ITP-Budapest 587, DESY 02-140

The QCD equation of state at finite T and μ

F. Csikor[a], G.I. Egri[a], Z. Fodor[a], S.D. Katz[b*], K.K. Szabó and A.I. Tóth[a]

[a]Institute for Theoretical Physics, Eötvös University, Pázmány P. 1/A, H-1117 Budapest, Hungary

[b]Deutsches Elektronen-Synchrotron DESY, Notkestr. 85, D-22607, Hamburg, Germany

We calculate the pressure (p), the energy density (ϵ) and the baryon density (n_B) of QCD at finite temperatures (T) and chemical potentials (μ). The recently proposed overlap improving multi-parameter reweighting technique is used to determine observables at nonvanishing chemical potentials. Our results are obtained by studying n_f=2+1 dynamical staggered quarks with semi-realistic masses on $N_t = 4$ lattices.

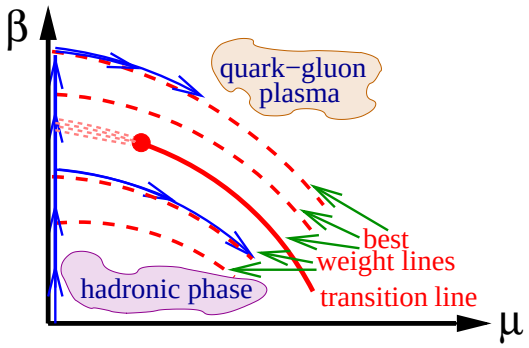

Figure 1. The best weight lines on the μ–β plane. In the middle we indicate the transition line. Its first dotted part is the crossover region. The blob represents the critical endpoint, after which the transition is of first order. The integration paths used to calculate p are shown by the arrows along the β axis and the best weight lines.

QCD at nonzero density is easily formulated on the lattice by multiplying the forward/backward links by $\exp(\pm\mu)$. However, standard importance sampling based Monte-Carlo techniques can not be used at $\mu \neq 0$. Up to now, no technique was suggested capable of giving the equation of state (EOS) at $\mu \neq 0$, which is essential to describe the quark gluon plasma (QGP) formation at heavy ion collider experiments. Results are only available for μ=0 (e.g. [1,2,3]) at T$\neq$0.

The overlap improving multi-parameter reweighting [4] opened the possibility to study

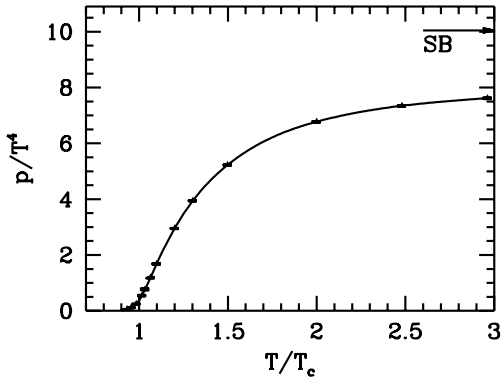

Figure 2. p normalised by T^4 as a function of T/T_c at $\mu = 0$ The N_t=4 SB limit is also shown.

lattice QCD at nonzero T and μ. First one produces an ensemble of QCD configurations at μ=0 and T$\neq$0. Then reweighting factors are determined at $\mu \neq 0$ and at a lowered T. The idea is expressed in terms of the partition function

$$Z(\beta,\mu,m) = \int \mathcal{D}U \exp[-S_g(\beta)] \det M(\mu,m)$$
$$= \int \mathcal{D}U \exp[-S_g(\beta_0)] \det M(\mu=0,m) \quad (1)$$
$$\left\{\exp[-S_g(\beta)+S_g(\beta_0)] \frac{\det M(\mu,m)}{\det M(\mu=0,m)}\right\},$$

where m is the quark mass, S_g is the action of the gluonic field (U), at $\beta = 6/g^2$ coupling. At nonzero μ one gets a complex fermion determinant $\det M$ which spoils importance sampling. Thus, the first line of eq. (1), $\mu \neq 0$, is rewritten in a way that the second line of eq. (1) is used as

*on leave from Institute for Theoretical Physics, Eötvös University, Pázmány P. 1/A, H-1117 Budapest, Hungary

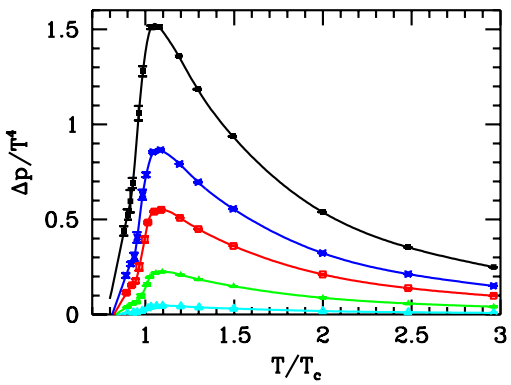

Figure 3. $\Delta p = p(\mu \neq 0, T) - p(\mu = 0, T)$ normalised by T^4 as a function of T/T_c for μ_B=100, 210, 330, 410 MeV and μ_B=530 MeV (from bottom to top).

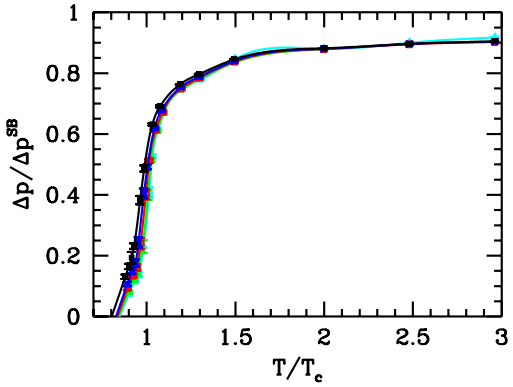

Figure 4. Δp of the QCD plasma normalised by Δp of the free gas (SB) as a function of T/T_c for the same μ_B values as in Fig. 3.

an integration measure (at μ=0 with importance sampling) and the curly bracket is measured on each independent configuration and is interpreted as a weight factor $\{w(\beta, \mu, m, U)\}$. In order to maximise the accuracy of Z the reweighting is performed along the best weight lines on the μ–β plane. These best weight lines are defined by minimising the spread of $\log w$.

Using the above technique, transition (or hadronic/QGP) configurations are reweighted to transition (or hadronic/QGP) configurations as illustrated by Fig. 1. The technique works for temperatures at, below and above the transition temperature (T_c). By using the reweighting technique, the phase diagram [4] and the location of the critical endpoint [5] was given. Using a Taylor expansion around μ=0, T$\neq$0 for small μ can be used to determine thermal properties [6]. A different method, analytic continuation from imaginary μ, confirmed results of [4,5] on the μ–T diagram [7].

We use $4 \cdot N_s^3$ lattices at $T \neq 0$ with N_s=8,10,12 for reweighting and we extrapolate to V$\to\infty$ using the available volumes (V). At T=0 lattices of $24 \cdot 14^3$ are taken for vacuum subtraction and to connect lattice parameters to physical quantities. 14 different β values are used, which correspond to $T/T_c = 0.8, \ldots, 3$. Our T=0 simulations provided R_0 and σ. The lattice spacing at T_c is $\approx$0.25–0.30 fm. We use 2+1 flavours of dynamical staggered quarks. While varying β (thus the temperature) we keep the physical quark masses constant at $m_{ud} \approx 65$ MeV and $m_s \approx 135$ MeV (the pion to rho mass ratio is $m_\pi/m_\rho \approx$0.66).

The determination of the equation of state at $\mu \neq 0$ needs several observables $\mathcal{O}$ at non-vanishing μ values. This can be calculated by using the weights of eq. (1)

$$\overline{\mathcal{O}}(\beta, \mu, m) = \frac{\sum\{w(\beta, \mu, m, U)\}\mathcal{O}(\beta, \mu, m, U)}{\sum\{w(\beta, \mu, m, U)\}}. \quad (2)$$

p can be obtained from the partition function as $p = T \cdot \partial \log Z / \partial V$ which can be written as $p = (T/V) \cdot \log Z$ for large homogeneous systems. On the lattice we can only determine the derivatives of $\log Z$ with respect to the parameters of the action (β, m, μ). Using the following notation $\langle \mathcal{O}(\beta, \mu, m) \rangle = \overline{\mathcal{O}}(\beta, \mu, m)_{T \neq 0} - \overline{\mathcal{O}}(\beta, \mu = 0, m)_{T=0}$. p can be written as an integral [8]:

$$\frac{p}{T^4} = \frac{1}{T^3 V} \int d(\beta, m, \mu) \quad (3)$$
$$\left(\left\langle \frac{\partial(\log Z)}{\partial \beta} \right\rangle, \left\langle \frac{\partial(\log Z)}{\partial m} \right\rangle, \left\langle \frac{\partial(\log Z)}{\partial \mu} \right\rangle \right).$$

The integral is by definition independent of the integration path. The chosen integration paths are shown on Fig 1.

The energy density can be written as $\epsilon = (T^2/V) \cdot \partial(\log Z)/\partial T + (\mu T/V) \cdot \partial(\log Z)/\partial \mu$. By changing the lattice spacing T and V are simul-

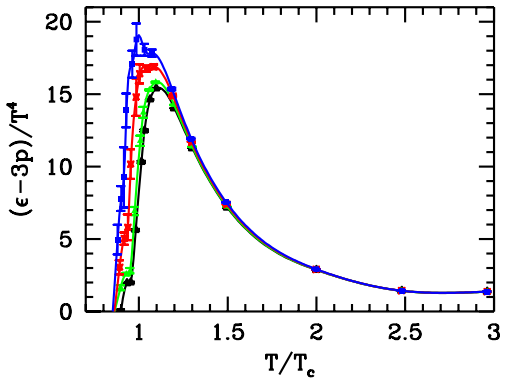

Figure 5. $(\epsilon - 3p)/T^4$ at μ_B=0, 210, 410 MeV and 530 MeV versus T/T_c (from bottom to top).

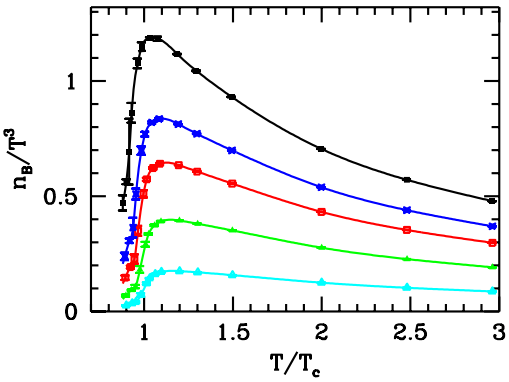

Figure 6. n_B/T^3 versus T/T_c for the same μ_B values as in Fig. 3 (from bottom to top).

taneously varied. The special combination $\epsilon - 3p$ contains only derivatives with respect to a and μ:

$$\frac{\epsilon - 3p}{T^4} = -\frac{a}{T^3 V}\frac{\partial \log(Z)}{\partial a}\bigg|_{\mu} + \frac{\mu}{T^3 V}\frac{\partial(\log Z)}{\partial \mu}\bigg|_{a} . \quad (4)$$

The quark number density is $n = (T/V) \cdot \partial \log(Z)/\partial\mu$ which can be measured directly or obtained from p (baryon density is n_B=$n/3$ and baryonic chemical potential is μ_B=3μ).

We present direct lattice results on $p(\mu = 0, T)$, $\Delta p(\mu, T) = p(\mu \neq 0, T) - p(\mu = 0, T)$, $\epsilon(\mu, T)$-$3p(\mu, T)$ and $n_B(\mu, T)$. Note, that in [9] additional overall factors were used to help the phenomenological interpretation. Our statistical errorbars are rather small, sometimes smaller than the thickness of the lines.

Fig. 2 shows p at μ=0. On Fig. 3 we present $\Delta p/T^4$ for five different μ values. Fig. 4 gives $\Delta p(\mu, T/T_c)$ normalised by $\Delta p^{SB} \equiv \Delta p(\mu, T \to \infty)$. Notice the interesting scaling behaviour. $\Delta p/\Delta p^{SB}$ depends only on T and it is practically independent of μ in the analysed region. Fig. 5 shows ϵ-$3p$ normalised by T^4, which tends to zero for large T. Fig. 6 gives the baryonic density as a function of T/T_c for different μ-s. As it can be seen the densities exceed the nuclear density by up to an order of magnitude.

An important finding concerns the applicability of our reweighting method: the maximal μ scales with the volume as $\mu_{\rm max} \cdot a \sim (N_t \cdot N_s^3)^{-0.25}$. If this behaviour persists, one could –in principle– approach the true continuum limit ($a \sim 1/N_t \sim (N_t \cdot N_s^3)^{-0.25}$, thus $\mu_{\rm max}$≈const.).

Future analyses should be done at smaller lattice spacings and quark masses. A detailed version of this work can be found elsewhere [9].

Acknowledgements: This work was partially supported by Hungarian Scientific grants OTKA-T37615/T34980/T29803/M37071/OMFB1548/-OMMU-708. For the simulations a modified version of the MILC public code was used (see http://physics.indiana.edu/∼sg/milc.html). The simulations were carried out on the Eötvös Univ., Inst. Theor. Phys. 163 node parallel PC cluster.

REFERENCES

1. S. Gottlieb *et al.*, Phys. Rev. D **55** (1997) 6852.
2. F. Karsch, E. Laermann and A. Peikert, Phys. Lett. B **478** (2000) 447.
3. A. Ali Khan *et al.* [CP-PACS collaboration], Phys. Rev. D **64** (2001) 074510.
4. Z. Fodor and S. D. Katz, Phys. Lett. B **534** (2002) 87.
5. Z. Fodor and S. D. Katz, JHEP **0203** (2002) 014.
6. C. R. Allton *et al.*, hep-lat/0204010; S. Ejiri *et al.*, hep-lat/0209012; C. Schmidt *et al*, hep-lat/0209009.
7. P. de Forcrand and O. Philipsen, hep-lat/0205016, hep-lat/0209084.
8. J. Engels *et al.*, Phys. Lett. B **252** (1990) 625.
9. Z. Fodor, S.D. Katz and K.K. Szabó, hep-lat/0208078; F. Csikor et al., in preparation.