PERIODICALLY REVERSE-SWITCHED CAPACITORS IN AN RC NETWORK

by

WILLIAM J. HARDENBURGER

B. S., Kansas State University, 1950
B. S. (E. E.), Kansas State University, 1961

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Approved by:

Charles A. Halijak
Major Professor

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INTRODUCTION

The principle of periodically switching the inputs to a capacitor was used by Hosenthien (1) in a ring modulator device patented in 1960. The reverse-switched capacitor was introduced and defined by McKinney (2) in 1964. He described the physical construction of the reverse-switched capacitor and presented the mathematical description for this device when used to replace the capacitor in a simple series resistance-capacitance network.

The purpose of this investigation is to extend derivations and procedures developed by McKinney for the series RC case to more complicated networks. Some detailed examples and counter examples are presented. A symmetric balanced RC lattice network terminated in a reverse-switched capacitor is used as the fundamental building block for this investigation. The lattices are cascaded to determine the effect of increasing complexity on the system. Finally, the symmetric lattice network terminated with a reverse-switched capacitor is described by a state space model and the investigation concluded.

To facilitate calculations all admittances and impedances are normalized and all initial conditions are assumed to be zero. This in no way detracts from the generality of the method.

PREVIOUS WORK

J. A. McKinney's Thesis

J. A. McKinney (5) gave a qualitative description of the reverse-switched capacitor. A mathematical analysis using Z- and Z_m-transforms was developed to describe the device in certain simple circuits.

The physical system (see Fig. 1) consisted of an ordinary capacitor whose leads are attached to a double-pole double-throw switch. This switch acts as a reversing switch and in use is controlled by a periodic timing mechanism. For the purposes of this analysis, the switching time is considered to be zero.

Using current as a driving source, McKinney found a relation occurs between this current and the voltage across the reverse-switched capacitor alone. In Laplace transform notation this relationship is

$$\bar{i} = Cs\bar{e} - 2CZ_m\bar{e} + Ce_0 \qquad (1)$$

where i(t) is the driving current, C is the capacitance of the reverse-switched capacitor, e(t) is the voltage across the reverse-switched capacitor, and e_0 is the initial condition on C. This equation is called the node pair voltage equation.

By similar reasoning, McKinney developed a relationship between the driving voltage and the current through the reverse-switched capacitor. This relationship is

$$\bar{e} = \bar{i}/Cs - \frac{2(1 - z^m)}{s(1 + z^m)} Z_m(\bar{i}/Cs) + e_0 \frac{(1 - z^m)}{s(1 + z^m)} \qquad (2)$$

where e(t) represents the driving voltage and i(t) the current through the reverse-switched capacitor. This equation is called the loop current equation.

McKinney reasoned that these equations could be used in the analysis of networks containing reverse-switched capacitors. The procedure was simply to express the network to be analyzed in Laplace transform notation. At the point(s) in the equation(s) where there is a voltage drop due to a reverse-switched capacitor, the node pair voltage equation is inserted. The same procedure is followed for the loop current equation. The equation(s) obtained can then be manipulated in much the same way as those used in sampled-data feedback systems.

McKinney demonstrated that this procedure does work for a simple series RC network, where the resistor is in series with one reverse-switched capacitor and a voltage generator used as a source.

THE Z- AND Z_m-TRANSFORMS

The Z- and Z_m-transforms are used extensively in this paper and are introduced at this time for completeness. The Z-transform is described thoroughly by Jury (2) and others, and is the basis for McKinney's investigation of the reverse-switched capacitor. For the work in this investigation, the Z- and Z_m-transforms are defined as

$$Z\bar{f} = \sum_{n=0}^{\infty} z^n f(nT) \qquad (3)$$

$$Z_m\overline{f} = \sum_{n=0}^{\infty} z^{nm} f(nmT) \qquad (4)$$

where $z \equiv e^{-sT}$.

The Z_m-transform is the ordinary Z-transform where the sampling interval T is replaced with mT, the duration of the switching period. In general, the theorems that apply to the ordinary Z-transform apply also to the Z_m-transform. One important exception to this is the case as follows.

For the ordinary Z-transform

$$Z(\overline{f}\ Z\ \overline{g}) = (Z\ \overline{f})\ (Z\ \overline{g}).$$

For mixed Z- and Z_m-transforms

$$Z_1(\overline{f}\ Z_m\overline{g}) = (Z_1\overline{f})\ (Z_m\overline{g})$$

and

$$Z_m(\overline{f}\ Z_1\overline{g}) = (Z_m\overline{f})\ (Z_m\overline{g}).$$

SECTION I

Application of McKinney's Procedure to a More Complicated Network

The circuit to be considered is a balanced symmetric resistance-capacitance lattice network terminated with a reverse-switched capacitor and with a voltage generator as a source. The circuit diagram for this network is displayed in Fig. 2.

The symbol —)(— will be used to denote a reverse-switched capacitor. The general block diagram for this circuit is represented in Fig. 3, where

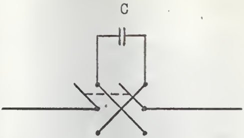

Fig. 1. Reverse-switched capacitor.

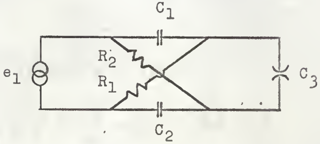

Fig. 2. A balanced symmetric RC lattice network with source and termination.

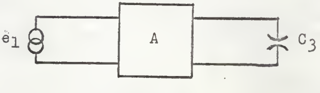

Fig. 3. Block diagram representation of network in Fig. 2.

$$A = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} . \tag{5}$$

The equations for this circuit written directly in matrix Laplace transform notation are

$$\begin{pmatrix} \bar{e}_1 \\ \bar{i}_1 \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} \bar{e}_2 \\ \bar{i}_2 \end{pmatrix} . \tag{6}$$

By normalizing $R_1 = R_2 = 1$ ohm and $C_1 = C_2 = 1$ farad, then

$$A = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} = \begin{pmatrix} \dfrac{s + 1}{s - 1} & \dfrac{2}{s - 1} \\ \dfrac{2\ s}{s - 1} & \dfrac{s + 1}{s - 1} \end{pmatrix} . \tag{7}$$

In this network the current through the reverse-switched capacitor (C_3) is $\bar{i}_2$; therefore (using McKinney's procedure) in place of $\bar{i}_2$, the node pair voltage equation (1) may be written.

$$\bar{e}_1 = \alpha\bar{e}_2 + \beta(C_3\ s\ \bar{e}_2 - 2\ C_3\ Z_m\ \bar{e}_2 + C_3\ e_{2\ 0}) \tag{8}$$

$$\bar{i}_1 = \gamma\bar{e}_2 + \delta(C_3\ s\ \bar{e}_2 - 2\ C_3\ Z_m\ \bar{e}_2 + C_3\ e_{2\ 0}) \tag{9}$$

Normalizing $C_3 = 1$ farad, letting the initial conditions be equal to zero, and collecting terms gives

$$\bar{e}_1 = (\alpha + s\beta)\bar{e}_2 - 2\ \beta\ Z_m\bar{e}_2 \tag{10}$$

$$\bar{i}_1 = (\gamma + s\delta)\bar{e}_2 - 2\ \delta\ Z_m\bar{e}_2 . \tag{11}$$

Dividing equation (10) by $\alpha + s\beta$ and Z_m-transforming yields

$$Z_m\left(\frac{\bar{e}_1}{\alpha + s\beta}\right) = Z_m\bar{e}_2\left(1 - Z_m\ \frac{2\beta}{\alpha + s\beta}\right) . \tag{12}$$

Solving for $Z_m\ \bar{e}_2$ gives

$$Z_m\bar{e}_2 = \frac{Z_m\left(\dfrac{\bar{e}_1}{\alpha + s\beta}\right)}{1 - Z_m\left(\dfrac{2\beta}{\alpha + s\beta}\right)} \; . \tag{13}$$

Solving for $\bar{e}_2$ in equation (10) gives

$$\bar{e}_2 = \frac{\bar{e}_1}{\alpha + s\beta} + \frac{2\beta}{\alpha + s\beta}\left(\frac{Z_m\left(\dfrac{\bar{e}_1}{\alpha + s\beta}\right)}{1 - Z_m\left(\dfrac{2\beta}{\alpha + s\beta}\right)}\right) \; . \tag{14}$$

Substituting $Z_m\bar{e}_2$ and $\bar{e}_2$ into equation (11) and simplifying yields

$$\bar{i}_1 = \frac{\gamma + s\delta}{\alpha + s\beta}\,\bar{e}_1 - \frac{2\,Z_m\left(\dfrac{\bar{e}_1}{\alpha + s\beta}\right)}{\alpha + s\beta\left[1 - Z_m\left(\dfrac{2\beta}{\alpha + s\beta}\right)\right]} \; . \tag{15}$$

For purposes of calculations choose $\bar{e}_1 = \frac{1}{s}$. Now substituting for $\bar{e}_1$, α, β, γ, and δ in equation (15) and simplifying

$$\bar{i}_1 = \frac{s + 3}{3s + 1} - \frac{2(s - 1)}{3s + 1}\left[\frac{Z_m\left(\dfrac{s - 1}{s(s + 1)}\right)}{1 - Z_m\left(\dfrac{4}{3s + 1}\right)}\right] \; . \tag{16}$$

Taking the Z-transform and simplifying gives

$$Z\bar{i}_1 = \frac{1 - ze^{-T/3} - z^m(\frac{1}{3} - 3e^{-(m/3)T}) + 3z^{m+1}(e^{-T/3} - e^{-(m+1/3)T}) - \frac{11}{3} z^{2m}e^{-(m/3)T} + z^{2m+1}\, e^{-(m+1/3)T}}{1 - ze^{-T/3} - z^m(1 - 3e^{-(m/3)T}) + z^{m+1}(e^{-T/3} - 3e^{-(m+1/3)T}) - 3z^{2m}e^{-(m/3)T} + 3z^{2m+1}\, e^{-(m+1/3)T}} \quad . \tag{17}$$

Equation (17) can be put into a recurrence relation as follows.

$$\begin{aligned} A_{(n)} = {} & A_{(n-1)}e^{-T/3} + A_{(n-m)}(1 - 3e^{-(m/3)T}) \\ & - A_{(n-m-1)}(e^{-T/3} - 3e^{-(m+1/3)T}) + 3A_{(n-2m)}e^{-(m/3)T} \\ & - 3A_{(n-2m-1)}e^{-(m+1/3)T} + x_n \end{aligned} \tag{18}$$

where

$$\begin{aligned} \{x_n\} = \Big\{ & 1,\ -e^{-T/3},\ 0,\ \ldots,\ -(\frac{1}{3} - 3e^{-(m/3)T})\ , \\ & 3(e^{-T/3} - e^{-(m+1/3)T}),\ 0,\ \ldots, \\ & -\frac{11}{3}\, e^{-(m/3)T},\ e^{-(m+1/3)T},\ 0,\ \ldots \Big\} \ . \end{aligned}$$

The digital computer program for equation (18) is displayed in Appendix A. Some results of this solution with $m = 10$ and $T = .4$ are given in Table 1. The data from Table 1 are plotted in Fig. 4.

Discussion of Results

Results from Table 1 indicate that the output (current) takes the form of an ultimately periodic function as predicted by McKinney. However, further investigation has demonstrated that as the sampling time (T) becomes smaller, the time necessary for the output (current) to become periodic is increased and for values of T less than about .3 second, with m held constant at 10, the output current becomes unstable. The data for the computer solution with m = 10 and T = .2 second are plotted in Fig. 5 and displays the instability of the output current for this value of sampling time.

Further Investigation Using McKinney's Procedure

The investigation was carried further using, first, two symmetric RC lattices cascaded, and, then, three lattices cascaded. In each case the cascaded lattices were terminated in a reverse-switched capacitor and driven by a unit step input (1/s). The computer results obtained from each case displayed the same type of output as was evidenced in the single lattice case presented. That is, for relatively long sampling times (T) the output would become ultimately periodic, but as the sampling time was reduced the output would become unstable. No significant improvement in output stability was experienced as more sections of the lattices were cascaded. The computer program used for the solution of the two-lattice cascaded case is given in Appendix B.

Verification of Results by a Physical Model

At this point in the investigation, the need for physical verification of the calculated results became apparent. Intuition did not indicate that the device under investigation should become unstable when a unit step of voltage was applied at the input.

A physical model of the balanced symmetric RC lattice network was constructed by the author. The device was terminated in a reverse-switched capacitor. The reverse-switched capacitor was operated by an electromechanical switch. This device was wired into an analog computer program and its behavior investigated at various conditions of inputs and switching times. In all cases the device behaved in a stable manner and exhibited a tendency to ultimate periodicity, thus indicating that the calculated output was not properly describing the actual output of the device.

Conclusion of Section I

The conclusion gathered from this investigation was that the procedure for analyzing a network containing a reverse-switched capacitor as outlined by McKinney does not hold in general for networks more complicated than a simple series

TABLE 1. SYMMETRIC LATTICE WITH REVERSE SWITCHED CAPACITOR

T. = .4 M = 10 E0 =0

N	I(N)	N	I(N)	N	I(N)
	1.000000	50	.666667	100	.666666
1	0.000000	51	1.706241	101	1.178589
2	0.000000	52	1.493257	102	1.031470
3	0.000000	53	1.306859	103	.902715
4	0.000000	54	1.143728	104	.790032
5	0.000000	55	1.000961	105	.691416
6	0.000000	56	.876014	106	.605109
7	0.000000	57	.766665	107	.529575
8	0.000000	58	.670965	108	.463470
9	0.000000	59	.587211	109	.405617
10	.666667	60	.666668	110	.666669
11	2.333795	61	.984516	111	1.401782
12	2.042476	62	.861622	112	1.226803
13	1.787520	63	.754069	113	1.073665
14	1.564390	64	.659941	114	.939644
15	1.369113	65	.577563	115	.822352
16	1.198211	66	.505468	116	.719701
17	1.048642	67	.442372	117	.629863
18	.917744	68	.387152	118	.551240
19	.803185	69	.338825	119	.482431
20	.666667	70	.666667	120	.666669
21	.488250	71	1.555250	121	1.225283
22	.427304	72	1.361114	122	1.072335
23	.373965	73	1.191211	123	.938480
24	.327284	74	1.042516	124	.821333
25	.286430	75	.912383	125	.728809
26	.250676	76	.798493	126	.629083
27	.219335	77	.698821	127	.550557
28	.191999	78	.611589	128	.481833
29	.168033	79	.535247	129	.421687
30	.666667	80	.666669	130	.666669
31	1.947691	81	1.103920	131	1.364857
32	1.734568	82	.966121	132	1.194487
33	1.491793	83	.845524	133	1.045384
4	8 05577	84	.739980	134	.914892
35	1.142607	85	.647611	135	.800690
36	.999979	86	.566772	136	.700743
37	.875155	87	.496024	137	.613272
38	.765913	88	.434107	138	.536721
39	.670306	89	.379919	139	.469723
40	.666668	90	.666668	140	.666667
41	.793579	91	1.460829	141	1.254484
42	.694519	92	1.278479	142	1.097891
43	.607824	93	1.118891	143	.960846
44	.531952	94	.979224	144	.849074
45	.465550	95	.856991	145	.735940
46	.437437	96	.750016	146	.644075
47	.356578	97	.656394	147	.563678
48	.312067	98	.574459	148	.493316
49	.273113	99	.502752	149	.431737

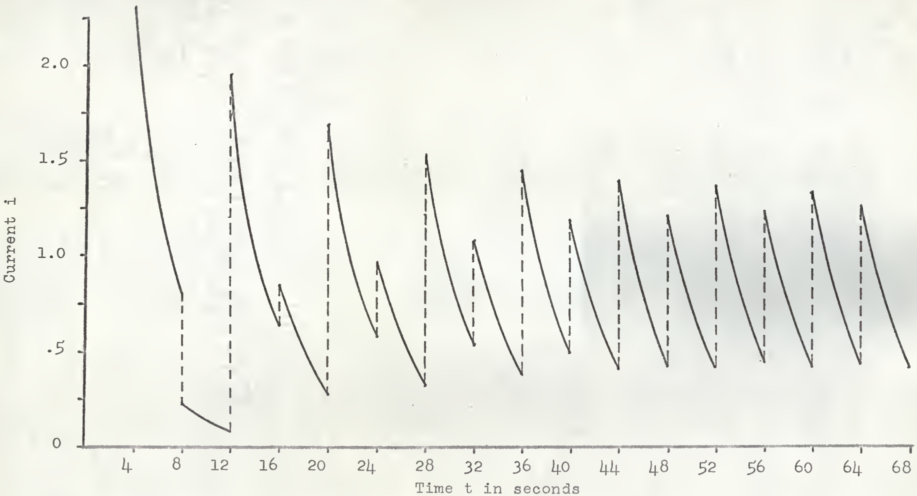

Fig. 4. Current versus time in reverse-switched RC lattice network with unit step input, m = 0, T = .4 second.

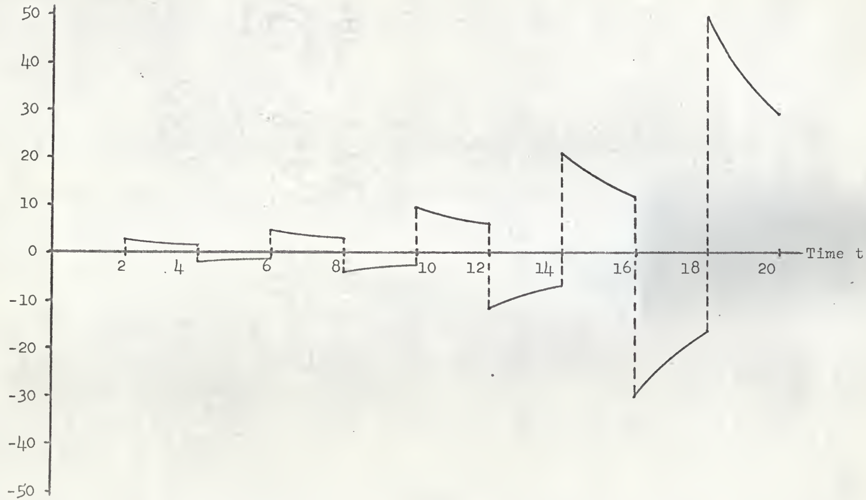

Fig. 5. Current versus time in reversed-switched RC lattice network with unit step input, m = 10, and T = .2 second.

resistance-capacitance circuit.

The investigation in the next section will attempt to develop new procedures and techniques for this analysis.

SECTION II

The Reverse-switched Capacitor in a Parallel RC Circuit

All previous work in Z-transform analysis of the reverse-switched capacitor in a network has involved series circuits and voltage driving sources with the behavior of the input current the point of interest. Investigation of a reverse-switched capacitor in a parallel resistance-capacitance network with a current driving source now appears useful. The associated input voltage will be investigated.

The circuit to be considered is the parallel RC circuit displayed in Fig. 6. This network is driven by a current source (i) and the capacitor (C) is a reverse-switched capacitor.

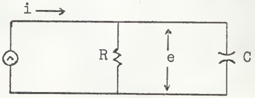

Fig. 6. Parallel RC circuit.

Application of the Kirchhoff current law yields the circuit equation

$$i = Re + C\dot{e} \; . \tag{19}$$

R and C are normalized to 1, yielding

$$i = e + \dot{e} \; , \tag{20}$$

The finite-time Laplace transform (P_n-transform) will be used in this development and is defined by

$$P_n f \equiv \int_{nmT}^{(n+1)mT} f(t) e^{-st} \, dt \; . \tag{21}$$

Summing the P_n-transform from 0 to ∞ yields the ordinary Laplace transform

$$\sum_{n=0}^{\infty} P_n f = \overline{f} \tag{22}$$

Furthermore

$$\sum_{n=0}^{\infty} P_n \dot{f} = s\overline{f} - f_o \tag{23}$$

where $\dot{f}$ is used to denote df/dt and f_o is the initial condition.

The symbol $\hat{P}_n$ will be used when the P_n-transform is applied to a reverse-switched capacitor.

Summing the $\hat{P}_n$-transform yields

$$\sum_{n=0}^{\infty} \hat{P}_n f = \overline{f} \tag{24}$$

and

$$\sum_{n=0}^{\infty} P_n \dot{f} = s\overline{f} - 2z_m\overline{f} + f_o \; . \tag{25}$$

The P_n- and $\hat{P}_n$-transforms have been developed thoroughly by McKinney (5). Note that $P_n f$ and $\hat{P}_n f$ when summed from 0 to ∞ both equal the ordinary Laplace transform of f. $\hat{P}_n f$ holds only for conditions across a resistive element and not across a voltage generator.

Transforming equation (20) by P_n and $\hat{P}_n$ yields

$$P_n i = P_n e + \hat{P}_n \dot{e} \tag{26}$$

$$\sum_{n=0}^{\infty} P_n i = \sum_{n=0}^{\infty} P_n e + \sum_{n=0}^{\infty} \hat{P}_n \dot{e} \, . \tag{27}$$

Substituting equations (22) and (25) into (27) yields the equation

$$\bar{i} = \bar{e} - s\bar{e} - 2Z_m\bar{e} + e_0 \, . \tag{28}$$

Letting $e_0 = 0$ and Z_m-transforming equation (28) yields

$$Z_m\bar{e} = \frac{Z_m(\bar{i}/1 + s)}{1 - Z_m(2/1 + s)} \, . \tag{29}$$

Substituting equation (29) into (28) and solving for $\bar{e}$ gives

$$\bar{e} = \frac{\bar{i}}{(1 + s)} + \frac{2}{(1 + s)} \left\{ \frac{Z_m(\bar{i}/s)}{1 - Z_m\left(\frac{2}{1 + s}\right)} \right\} . \tag{30}$$

Letting $\bar{i} = 1/s$ and Z-transforming yields

$$Z\bar{e} = \frac{z - ze^{-T}}{(1-z)(1-ze^{-T})} - \frac{2(z^m - z^m e^{-mT})}{(1+z^m e^{-mT})(1-z^m)(1-z^m e^{-T})} \, . \tag{31}$$

After expanding, equation (31) becomes

$$Z\bar{e} = \frac{z(1-e^{-T}) + z^m(2e^{-mT}-2) + z^{m+1}(1+e^{-T}-3e^{-mT}+ e^{-(m+1)T}}{1+z(1+e^{-T})+z^2e^{-T}+z^m(e^{-mT}-1)+z^{m+1}(1+e^{-T}-e^{-mT}-e^{-(m+1)T})}$$

$$\frac{+ z^{2m+1}(e^{-mT} - e^{-(m+1)T})}{+ z^{m+2}(e^{-(m+1)T}-e^{-T})-z^{2m}e^{-mT}+z^{2m+1}(e^{-mT}+e^{-(m+1)T}) - z^{2m+2}e^{-(m+1)T}} \tag{32}$$

The recurrence relation for equation (32) may be written as

$$V_{(n)} = -V_{(n-1)}(B1) - V_{(n-2)}(B2) - V_{(n-m)}(B3) - V_{(n-m-1)}(B4) - V_{(n-m-2)}(B5) - V_{(n-2m)}(B6) - V_{(n-2m-1)}(B7) - V_{(n-2m-2)}(B8) - X_{(n)} \quad (33)$$

where

$$\left\{X_{(n)}\right\} = \left\{0,\ (A1),\ 0,\ 0,\ \ldots,\ (A2),\ (A3),\ 0,\ 0,\ \ldots,\ 0,\ (A4),\ 0,\ \ldots\right\}.$$

The coefficients of powers of z in equation (33) are defined from equation (32) and are

$$A1 = 1 - e^{-T}$$

$$A2 = 2e^{-mT} - 2$$

$$A3 = 1 + e^{-T} - 3e^{-mT} + e^{-(m+1)T}$$

$$A4 = e^{-mT} - e^{-(m+1)T}$$

$$B1 = 1 + e^{-T}$$

$$B2 = e^{-T}$$

$$B3 = e^{-mT} - 1$$

$$B4 = 1 + e^{-T} - e^{-mT} - e^{-(m+1)T}$$

$$B5 = e^{-(m+1)T} - e^{-T}$$

$$B6 = e^{-mT}$$

$$B7 = e^{-mT} + e^{-(m+1)T}$$

$$B8 = e^{-(m+1)T}.$$

Discussion of Results

The computer program used for the solution of equation (33) is in Appendix C. The solution of this recurrence relation shows that the input voltage to this circuit is periodic in nature as would be expected from previous work. The next section will extend this type of analysis to a more complicated network.

SECTION III

A State Space Approach

The state-variable approach to network analysis has been thoroughly reviewed by Kuh and Rohrer (4). This section is devoted to an exploitation of state-variable analysis in terms of the symmetric balanced resistance-capacitance network displayed in Fig. 7.

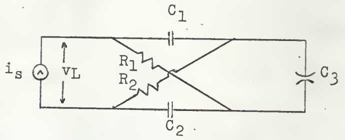

Fig. 7. Symmetric RC network.

In the network of Fig. 7, the voltages across the capacitors C_1, C_2, and C_3 are chosen as the state variables and are denoted as v_1, v_2, and v_3, respectively. Application of the Kirchhoff current law to obtain the capacitor current yield the right-hand side of the equations of dynamic equilibrium:

$$C_1(dv_1/dt) = C_1\dot{v}_1 = i_s - \left(\frac{v_1 - v_3}{R_1}\right) \tag{34}$$

$$C_2(dv_2/dt) = C_2\dot{v}_2 = i_s - \left(\frac{v_2 - v_2}{R_2}\right) \tag{35}$$

$$C_3(dv_3/dt) = C_3\dot{v}_3 = \left(\frac{v_1 - v_3}{R_2}\right) - C_2\dot{v}_2$$

$$= \left(\frac{v_1 - v_3}{R_2}\right) + \left(\frac{v_2 - v_3}{R_1}\right) - i_s \tag{36}$$

The input voltage v_L in Fig. 7 can be related to the state variables by Ohm's law as

$$v_L = v_1 + v_2 - v_3 \; . \tag{37}$$

Equations developed are almost in standard state equation form,

$$\dot{x} = Ax + Bu \tag{38}$$

and

$$y = Cx + Du \tag{39}$$

where x(t) represents the state vector, u(t) the input vector, and y(t) the output vector. Therefore we choose

$$x = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \tag{40}$$

for the state vector,

$$u = (i_s(t)) \tag{41}$$

for the input vector, and

$$y = (v_L(t)) \tag{42}$$

for the output vector.

This leads to the following set of A, B, C, and D matrices:

$$A = \begin{pmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & -2 \end{pmatrix} \tag{43}$$

$$B = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \tag{44}$$

$$C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \tag{45}$$

$$D = \begin{pmatrix} 0 \end{pmatrix} \tag{46}$$

If we consider the specific element values to be normalized to $R_1 = R_2 = 1$ ohm and $C_1 = C_2 = C_3 = 1$ farad, the input-output state relation assumes the form,

$$\begin{pmatrix} \dot{v}_1 \\ \dot{v}_2 \\ \dot{v}_3 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & -2 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \begin{pmatrix} i_s \end{pmatrix} \tag{47}$$

and

$$\begin{pmatrix} v_L \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} . \tag{48}$$

This specific network description will be employed in the subsequent development of the reverse-switched capacitor analysis in this section. Note that capacitor C_3 in Fig. 7 is a reverse-switched capacitor and C_1 and C_2 are normal unswitched capacitors.

If we consider all initial conditions in the network elements to be zero, we can rewrite equation (47) in Laplace transform notation as

$$\begin{pmatrix} s\bar{v}_1 \\ s\bar{v}_2 \\ s\bar{v}_3 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & -2 \end{pmatrix} \begin{pmatrix} \bar{v}_1 \\ \bar{v}_2 \\ \bar{v}_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \left(\bar{I}_s \right) . \qquad (49)$$

To obtain the proper action of the reverse-switched capacitor in the network, premultiply both sides of equation (47) by the diagonal operator matrix

$$\begin{pmatrix} P_n & 0 & 0 \\ 0 & P_n & 0 \\ 0 & 0 & \hat{P}_n \end{pmatrix} \qquad (50)$$

where

$$\sum_{n=0}^{\infty} P_n \, v = \bar{v}$$

$$\sum_{n=0}^{\infty} P_n \, \dot{v} = s\bar{v} - v_o$$

$$\sum_{n=0}^{\infty} \hat{P}_n \, v = \bar{v}$$

$$\sum_{n=0}^{\infty} \hat{P}_n \, \dot{v} = s\bar{v} - 2Z_m\bar{v} + v_o$$

obtaining

$$\begin{pmatrix} P_n & \dot{v}_1 \\ P_n & \dot{v}_2 \\ \hat{P}_n & \dot{v}_3 \end{pmatrix} = \begin{pmatrix} -P_n & 0 & P_n \\ 0 & -P_n & P_n \\ \hat{P}_n & \hat{P}_n & -2\hat{P}_n \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} + \begin{pmatrix} P_n \\ P_n \\ -\hat{P}_n \end{pmatrix} \begin{pmatrix} i_s \end{pmatrix} . \quad (51)$$

Taking the P_n- and $\hat{P}_n$-transforms and letting the initial conditions be equal to zero yields

$$\begin{pmatrix} s\bar{v}_1 \\ s\bar{v}_2 \\ s\bar{v}_3 - 2Z_m\bar{v}_3 \end{pmatrix} = \begin{pmatrix} -\bar{v}_1 + \bar{v}_3 \\ -\bar{v}_2 + \bar{v}_3 \\ \bar{v}_1 + \bar{v}_2 - 2\bar{v}_3 \end{pmatrix} + \begin{pmatrix} \bar{i}_s \\ \bar{i}_s \\ -\bar{i}_s \end{pmatrix} . \quad (52)$$

Simplifying this gives

$$\begin{pmatrix} (s+1) & 0 & -1 \\ 0 & (s+1) & -1 \\ -1 & -1 & (s+2-2Z_m) \end{pmatrix} \begin{pmatrix} \bar{v}_1 \\ \bar{v}_2 \\ \bar{v}_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \begin{pmatrix} \bar{i}_s \end{pmatrix} . \quad (53)$$

Solving equation (53) for $\bar{v}_3$ yields

$$\bar{v}_3 = \frac{2(s + 1)Z_m\bar{v}_3 - \bar{i}_s(s - 1)}{s(s + 3)} . \quad (54)$$

Now take the Z_m-transform of equation (54)

$$Z_m\bar{v}_3 = Z_m\left(\frac{2(s+1)}{s(s+3)}\right) Z_m\bar{v}_3 - Z_m\left(\frac{\bar{i}_s(s-1)}{s(s+3)}\right) \quad (55)$$

and solving for $Z_m\bar{v}_3$ gives

$$Z_m\bar{v}_3 = \frac{-Z_m\left(\dfrac{\bar{i}_s(s - 1)}{s(s + 3)}\right)}{1 - Z_m\left(\dfrac{2(s + 1)}{s(s + 3)}\right)} . \quad (56)$$

From equation (53),

$$\overline{v}_1 = \frac{\overline{v}_3 + \overline{i}_s}{s + 1} \tag{57}$$

and

$$\overline{v}_2 = \frac{\overline{v}_3 + \overline{i}_s}{s + 1} \quad . \tag{58}$$

The output voltage to be investigated from equation (48) is

$$\overline{v}_L = \overline{v}_1 + \overline{v}_2 - \overline{v}_3 \; . \tag{59}$$

Substituting equations (54), (57), and (58) into equation (59) gives

$$\overline{v}_L = \frac{(-2s^2 - 2s + 4)Z_m\overline{v}_3 + (s^2 + s + 2)\overline{i}_s}{s(s + 3)(s + 2)} \quad . \tag{60}$$

Substituting equation (56) for $Z_m\overline{v}_3$ into equation (60) yields

$$\overline{v}_L = \frac{s^2 + s + 2}{s(s+3)(s+2)}\,\overline{i}_s + \frac{2(s-1)}{s(s+3)}\left[\frac{Z_m\left(\dfrac{\overline{i}_s(s-1)}{s(s+3)}\right)}{1-Z_m\left(\dfrac{2(s+1)}{s(s+3)}\right)}\right] \quad . \tag{61}$$

Again, for ease of calculation, let $\overline{i}_s$ be 1/s in equation (61) and Z-transforming gives

$$Z\overline{v}_L = Z\left(\frac{s^2 + s + 2}{s^2(s+3)(s+2)}\right) + Z\left(\frac{2(s-1)}{s(s+3)}\right)\left[\frac{Z_m\left(\dfrac{s - 1}{s^2(s+3)}\right)}{1-Z_m\left(\dfrac{2(s+1)}{s(s+3)}\right)}\right] \; . \tag{62}$$

Executing the Z-transform and performing a large amount of algebra, equation (62) becomes

$$Z\bar{v}_L = \frac{(F1)z+(F2)z^2+(F3)z^3+(F4)z^4+(F5)z^5-(F6)z^m+(F7)z^{m+1}}{1-(D1)z+(D2)z^2-(D3)z^3+(D4)z^4-(D5)z^5+(D6)z^6+(D7)z^m}$$

$$\frac{+(F8)z^{m+2}+(F9)z^{m+3}+(F10)z^{m+4}+(F11)z^{m+5}+(F12)z^{2m}}{-(D8)z^{m+1}+(D9)z^{m+2}-(D10)z^{m+3}+(D11)z^{m+4}-(D12)z^{m+5}}$$

$$\frac{+(F13)z^{2m+1}+(F14)z^{2m+2}+(F15)z^{2m+3}+(F16)z^{2m+4}}{+(D13)z^{m+6}+(D14)z^{2m}-(D15)z^{2m+1}+(D16)z^{2m+2}-(D17)z^{2m+3}}$$

$$\frac{+(F17)z^{2m+5}+(F18)z^{3m+1}+(F19)^{3m+2}+(F20)z^{3m+3}}{+(D18)z^{2m+4}-(D19)z^{2m+5}+(D20)z^{2m+6}+(D21)z^{3m}-(D22)z^{3m+1}}$$

$$\frac{+(F21)z^{3m+4}+(F22)z^{3m+5}}{+(D23)z^{3m+2}-(D24)z^{3m+3}+(D25)z^{3m+4}-(D26)z^{3m+5}+(D27)z^{3m+6}} \qquad (63)$$

The coefficients of the powers of z in equation (63) are defined as follows:

$E1 = e^{-3mT}$

$E2 = e^{-2T}$

$E3 = e^{-3T}$

$E4 = e^{-5T}$

$A1 = 4 - 4(E1) - 3mT$

$A2 = 4 - 4(E1) - 3mT(E1)$

$A3 = (E1) - 4$

$A4 = 1 - 4(E1)$

$A5 = 3(E1)$

$A6 = 3T - 1 + 9(E2) - 8(E3)$

$A7 = 1+17(E3)-17(E2)-3T(E2)-3T(E3)-(E4)$

$A8 = 3T(E4)+(E4)-9(E3)+8(E2)$

A9 = (E3) + (E2)

A10 = (E4)+2(E3)+2(E2)+1

A11 = 2(E4)+(E3)+(E2)

A12 = 1+(E3)

B1 = 6(A1)

B2 = 2(A1)(A13)-8(A1)

B3 = 6(A2)

B4 = 8(A2)-2(A2)(A13)

B5 = 3(A12)

B6 = 3(A13)

B7 = (A3)(A12)

B8 = (A3)(A13)

B9 = (A4)(A12)

B10 = (A4)(A13)

B11 = (A5)(A12)

B12 = (A5)(A13)

F1 = (A6)/9

F2 = (3(A7)-(B5)(A6))/27

F3 = ((B6)(A6)-(A7)(B5)+3(A8))/27

F4 = ((A7)(B6)-(A8)(B5))/27

F5 = ((A8)(B6))/27

F6 = (B1)/27

F7 = ((A6)(A3)-(B2)+(A9)(B1))/27

F8 = ((A7)(A3)-(A6)(B7)-(A10)(B1)+(A9)(B2))/27

F9 = ((A6)(B8)-(A7)(B7)+(A8)(A3)-(A10)(B2)+(A11)(B1))/27

F10 = ((A7)(B8)-(A8)(B7)-(A14)(B1)+(A11)(B2))/27

F11 = ((A8)(B8)+(B2)(A14))/27

F12 = (B3)/27

F13 = ((A6)(A4)-(B4)-(B3)(A9))/27

F14 = ((A7)(A4)-(A6)(B9)-(A9)(B4)-(A10)(B3))/27

F15 = ((A6)(B10)-(A7)(B9)+(A8)(A4)-(B4)(A10)-(B3)(A11))/27

F16 = ((A7)(B10)-(A8)(B9)-(A11)(B4)-(A14)(B3))/27

F17 = ((A8)(B10)-(B4)(A14))/27

F18 = ((A6)(A5))/27

F19 = ((A7)(A5)-(A6)(B11))/27

F20 = ((A6)(B12)-(A7)(B11)+(A8)(A5))/27

F21 = ((A7)(B12)-(A8)(B11))/27

F22 = ((A8)(B12))/27

D1 = (3(A9)+(B5))/3

D2 = ((B6)+(B5)(A9)+3(A10))/3

D3 = ((A9)(B6)+(A10)(B5)+3(A11))/3

D4 = ((A10)(B6)+(A11)(B5)+3(A14))/3

D5 = ((A11)(B6)+(A14)(B5))/3

D6 = ((A14)(B6))/3

D7 = (A3)/3

D8 = ((B7)+(A9)(A3))/3

D9 = ((B8)+(A9)(B7)+(A10)(A3))/3

D10 = ((A9)(B8)+(A10)(B7)+(A11)(A3))/3

D11 = ((A10)(B8)+(A11)(B7)+(A14)(A3))/3

D12 = ((A11)(B8)+(A14)(B7))/3

D13 = ((A14)(B8))/3

D14 = (A4)/3

D15 = ((B9)+(A9)(A4))/3

D16 = ((B10)+(A9)(B9)+(A10)(A4))/3

D17 = ((A9)(B10)+(A10)(B9)+(A11)(A4))/3

D18 = ((A10)(B10)+(A11)(B9)+(A14)(A4))/3

D19 = ((A11)(B10)+(A14)(B9))/3

D20 = ((A14)(B10))/3

D21 = (A5)/3

D22 = ((B11)+(A9)(A5))/3

D23 = ((B12)+(A9)(B11)+(A10)(A5))/3

D24 = ((A9)(B12)+(A10)(B11)+(A11)(A5))/3

D25 = ((A10)(B12)+(A11)(B11)+(A14)(A5))/3

D26 = ((A11)(B12)+(A14)(B11))/3

D27 = ((A14)(B12))/3

Equation (63) can be put into a recurrence relation as follows:

$$
\begin{aligned}
R_{(n)} = {} & R_{(n-1)}(D1) - R_{(n-2)}(D2) + R_{(n-3)}(D3) - R_{(n-4)}(D4) \\
& + R_{(n-5)}(D5) - R_{(n-6)}(D6) - R_{n-m)}(D7) + R_{(n-m-1)}(D8) \\
& - R_{(n-m-2)}(D9) + R_{n-m-3)}(D10) - R_{(n-m-4)}(D11) \\
& + R_{(n-m-5)}(D12) - R_{(n-m-6)}(D13) - R_{(n-2m)}(D14) \\
& + R_{(n-2m-1)}(D15) - R_{(n-2m-2)}(D16) + R_{(n-2m-3)}(D17) \\
& - R_{(n-2m-4)}(D18) + R_{(n-2m-5)}(D19) - R_{(n-2m-6)}(D20) \\
& - R_{(n-3m)}(D21) + R_{(n-3m-1)}(D22) - R_{(n-3m-2)}(D23) \\
& + R_{(n-3m-3)}(D24) - R_{(n-3m-4)}(D25) + R_{(n-3m-5)}(D26) \\
& - R_{(n-3m-6)}(D27) + X_{(n)} \qquad (64)
\end{aligned}
$$

where

$$
\left\{X_{(n)}\right\} = \{0, (F1), (F2), (F3), (F4), (F5), 0, \ldots, (F6), (F7), (F8), (F9), (F10), (F11), 0, \ldots, (F12), (F13), (F14), (F15), (F16), (F17), 0, 0, \ldots\} .
$$

Discussion of Results

The computer program for the solution of equation (64) is displayed in Appendix D. Some results from this program with m = 10 and T = .1 second are given in Table 2. No attempt is made to present a plot of the data in Table 2 because of the obvious instability of the solution.

The output displays an ultimate periodicity for fairly large sampling times T. Decreasing the sampling time increases the time for the output to become periodic, and further decrease will cause the output to become unstable. Again the calculated solution does not exhibit the actual operation of the circuit being analyzed.

The analysis in this section made use of the Z-transform techniques and procedures that were used successfully in Section II for the analysis of a parallel RC network. This section serves as an example that the procedures developed in Section II do not hold in general for the analysis of more complicated networks. A different approach and new techniques are needed to accomplish this generalization.

SUMMARY

This report compiles the results of an investigation into the Z-transform method of analysis of networks containing reverse-switched capacitors. Work by McKinney in this area resulted in a Z-transform description for the reverse-switched

TABLE 2. SYMMETRIC LATTICE WITH REVERSE SWITCHED CAPACITOR

T = .1 M = 10 E0 = 0

N	I(N)	N	I(N)	N	I(N)
0	0.000	33	2.987E+08	66	2.749E+18
1	8.245E-02	34	5.140E+08	67	2.373E+18
2	-2.685E-02	35	-7.274E+08	68	-8.808E+18
3	-3.569E-01	36	-2.698E+09	69	-1.734E+19
4	-2.012E-01	37	4.740E+08	70	1.949E+19
5	1.249	38	1.116E+10	71	8.649E+19
6	1.925	39	8.125E+09	72	4.667E+16
7	-3.240	40	-3.711E+10	73	-3.441E+20
8	-1.056E+01	41	-6.562E+10	74	-3.089E+20
9	3.415	42	8.880E+10	75	1.092E+21
10	4.492E+01	43	3.408E+11	76	2.209E+21
11	2.675E+01	44	-4.763E+10	77	-2.364E+21
12	-1.554E+02	45	-1.399E+12	78	-1.091E+22
13	-2.463E+02	46	-1.065E+12	79	-3.806E+20
14	3.971E+02	47	4.610E+12	80	4.306E+22
15	1.336E+03	48	8.373E+12	81	4.014E+22
16	-3.824E+02	49	-1.083E+13	82	-1.353E+23
17	-5.659E+03	50	-4.303E+13	83	-2.811E+23
18	-3.556E+03	51	4.484E+12	84	2.862E+23
19	1.933E+04	52	1.752E+14	85	1.375E+24
20	3.149E+04	53	1.394E+14	86	9.489E+22
21	-4.865E+04	54	-5.722E+14	87	-5.387E+24
22	-1.689E+05	55	-1.068E+15	88	-5.210E+24
23	4.200E+04	56	1.319E+15	89	1.676E+25
24	7.098E+05	57	5.432E+15	90	3.577E+25
25	4.696E+05	58	-3.747E+14	91	-3.460E+25
26	-2.403E+06	59	-2.195E+16	92	-1.734E+26
27	-4.024E+06	60	-1.820E+16	93	-1.783E+25
28	5.952E+06	61	7.101E+16	94	6.738E+26
29	2.135E+07	62	1.361E+17	95	6.754E+26
30	-4.527E+06	63	-1.604E+17	96	-2.075E+27
31	-8.902E+07	64	-6.855E+17	97	-4.549E+27
32	-6.184E+07	65	2.336E+16	98	4.176E+27

capacitor and a procedure for network analysis. He demonstrated that this procedure could successfully be used in the analysis of a simple series RC circuit. The author of this previous work predicted that this procedure could be applied directly to more complicated networks.

Section I of this report is devoted to using McKinney's procedure to analyze a balanced symmetric RC lattice network terminated with a reverse-switched capacitor and driven with a unit step voltage source. The solution to the calculations for this analysis produced an unstable output current for all reasonable values of sampling intervals. This result led the writer to construct a physical model of the network and its reverse-switched capacitor termination. Subsequent study of the network model on the analog computer revealed that output instability did not exist for any value of sampling time. Therefore the results of Section I serve as a counter example of the analysis procedure previously developed. The statement can be made that McKinney's procedure does not hold in general for analysis of networks more complicated than simple series RC circuits.

In Section II of this report the finite time Laplace transform (P_n-transform) is introduced, and this method of analysis is applied to the analysis of a simple parallel RC network using a current generator as the driving source. This calculation procedure proved to be a correct analysis of the network. The solution to the calculations exhibited a periodic output voltage as was expected.

Section III of this report is an attempt to use the P_n-transform analysis of Section II for the analysis of a balanced symmetric RC lattice network driven by a current generator. The state variable approach to network analysis is used in Section III. Results of the analysis displays an unstable output voltage associated with a unit step current driving source. This result, of course, is not physically possible. The direct extension of the P_n-transform method of analysis to a more complicated network was not realized.

The attempt by this author to extend the Z-transform method of analysis to complicated networks containing reverse-switched capacitors has, as a whole, been unsuccessful. However, considerable insight into the problem has been obtained and because of successful application of the method to simple cases, the extension still appears feasible. Further research in this area is recommended.

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APPENDIX

APPENDIX A

```
C  C  SYMMETRIC LATTICE WITH REVERSE SWITCHED CAPACITOR WJH
C     SAMPLING TIME T, SWITCHING TIME MT, NO INITIAL CONDITIONS
      DIMENSION X(622), A(622), NT(622)
    1 READ, T,M,NPTS
      PUNCH 5, T, M
    5 FORMAT(,//,2X,3HT =, F8.6, 2X, 3HM =,I5/)
      AM=M
      PUNCH 2
    2 FORMAT (//3(2X,1HN,5X,4HI(N),5X)//)
C     N-(2M+2) IS EQUIVALENT TO REAL TIME N=0
      E1=EXP(-T/3.)
      E1=EYP(-T/3.)
      E2=EXP(-AM*T/3.)
      E3=EXP (-(AM+1.)*T/3.)
      NPTS=NPTS/3*3
      LAST = 2*M+1+NPTS
      DO 10 I=1, LAST
   10 X(I)=0
      X(2*M+2)=1
      X(2*M+3)=-E1
      X(3*M+2)=-1./3.+3.*E2
      X(3*M+3)=3.*E1-3.*E3
      X(4*M+2)=-11./3.*E2
      X(4*M+3)=E3
      I1=2*M+1
      I2=2*M+2
      DO 20 I=1, I1
   20 A(I)=0
      DO 30 N=I2, LAST
      N1=N-M
      N2=N1-M
      A(N)=A(N-1)*E1+A(N1)*(1.-3.*E2)-A(N1-1)*(E1-3.*E3)
      A(N)=A(N)+A(N2)*3.*E2-3.*A(N2-1)*E3+X(N)
   30 NT(N)=N-I2
      I3=NPTS/3
      I4=I2+I3-1
      DO 50 I=I2, I4
      J=I+I3
      K= J+I3
   50 PUNCH 40, NT(I), A(I), NT(J), A(J), NT(K), A(K)
   40 FORMAT (3(I4,2X,F10.7,3X))
      GO TO 1
      END
```

APPENDIX B

```
C  C  CASCADED LATTICE WITH REVERSED SWITCHED CAPACITOR WJH (R1)
C     SAMPLING TIME T, SWITCHING TIME MT, NO INITIAL CONDITIONS
      DIMENSION X(433), R(433), NT(433)
    1 READ, T, M, NPTS
      PUNCH 5, T, M
    5 FORMAT (//,2X,3HT=  F8.4,2X,3HM= I5/)
      CM=M
      A= .1056
      B=1.8944
      PUNCH 2
    2 FORMAT (//3(2X,1HN,5X,4HI(N),5X)//)
C     N-(3M+3) IS EQUIVALENT TO REAL TIME N=0
      E1=EXP (-A*T)
      E2= EXP (-B*T)
      E3= EXP (-2.*T)
      E4= EXP (-A*CM*T)
      E5= EXP (-B*CM*T)
      E6= EXP (-(CM+1.)*A*T)
      E7= EXP (-(CM+1.)*B*T)
      E8= EXP (-(A+B*CM)*T)
      E9= EXP (-(B+A*CM)*T)
      E10= EXP (-(A*CM+2.)*T)
      E11= EXP (-(B*CM+2.)*T)
      E12= EXP (-2.*CM*T)
      E13= EXP (-(B+2.*CM)*T)
      E14= EXP (-(A+2.*CM)*T)
      E15= EXP (-(CM+1.)*2.*T)
      D= 1.148
      F= 1.268
      NPTS= NPTS/3*3
      LAST= 3*M+2+NPTS
      DO 10 I=1, LAST
   10 X(I)= 0
      X(3*M+3)= 2.693
      X(3*M+4)= -(.507*E1+2.434*E2)
      X(3*M+5)=-(.096*E3)
      X(4*M+3)=(.388*E4+.078*E5)
      X(4*M+4)=(2.528*E1-3.728*E2-2.468*E6-3.76*E7+3.107*E8+4.046*E9)
      X(4*M+5)=-(.599*E3-.588*E10+.730*E11)
      X(5*M+3)=-(2.066*E4-.176*E5-.057*E12)
      X(5*M+4)=-(1.347*E6-3.44*E9-.694*E8-3.774*E13+1.44*E14+2.455*E7)
      X(5*M+5)=-(.331*E10-.176*E11+.355*E15)
      X(6*M+3)= (2.282*E12)
      X(6*M+4)= (E14-3.535*E13)
      X(6*M+5)= (.254*E15)
      I1= 3*M+2
      I2= 3*M+3
      DO 20 I=1, I1
   20 R(I)= 0
      DO 30 N=I2, LAST
      N1= N-M
```

```
      N2= N1-M
      N3= N2-M
        R1=R(N-1)*(E1+E2)-R(N-2)*E3+R(N1)*(1.-D*E4+.88*E5)
        R1=  R1-R(N1-1)*(E1-D*E6+.88*E8+E2-D*E9+.88*E7)
        R1=  R1+R(N1-2)*(E3-D*E10+.88*E11)
        R1=  R1+R(N2)*(D*E4-.88*E5+F*E3)
        R1=  R1-R(N2-1)*(D*E6-.88*E8+F*E14+D*E9-.88*E7+.254*E13)
        R1=  R1+R(N2-2)*(D*E10-.88*E11+F*E15)
        R1=  R1-R(N3)*(F*E12)-R(N3-1)*(F*E14+F*E13)
      R(N)=  R1-(R(N3-2)*.254*E15)+ X(N)
   30 NT(N)= N-I2
      I3= NPTS/3
      I4= I2+I3-1
      DO 50 I=I2, I4
      J= I+I3
      K=J+I3
   50 PUNCH 40, NT(I), R(I), NT(J), R(J), NT(K), R(K)
   40 FORMAT (3(I4, 1PE10.3, 3X))
      GO TO 1
      END
```

APPENDIX C

```
C  C  RC CIRCUIT WITH REVERSE-SWITCHED CAPACITOR  WJH
C     SAMPLING TIME T, SWITCHING TIME MT, NO INITIAL CONDITIONS
      DIMENSION X(623), A(623), NT(623)
    1 READ, T, M
      PUNCH 5, T, M
    5 FORMAT (//,2X,3HT=F8.6,2X,3HM=I5/)
      CM=M
      PUNCH 2
    2 FORMAT (//3(2X,1HN,5X,4HI(N),5X)//)
C     N-(2M+3) IS EQUIVELENT TO REAL TIME N=0
      E1=EXP(-T)
      E2=EXP(-CM*T)
      E3=EXP(-(CM+2.)*T)
      A1=(1.-E1)
      A2=(2.*E2-2.)
      A3=(1.+E1-3.*E2+E3)
      A4=(E2-E3)
      B1=1.+E1
      B2=E1
      B3=E2-1.
      B4=1.+E1-E2-E3
      B5=E3-E1
      B6=E2
      B7=E2+E3
      B8=E3
      NPTS=NPTS/3*3
      LAST=2*M+2+NPTS
      DO 10 I=1, LAST
   10 X(I)=0
      X(2*M+3)=0
      X(2*M+4)=A1
      X(3*M+3)=A2
      X(3*M+4)=A3
      X(4*M+4)=A4
      I1=2*M+2
      I2=2*M+3
      DO 20 I=1, I1
   20 R(I)=0
      DO 30 N=I2, LAST
      N1=N-M
      N2=N1-M
      R1=-R(N-1)*B1-R(N-2)*B2-R(N1)*B3-R(N1-1)*B4
      R(N)=R1-R(N1-2)*B5+R(N2)*B6-R(N2-1)*B7+R(N2-2)*B8+X(N)
   30 NT(N)=N-I2
      I3=NPTS/3
      I4=I2+I3-1
      DO 50 I=I2,I4
      J=I+I3
      K=J+I3
   50 PUNCH 40,NT(I),R(I),NT(J),R(J),NT(K),R(K)
   40 FORMAT (3(I4,2X,F10.7,3X))
      GO TO 1
      END
```

APPENDIX D

```
C  C  SYMMETRIC LATTICE WITH REVERSED SWITCHED CAPACITOR WJH
C     SAMPLING TIME T, SWITCHING TIME MT, NO INITIAL CONDITIONS
      DIMENSION X(300), R(300), NT (300)
    1 READ , T, M, NPTS
      PUNCH 5, T, M
    5 FORMAT (//,2X,3HT= F8.4,2X,3HM= I5/)
      CM=M
      E1=EXP (-3.*CM*T)
      E2=EXP (-2.*T)
      E3=EXP (-3.*T)
      E4=EXP (-5.*T)
      A1= 4.-4.*E1-3.*CM*T
      A2= 4.-4.*E1-3.*CM*T*E1
      A3= E1-4.
      A4= 1.-4.*E1
      A5= 3.*E1
      A6= 3.*T-1.+9.*E2-8.*E3
      A7= 1.+17.*E3-17.*E2-3.*T*E2-3.*T*E3-E4
      A8= 3.*T*E4+E4-9.*E3+8.*E2
      A9= E3+E2
      A10= E4+2.*E3+2.*E2+1.
      A11= 2.*E4+E3+E2
      A12= 1.+E3
      A13= E3
      A14= E4
      PUNCH 2
    2 FORMAT (//3(2X,1HN,5X,4HI(N),5X)//)
C     N-(3M+6) IS EQUIVALENT TO REAL TIME N=0
      B1= 6.*A1
      B2= 2.*A1*A13-8.*A1
      B3= 6.*A2
      B4= 8.*A2-2.*A2*A13
      B5= 3.*A12
      B6= 3.*A13
      B7= A3*A12
      B8= A3*A13
      B9= A4*A12
      B10= A4*A13
      B11= A5*A12
      B12= A5*A13
      F1= (3.*A6)/27.
      F2= (3.*A7-B5*A6)/27.
      F3=(B6*A6-A7*B5+3.*A8)/27.
      F4= (A7*B6-A8*B5)/27.
      F5= (A8*B6)/27.
      F6= B1/27.
      F7=(A6*A3-B2+A9*B1)/27.
      F8=(A7*A3-A6*B7-A10*B1+A9*B2)/27.
      F9=(A6*B8-A7*B7+A8*A3-A10*B2+A11*B1)/27.
      F10=(A7*B8-A8*B7-A14*B1+A11*B2)/27.
      F11=(A8*B8+B2*A14)/27.
      F12= B3/27.
```

```
      F13=(A6*A4-B4-B3*A9)/27.
      F14=(A7*A4-A6*B9-A9*B4-A10*B3)/27.
      F15=(A6*B10-A7*B9+A8*A4-B4*A10-B3*A11)/27.
      F16=(A7*B10-A8*B9-A11*B4-A14*B3)/27.
      F17=(A8*B10-B4*A14)/27.
      F18=(A6*A5)/27.
      F19=(A7*A5-A6*B11)/27.
      F20=(A6*B12-A7*B11+A8*A5)/27.
      F21=(A7*B12-A8*B11)/27.
      F22=(A8*B12)/27.
      D1=(3.*A9+B5)/3.
      D2=(B6+B5*A9+3.*A10)/3.
      D3=(A9*B6+A10*B5+3.*A11)/3.
      D4=(A10*B6+A11*B5+3.*A14)/3.
      D5=(A11*B6+A14*B5)/3.
      D6=(A14*B6)/3.
      D7= A3/3.
      D8=(B7+A9*A3)/3.
      D9=(B8+A9*B7+A10*A3)/3.
      D10=(A9*B8+A10*B7+A11*A3)/3.
      D11=(A10*B8+A11*B7+A14*A3)/3.
      D12=(A11*B8+A14*B7)/3.
      D13=(A14*B8)/3.
      D14= A4/3.
      D15=(B9+A9*A4)/3.
      D16=(B10+A9*B9+A10*A4)/3.
      D17=(A9*B10+A10*B9+A11*A4)/3.
      D18=(A10*B10+A11*B9+A14*A4)/3.
      D19=(A11*B10+A14*B9)/3.
      D20=(A14*B10)/3.
      D21= A5/3.
      D22=(B11+A9*A5)/3.

      D23=(B12+A9*B11+A10*A5)/3.
      D24=(A9*B12+A10*B11+A11*A5)/3.
      D25=(A10*B12+A11*B11+A14*A5)/3.
      D26=(A11*B12+A14*B11)/3.
      D27=(A14*B12)/3.
      NPTS=NPTS/3*3
      LAST=3*M+6+NPTS
      DO 10 I=1, LAST
   10 X(I)=0
      X(3*M+7)=0
      X(3*M+8)= F1
      X(3*M+9)= F2
      X(3*M+10)= F3
      X(3*M+11)= F4
      X(3*M+12)= F5
      X(4*M+7)= -F6
      X(4*M+8)= F7
      X(4*M+9)= F8
      X(4*M+10)= F9
      X(4*M+11)= F10
      X(4*M+12)= F11
      X(5*M+7)= F12
```

```
      X(5*M+8)= F13
      X(5*M+9)= F14
      X(5*M+10)= F15
      X(5*M+11)= F16
      X(5*M+12)= F17
      X(6*M+8)= F18
      X(6*M+9)= F19
      X(6*M+10)= F20
      X(6*M+11)= F21
      X(6*M+12)= F22
      I1=3*M+6
      I2=3*M+7
      DO 20 I=1, I1
   20 R(I)=0
      DO 30 N=I2, LAST
      N1= N-M
      N2= N1-M
      N3= N2-M
      R1=R(N-1)*D1-R(N-2)*D2+R(N-3)*D3-R(N-4)*D4+R(N-5)*D5
      R1=R1-R(N-6)*D6-R(N1)*D7+R(N1-1)*D8-R(N1-2)*D9+R(N1-3)*D10
      R1=R1-R(N1-4)*D11+R(N1-5)*D12-R(N1-6)*D13-R(N2)*D14
      R1=R1+R(N2-1)*D15-R(N2-2)*D16+R(N2-3)*D17-R(N2-4)*D18
      R1=R1+R(N2-5)*D19-R(N2-6)*D20-R(N3)*D21+R(N3-1)*D22
      R1=R1-R(N3-2)*D23+R(N3-3)*D24-R(N3-4)*D25+R(N3-5)*D26
      R(N)=R1-R(N3-6)*D27+X(N)
   30 NT(N)=N-I2
      I3=NPTS/3
      I4=I2+I3-1
      DO 50 I=I2,I4
      J=I+I3
      K=J+I3
   50 PUNCH 40,NT(I),R(I),NT(J),R(J),NT(K),R(K)
   40  FORMAT (3(I4, 1PE10.3,3X))
      GO TO 1
      END
```

PERIODICALLY REVERSE-SWITCHED CAPACITORS IN AN RC NETWORK

by

WILLIAM J. HARDENBURGER

B. S., Kansas State University, 1950
B. S. (E. E.), Kansas State University, 1961

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KANSAS STATE UNIVERSITY
Manhattan, Kansas

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The Z-transform method of analyzing an RC network containing reverse-switched capacitors has been developed. This method of analysis gives recurrence relations which adapt easily to digital computer solutions and effects easy and straightforward solutions to problems involving the reverse-switched capacitor in a series RC network.

This paper investigates the application of the Z-transform method of analysis to more complicated networks. The basic network used in this investigation is the balanced symmetric RC lattice network terminated in a reverse-switched capacitor. The unsuccessful application of the previously developed Z-transform procedures to such a network shows that a direct extension of the method is not possible.

The finite time Laplace transform is introduced and its use is demonstrated in the solution of a parallel RC circuit containing a reverse-switched capacitor. Extension of this method to a more complicated network using state variable techniques of network analysis proved to be unsuccessful.

The attempt in this report to generalize the Z-transform method of analysis has been unsuccessful. However, considerable insight into the problem has been developed and provides a basis for further research.