Negative length orbits in normal-superconductor billiard systems

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The Path-Length Spectra of mesoscopic systems including diffractive scatterers and connected to superconductor is studied theoretically. We show that the spectra differs fundamentally from that of normal systems due to the presence of Andreev reflection. It is shown that negative path-lengths should arise in the spectra as opposed to normal system. To highlight this effect we carried out both quantum mechanical and semiclassical calculations for the simplest possible diffractive scatterer. The most pronounced peaks in the Path-Length Spectra of the reflection amplitude are identified by the routes that the electron and/or hole travels.

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In recent years, semiclassical methods have become a popular tool for describing devices operating in the mesoscopic regime. Advances in manufacturing and material design have made possible the creation of clean mesoscopic devices, whose properties depend on the microscopic details of individual samples. For example, in recent experiments [1,2] involving semiconductor microjunctions, both the quantum coherence length and the mean free path of elastic collisions are large compared to the size of the junction. In such devices electrons can be described as a two-dimensional ideal Fermi gas of noninteracting particles. The conductance of such junctions has been measured and found to oscillate strongly as the Fermi energy is varied.

Semiclassical methods have proved to be very effective for understanding conductance fluctuations in normal microjunctions. On the one hand, methods based on random matrix theory [3–5] successfully predict statistical properties of transport properties. On the other, short-wavelength semiclassical descriptions able to explore geometry-induced interference effects in weakly disordered or clean mesoscopic devices. In particular, it has been shown that the *Path-Length Spectra* (PLS), defined as the power spectrum of the reflection (transmission) amplitudes with respect to the Fermi wavelength

$$\hat{r}_{mn}(L) = \left| \int_{k_{min}}^{k_{max}} e^{-ik_F L} r_{mn}(k_F) dk_F \right|^2, \qquad (1)$$

possesses peaks at lengths corresponding to classical trajectories of electrons starting and ending at the external contacts [6–12]. Here $r_{mn}(k_F)$ ($t_{mn}(k_F)$) is the reflection (transmission) amplitude at the Fermi wavenumber k_F for scattering from mode n of the entrance lead to mode m of the entrance (exit) lead in a two probe conductance measurement.

Mesoscopic devices connected to a superconductor present a new challenge for semiclassics. In such normal-superconductor (NS) systems Andreev reflection [13,5,14,15] plays an important role, whereby electrons at the Fermi energy in the normal metal are retro-reflected as holes at the NS interface. Such a process might be expected to dramatically affect the PLS, but to-date, no investigations of the PLS in NS systems have been carried out.

In this Letter we present the first such investigation, by examining a mesoscopic device connected to a superconductor and show that the PLS of NS systems differ from those of normal systems in a fundamental way. In particular for NS systems containing diffractive scatters, negative path-lengths can arise in the PLS which are absent from the corresponding normal systems.

Before turning to the NS system we shortly discuss the role of diffraction in the PLS of normal systems, following Ref. [16]. In the semiclassical approximation, particles hitting a diffractive scatterer may scatter in any direction since the classical dynamics is not uniquely defined [17,18]. In Ref. [16] a two-dimensional wave guide with a small point-like diffractive scatterer has been analyzed [19] (see Fig 1a.). It has been shown that the PLS of the reflection amplitude r_{nm} has peaks at path-lengths corresponding to classical trajectories starting and returning to the entrance of the lead, either diffracted once or several times by the scatterer. At large Fermi wavelengths a typical trajectory with multiple bounces is shown in Fig. 1a. Such trajectories consist of two parts: segments (along the z axis) connecting the lead and the scatterer with total length $2 \times z_0$, and multiple diffraction trajectories starting and ending on the scatterer. A multiple diffraction trajectory can be decomposed into loops that start and end on the scatterer, making bounces on the walls of the waveguide. The possible lengths of the loops are

$$l_r = \begin{cases} 2Wr \\ 2x_0 + 2Wr \\ 2(W - x_0) + 2Wr \end{cases}, \quad (2)$$

where $r = 0, 1, 2, ...$ is the repetition number and W is the width of the waveguide [16].

In Figure 2 the PLS calculated quantum mechanically and semiclassically using the method developed in Ref. [16] are shown. One can see that the agreement between the quantum and the semiclassical calculation is excellent and the peaks are located at lengths which are linear combinations of (2) plus $2z_0$. It is obvious that the PLS has peaks *only* for positive lengths L. The amplitude of the peaks is decreased by multiple diffraction and therefore the most pronounced peaks correspond to paths diffracted only once on the scatterer.

Now consider the effect of replacing one of the exit leads by a superconductor. In this case a new contribution to the reflection amplitude r_{mn} has to be taken into account, namely the one coming from Andreev reflection at the NS interface. By solving the Bogoliubov-de Gennes equation for a NS interface it is possible to show that, at the Fermi energy, electron-like excitations impinging onto the superconducting interface are coherently retro-reflected as hole-like excitations. In a semiclassical description, the classical action associated with the path connecting points q' and q'' is given by

$$S(q', q'') = \int_{q'}^{q''} p(q) dq, \quad (3)$$

where $p(q)$ is the momentum of the electron or the hole along the path (note that if the path touches the superconductor at least once, it will contain both electron and hole parts). In particular the action associated with an Andreev reflection process in which an electron starting at point q' returns to the same point as a hole can be written as

$$S(q', q') = k_F L(q', q^*) - k_F L(q^*, q') = 0, \quad (4)$$

where $L(q', q^*)$ denotes the length of the path of the electron until it hits the superconductor at q^* and $L(q^*, q')$ is the path length of the hole from q^* to q'. The minus sign in the second term is due to the fact that the directions of the momentum and the velocity of the hole are opposite. The total action of an electron-hole (e-h) trajectory returning to its starting point is always zero, since the hole retraces the path of the electron. As a consequence of this result, the PLS of the reflection amplitude for electron-electron (e-e) has peaks at positive lengths L while for e-h it has a pronounced peak at $L = 0$.

We now consider the case where *diffractive scattering* is possible in the system (Fig. 1b). The new feature is that the trajectory of electrons or holes hitting a diffractive point is not uniquely defined. Therefore, a hole retracing an electron, which scattered on a diffractive center, will not necessarily retrace the trajectory of the electron beyond the diffractive center. The hole may leave the diffraction center at a different angle than that of the incident electron. This effect has already been pointed out by Beenakker [20] in connection with normal-metal-superconductor junction containing a point contact. Consequently, in the presence of diffraction, complicated trajectories consisting of several electron and hole segments may arise. The classical action for this case can be written as a sum of actions of the segments

$$S = \sum k_F(\pm L_i), \quad (5)$$

where L_i is the length of the segment i and the + or − correspond to cases when electron or hole traverses the segment i, respectively. Unlike in the diffractionless case, the sum of positive and negative terms in Eq. (5) is not necessarily zero. Moreover, the total length of the hole segments may exceed those of the electrons. Thus, in the PLS of the e-e reflection amplitude, peaks at *negative lengths* may appear, which are completely absent from the PLS of the corresponding normal systems.

To observe negative lengths in the PLS we note that when the exit lead is replaced by a superconductor [5] the e-e reflection amplitude can be expressed in terms of the transmission and reflection amplitudes of the corresponding normal system (see Eq. 266a in Ref. [5]). For the system of Fig. 1b, the latter amplitudes have been derived in Ref. [16] and, after lengthy but straightforward algebra, we find the following expression for the n, m matrix element of the e-e reflection amplitude at the Fermi energy:

$$s_{nm}^{e-e}(k_F) = \frac{2r_{nm}(k_F)}{1 + |\mathcal{D}|^2 \left[Im\, G_0(x_0, z_0|x_0, z_0)\right]^2}, \quad (6)$$

where $r_{nm}(k_F)$ is the matrix of reflection amplitudes of the normal system [16] for the entrance lead, $G_0(x, z|x', z')$ is the Green's function of the empty waveguide. $\mathcal{D} = -i\tilde{\lambda}/\left[1 - \tilde{\lambda} G_0(x_0, z_0|x_0, z_0)\right]$ can be regarded as the diffraction constant of the scatterer, where $\tilde{\lambda}$ is the renormalized strength of the scatterer [21].

On the upper part of Fig. 3 the PLS of the reflection amplitude of (6) (the exact quantum mechanical result) is plotted as a function of the path length. Peaks at negative path lengths are clearly visible here as opposed to normal system shown in Fig. 2.

The semiclassical approximation of Eq. (6) can be obtained from the semiclassical form of the Green's function which is given by

$$G_0(x_0, z_0|x_0, z_0) = \sum \frac{(-1)^{n_r}}{\sqrt{8\pi k_F l_r}} e^{ik_F l_r - i3\pi/4}, \quad (7)$$

where the summation is over all possible loops with lengths l_r given in (2) and n_r is the number of bounces on the walls of the waveguide. One can see that $Im\, G_0(x_0, z_0|x_0, z_0)$ will contain the complex conjugate of Eq. (7) and is therefore a sum with terms proportional to $e^{\pm i k_F l_r}$. Here, terms with $e^{-i k_F l_r}$ correspond to loops traversed by holes. The e-e reflection amplitude given in (6) can be rewritten as a multiple diffraction series:

$$s_{nm}^{e-e}(k_F) = 2 r_{nm}(k_F) \Big\{ 1 - |\mathcal{D}|^2 \left[Im\, G_0(x_0, z_0|x_0, z_0) \right]^2 + |\mathcal{D}|^4 \left[Im\, G_0(x_0, z_0|x_0, z_0) \right]^4 \cdots \Big\} . \qquad (8)$$

Expressing powers of $Im\, G_0$ with Eq. (7) and its complex conjugate $s_{nm}^{e-e}(k_F)$ can be written as an oscillating sum

$$s_{nm}^{e-e}(k_F) = \sum_j A_j e^{i S_j}, \qquad (9)$$

where S_j is of form Eq. (5). The amplitudes A_j can be determined exactly by using the above formulas. In the lower part of Fig. 3 the PLS of the reflection amplitude $s_{nm}^{e-e}(k_F)$ is plotted using Eq. (9). Again one can see a very good agreement between the quantum and semiclassical calculations. The most pronounced peaks with negative lengths (see Fig. 4) come from the family $L = 2z_0 - l_r$, where l_r is given in Eq. (2), namely $L = -0.4, -1.0, -1.6, -2.4, -3.0, -3.6, -4.4, -5, \cdots$. These lengths can be associated with the following routes: First the electron hits the superconductor then the retro-reflected hole diffracts on the scatterer and makes a loop with repetition number r as described before Eq. (2). Next, the hole diffracts off the scatterer then hits the superconductor on which it converts back to an electron. Finally, the electron goes back to the entrance lead.

The above results represent the first theoretical study of the path-length spectra of a normal superconductor mesoscopic system with diffractive scattering. We have demonstrated that the PLS of such systems differs fundamentally from that of normal systems, due to the appearance of peaks at negative lengths. To highlight this effect, we have analyzed the simplest possible diffractive scatterer. Since the appearance of negative lengths in PLS is the *direct* consequence of the presence of diffractive scatterers it is desirable to study other types of diffractive centers such as corners. There is a growing interest in studying the role of diffractive scatterers in normal mesoscopic systems [22], therefore the extension to normal superconductor systems may become a new playground both in the semiclassical theory of scattering processes and level statistics of these systems. For the future it will also be of interest to examine the amplitudes of the negative-length peaks in more complex geometries, such as those of [15], to examine the effect of a tunnel junction at the NS interface and the role of order parameter symmetry.

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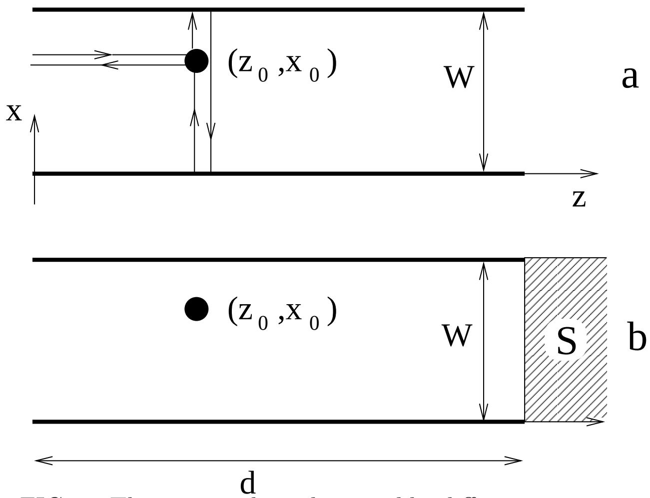

FIG. 1. The waveguide with point-like diffractive scatterer (a) opened and (b) a superconductor attached to the exit lead (left side of the waveguide). The origin of the coordinate system (z, x) is at the left bottom corner of the waveguide. In Fig. 1a one possible trajectory is shown. For clarity the forward and backward paths are shifted.

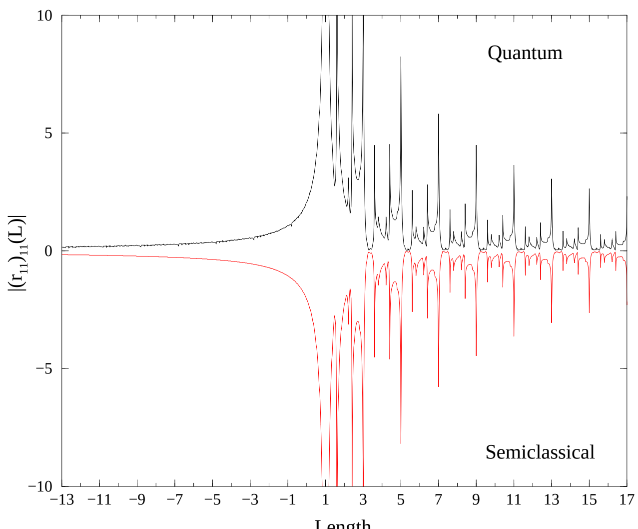

FIG. 2. The PLS of the reflection amplitude defined by Eq. (1) is obtained from quantum mechanical and semiclassical calculations. The width of the waveguide, $W = 1.0$. The strength of the Dirac delta potential, $\lambda = 10.0$, and its position is located at $(z_0, x_0) = (0.5, 0.7)$.

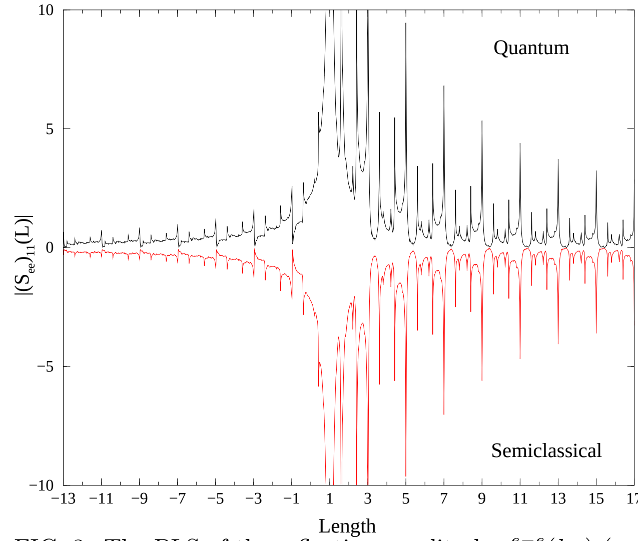

FIG. 3. The PLS of the reflection amplitude $s_{11}^{e-e}(k_F)$ (see Eq. (6)) is obtained from quantum and semiclassical calculations. For parameters see Fig. 2.

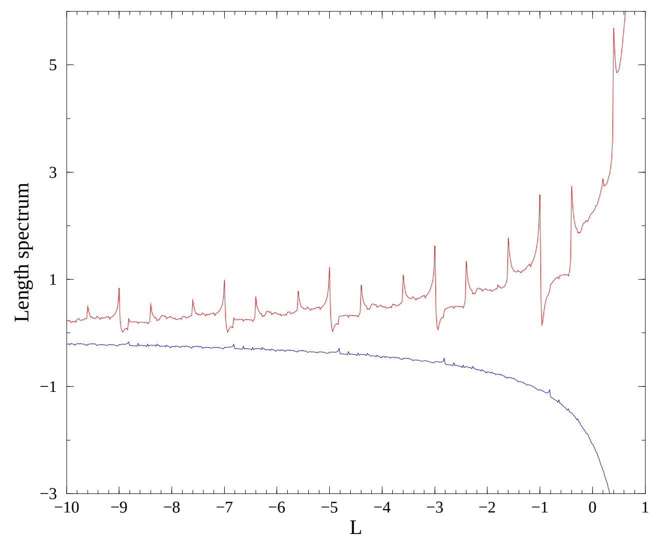

FIG. 4. The PLS of the reflection amplitude $r_{11}(k_F)$ of the normal system (lower part) and $s_{11}^{e-e}(k_F)$ given in Eq. (6) (upper part) are plotted. The most pronounced peaks with negative lengths come from the family $L = 2z_0 - l_r$, where l_r is given in Eq. (2), namely $L = -0.4, -1.0, -1.6, -2.4, -3.0, -3.6, -4.4, -5, \cdots$. For parameters see Fig. 2.