life factor K_ℓ	Table 7	1.000	B.h.n. = 300.0. Assume contact cycle 10^7
Surface condition factor K_f	Assumed	1.300	--
Modulus of elasticity E_p, psi	Table 2	29.7×10^6	Steel SAE 1095
Modulus of elasticity E_g, psi	Assumed	16.0×10^6	Cast iron class 40
Poisson's ratio	Assumed	0.300	--
Allowable contact stress S_{ac}, psi	Table 2	65,000	Steel SAE 1095
Allowable tensile stress S_{at}, psi	Table 2	40,000	Steel SAE 1095

Assuming the peak operating temperature equal to 80.0 degrees F., the temperature factor

$$K_t = \frac{460.0 + 80.0}{620.0} = 0.871.$$

Hence $$S_t = \frac{40{,}000(1.00)}{0.871(1.20)} = 38{,}300 \text{ psi.}$$

But the calculated tensile stress is S_t = 40,300 psi, which is greater than the allowable stress. Calculated stress can be reduced by increasing the face width. Modifying the face width F to 4.00 inches

$$S_t = (3/4)40{,}300 = 30{,}300 \text{ psi}$$

which is considerably less than allowable stress.

Power Capacity (Based on Strength)

$$P_{at} = \frac{n_p\ d_1\ K_v}{126{,}000\ K_o}\ \frac{F}{K_m}\ \frac{J}{K_s\ P_d}\ \frac{S_{at}\ K}{K_r\ K_t}$$

$$= \frac{(10{,}000)(3.7)(0.664)}{(126{,}000)(1.25)}\ \frac{(4.0)}{1.24}\ \frac{(0.52)}{(1.00)(9.2050)}\ \frac{40{,}000(1.00)}{(0.871)(1.20)}$$

so that P_{at} = 650.25 hp, which is not permissible.

Power capacity of the gear set should not be less than 860.0 hp, so face width F still needs to be increased.

Redesigning the face width to 8.00 inches, the new P_{at} = 650.25(8/4) = 1300.5 hp, which is quite safe.

Surface Durability

$$S_c = K_p \sqrt{\frac{W_t\ K_o}{K_v}\ \frac{K_s}{d_1\ F}\ \frac{K_m\ K_f}{J}}$$

where $$K_p = \sqrt{\frac{1.0}{3.14\left[\frac{1 - \mu_p{}^2}{E_p} + \frac{1 - \mu_g{}^2}{E_g}\right]}}$$

$$= \sqrt{\frac{1.0}{3.14\left[\frac{1 - 0.3^2}{29.7(10^6)} + \frac{1 - 0.3^2}{16.0(10^6)}\right]}}$$

$$= 1813.0$$

so $$S_c = 1813.0\sqrt{\frac{2939.2(1.25)}{(0.664)}\ \frac{(1.00)}{3.7(8.0)}\ \frac{(1.24)(1.30)}{(0.52)}}$$

$$= 43{,}500 \text{ psi.}$$

The allowable contact stress

$$= S_{ac}\ \frac{K_\ell\ K_h}{K_t\ K_r} = \frac{65{,}000(1.0)(1.04)}{(1.20)(0.871)} = 64{,}600 \text{ psi}$$

which is greater than the calculated contact stress; hence it is safe.

Power Capacity (Based on Surface Durability)

$$P_{ac} = \frac{n_p\ F}{126{,}000}\ \frac{J\ K_v}{K_s\ K_m\ K_f\ K_o}\left[\frac{S_{ac}\ d_1}{K_p}\ \frac{K_\ell\ K_h}{K_t\ K_r}\right]^2$$

$$= \frac{(10{,}000)(8.0)}{126{,}000} \quad \frac{(0.664)(0.52)}{(1.0)(1.24)(1.3)(1.25)} \left[\frac{65{,}000(3.70)}{(1813.0)} \quad \frac{(1.00)(1.04)}{(1.24)(.871)}\right]^2$$

$$= 1960.0 \text{ hp}$$

which is quite safe.

Dynamic Load W_d

$$W_d = W_t + \frac{0.05\ F_1\ V_t\ \cos\ \psi}{0.05\ V_t + \sqrt{F_1}}$$

where $F_1 = F\ C\ \cos^2\ \psi + W_t$

and
$$C = \frac{e}{c_1\ [1/E_p + 1/E_g]}$$

$$= \frac{(0.002)}{9.00\left[\frac{1}{29.7(10^6)} + \frac{1}{16.0(10^6)}\right]}$$

$$= 2310.0$$

$$F_1 = 8.0(2310.0)(\cos^2 23^\circ) + 2939.2 \text{ lbs}$$

$$= 15{,}600 + 2939.2 \text{ lbs}$$

$$= 18{,}539.2 \text{ lbs.}$$

So

$$W_d = 2939.2 + \frac{(0.05)(18539.2)(9707.9)(\cos 23^\circ)}{(0.05)(9707.9) + \sqrt{18539.2}}$$

$$= 16{,}339.2 \text{ lbs.}$$

Using equation (18),

$$W_d = W_t + \sqrt{F_a(2\ F_1 - F_a)}$$

$$F_1 = \frac{F_1\ F_2}{F_1 + F_2}$$

$$F_2 = H\ m\ V_t^2\ \cos^2\Psi$$

$$H = c_2\left[\frac{1}{R_1} + \frac{1}{R_2}\right] = 0.0012\left[\frac{2}{(3.70)} + \frac{2}{(30.9)}\right]$$

$$= 0.000727\ .$$

To find effective mass m, all dimensions of the pinion and gear must be known.

<u>Pinion</u> <u>and</u> <u>Gear</u> <u>Design</u>. Consider the shaft of the pinion and the gear in torsion. Mild steel can be used for the shaft.

$$T = W_t(d/2) = 2939.2(3.70/2) = 5430.0 \text{ in lb}$$

$$= \frac{S_s\ I_p}{d_s} = \frac{S_s\ \pi\ d_s^3}{16}$$

so $$d_s^3 = \frac{16\ T}{\pi\ S_s}\ .$$

For mild steel

S_s = working shear stress = 6,000.0 psi, so that

$$d_s^3 = \frac{(16)(5,430.0)}{(3.14)(6,000)} = 4.60$$

which gives d_s = 1.663 inches.

For key allowance, consider d_s = 1.70 inches.

The diameter of hub D = 2.0 d_s = 2.0(1.70) = 3.40 inches, and the length of hub L = 1.2 F = 1.2(8.00) = 10.00 inches.

The rim thickness of gear can be calculated by formula (28).

$$h = \frac{1}{P_n}\sqrt[3]{\frac{N}{2\ j}} = \frac{1}{10.00}\sqrt[3]{\frac{309.0}{(2.0)(6.0)}} = 0.295 \text{ inch.}$$

For casting allowance, consider $h = 0.05$ inch.

$t = (\text{face width})/4 = 8.0/4 = 0.05$ inch.

Assuming the rectangular cross section of the arm with $a = 2t$, check:

Bending moment on each arm,

$$\text{B.M.} = \frac{(2939.2)(3.70)}{2(6)} = 905.0 \text{ in.lb.}$$

Bending stress = 905.0/5.33 = 170.0 psi, which is permissible.

Now effective mass m can be calculated.

Pinion (Fig. 4).

$$\rho = \text{density of pinion}$$
$$= 0.282 \text{ lb/cu.in. for steel SAE 1095.}$$

$$\text{Mass of pinion} = \frac{(\text{density})(\text{volume})}{g}$$
$$= \frac{(0.282)(3.14)(3.7)(3.7)(8.0)}{4(32.2)}$$
$$= 0.745 \text{ slug.}$$

Effective mass of pinion

$$m_1 = \text{mass moment of inertia}/R_1^2$$
$$= \frac{(\text{mass of pinion})\ d_1^2/8}{d_1^2/4}$$
$$= (\text{mass of pinion})/2.$$

So the effective mass of pinion $m_1 = (0.745)/2$

$= 0.3725$ slug.

Effective Mass of Gear m_2 (Figs. 5 and 6).

ρ = 0.26 lb/cu.in. for cast iron class 40

$$m_a = \frac{\rho \pi}{4\ g} \text{(outer diameter}^2 \text{ - inner diameter}^2\text{)(length)}$$

$$= \frac{(0.26)(3.14)(30.9^2 - 29.9^2)(8.0)}{4(32.2)}$$

$$= 3.048 \text{ slugs}$$

$$m_b = \frac{(0.26)(3.14)(29.9^2 - 3.40^2)(2.0)}{4(32.2)}$$

$$= 11.34 \text{ slugs}$$

$$m_c = \frac{(0.26)(3.14)(3.4^2)(10.0)}{4(32.2)}$$

$$= 0.735 \text{ slug.}$$

Since the moment of inertia of entire gear is

$$= \frac{m_a(30.9^2 + 29.9^2) + m_b(29.9^2 + 3.4^2) + m_c(3.4)(3.4)}{8.0}$$

$$= \frac{(3.048)(1852) + (11.34)(907.6) + (0.735)(11.6)}{8.0}$$

$$= 1996.0 \text{ slugs/sq.in.,}$$

then the effective mass of the gear is

$$m_2 = \frac{(1996)}{R_2{}^2} = \frac{1996}{(25.45)^2} = 8.20 \text{ slugs,}$$

and the effective mass

$$m = \frac{m_1\ m_2}{m_1 + m_2} = \frac{(0.3725)(8.2)}{0.3725 + 8.2} = 0.356 \text{ slug.}$$

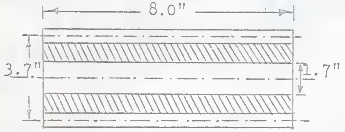

Fig. 4. Pinion dimensions.

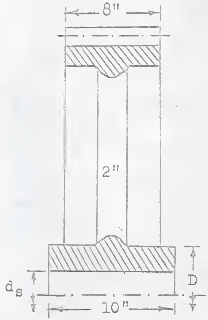

Fig. 5. Gear dimensions.

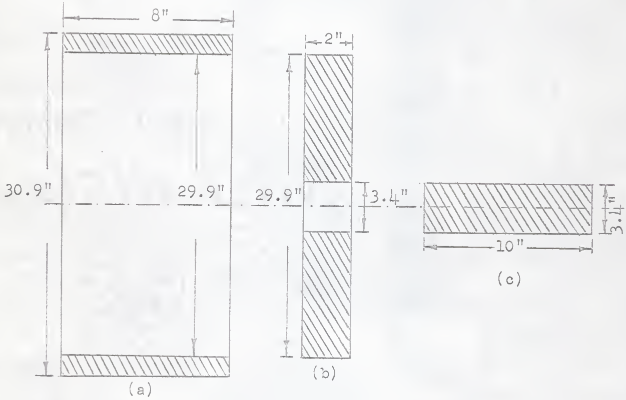

Fig. 6. Division of gear into three parts.

Now $$F_2 = H\ m\ V_t^2 \cos^2 \psi$$

$$= (0.000727)(0.356)(9707.9^2)(\cos^2 23^\circ)$$

$$= 208.0 \text{ lbs.},$$

and $$F_a = \frac{F_1\ F_2}{F_1 + F_2} = \frac{(208.0)(18539.2)}{208.0 + 18539.2}$$

$$= 206.0 \text{ lbs.}$$

Now the dynamic load

$$W_d = W_t + \sqrt{F_a(2\ F_1 - F_a)}$$

$$= 2939.2 + \sqrt{206(2\ 18539.2 - 206)}$$

$$= 5764.2 \text{ lbs.},$$

which is permissible.

Limiting Load for Wear W_w

Limiting load for wear W_w is given by formula (23).

$$W_w = \frac{2\ R_1\ F\ Q\ K}{\cos^2\ \psi}$$

where $$Q = \frac{2R_2}{R_1 + R_2} = \frac{2(30.9)}{3.70 + 30.9} = 1.79$$

and $$K = \frac{S_{ac}^2 \sin \phi \left[\frac{1}{E_p} + \frac{1}{E_g}\right]}{1.4}$$

$$= \frac{65{,}000^2(\sin 20^\circ)\left[\frac{1}{29.7 \times 10^6} + \frac{1}{16.0 \times 10^6}\right]}{1.4}$$

$$= 91.0.$$

$$\text{Therefore} \quad W_w = \frac{(2.0)(3.7)(8.0)(1.79)(91.0)}{(2.0)(\cos^2 23^o)}$$

$$= 5670.0 \text{ lbs.}$$

which is quite safe.

Since the calculated limiting load for wear is greater than the transmitted tangential load of 2939.2 pounds, the design conditions for wear are satisfied. Wear is likely to be rapid in cases where the transmitted load is greater than the calculated limiting load for wear.

Results

	Factors	Values
1.	Center distance C_p, in.	17.30
2.	Pitch diameter of pinion d_1, in.	3.70
3.	Pitch diameter of gear d_2, in.	30.90
4.	Number of teeth on pinion N_p	37.00
5.	Number of teeth on gear N_g	309.00
6.	Face width F, in.	8.00
7.	Normal diametral pitch P_n	10.00
8.	Transmitted tangential load W_t, lbs.	2,939.20
9.	Calculated tensile stress S_t, psi	30,300.00
10.	Calculated contact stress S_c, psi	43,500.00
11.	Power capacity P_{at}, hp	1,300.50
12.	Power capacity P_{ac}, hp	1,960.00
13.	Thickness of the rim of gear h, in.	0.50

Factors		Values
14.	Number of arms of gear	6.00
15.	Hub diameter of gear D, in.	3.40
16.	Hub length of gear L, in.	10.00
17.	Dynamic load W_d, lbs.	5,764.20
18.	Limiting load for wear W_w, lbs.	5,670.00

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HIGH-SPEED HELICAL GEAR DESIGN

by

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AN ABSTRACT OF A MASTER'S REPORT

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This report presents a method of designing high-speed helical gears for any given reduction gear ratio.

In helical gears, the line of contact is diagonally across the face of the tooth that gives gradual engagement of the tooth and smooth transfer of load from one tooth to another. This helps in transmitting heavy load at high speed with less noise.

The selection of gear materials depends upon the type of load which occurs by driving and driven machines. At the same time, it is necessary to know how long the required gear set is expected to last. Gears are commonly made of steel, cast iron, and bronze. Cast iron is the most popular gear material, and is easy to cast and machine. It also has good wearing characteristics and makes less noise than the steel. To keep the pitch line velocity down, the smallest possible center distance should be used. By selecting the standard normal diametral pitch, and the number of teeth on pinion and gear, we can easily find the transmitted tangential load at operating pitch diameter.

The cantilever-plate theory for determining the bending strength of gear teeth, reported by E. J. Wellauer, is an improvement over the older Lewis formula which considered the tooth as a cantilever beam. Calculated bending stress must always be less than the allowable bending stress. Also calculated contact stress should not be higher than allowable contact stress. The power capacity of a gear set, based on strength and surface durability, must be at least 25 per cent or more higher than the required power.

The designed gear tooth should also be checked under dynamic load. According to the Buckingham formula, the analysis of the dynamic load on helical gear teeth requires the determination of the effective mass acting at pitch line of gear. The effective mass of a gear is calculated by determining the moment of inertia and then dividing by the pitch radius required. It is necessary that the limiting load for wear must be higher than the transmitted tangential load, since the limiting load for wear is the load beyond which wear is likely to be rapid.

Gear blanks are made by casting, forging, and machining. Small pinions are made integral with the shaft. For any gear the length of the hub should never be less than the gear face width. The Westinghouse-Nuttall formula can be used to calculate the thickness of the rim. The usual cross section of the arms is an ellipse, a cross, and an I-section or an H-section.