

Coherent center domains from local Polyakov loops

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Abstract. We analyze properties of local Polyakov loops using quenched as well as dynamical SU(3) gauge configurations for a wide range of temperatures. It is demonstrated that for both, the confined and the deconfined regime, the local Polyakov loop prefers phase values near the center elements $1, e^{\pm i2\pi/3}$. We divide the lattice sites into three sectors according to these phases and show that the sectors give rise to the formation of clusters. For a suitable definition of these clusters we find that in the quenched case deconfinement manifests itself as the onset of percolation of the clusters. A possible continuum limit of the center clusters is discussed.

1. Motivation and general framework

With the running and upcoming experiments at LHC, RHIC and GSI the analysis of the QCD phase diagram has become an important focus of research. Not only the transition curves which separate different phases are of interest, but one would also like to understand from first principles the physical mechanisms that drive the various transitions.

In this project we probe the finite temperature transition of QCD using static quark sources. In the framework of lattice QCD these can be implemented using local Polyakov loops. The local Polyakov loop $L(\vec{x})$ is given by the trace of the product of temporal gauge links $U_4(\vec{x}, t)$ at a fixed spatial position \vec{x} (N_t is the number of lattice points in time direction):

$$L(\vec{x}) = \text{Tr} \prod_{t=0}^{N_t-1} U_4(\vec{x}, t), \quad (1)$$

i.e., the Polyakov loop is a gauge transporter that propagates a static quark at position \vec{x} forward in time. For later use we also define the spatially averaged Polyakov loop P as $P = V^{-1} \sum_{\vec{x}} L(\vec{x})$, where V is the spatial volume. Due to translation invariance P and $L(\vec{x})$ have the same vacuum expectation value. After suitable renormalization the Polyakov loop may be related to the free energy F_q of a single quark, i.e., $\langle P \rangle \propto \exp(-F_q/T)$, where T is the temperature. Below T_c the free energy is infinite, thus $\langle P \rangle = 0$, and the quarks are confined. Above T_c we have a finite free energy and thus $\langle P \rangle \neq 0$, signaling deconfinement. Hence the Polyakov loop acts as an order parameter for deconfinement.

In the deconfined phase of pure SU(3) gauge theory the phases of the summed Polyakov loops P assume values in the vicinity of the three center elements $1, e^{\pm i2\pi/3}$ of SU(3). This behavior shows the underlying center symmetry, a symmetry of the action and the path integral measure, which in pure gauge theory becomes broken spontaneously above T_c . The Polyakov loop P transforms non-trivially under the center transformation and thus is also an order parameter

for the symmetry breaking. Above T_c the Polyakov loop $\langle P \rangle$ is non-vanishing with phases near $1, e^{\pm i2\pi/3}$. For full QCD the fermion determinant breaks the center symmetry explicitly and acts as an external magnetic field favoring the real sector.

Except for the interchange of low and high temperature the situation is equivalent to simple spin systems with spins $s(\vec{x})$. Without external field the corresponding Hamiltonian has a symmetry which may be broken spontaneously. The symmetry breaking can be analyzed with observables that transform non-trivially under the symmetry group, e.g., the magnetization $M = V^{-1} \sum_{\vec{x}} s(\vec{x})$. To obtain the equivalence between the spin system and the gauge theory one has to identify the local loops $L(\vec{x})$ with the spins $s(\vec{x})$ and the spatially averaged Polyakov loop P with the magnetization M . In the absence of an external field (the fermion determinant) the phase of the magnetization $\langle M \rangle$ (the spatially averaged Polyakov loop $\langle P \rangle$) spontaneously selects one of the phases according to the underlying symmetry. If the external field (the fermion determinant) is turned on, the previously sharp transition becomes a crossover, and the phase of the magnetization (the Polyakov loop) is determined by the symmetry breaking term.

For the case of pure gauge theory these arguments are the basis of the Svetitsky-Yaffe conjecture [1] which states that at T_c pure SU(3) gauge theory in 4 space-time dimensions can be described by a 3-d effective spin system with an effective action which is symmetric under the center group \mathbb{Z}_3 . The spin degrees of freedom are related to the local loops $L(\vec{x})$. The leading term of the effective action is given by (τ and κ are real non-negative couplings)

$$S[s] = -\tau \sum_{\langle x,y \rangle} \left[s(\vec{x})s(\vec{y})^* + s(\vec{y})s(\vec{x})^* \right] - \kappa \sum_{\vec{x}} \left[s(\vec{x}) + s(\vec{x})^* \right], \quad (2)$$

where for illustration purposes we also included a symmetry breaking term which can be turned off when $\kappa = 0$. In the simplest version the effective spins $s(\vec{x})$ have values $s(\vec{x}) \in \{1, e^{\pm i2\pi/3}\}$.

Magnetic finite temperature transitions for spin systems are well understood phenomena. For discrete symmetry groups the transition is accompanied by the formation of locally spin coherent Weiss domains near T_c , before one spin orientation wins out in the symmetry broken phase. Even if a modest external magnetic field is applied one can still observe local clusters of aligned spins different from the direction preferred by the magnetic field.

One can go a step further and analyze the connectedness properties of the Weiss domains. One may define clusters of aligned spins and switch to a cluster description of magnetic systems [2, 3, 4]. For several spin systems one finds that the magnetic transition may be characterized as a percolation phenomenon of suitably defined clusters.

With the Svetitsky-Yaffe conjecture in mind, which describes the deconfinement transition with an effective spin system, we can formulate the central questions we explore in our project:

- Can one identify (at least near T_c) characteristic properties of spin-like behavior in an ab-initio lattice simulation of pure gauge theory and/or full QCD?
- Is it possible to identify spatial structures (clusters) that correspond to Weiss domains?
- How do the domains behave near T_c ? Do suitably defined clusters percolate?
- What is the role of the fermion determinant which breaks the underlying center symmetry?
- Can the clusters (Weiss domains) be given a physical meaning also in the continuum limit?

For SU(2) gauge theory similar questions were addressed in [5] – [8] (see also [9]). First results for SU(3) gauge theory were presented in [10].

2. Distribution properties of local Polyakov loops

We study the Polyakov loop (1) using quenched configurations as well as configurations from full QCD. For our quenched analysis we use the Lüscher-Weisz gauge action with lattice sizes

from $20^3 \times 6$ to $40^3 \times 12$ and temperatures ranging from $T = 0.63 T_c$ to $1.32 T_c$ [10]. In full QCD we use configurations generated with a Symanzik improved gauge action and 2 + 1 flavors of stout-link improved staggered quarks at physical masses [11, 12]. We study lattices with sizes $18^3 \times 6$, $36^3 \times 6$ and $24^3 \times 8$ in a temperature range from $T = 110$ MeV to 320 MeV.

To analyze the properties of local Polyakov loops we evaluate the $L(\vec{x})$ and write them as

$$L(\vec{x}) = \rho(\vec{x}) e^{i\varphi(\vec{x})}, \quad (3)$$

i.e., we decompose the local loops into modulus and phase. We begin with studying histograms $H[\rho(\vec{x})]$ and $H[\varphi(\vec{x})]$ for the distribution of the local modulus $\rho(\vec{x})$ and the local phase $\varphi(\vec{x})$, shown in Figs. 1 and 2. In both figures the lhs. is for the quenched case, while the rhs. is for full QCD and we compare results in the confining (low T) and the deconfining (high T) phase.

Fig. 1 shows that the distribution of the modulus is essentially independent of T , and furthermore is the same for both quenched and full QCD. The distribution of the modulus almost perfectly follows the Haar measure distribution (full curves in Fig. 1), $P[\rho] = \int dU \delta(\rho - |\text{Tr } U|)$, where dU is the SU(3) Haar measure. Comparing the distributions below and above the transition/crossover temperature clearly shows that the local modulus is not involved in the transition, which in the case of pure SU(3) gauge theory is even manifest as a first order jump of $\langle P \rangle$. Obviously it must be the local phases $\varphi(\vec{x})$ which drive the transition.

The histograms for the local phase in Fig. 2 show a pronounced peak structure, with maxima at the three center angles 0 , $2\pi/3$ and $-2\pi/3$. In the confining phase (top plots) for all three maxima have the same height and we again observe no difference between the quenched and the dynamical case, both of which can be described by the Haar measure distribution $P[\varphi] = \int dU \delta(\varphi - \arg \text{Tr } U)$ (full curves in the top plots). In the deconfined phase (bottom plots) the situation is different. One of the three center phases is more populated. For full QCD, where the fermion determinant acts like an external field, it is always the real center sector (phase values near 0) which becomes enhanced. In the quenched case any of the three center sectors may be selected spontaneously (similar to, e.g., the Ising system where the magnetization has two signs to choose from in the symmetry broken phase). In the quenched distribution we show here we have chosen configurations where the phase angle of the summed Polyakov loop P is near $-2\pi/3$ and obviously the local phases are enhanced for that value. If one switches to any of the other equivalent center sectors, the distribution is shifted by $\pm 2\pi/3$.

3. Cluster properties

In the previous section we have seen that the transition to the deconfined phase is accompanied by an increasing population of the histograms for the phase $\varphi(\vec{x})$ of the local Polyakov loop $L(\vec{x})$

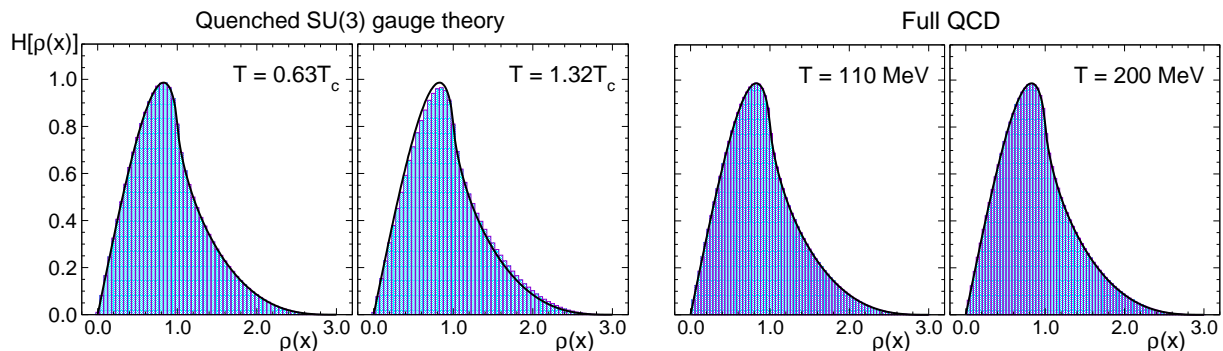


Figure 1. Histograms for the distribution of the local modulus $\rho(\vec{x})$. We compare quenched (lhs.) and full QCD (rhs.) at low and high T . The full curve is the Haar measure distribution.

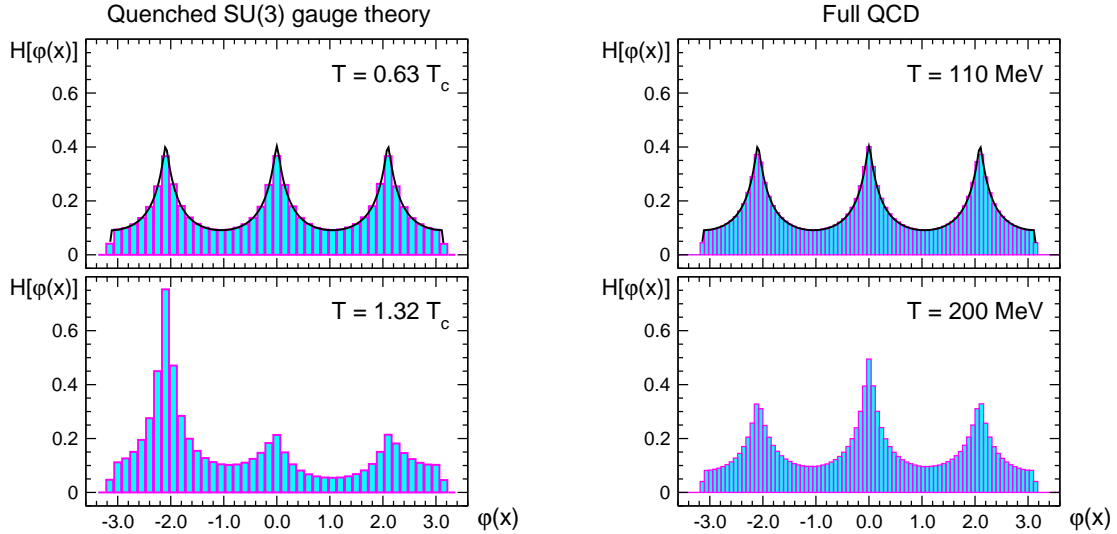


Figure 2. Histograms for the distribution of the local phase $\varphi(\vec{x})$. We compare quenched (lhs.) and full QCD (rhs.) at low and high T . The full curve is the Haar measure distribution.

near one of the center angles. This accumulation of the local phases in one sector drives the increase of the expectation value $\langle P \rangle$, while the modulus of the local loops plays no role. It is interesting to note that at high temperatures (bottom plots in Fig. 2) there are still pronounced peaks also in the subdominant sectors. The question is whether the phase values at different positions \vec{x} are distributed independently, or if there are spatial domains where the phases of the local Polyakov loops tend to align in the same center sector. The latter case is what is suggested by the effective action (2), where the first term favors parallel spins.

In order to study the formation of domains we assign sector numbers $n(\vec{x})$ to the sites \vec{x} ,

$$n(\vec{x}) = \begin{cases} -1 & \text{for } \varphi(\vec{x}) \in [-\pi + \delta, -\pi/3 - \delta], \\ 0 & \text{for } \varphi(\vec{x}) \in [-\pi/3 + \delta, \pi/3 - \delta], \\ +1 & \text{for } \varphi(\vec{x}) \in [\pi/3 + \delta, \pi - \delta]. \end{cases} \quad (4)$$

Here δ is a free real and positive parameter which allows to cut lattice points \vec{x} where the corresponding phase $\varphi(\vec{x})$ is near a minimum of the distributions in Fig. 2. These points do not have a clear preference for one of the center sectors and the parameter δ allows one to remove them from the cluster analysis. The remaining lattice points \vec{x} can now be organized in clusters according to the sector numbers: We put two neighboring points \vec{x}, \vec{y} into the same cluster if $n(\vec{x}) = n(\vec{y})$. For illustration purposes in Fig. 3 we show the largest cluster for two quenched configurations, one below T_c (lhs.), the other one above (rhs.). The plot is for lattice size $30^3 \times 6$ and a value of δ chosen such that 39 % of the lattices points are cut. It is obvious, that above T_c the largest cluster percolates (stretches over all of the lattice), while it is finite below T_c .

Of course the cutoff parameter δ will influence the size of the clusters, since with increasing δ less sites are available. We stress at this point that also for the characterization of the magnetic transitions in spin systems a similar reduction step is necessary in the construction of the clusters. This may be a reduced linking probability between neighbors with equal spins [4], but also more general approaches similar to the one used here were considered [8]. As a consistency check of our cluster construction one may show [13] that the points that survive the cut carry most of the signal of a rising Polyakov loop above the transition/crossover temperature.

In order to quantify the dependence of the cluster size on the temperature, in Fig. 4 we show the expectation value of the weight W (i.e., the number of lattice points) of the largest cluster

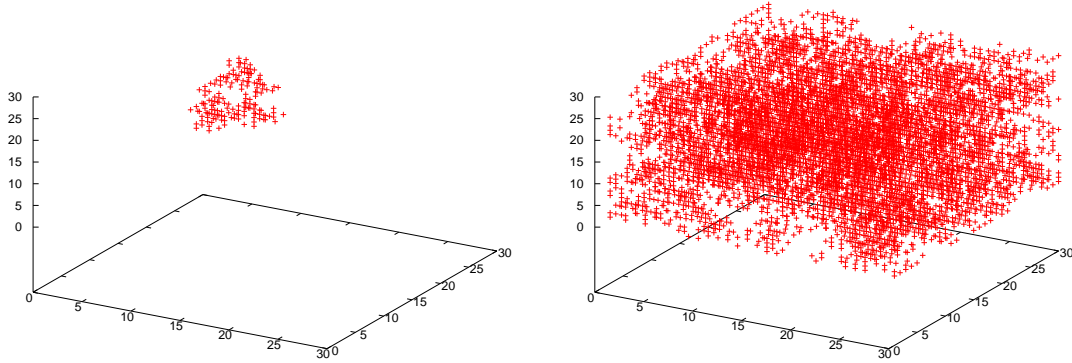


Figure 3. Largest clusters for two quenched configurations below T_c (lhs.) and above (rhs.).

normalized by the total number V of sites as a function of temperature. Again the lhs. is for the quenched case (at a cut of 39% for all volumes and temperatures), while the rhs. is for full QCD (19 % cut), and we compare three (two) different spatial volumes. Both in the quenched and the dynamical case we observe that the clusters start out small below T_c and grow quickly in size at the transition temperature. If one analyzes the percolation probability one finds that for the quenched case, where we have a true phase transition, this probability indeed approaches a step function as the volume is increased. For the dynamical case, where one observes only a crossover [14], the situation concerning percolation is not yet clear and further studies on several volumes will be necessary to settle this issue.

4. Towards a continuum limit for the center clusters

We have shown that the phases $\varphi(\vec{x})$ of the local Polyakov loops $L(\vec{x})$ have preferred values near the center angles, and that neighboring sites have a tendency towards aligning these phases. The corresponding clusters were found to grow quickly near the transition/crossover temperature, and at least for the quenched case the deconfinement transition may be characterized by the onset of percolation for suitably defined clusters. So far this picture is established only for a fixed lattice spacing a and some arbitrarily chosen value of the cutoff parameter δ which enters our cluster definition. If one wants to assign a physical significance to the center clusters, a way of constructing them such that a continuum limit can be taken is necessary. In particular this

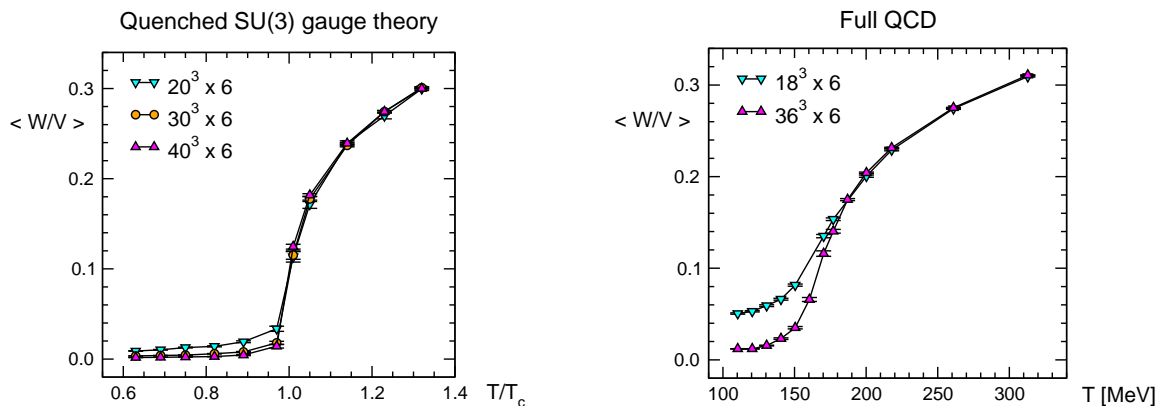


Figure 4. Weight W of the largest cluster normalized with the volume V as function of T .

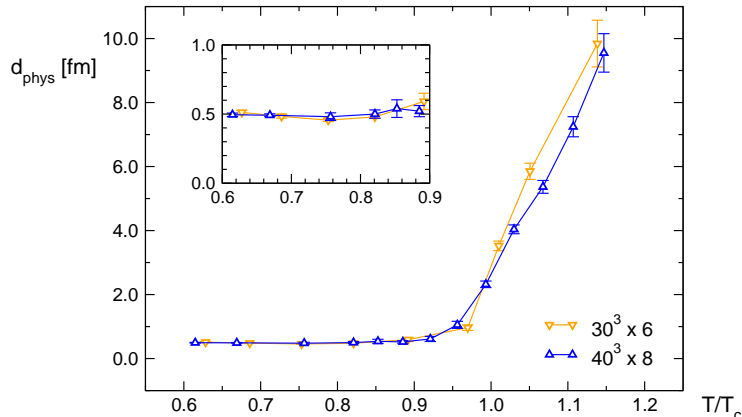


Figure 5. Average cluster diameter in fm versus T for two resolution scales (quenched case).

involves a prescription for connecting the scale a in physical units to the clusters.

We begin with defining the linear extension (radius) of a cluster by considering the expectation value of two-point correlators $C(|\vec{x} - \vec{y}|)$ for sites \vec{x}, \vec{y} within the same cluster. These 2-point functions decay exponentially, $C(|\vec{x} - \vec{y}|) \propto \exp(-|\vec{x} - \vec{y}|/r)$, and we use the parameter r to define the radius of the cluster in units of the lattice spacing a . Using the value of the lattice constant a in fm, the diameter of the clusters in physical units (fm) is then given by $d_{phys} = ra$. The result will depend on both, the lattice spacing a and the cutoff δ . In order to compare the physical size of the clusters for different lattice resolutions a , we always adjust δ such that at a low temperature ($T = 0.63 T_c$ for the quenched case where this analysis is done) we fix the physical diameter to a typical hadronic size, e.g., $d_{phys} = 0.5$ fm. The corresponding value of δ is then kept fixed for all other temperatures, and is used to study d_{phys} as a function of T . In Fig. 5 we show the result for the cluster diameter d_{phys} in physical units as a function of temperature for the quenched case, comparing two different resolution scales a (i.e., $N_t = 6$ and $N_t = 8$). It is obvious that also in physical units the phase transition is signaled by a sudden increase of the cluster size. Furthermore, the results for the two different scales fall onto a universal curve, which suggests that a continuum limit for the cluster size might exist. This question is analyzed in detail in a future publication [13], where we study the flow of the cutoff δ as a function of the resolution scale a , arguing that the center clusters indeed have a continuum limit.

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