To make a nanomechanical Schrödinger-cat mew

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Abstract. By an explicite calculation of Michelson interferometric output intensities in the optomechanical scheme proposed by Marshall *et al.* [1], an oscillatory factor is obtained that may go down to zero just at the time a visibility revival ought to be observed. Including a properly tuned phase shifter offers a simple amendment to the situation. By using a Pockels phase shifter with fast time-dependent modulation in one arm, one may obtain further possibilities to enrich the quantum state preparation and reconstruction abilities of the original scheme, thereby improving the chances to reliably detect genuine quantum behaviour of a nanomechanical oscillator.

Detecting genuine quantum effects in nanomechanical systems is a highly desirable scope, expected to be reached in the near future. The major challenges are: efficient cooling close to the ground state, sufficiently strong coupling to a well-identified quantum system, reliable preparation and identification of non-classical states of the nanomechanical component. Most of the experimental studies are done on mechanical oscillators, considered harmonic for the small displacements characteristic for the quantum domain. For those systems ground-state cooling means $k_B T < \hbar\omega$ which is relatively easy to fulfil for hard (high-frequency) oscillators [2]. For them, however, preparing and analyzing quantum states is overly demanding [3]. The efforts to reconcile these conflicting requirements have seen rapid and most competitive advance with the participation of a number of experimental groups. A still more ambitious, so far elusive prospect of related studies would be to detect deviations from standard quantum mechanics, predicted by several theoretical studies [4].

An original and promising project, using a soft (low-frequency) nanomechanical oscillator, coupled by a mirror to one of the paths of a Michelson interferometer, and using Fabry-Perot cavities (on both paths, to preserve interference) to strengthen the optomechanical coupling, has been started by Marshall *et al.* [1], and following extensive development of an optical feedback cooling technique [5], is now approaching the level of seeing quantum effects.

According to the argument of Ref. [1], for a single-photon source of frequency ω_c, and a vibrating mirror of effective mass M and vibration frequency ω_m, acting as one of the mirrors delimiting a Fabry-Perot resonator cavity of length L, the dimensionless coupling constant is $\kappa = (\omega_c/\omega_m)(\sqrt{(\hbar/2M\omega_m)}/L)$. With the vibrating-mirror cavity on interferometer arm A, and the cavity with two rigid mirrors on arm B, starting with a mirror in its quantum mechanical ground state $|0\rangle_m$, coupling during time t results in an entangled photon-mirror state

$$|\Psi(t)\rangle = \frac{e^{-i\omega_c t}}{\sqrt{2}} \left(e^{i\varphi(t)} |A\rangle |\alpha(t)\rangle_m + |B\rangle |0\rangle_m \right) \qquad (1)$$

where $|A\rangle = |1\rangle_A |0\rangle_B$ and $|B\rangle = |0\rangle_A |1\rangle_B$ denote one-photon states with the photon in arm A

and in arm B resp., whereas, $|\alpha(t)\rangle_m$ and $|0\rangle_m$ are two coherent states of respective complex amplitudes $\alpha(t)$ and 0 of the vibrating mirror, appearing as a pair of Schrödinger cat states entangled to the respective orthogonal photon states $|A\rangle$ and $|B\rangle$. Finally,

$$\varphi(t) = \kappa^2(\omega_m t - sin\omega_m t); \quad \alpha(t) = \kappa(1 - e^{-i\omega_m t}). \tag{2}$$

The density matrix $\hat{R}(t) = |\Psi(t)\rangle\langle\Psi(t)|$ corresponding to the pure state $|\Psi\rangle$ can be expanded on the photon state basis $|A\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $|B\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ in the form of a two-by-two matrix

$$\hat{R}(t) = \begin{pmatrix} \hat{\rho}_{AA}(t) & \hat{\rho}_{AB}(t) \\ \hat{\rho}_{BA}(t) & \hat{\rho}_{BB}(t) \end{pmatrix}, \tag{3}$$

where the matrix elements are operators acting on variables of the vibrating mirror. It has been pointed out [1, 6] that the crucial quantity characterizing the interference pattern to be observed is the nondiagonal element on that basis, $\hat{\rho}_{AB}(t) = \langle A|\Psi(t)\rangle\langle\Psi(t)|B\rangle$, which carries extra information about the motion of the vibrating mirror, missing from the more usual reduced density matrix $\hat{\rho}_m(t) = \hat{\rho}_{AA}(t) + \hat{\rho}_{BB}(t)$, obtained by tracing out $\hat{R}(t)$ over photon variables.

Interference visibility, as expressed by $\hat{\rho}_{AB}(t)$, carries a signature of the motion of the vibrating mirror, but the usefulness of that signature is seriously limited by interactions of the mirror with the environment. Neglecting direct environmental effects on the photon subsystem, the above expansion remains valid, allowing decoherence and friction of the vibrating mirror to be included in the dynamics of $\hat{\rho}_{AB}(t)$ and analyzed by standard tools [6, 7].

Returning for a moment to the decoherence-free case covered by Eq. (1), let us quote the result of Ref. [1]:

$$\mathrm{Tr}_m\, \hat{\rho}_{AB}(t) = \frac{e^{i\varphi(t)}}{2}\, \mathrm{Tr}_m\, [|\alpha(t)\rangle_m\, {}_m\langle 0|] = \frac{1}{2} e^{i\varphi(t) - |\alpha(t)|^2/2}, \tag{4}$$

where Tr_m means trace over mirror variables. From here one concludes that the crucial quantity to keep under control is $|\alpha(t)|$, and - using Eq. (2) - the observation should be carried out at multiples of time $t = 2\pi/\omega_m$, where $|\alpha(t)|$ returns to 0 and the interference signal can be large enough to be observed, under the extremely stringent condition that decoherence could be suppressed by sufficient cooling and isolation from mechanical supports.

That treatment catches the essential point of the phenomenon but fails to give a correct description of the underlying oscillatory behaviour, which is relevant for a reliable identification of superposition states of the mirror. To show that, we proceed by noticing that Eq. (1) is not the final state observed by the photon detectors, since the photons pass once more through the beam splitter, before reaching output ports C (facing A) and D (facing B). On the time scale of whatever happens to the vibrating mirror, this is a fast and coherence-preserving unitary operation on the photon subsystem, transforming the intermediate state (1) into the final one

$$\frac{1}{2} e^{-i\omega_c t} \Big[|C\rangle \left(|0\rangle_m + i e^{i\varphi(t)} |\alpha(t)\rangle_m \right) + i\, |D\rangle \left(|0\rangle_m - i e^{i\varphi(t)} |\alpha(t)\rangle_m \right) \Big] \tag{5}$$

with $|C\rangle = |1\rangle_C |0\rangle_D = (|A\rangle + i|B\rangle)/\sqrt{2}$ and $|D\rangle = |0\rangle_C |1\rangle_D = (i|A\rangle + |B\rangle)/\sqrt{2}$. Carrying out the above unitary transformation on the full density matrix, now we can trace over mirror (and eventually, for a full analysis, environmental) variables to obtain directly the density matrix of the photons to be detected, in the form

$$\mathbf{R}^{phot} = \begin{pmatrix} \mathrm{Tr}_m\, \hat{\rho}_{CC} & \mathrm{Tr}_m\, \hat{\rho}_C \\ \mathrm{Tr}_m\, \hat{\rho}_{DC} & \mathrm{Tr}_m\, \hat{\rho}_{DD}, \end{pmatrix} \tag{6}$$

with the diagonal elements

$$\left.\begin{array}{c} I_C \\ I_D \end{array}\right\} = \left.\begin{array}{c} \mathrm{Tr}_m\,\hat\rho_{CC} \\ \mathrm{Tr}_m\,\hat\rho_{DD} \end{array}\right\} = \frac{1}{2} \mp \mathrm{Im}\,(\mathrm{Tr}_m\,\hat\rho_{AB}) \tag{7}$$

describing the output intensities on the two respective ports, and the off-diagonal elements

$$\mathrm{Tr}_m\,\hat\rho_{CD} = [\mathrm{Tr}_m\,\hat\rho_{DC}]^* = \frac{1}{2}\mathrm{Re}\,(\mathrm{Tr}_m\,\hat\rho_{AB}) + i\left(\frac{1}{2} - \mathrm{Tr}_m\,\hat\rho_{AA}\right) \tag{8}$$

describing cross-correlations between the two detectors.

Of particular interest are the output intensities, and we see from Eq. (7) that, instead of the modulus of $\mathrm{Tr}_m\,\hat\rho_{AB}(t)$, they display the *imaginary* part of the same quantity. Evaluating, like before, the decoherence-free case, we obtain

$$\begin{aligned} \left.\begin{array}{c} I_C \\ I_D \end{array}\right\} &= \frac{1}{2}\left(1 \mp e^{-|\alpha(t)|^2/2}\,\sin\left[\varphi(t) + \chi(t)\right]\right) &(9)\\ &= \frac{1}{2}\left(1 \mp e^{-\kappa^2(1-\cos\omega_m t)}\,\sin\left[\kappa^2(\omega_m t - \sin\omega_m t) + \chi(t)\right]\right), &(10) \end{aligned}$$

where we have used Equation (2), and for the discussion to follow, included an additional phase shift $\chi(t)$ created by a phase shifter device on arm A of the Michelson interferometer, not present in the original setup [1].

Having to measure the imaginary part of $\mathrm{Tr}_m\,\hat\rho_{AB}(t)$ makes a huge difference. Without the phase shifter, that factor oscillates as $\sin\kappa^2(\omega_m t - \sin\omega_m t)$, which - depending on the actual value of the optomechanical coupling constant κ - may go down close to zero just about time $(2\pi/\omega_m)n$, when the nth visibility revival is expected. There is a simple amendment though: one should include a constant phase shifter and tune it to

$$\chi_n = 2\pi\left(\frac{2n+1}{4} - \kappa^2\right), \tag{11}$$

to match the nth maximum of the oscillatory factor with the expected revival time. With that, revivals become observable in the interferometric intensity patterns, as soon as decoherence effects are suppressed to the desired level [8]; of course, to see at least $n = 1$ would be the minimum requirement. Alternatively, if noise level of data allows, scanning with χ to locate the maximum signal can be a way to measure the coupling constant κ.

The above reasoning connects the Marshall *et al.* scheme to a simple token introduced by Yurke and Stoler [9], used in various experiments [10, 11] to overcome some of the difficulties associated with the entanglement present e.g. in Eq. (1), and create a visible interference pattern of the two Schrödinger cat components, as apparent in Eq. (5), observable through straightforward intensity measurement of a chosen component of the associated two-state quantum system. In the Marshall *et al.* scheme it is the last passage through the Michelson beamsplitter that - acting as a $\pi/2$ quasi-spin rotation of the two-state photon basis - offers a kind of gratis implementation of the Yurke and Stoler procedure; this was the original motivation for the present work.

Alas, just seing the visibility revival would not prove quantumness [7]. Therefore to reach that final scope it may be necessary to add a time-dependent phase shifter $\chi(t)$ to the photon subsystem. That can strongly enhance the possibilities of preparing and analysing quantum states of an oscillator, as demonstrated by the successful application of phase controlling protocols in ion-trap Schrödinger cat experiments [10]. Optical phase shifters based on the Pockels effect are commercially available, and admit electric modulation at frequencies up to a few GHz [12], with the potentiality to make the Marshall *et al.* nanomechanical Schrödinger cat mew more clearly about his/her quantum behaviour. Some of the possibilities offered by that token will be analyzed in a forthcoming paper.

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